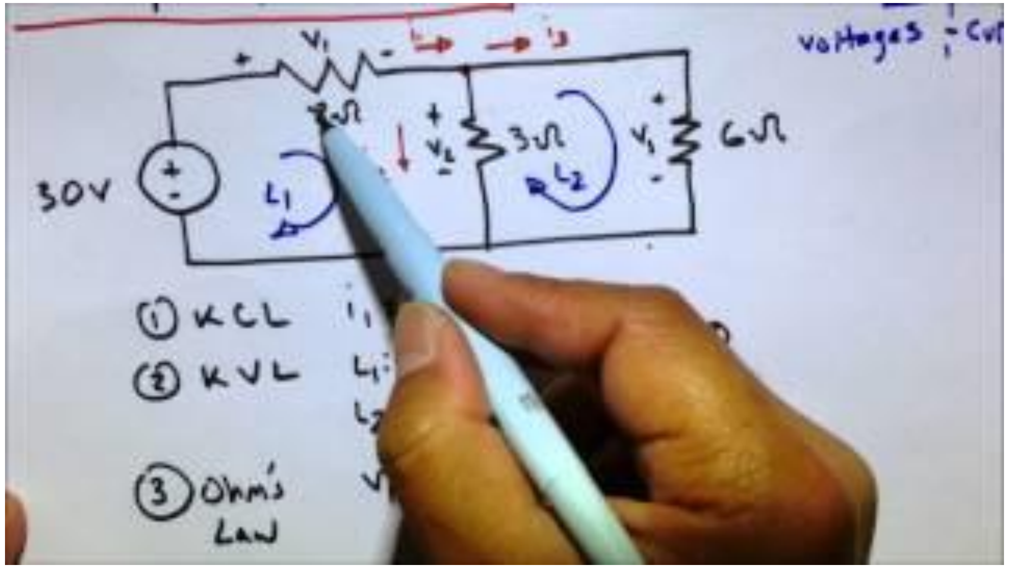
DR. GYURCSEK ISTVÁN

Methods of Circuit Analysis

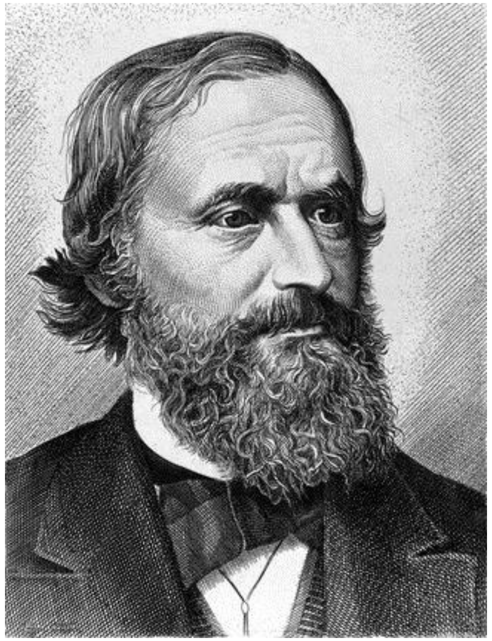
Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*



KCL + KVL + CHAR

KCL + KVL + Characteristics → WORKS … *(but difficult)*



‚Alternatives'…

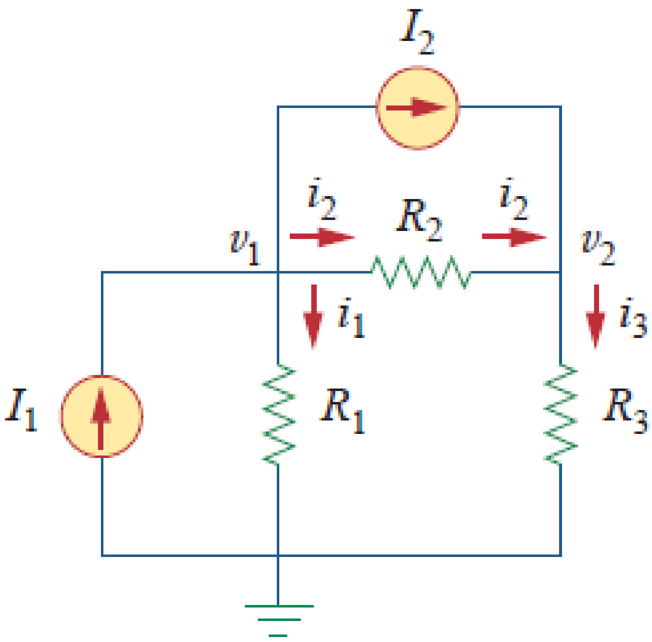
- ❑ **METHODS**
  - Nodal analysis
  - Mesh analysis
- ❑ ***THEOREMS***
  - *Superposition theorem*
  - *Thevenin's theorem*
  - *Norton's theorem*
  - *Source transform*

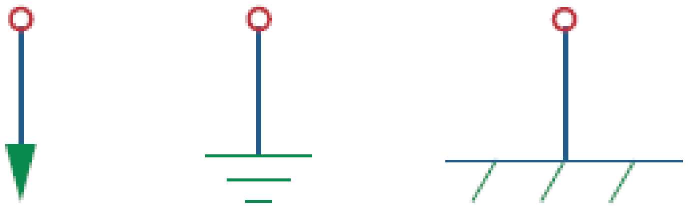
Progress in Content

- **Nodal Analysis**
- Mesh Analysis
- Applications *(DC Transistor Circuits)*

Nodal Analysis

- Goal → determining each nodal voltage *(one is reference = 0 V)*
- Simbols for reference node (GND)





$$principle \rightarrow i = \frac{v_{higher} - v_{lower}}{R}$$

$$\textbf{node 1} \rightarrow I_1 = I_2 + i_1 + i_2$$

$$\textbf{node 2} \rightarrow I_2 + i_2 = i_3$$

$$\textbf{node 1} \rightarrow I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$\rightarrow I_1 = I_2 + v_1 G_1 + G_2(v_1 - v_2)$$

$$\textbf{node 2} \rightarrow I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

$$\rightarrow I_2 + G_2(v_1 - v_2) = G_3 v_2$$

$$(G_1 + G_2)v_1 - G_2 v_2 = I_1 - I_2$$

$$-G_2 v_1 + (G_2 + G_3)v_2 = I_2$$

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

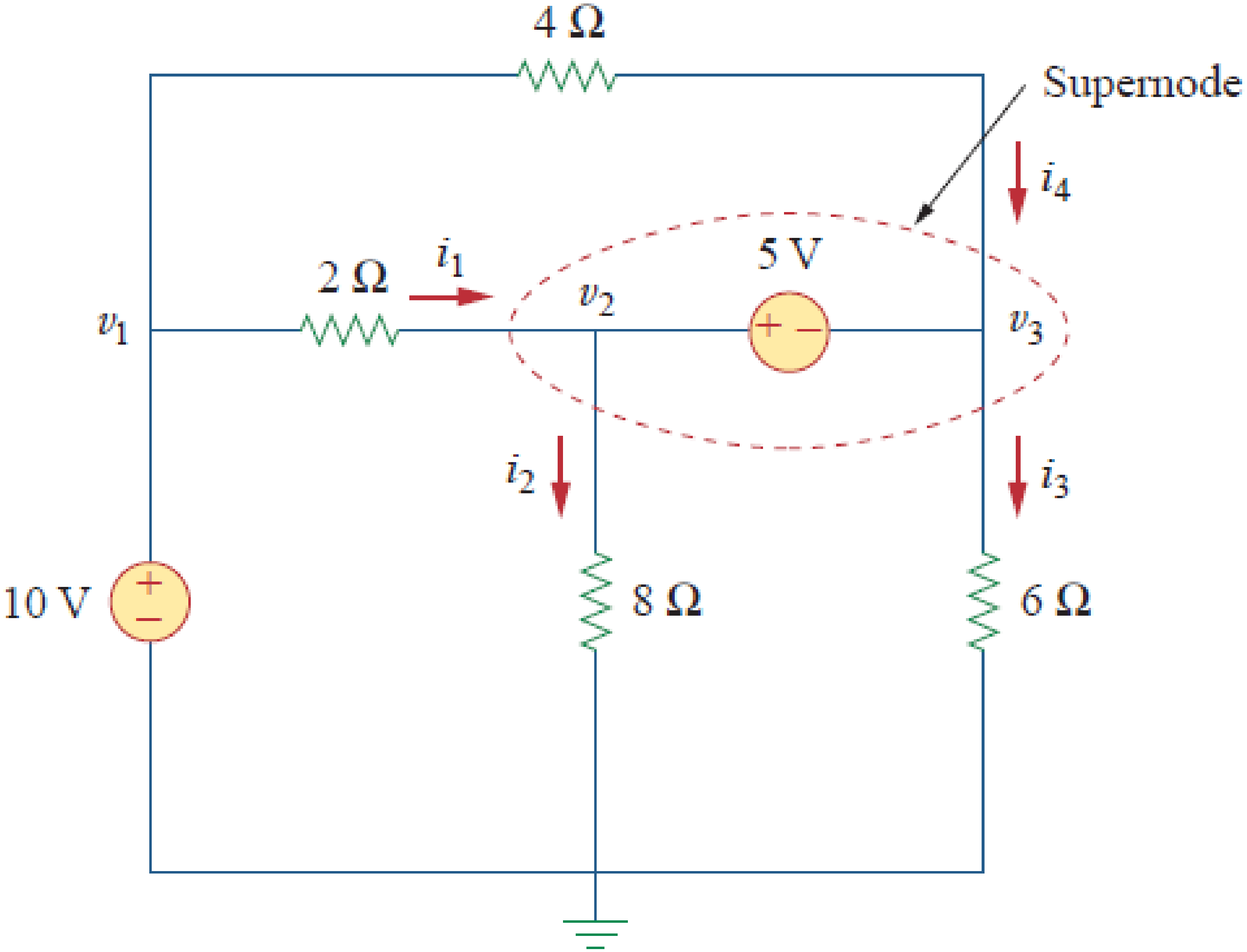
$$Cramer's\ rule \rightarrow v_1 = \frac{\Delta_1}{\Delta}, v_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta = det \begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix}$$

$$\Delta_1 = det \begin{bmatrix} I_1 - I_2 & -G_2 \\ I_2 & G_2 + G_3 \end{bmatrix}$$

$$\Delta_2 = det \begin{bmatrix} G_1 + G_2 & I_1 - I_2 \\ -G_2 & I_2 \end{bmatrix}$$

Nodal Analysis with Voltage Sources – Supernode



- CASE 1 → Volatege source bw. node and ref. node

  $(\boldsymbol{eq.1})\!: \; v_1 = 10\ V$

- CASE 2 → Voltage source bw. two nodes

  $v_2; v_3 \rightarrow form\ super\ node$

  $i_1 + i_4 = i_2 + i_3 \rightarrow$

  $(\boldsymbol{eq.2})\!: \; \frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$

  $(\boldsymbol{eq.3})\!: \; v_2 - v_3 = 5$ *(additional KVL!)*

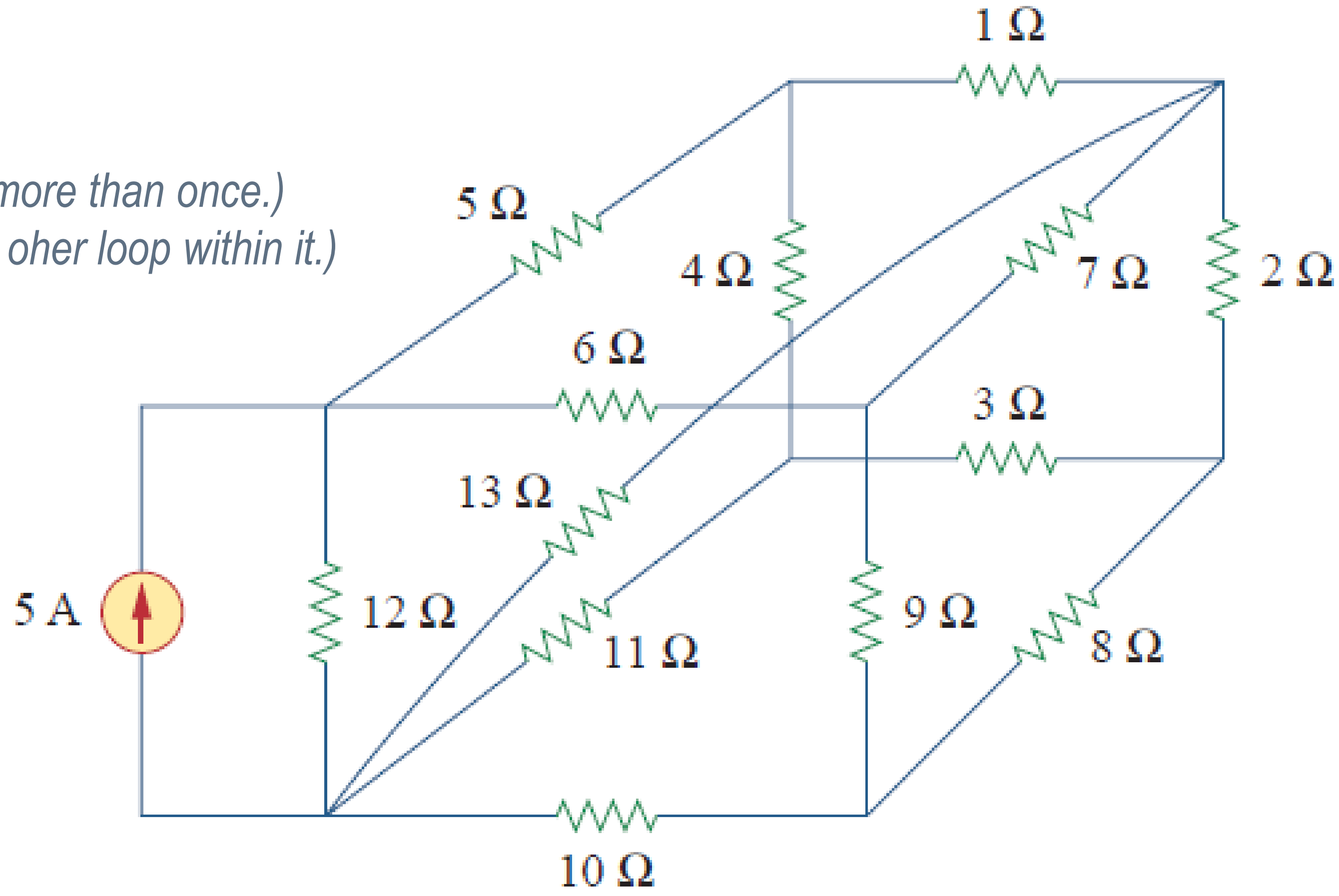
Progress in Content

- Nodal Analysis
- **Mesh Analysis**
- Applications *(DC Transistor Circuits)*

Mesh Analysis

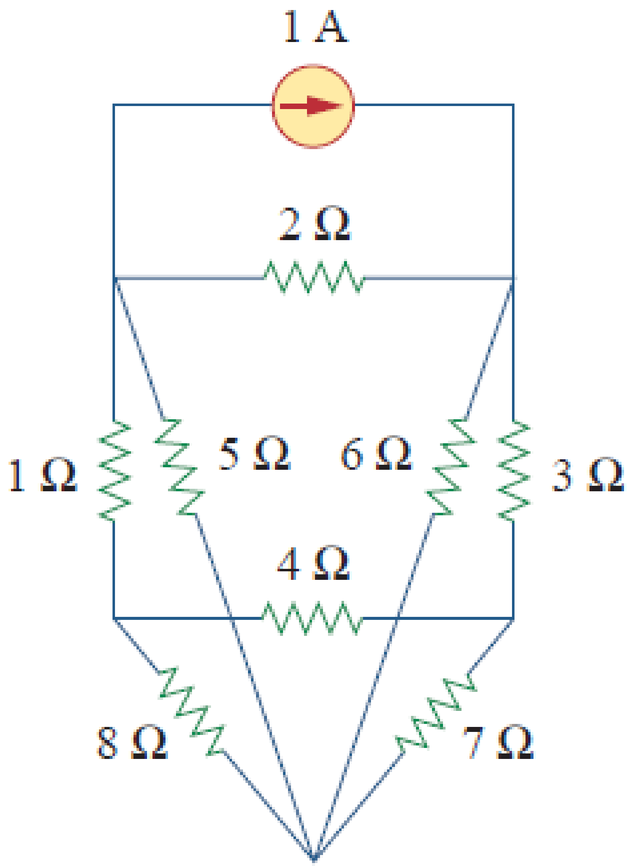
- ❑ Mesh current → circuit variable
- ❑ *(Loop → closed path, no node passed more than once.)*
- ❑ *(Mesh → loop that does not contain any oher loop within it.)*

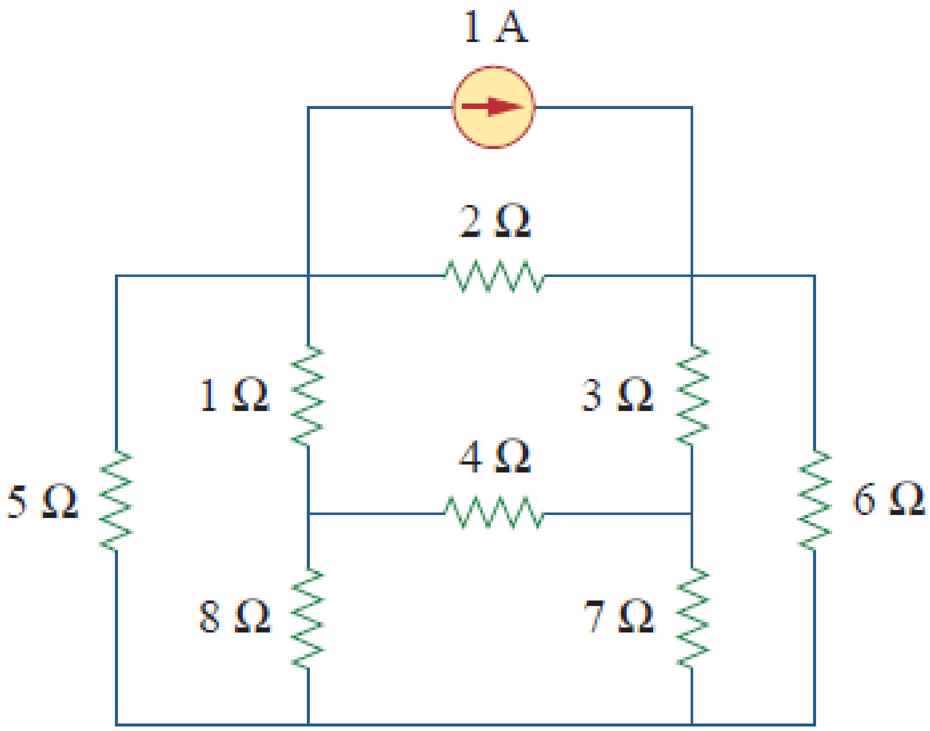
Mesh analysis →
easy to use for planar circuits!
(nonplanar: can't be drawn in plain w/o crossing branches)



Example for Planar Circuit

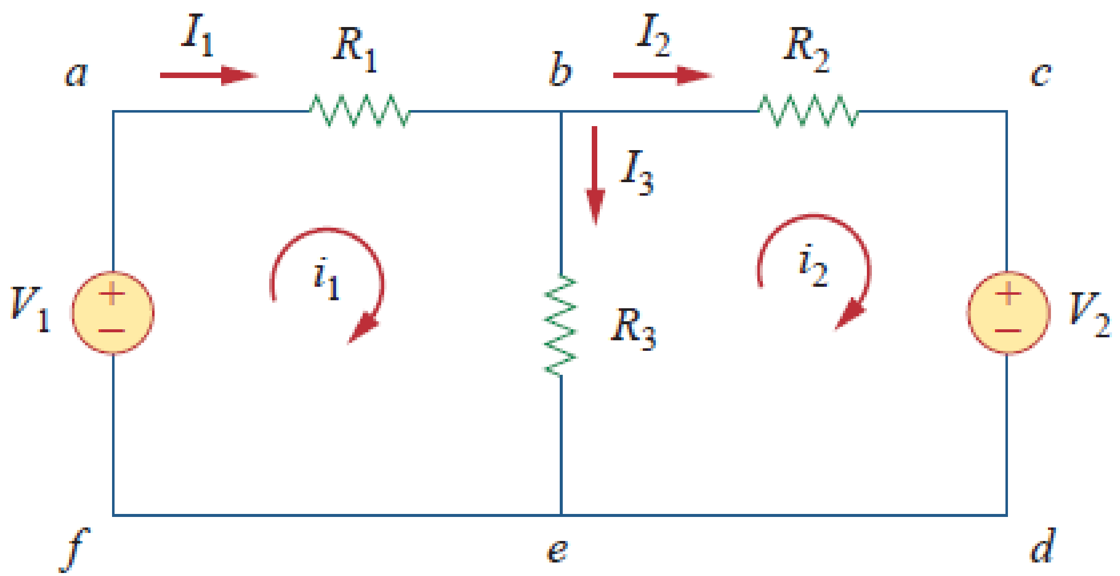
Seems to be nonplanar → REDRAW → planar





Mesh Analysis

Mesh → ‚window'



$$\begin{bmatrix} R_1+R_3 & -R_3 \\ -R_3 & R_2+R_3 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

$$Cramer's\ rule \ \rightarrow i_1 = \frac{\Delta_1}{\Delta}, i_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta = det \begin{bmatrix} R_1+R_3 & -R_2 \\ -R_2 & R_2+R_3 \end{bmatrix}$$

$$\Delta_1 = det \begin{bmatrix} V_1 & -R_2 \\ -V_2 & R_2+R_3 \end{bmatrix}$$

$$\Delta_2 = det \begin{bmatrix} R_1+R_3 & V_1 \\ -R_2 & -V_2 \end{bmatrix}$$

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0 \rightarrow (R_1+R_3)i_1 - R_3 i_2 = V_1$$

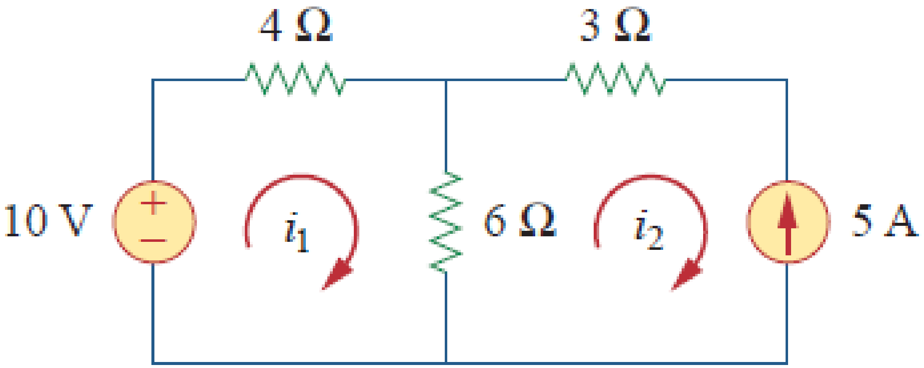
$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0 \rightarrow -R_3 i_1 + (R_2+R_3)i_2 = -V_2$$

Last step! $\rightarrow I_1 = i_1, I_2 = i_2, I_3 = i_1 - i_2$

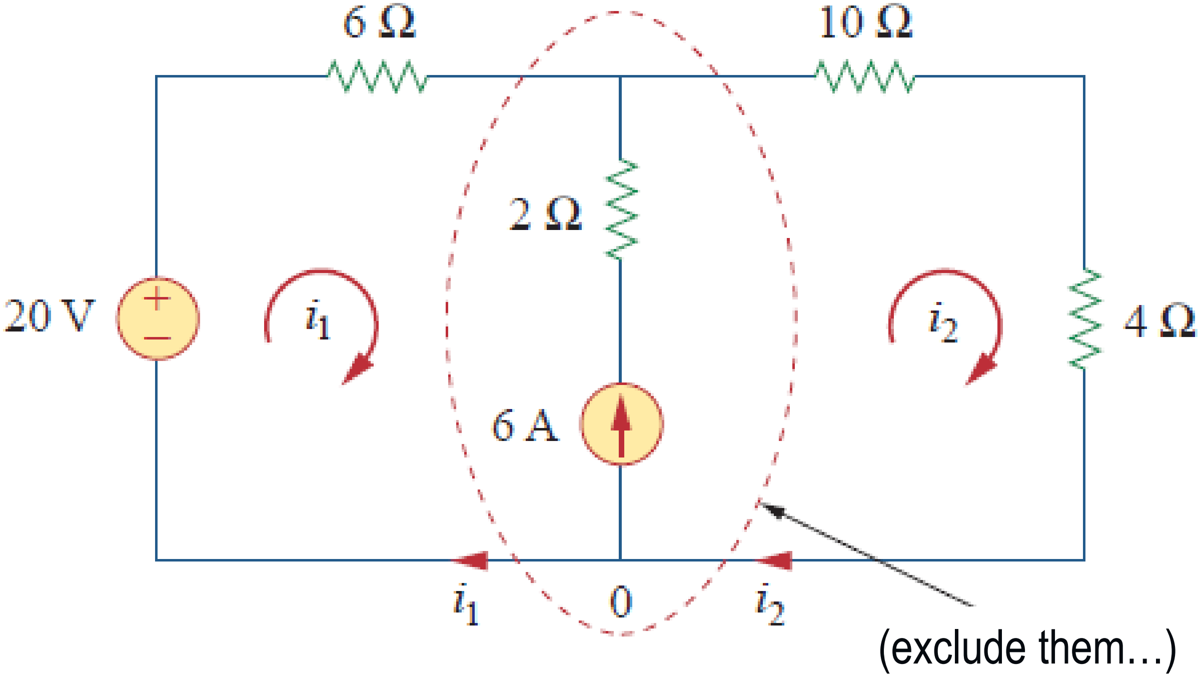
Mesh Analysis with Current Sources – Supermesh

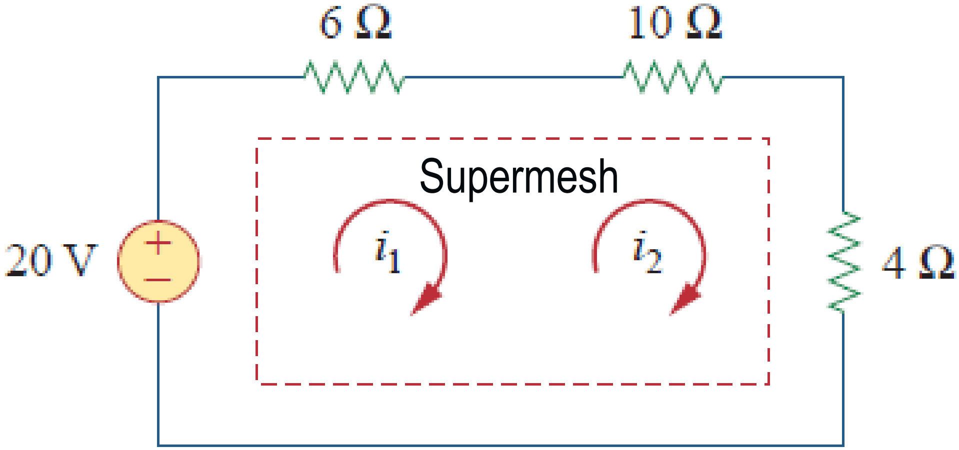
- CASE 1 → Current source in one mesh

$$i_2 = -5\,A\,, \qquad -10 + 4i_1 + 6(i_1 - i_2) = 0 \rightarrow i_1 = -2\,A$$



- CASE 2 → Common current source in two meshes





$$-20 + 6i_1 + 10i_2 + 4i_2 = 0\,, \qquad i_2 - i_1 = 6 \rightarrow i_1 = -3{,}2\,A\,, \qquad i_2 = 2.8\,A$$

WHICH ONE IS BETTER…

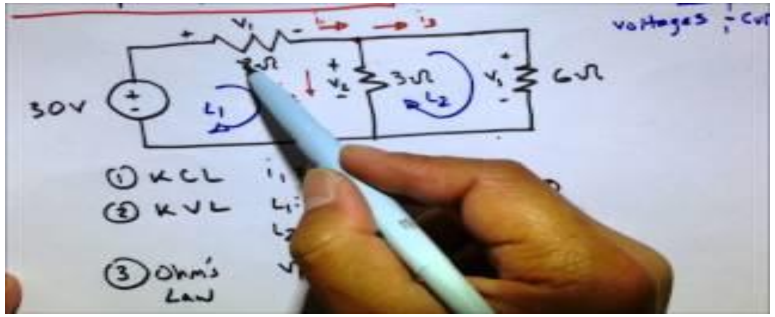
THE LESS EQUATIONS → THE EASIER SOLUTION

Advice-1

- ❑ Mesh analysis
  Many <u>series elements</u>, <u>voltage sources</u>, <u>supermeshes</u>, <u>fewer meshes than nodals</u>
- ❑ Nodal analysis
  Many <u>parallel elements</u>, <u>current sources</u>, <u>supernodes</u>, <u>fewer nodals than meshes</u>

Advice-2

- ❑ Nodal analysis
  Nodal <u>voltages required</u>
- ❑ Mesh analysis
  Branch / mesh <u>currents required</u>



Progress in Content

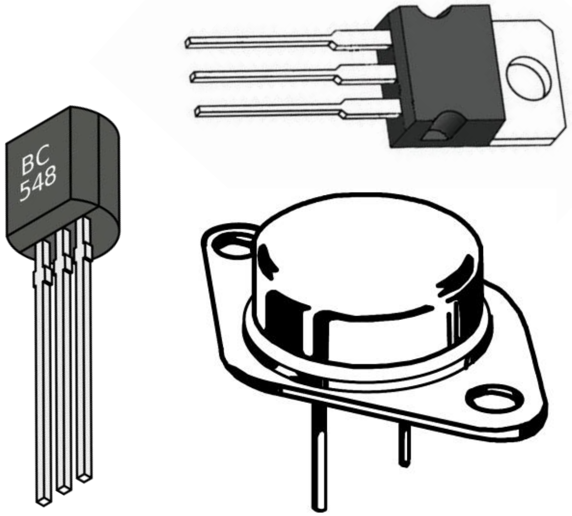
- ❑ Nodal Analysis
- ❑ Mesh Analysis
- ❑ **Applications *(DC Transistor Circuits)***

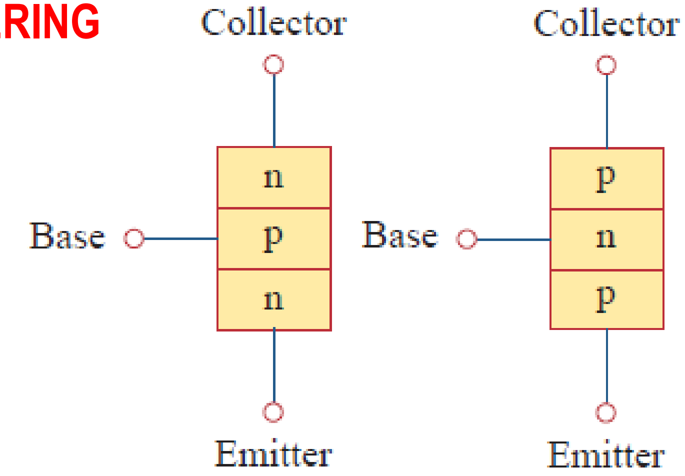
Applications: DC Transistor Circuits

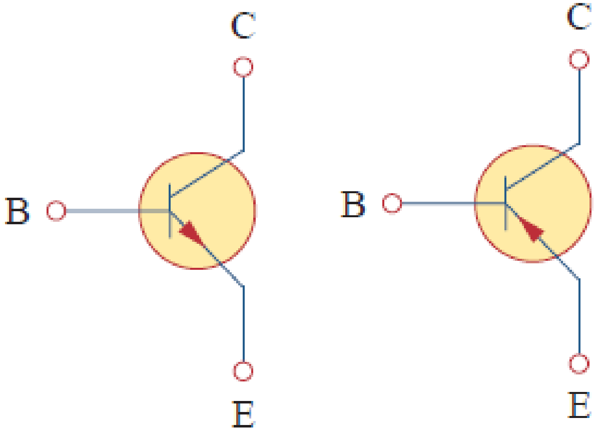
INDUSTRIAL AGE → AGE OF THE ENGINEERING

Bell Labs

- ❑ Bardeen, Brattain ('47) point contact trans.
- ❑ Schockley ('48) bip. junct. transistor (BJT)
- ❑ Schockley ('54) FET (widely used today)
- ❑ Nobel Prize in Phisics ('56) for 3 inventors







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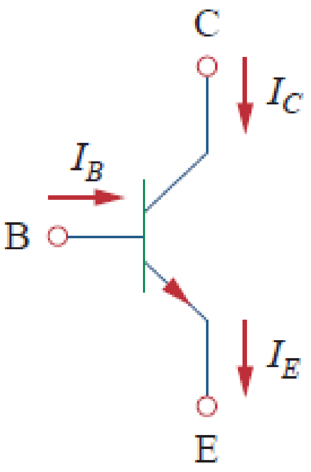
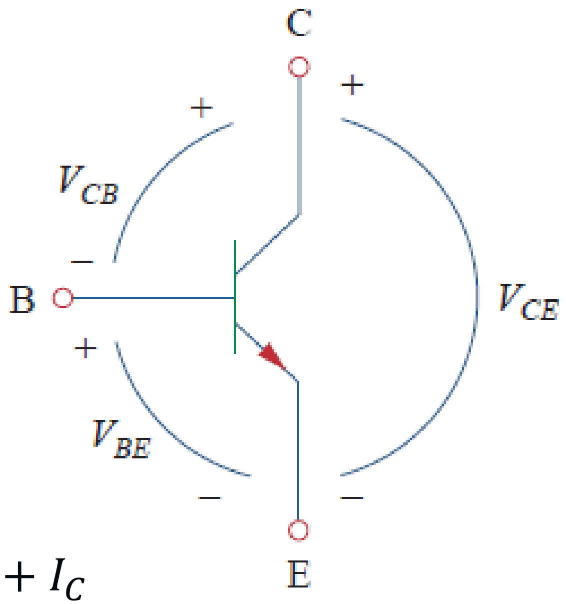
Transistor Model

Transistor → current dependent current source





$KCL \rightarrow I_E = I_B + I_C$

$KVL \rightarrow V_{CE} = V_{BE} + V_{BC}$

Characteristic properties

- Common-base current gain

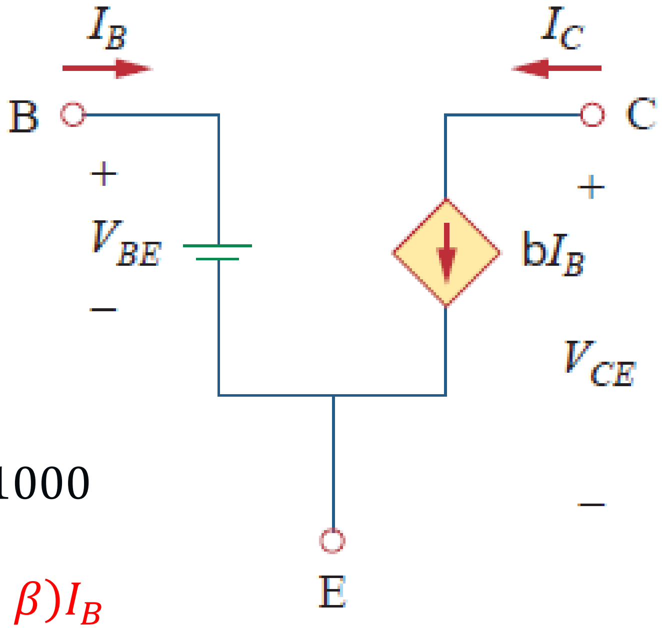
$(\alpha) \rightarrow I_C = \alpha I_E, \qquad \alpha \approx 0.99$

- Commom-emitter current gain

$(\beta) \rightarrow I_C = \beta I_B, \qquad \beta \approx 50 \ldots 1000$

$I_E = I_B + I_C = I_B + \beta I_B = (1+\beta) I_B$

$\beta = \dfrac{I_C}{I_B} = \dfrac{\alpha I_E}{I_E - I_C} = \dfrac{\alpha I_E}{I_E - \alpha I_E} = \dfrac{\alpha}{1-\alpha}$



$Active\ mode$
$\rightarrow V_{BE} \approx 0.7\ V$

Operation modes → [active] , [cutoff] , [saturation]

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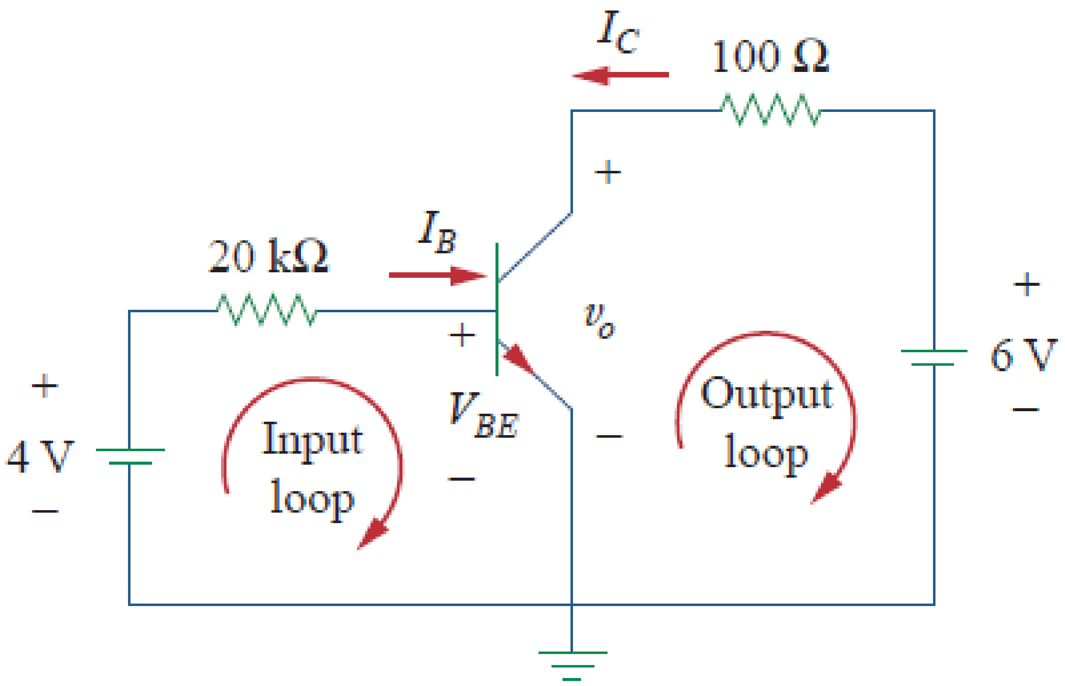
Example with DC Transistor Circuits

Find I_B. I_C, v_0 if β = 50 (transistor operates in active mode)



$$Input\ loop: -4 + I_B \cdot 20 \cdot 10^3 + V_{BE} = 0$$

$$V_{BE} = 0.7 \rightarrow I_B = \frac{4 - 0.7}{20 \cdot 10^3} = 165\ \mu A$$

$$I_C = \beta \cdot I_B = 50 \cdot 165\ \mu A = 8.25\ mA$$

$$Output\ loop: -v_0 - 100 I_C + 6 = 0$$

$$v_0 = 6 - 100 I_C = 6 - 0.825 = 5.175\ V$$

Questions