DR. GYURCSEK ISTVÁN

Basic Laws of Electrical Circuits

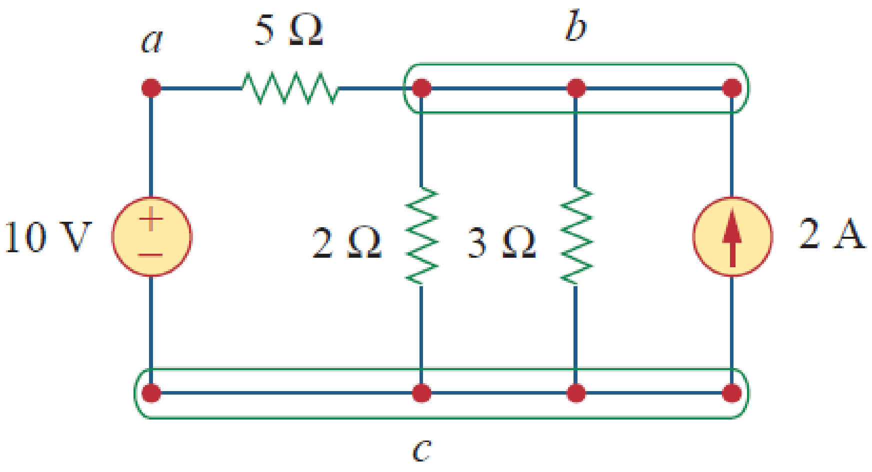
Sources and additional materials (recommended)

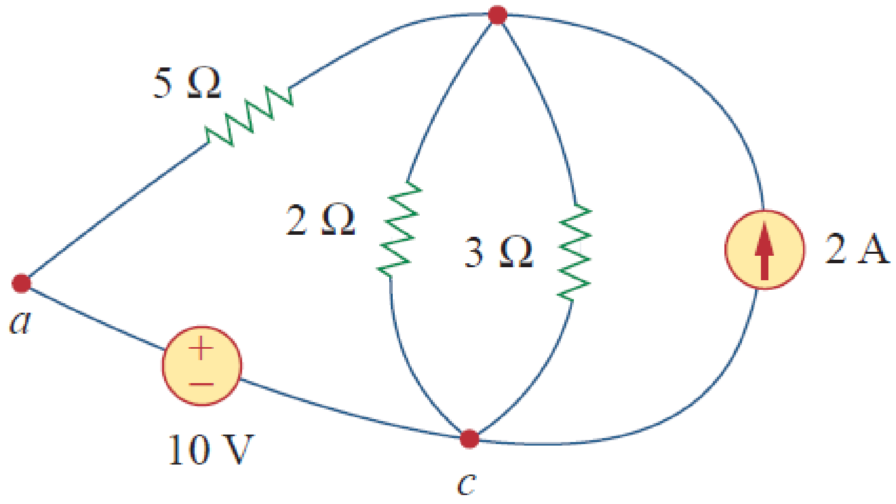
- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*

- **Network topology, KCL, KVL**
- Characteristics of Circuit Elements
- Passive Equivalent Transformations

Nodes, Branches, Loops

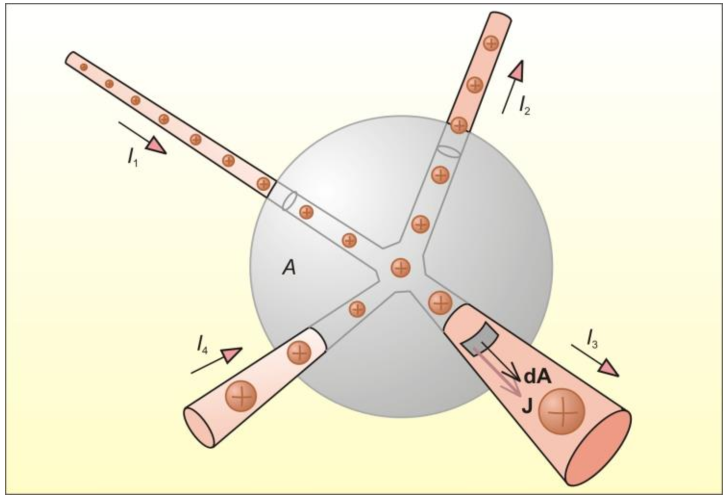
Fundamental theorem of network topology: $b = l + n - 1$





Recall:

Eq. of Continuity - Work Over a Closed Path



$$W = \oint_l \boldsymbol{F}\, d\boldsymbol{l} = \oint_l q\, \boldsymbol{E}\, d\boldsymbol{l} = 0$$

(E – conservative, curl-free vector field)

$$W = q\,V = q \oint_l \boldsymbol{E}\, d\boldsymbol{l} = 0$$

Equation of continuity

$$\oint_A \boldsymbol{J}\, d\boldsymbol{A} = 0 \;\dots (\leftarrow \operatorname{div} \boldsymbol{J} = 0)$$

$$\oint_l \boldsymbol{E}\, d\boldsymbol{l} = 0$$

***J** → curly, divergence-less (source-less)*

$$\oint_A \boldsymbol{J}\, d\boldsymbol{A} = \sum_{i=1}^{n} I_i = 0$$

$$\sum_{i=1}^{n} v_i = 0$$

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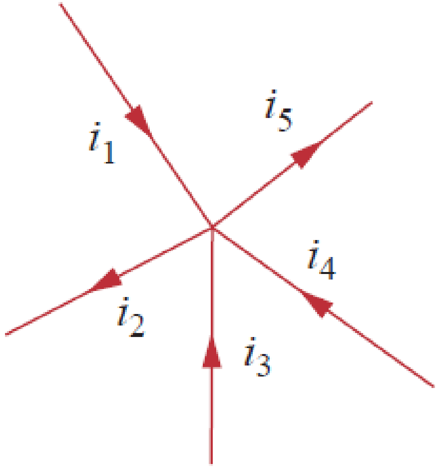
KCL, KVL

Algebraic sum of currents entering a node (or a closed boundary) is zero. *(Sum of the currents entering a node is equal to the sum of the currents leaving the node.)*

$$\sum_{n=1}^{N} i_n = 0 \qquad \sum i_{entering} - \sum i_{leaving} = 0$$

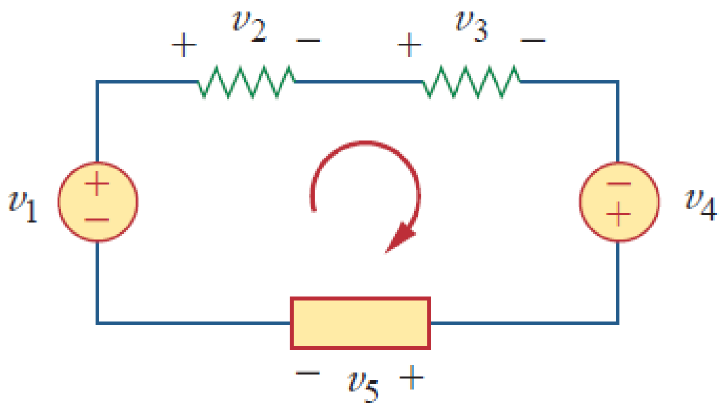
$$I_1 - I_2 + I_3 + I_4 - I_5 = 0$$



Algebraic sum of all voltages around a closed path (or loop) is zero. *(Sum of voltage drops = sum of voltage rises.)*

$$\sum_{m=1}^{M} v_m = 0 \qquad \sum v_{drops} - \sum v_{rises} = 0$$

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$



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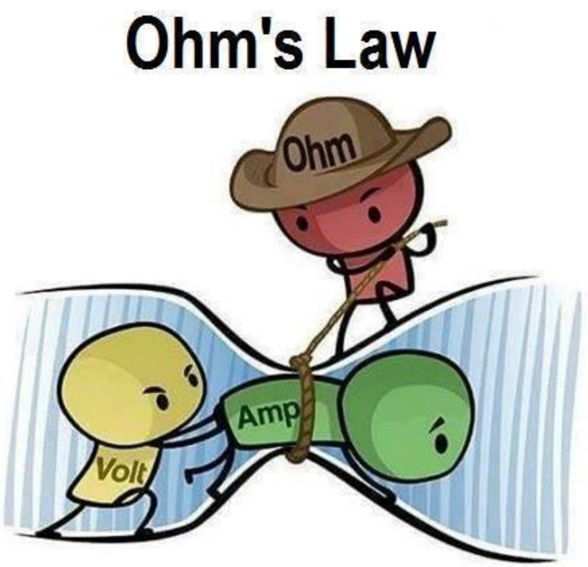
- **Characteristics of Circuit Elements**
- Passive Equivalent Transformations

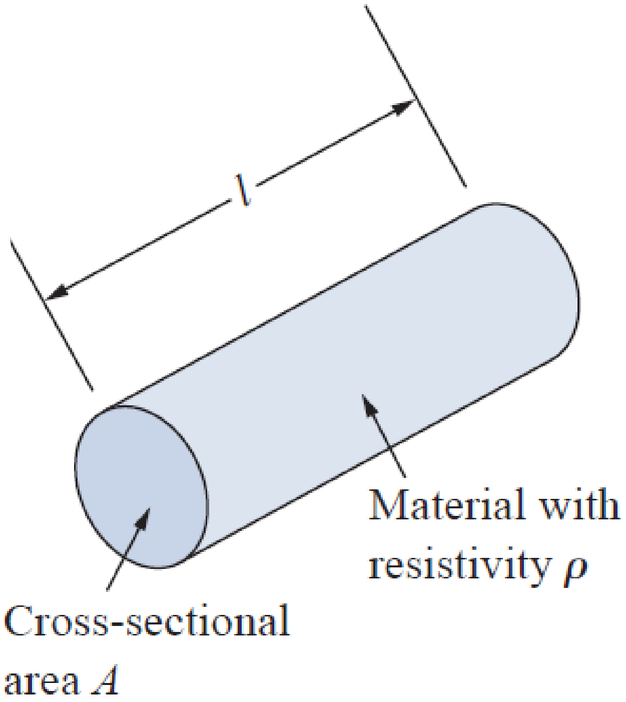
Ohm’s Law



$$J = \frac{1}{\rho} \cdot E \rightarrow i = \frac{1}{R} \cdot v$$

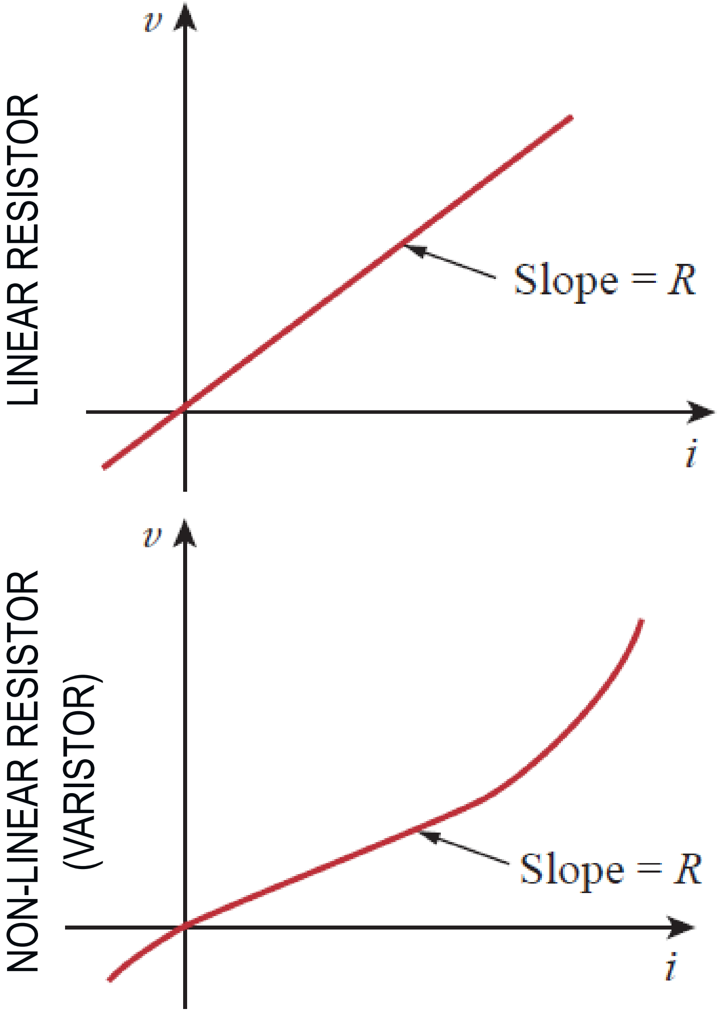
$$J = \sigma \cdot E \rightarrow i = G \cdot v$$

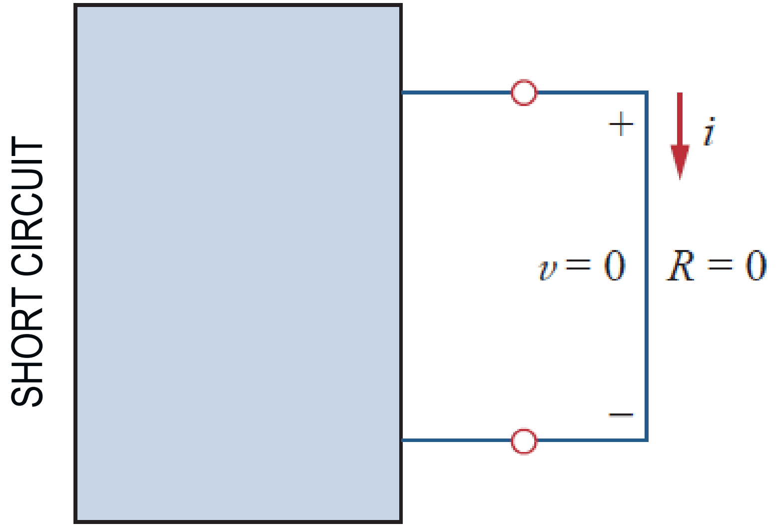
$$R = \rho \cdot \frac{l}{A}$$



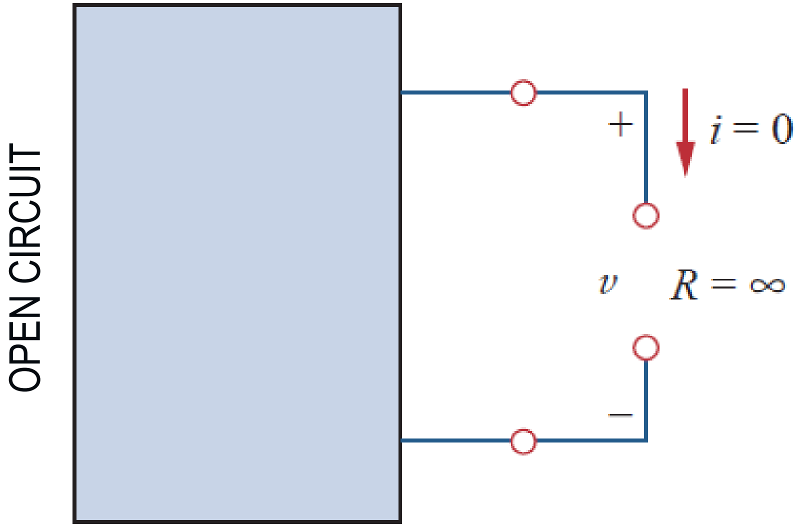
Material	Spec. Resistivity (Ω m)	Usage
Silver	$1.64\ 10^{-8}$	Conductor
Copper	$1.72\ 10^{-8}$	Conductor
Gold	$2.45\ 10^{-8}$	Conductor
Aluminum	$2.80\ 10^{-8}$	Conductor
Carbon	$4\ 10^{-5}$	Semiconductor
Germanium	$47\ 10^{-2}$	Semiconductor
Silicon	$6.4\ 10^{-2}$	Semiconductor
Paper	10^{10}	Insulator
Mica	$5\ 10^{11}$	Insulator
Glass	10^{12}	Insulator
Teflon	$3\ 10^{12}$	Insulator

Sort of Resistors



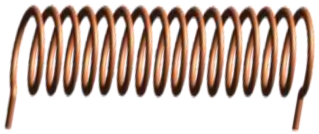


$$v = (R \cdot \mathrm{i})\Big|_{R=0} = 0$$

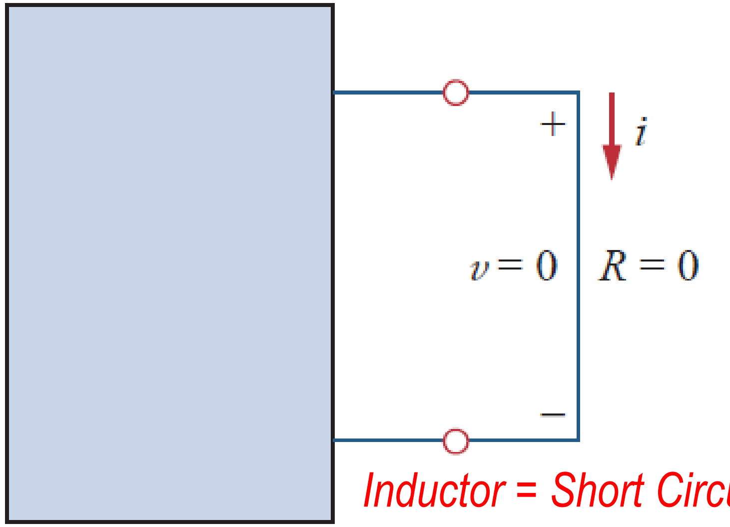


$$i = \lim_{R \to \infty} \frac{v}{R} = 0$$

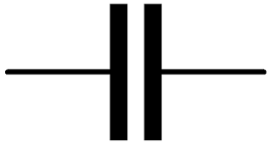
DC Behavior of L and C

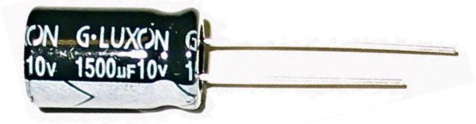


$$v(t) = L\frac{di(t)}{dt}, \qquad i(t) = constant \rightarrow v(t) = 0$$

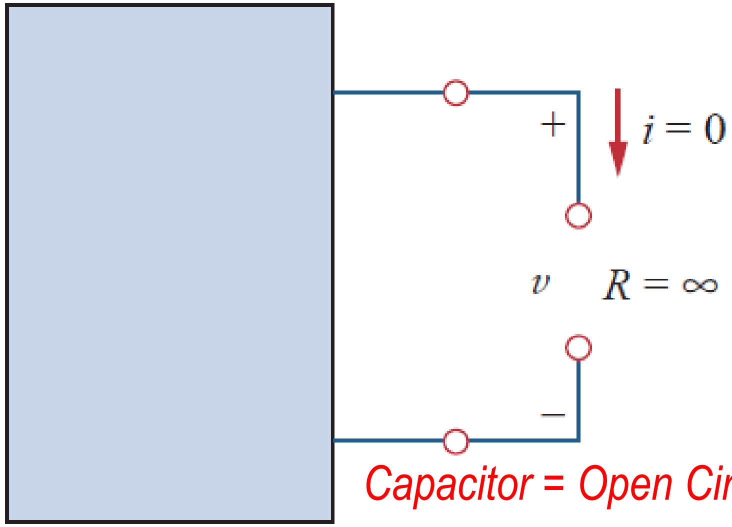


Inductor = Short Circuit for DC





$$i(t) = C\frac{dv(t)}{dt}, \qquad v(t) = constant \rightarrow i(t) = 0$$



Capacitor = Open Circuit for DC

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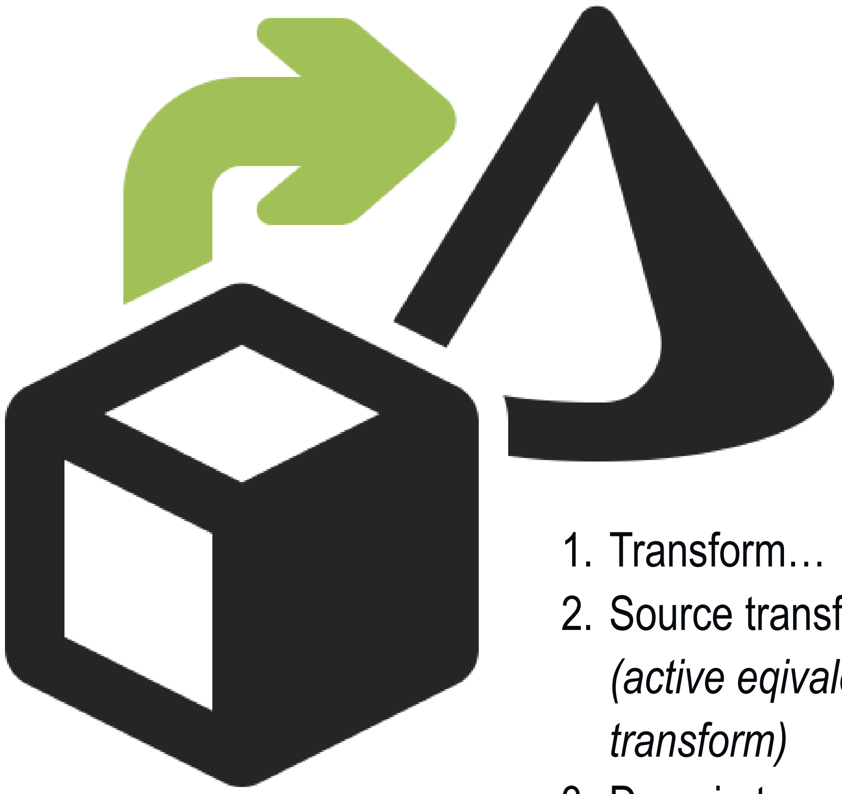
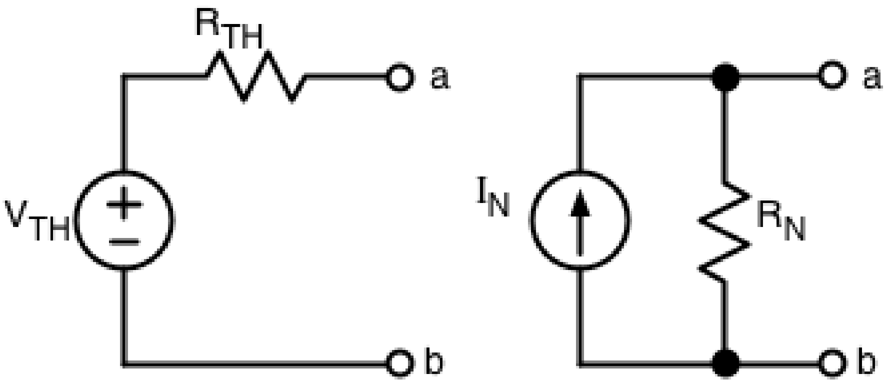
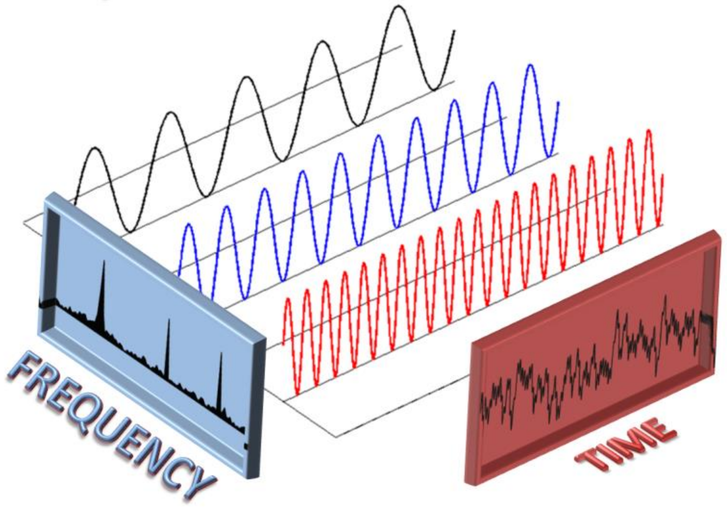
- **Passive Equivalent Transformations**

Equivalent Transform



1. Transform…
2. Source transform *(active eqivalent transform)*
3. Domain transform





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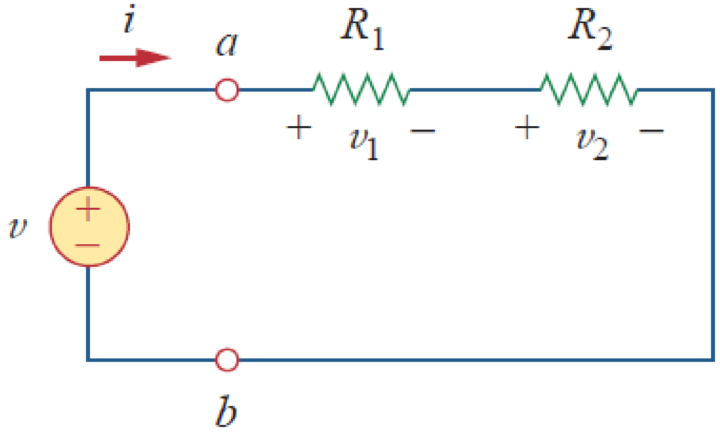
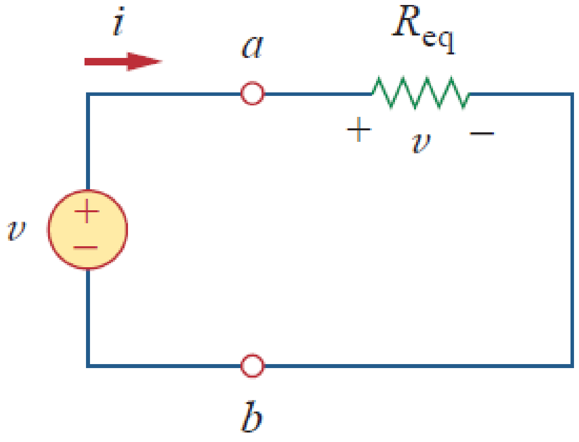
Series Resistors, Voltage Division





$$v_1 = i \cdot R_1\,, \qquad v_2 = i \cdot R_2$$

$$v = v_1 + v_2 = i \cdot R_1 + i \cdot R_2 = i \cdot (R_1 + R_2)$$

$$v = i \cdot R_{eq}$$

$$R_{eq} = R_1 + R_2$$

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

$$R_{eq} = R \cdot N$$

Voltage division

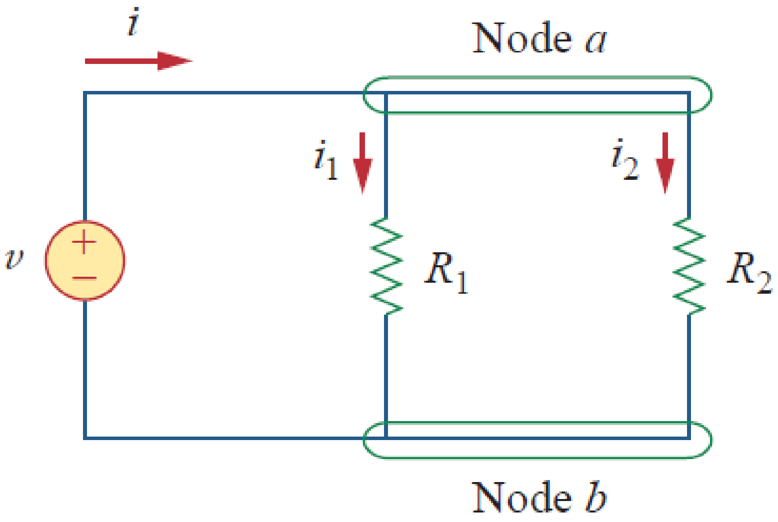
$$\frac{v_n}{v} = \frac{i \cdot R_n}{i \cdot R_{eq}}$$

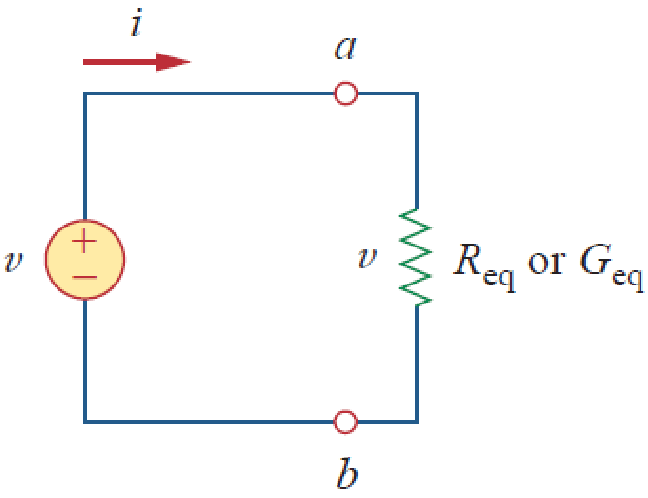
$$v_n = v \frac{R_n}{R_1 + R_2 + \cdots + R_N}$$

$$v_1 = v \frac{R_1}{R_1 + R_2}\,, \qquad v_2 = v \frac{R_2}{R_1 + R_2}$$

$$v_{SC} = v \frac{0}{R_1 + 0} = 0\,, \qquad v_{OC} = v \frac{\infty}{R_1 + \infty} = v$$

Parallel Resistors, Current Division





$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

$$i = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = v \cdot \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$i = v \cdot \frac{1}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \rightarrow G_{eq} = \sum_{n=1}^{N} G_n$$

$$R_{eq} = \frac{R}{N}$$

Current division

$$\frac{i_n}{i} = \frac{v \cdot G_n}{v \cdot G_{eq}}$$

$$i_n = i \cdot \frac{G_n}{G_1 + G_2 + \cdots + G_N}$$

$$i_1 = i \frac{R_2}{R_1 + R_2}$$

$$i_2 = i \frac{R_1}{R_1 + R_2}$$

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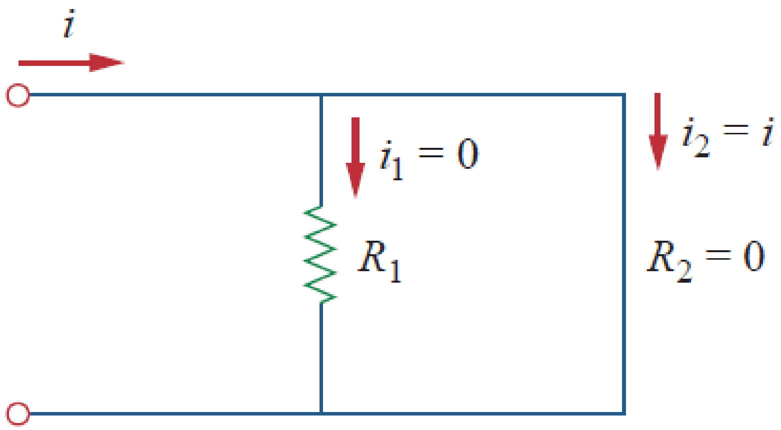
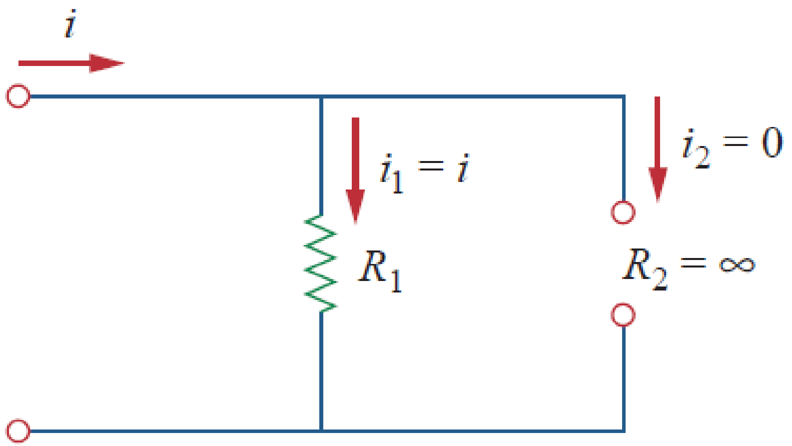
Current Division in OC and SC

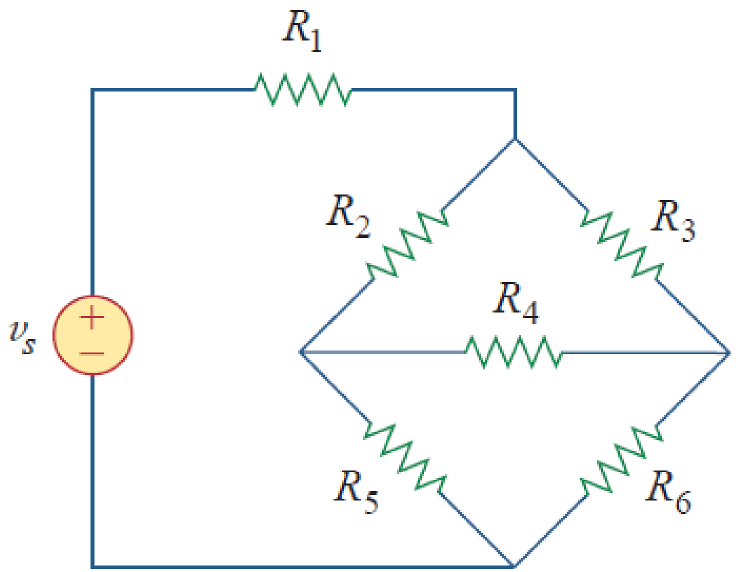




$$i_{SC} = i\frac{R_1}{R_1 + 0} = i \rightarrow i_1 = 0$$

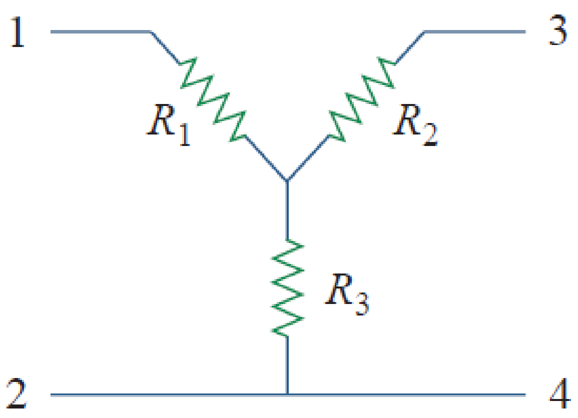
$$i_{OC} = i\frac{R_1}{R_1 + \infty} = 0 \rightarrow i_1 = i$$

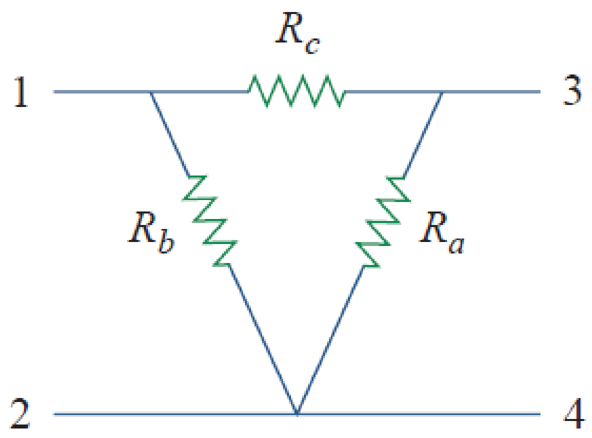
Wye-Delta Transformations



The problem - neither serial nor parallel elements

Solution - equivalent circuits





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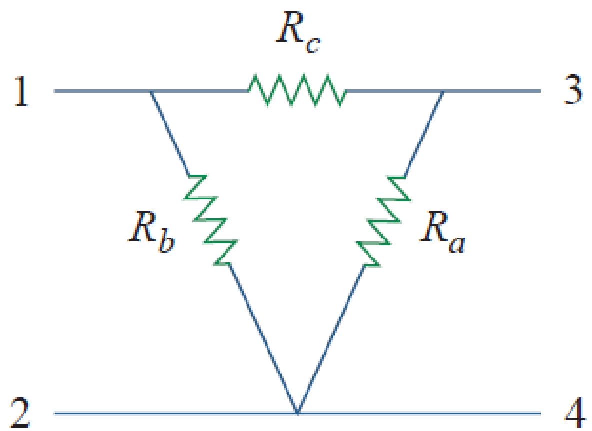
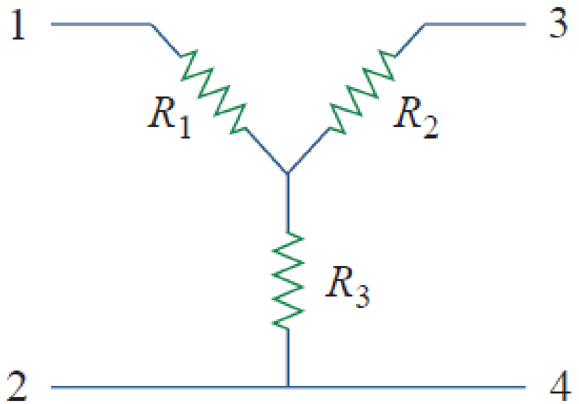
Delta to Wye Conversion

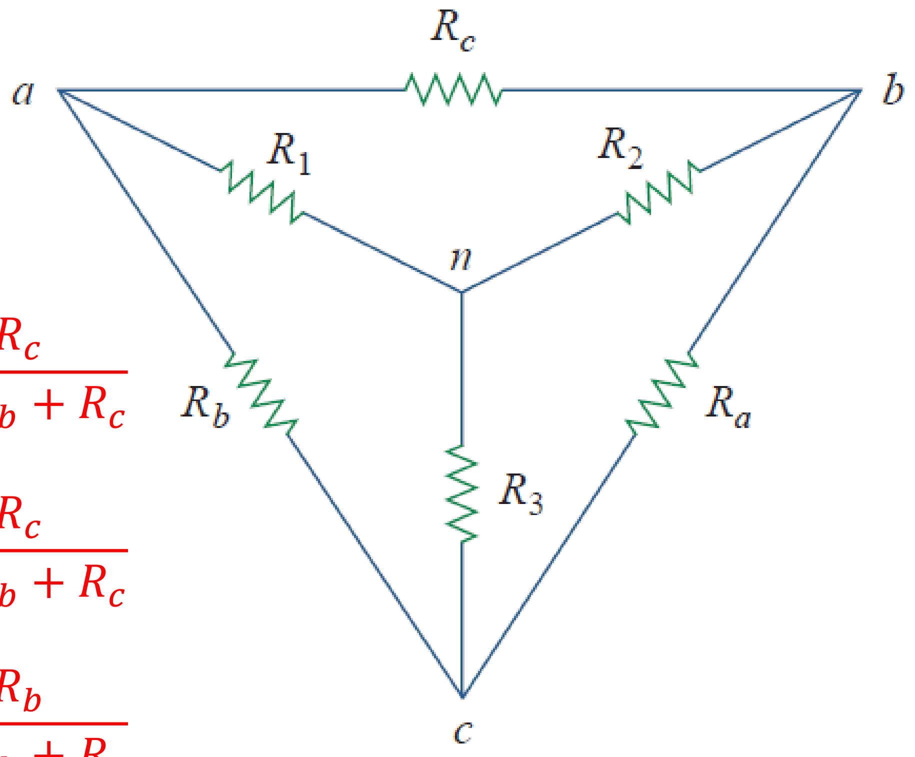


$$R_{12}(\Delta) = R_b \times (R_a + R_c)$$



$$R_{12}(Y) = R_1 + R_3$$

Each resistor in the wye network is the product of the resistors in the two adjacent delta branches, divided by the sum of the three delta resistors.



$$\left.\begin{aligned}
R_{12} &= R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \\
R_{13} &= R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \\
R_{34} &= R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}
\end{aligned}\right\} \to \cdots \to \left\{\begin{aligned}
R_1 &= \frac{R_b R_c}{R_a + R_b + R_c} \\
R_2 &= \frac{R_a R_c}{R_a + R_b + R_c} \\
R_3 &= \frac{R_a R_b}{R_a + R_b + R_c}
\end{aligned}\right.$$

Wye to Delta Conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

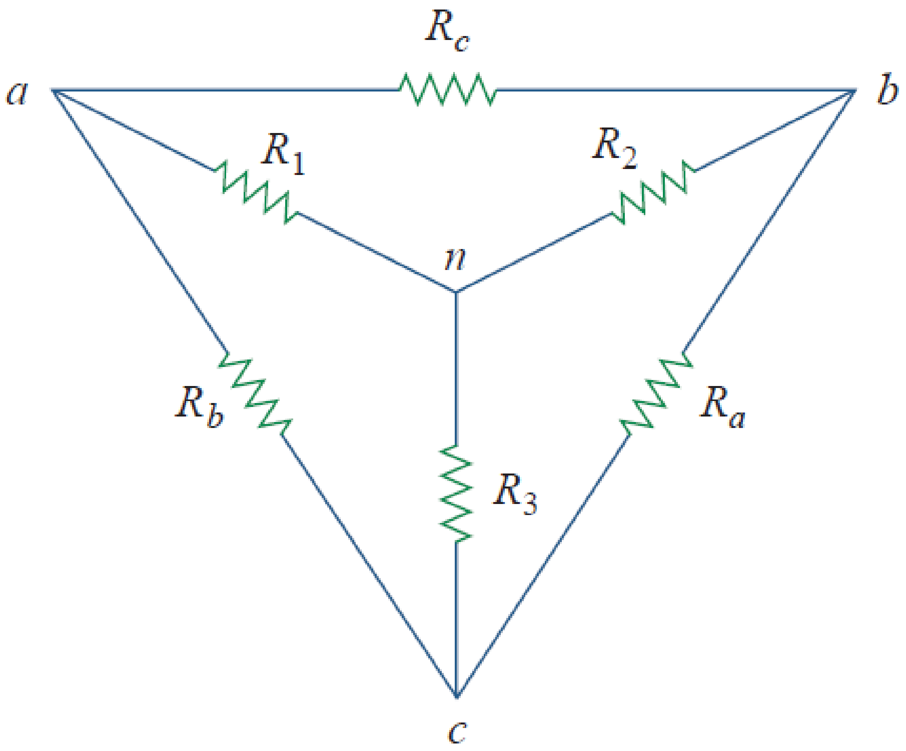
$$\rightarrow \cdots \rightarrow$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

Each resistor in the delta network is the sum of all possible product of Y resistors taken two at a time, divided by the oposite Y resistor.



OR … Each conductor in the delta network is the product of the conductors in the two adjacent wye branches, divided by the sum of the three wye conductors.

$$G_a = \frac{G_2 G_3}{G_1 + G_2 + G_3}$$

$$G_b = \frac{G_1 G_3}{G_1 + G_2 + G_3}$$

$$G_c = \frac{G_1 G_2}{G_1 + G_2 + G_3}$$

Balanced network →

$$R_Y = \frac{R_\Delta}{3}, \qquad R_\Delta = 3 \cdot R_Y$$

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Questions