DR. GYURCSEK ISTVÁN

Circuit Theorems

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*

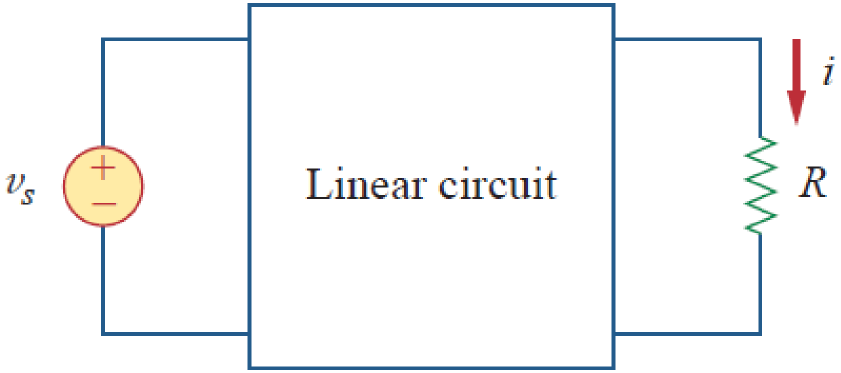
Progress in Content

- ❑ **Linearity Property**
- ❑ Superposition Principle
- ❑ Source Transformation
- ❑ Thevenin’s Theorem
- ❑ Norton’s Theorem
- ❑ Maximum Power Transfer
- ❑ Applications: Practical Sources

Linearity Property

Conditions of linearity

- ❑ *Homogenity* $y = g(x) \rightarrow a \cdot y = g(a \cdot x)$
- ❑ *Additivity* $y_1 = g(x_1), y_2 = g(x_2) \rightarrow y_1 + y_2 = g(x_1 + x_2)$



Linear circuit (no independent source internally)

- ❑ Linear relationship bw. v_S input (*excitation*) and i output (*response*)
- ▪ Homogenity $i = G \cdot v_S \rightarrow a \cdot i = G \cdot (a \cdot v_S)$
- ▪ Additivity $i_1 = G \cdot v_{S1}, \quad i_2 = G \cdot v_{S2} \quad \rightarrow \quad i_1 + i_2 = G \cdot v_{S1} + G \cdot v_{S2} = G \cdot (v_{S1} + v_{S2})$

WARNING! – The power relation is nonlinear!

$$\left.\begin{aligned} i_1 \rightarrow p_1 = i_1^2 \cdot R \\ i_2 \rightarrow p_2 = i_2^2 \cdot R \end{aligned}\right\} \; but \;\; i_1 + i_2 \rightarrow p_{12} = (i_1 + i_2)^2 \cdot R = i_1^2 \cdot R + i_2^2 \cdot R + 2 \cdot i_1 \cdot i_2 \cdot R \qquad p_{12} \neq p_1 + p_2$$

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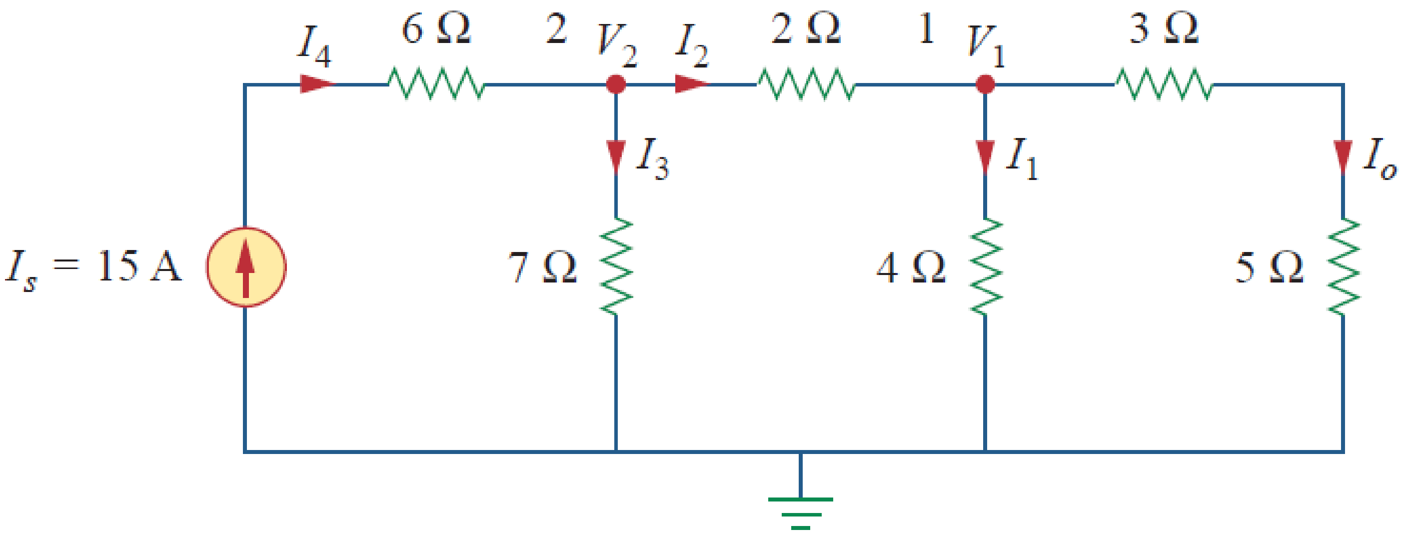
Calculation Example Using Linearity Property

Find I_0 in the circuit.



(1): $assume\ I_0 = 1\ A$

(2): $V_1 = (3+5) \cdot I_0 = 8\ V$

(3): $I_1 = \frac{V_1}{4} = 2\ A$

(4): $I_2 = I_1 + I_0 = 3\ A$

(5): $V_2 = I_2 \cdot 2 + V_1 = 14\ V$

(6): $I_3 = \frac{V_2}{7} = 2\ A$

(7): $I_4 = I_2 + I_3 = 5\ A$

(8): $but \rightarrow\ I_4^{REAL} = I_S = \mathbf{3} \cdot 5 = 15\ A$

(9): $thus \rightarrow I_0^{REAL} = \mathbf{3} \cdot I_0 = 3\ A$

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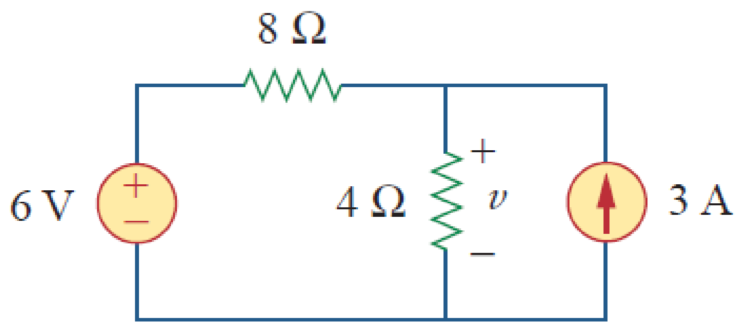
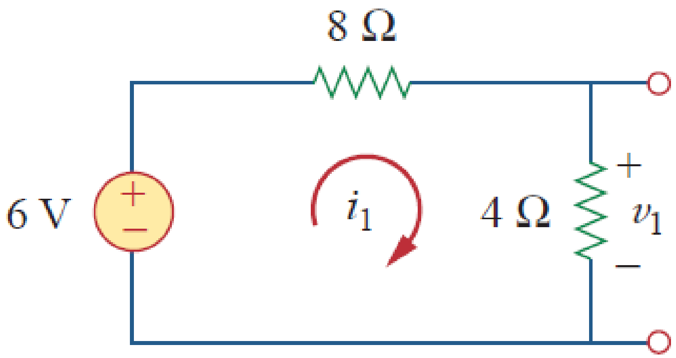
Superposition Principle

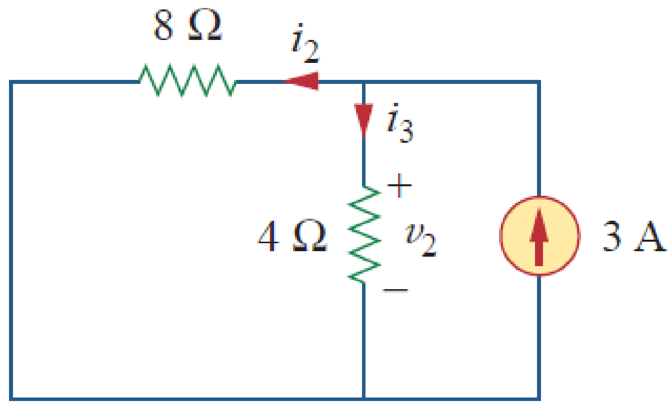
Steps to apply…

- Turn off independent sources except one and find output.
- Repeat it for each of independent sources.
- Find total by adding all the contributions.



Example – Find *v* by using superposition. $v = v_1 + v_2$





$12i_1 - 6 = 0 \rightarrow i_1 = 0.5\ A$

$v_1 = 4i_1 = 2\ V$ or $v_1 = \frac{4}{4+8}6 = 2\ V$

$i_3 = \frac{8}{4+8}3 = 2\ A \rightarrow v_2 = 4i_3 = 8\ V$

$v = v_1 + v_2 = 2 + 8 = 10\ V$

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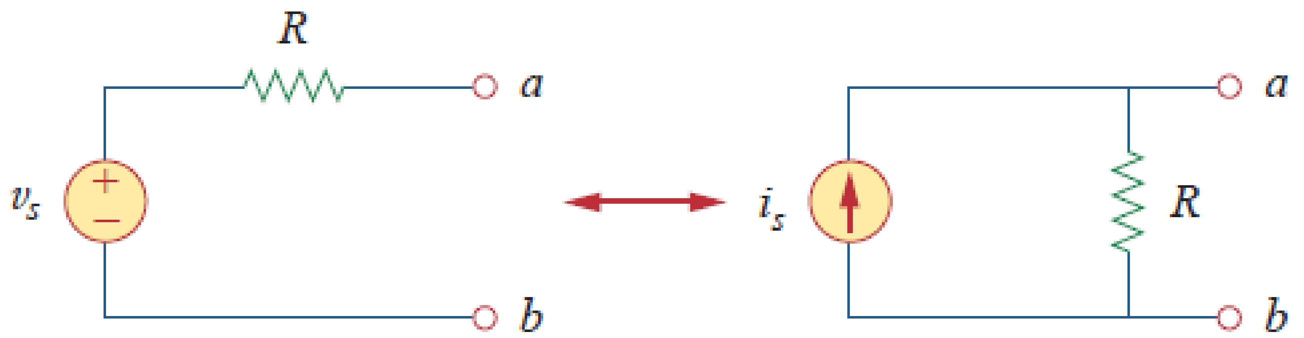
Source Transformation



Source transformation…

- ❑ Another tool for simplifying circuits
- ❑ Active equivalent transformation
- ❑ *For real generators only!* $(R \neq 0, \infty)$

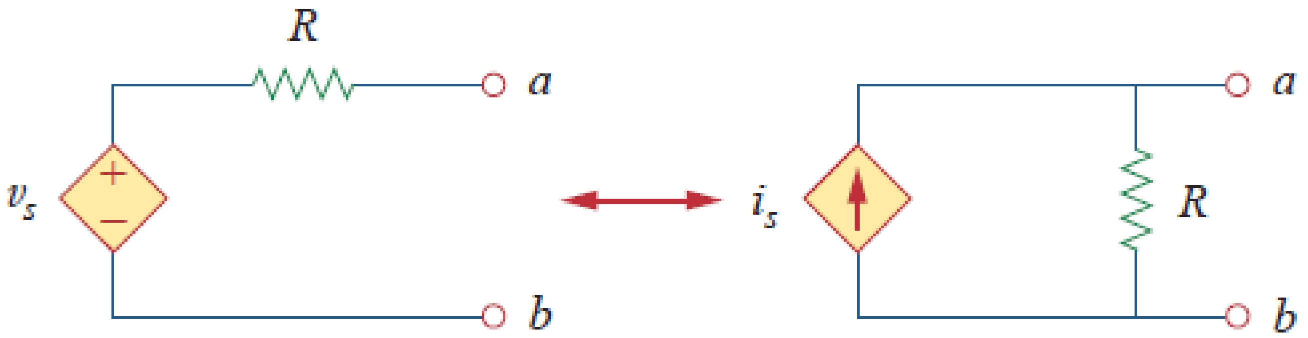
$$R_{Th} = R_N = R$$

$$Thevenin\,'SC' \rightarrow i_{ab} = \frac{v_S}{R}\,, \qquad Norton\,'SC' \rightarrow i_{ab} = i_S$$

$$Thevenin\,'OC' \rightarrow v_{ab} = v_S\,, \qquad Norton\,'OC' \rightarrow v_{ab} = i_S \cdot R$$

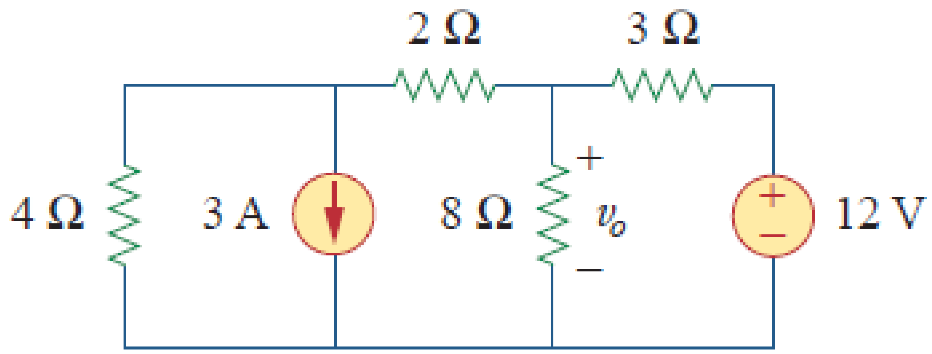
$$v_{Th} = i_N \cdot R$$

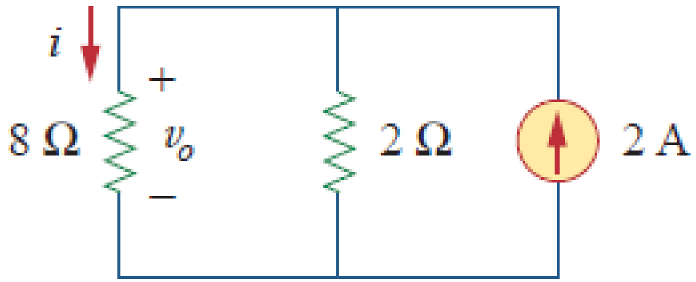
$$i_N = \frac{v_{Th}}{R}$$

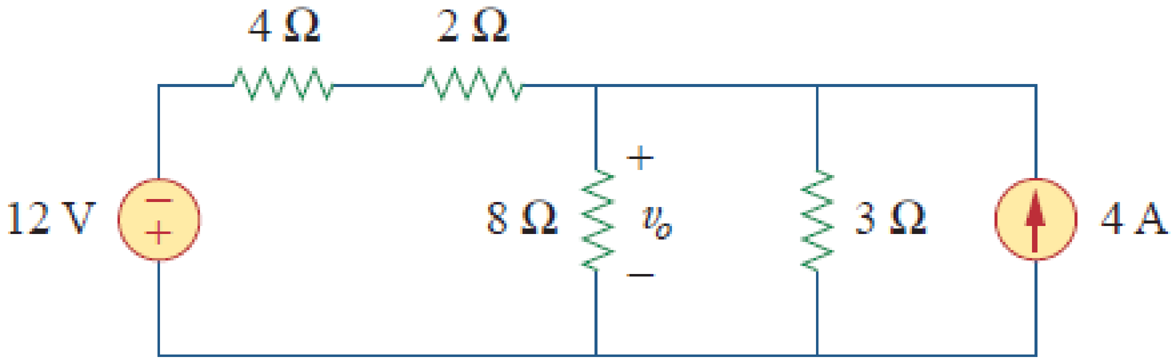


Source Transformation Example

$v_0 = ?$

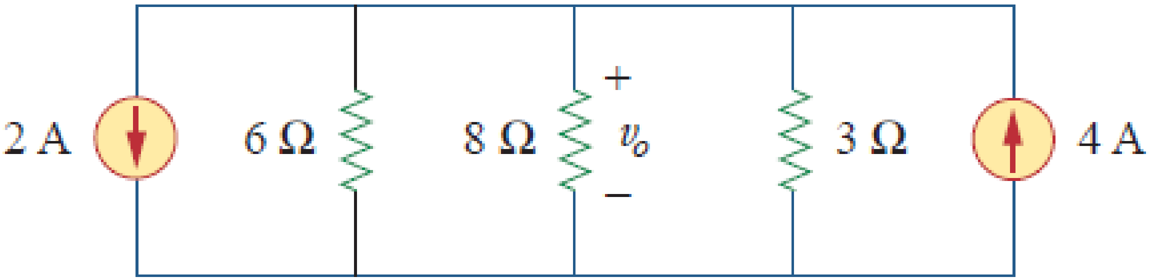






$$i = \frac{2}{2+8} \cdot 2 = 0.4\ A$$

$$v_0 = 8i = 3.2\ V$$



$$or \ldots\ v_0 = R_{eq} \cdot i = \frac{2 \cdot 8}{2+8} \cdot 2 = 3.2\ V$$

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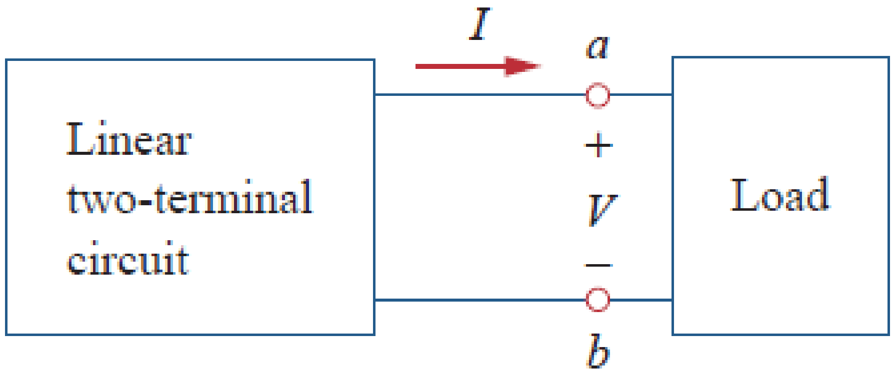
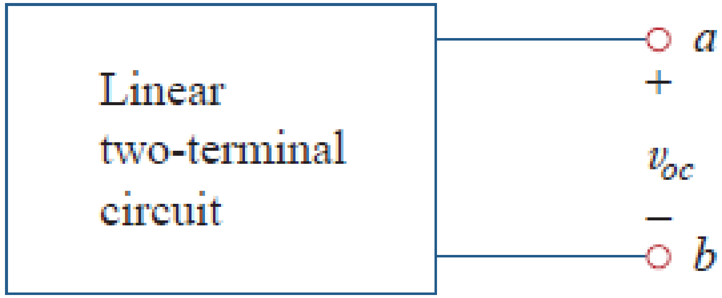
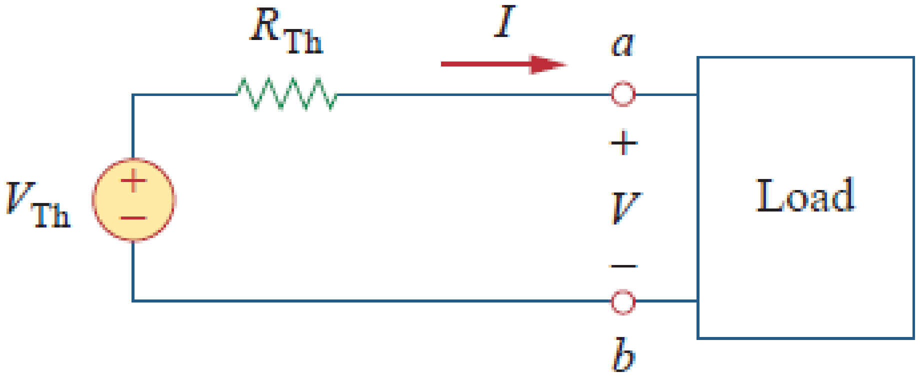
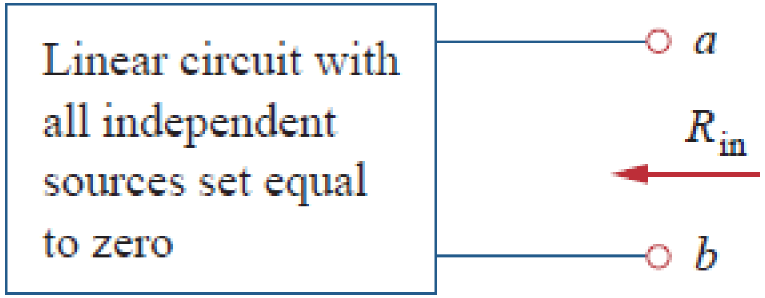




$$V_{Th} = v_{OC}$$





$$R_{Th} = R_{IN}$$

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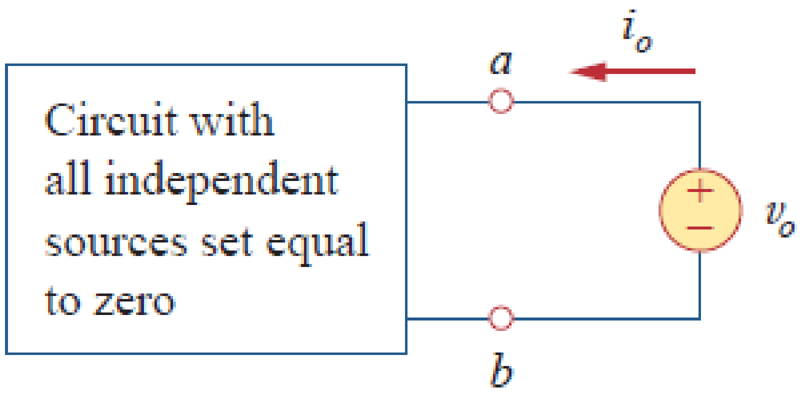
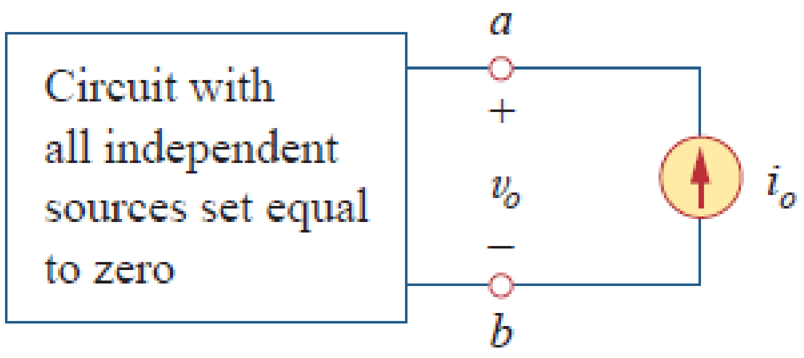
Determining R_{Th}

- CASE 1 – no dependent sources $R_{Th} = R_{IN}$
- CASE 2 – dependent sources also $R_{Th} = \frac{v_0}{i_0}$

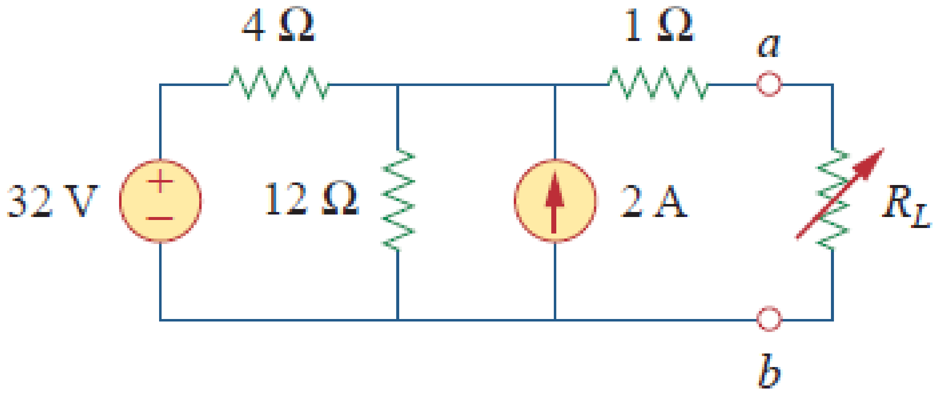


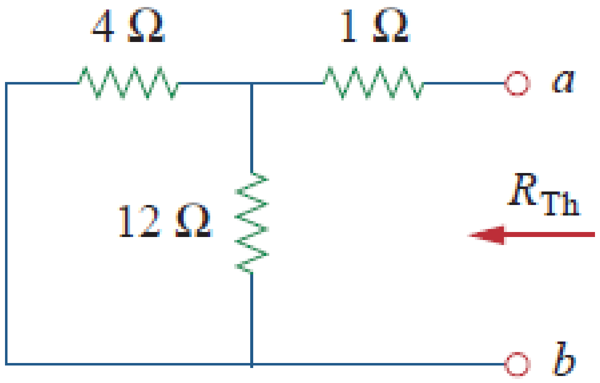


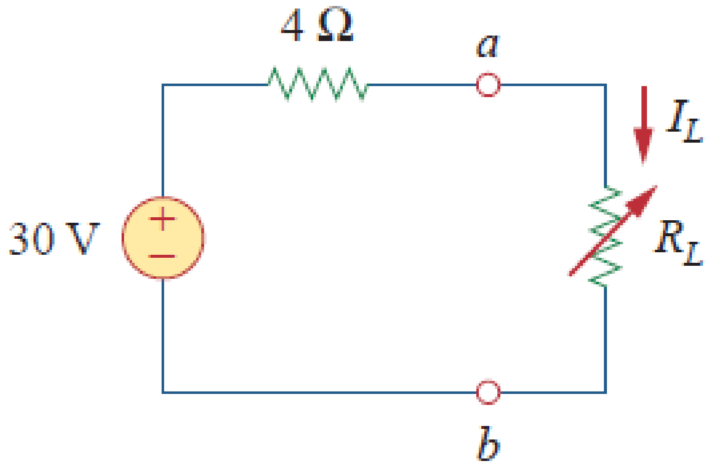
Another way (later on)… $R_{Th} = \frac{v_{OC}}{i_{SC}}$

Thevenin’s Theorem Example

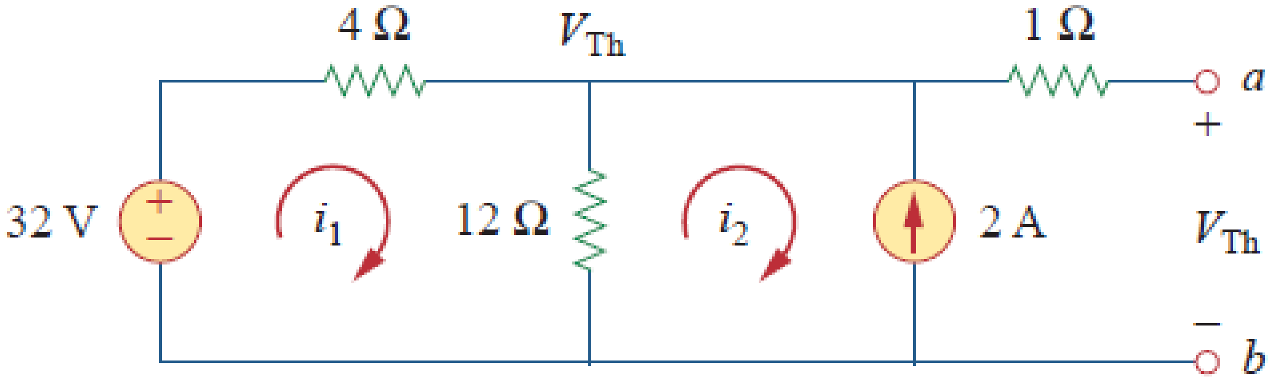
Find the current throug the 6, 16 and 36 ohms load.







$$R_{Th} = 4 \times 12 + 1 = \frac{4 \cdot 12}{16} + 1 = 4\,\Omega$$



$$-32 + 4i_1 + 12(i_1 - i_2) = 0$$

$$i_2 = -2\,A\,, \qquad i_1 = 0.5\,A$$

$$V_{Th} = 12(i_1 - i_2) = 12 \cdot 2.5 = 30\,V$$

$$or\ \ldots \frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12} \rightarrow V_{Th} = 30\,V$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L} = 3, 1.5, 0.75\,A$$

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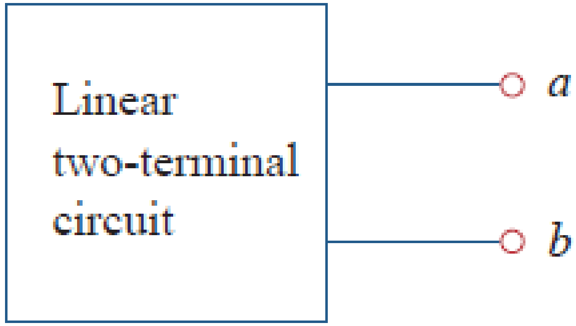
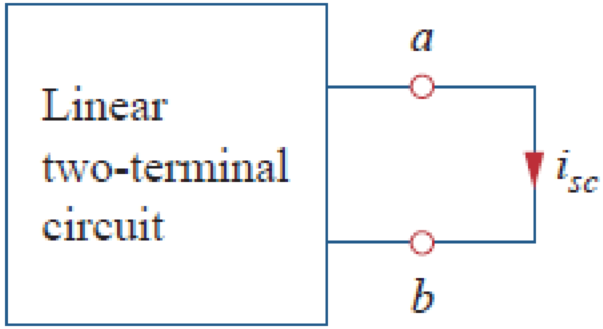
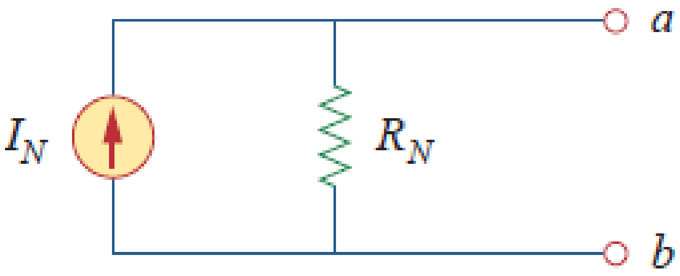
Norton’s Theorem





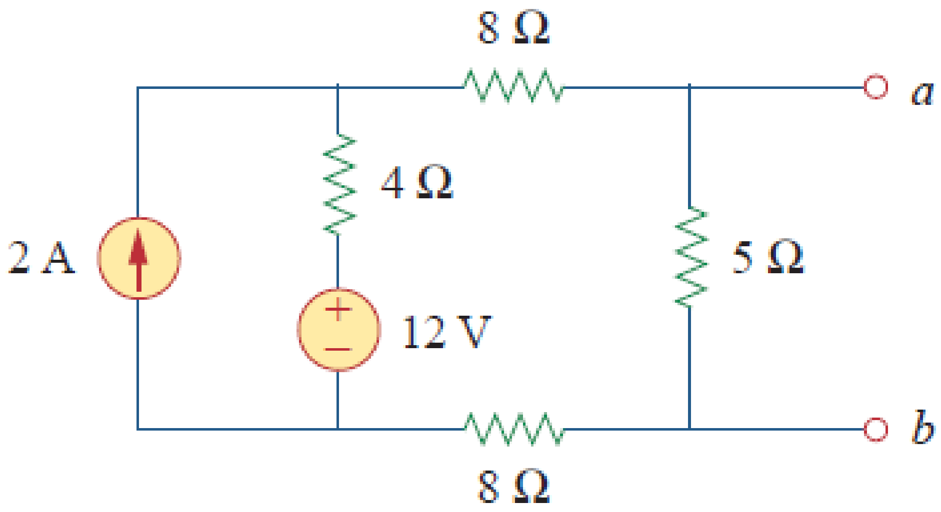


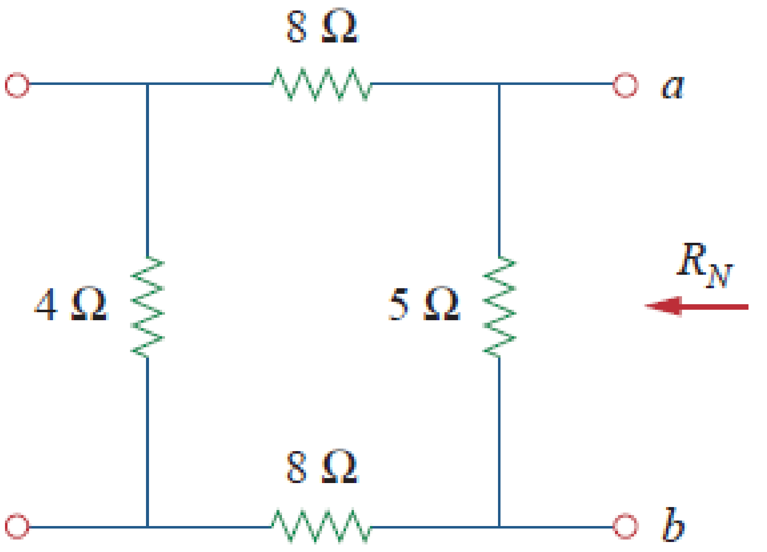
$$I_N = i_{SC} \qquad V_{Th} = v_{OC} \qquad I_N = \frac{V_{Th}}{R_{Th}}$$

$$R_{Th} = \frac{v_{OC}}{i_{SC}} = \frac{V_{Th}}{I_N} = R_N$$

Norton’s Theorem Example

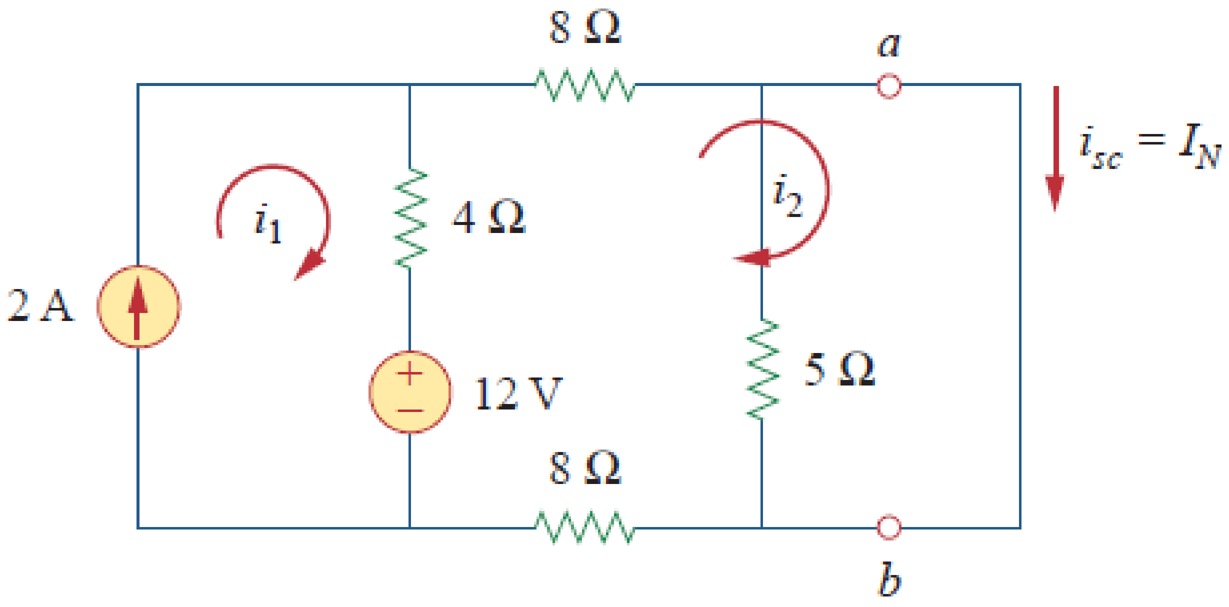
Find the Norton equivalent circuit.





$$R_N = 5 \times (8 + 4 + 8) = \frac{5 \cdot 20}{25} = 4\ \Omega$$

Norton’s Theorem Example



$$i_1 = 2\ A\,, \qquad 20i_2 - 4i_1 - 12 = 0 \rightarrow i_2 = 1\ A = i_{SC} = I_N$$

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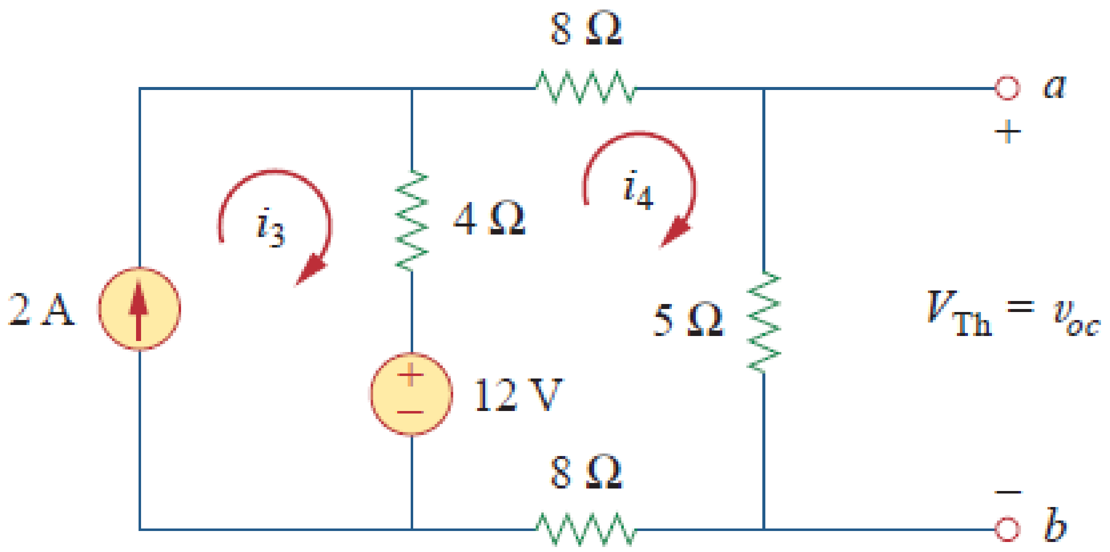
Norton's Theorem Example

Alternatively…

$$I_N = \frac{V_{Th}}{R_{Th}}$$

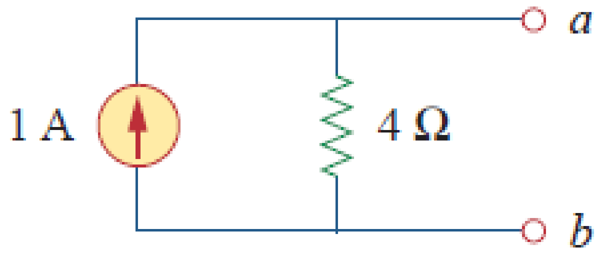


$$i_3 = 2\ A$$

$$25i_4 - 4i_3 - 12 = 0 \rightarrow i_4 = 0.8\ A$$

$$v_{OC} = V_{Th} = 5i_4 = 4\ V$$

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1\ A$$



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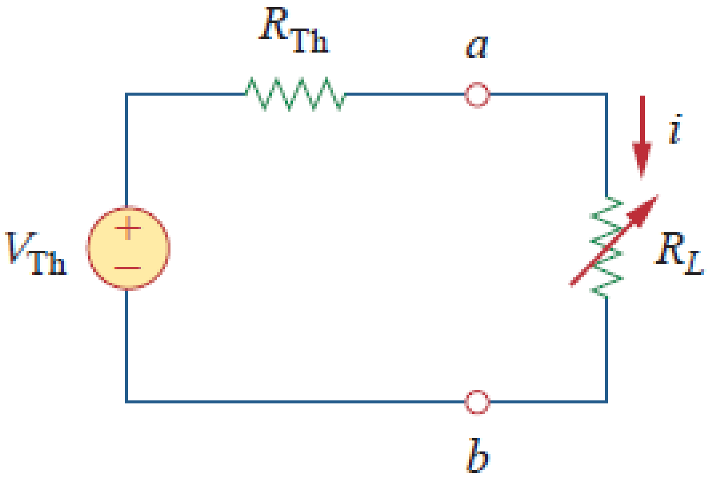
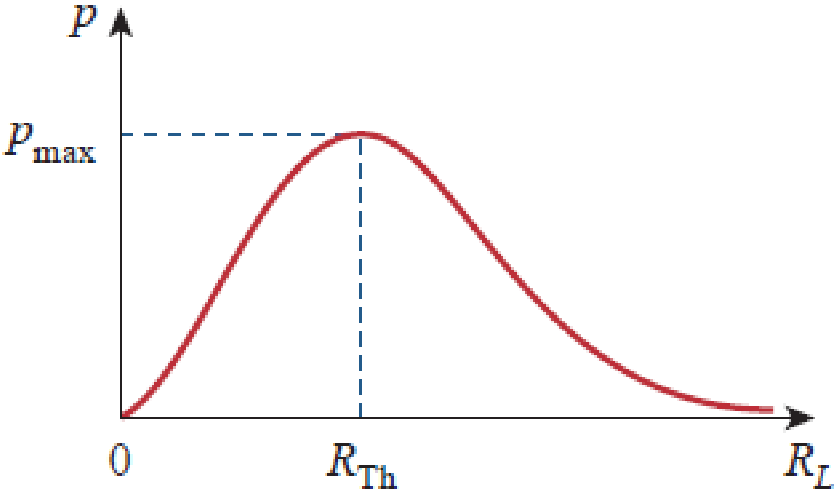
Maximum Power Transfer



$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right] = V_{Th}^2 \left[\frac{R_{Th} + R_L - 2R_L}{(R_{Th} + R_L)^3}\right] = 0$$

$$\rightarrow R_{Th} = R_L$$



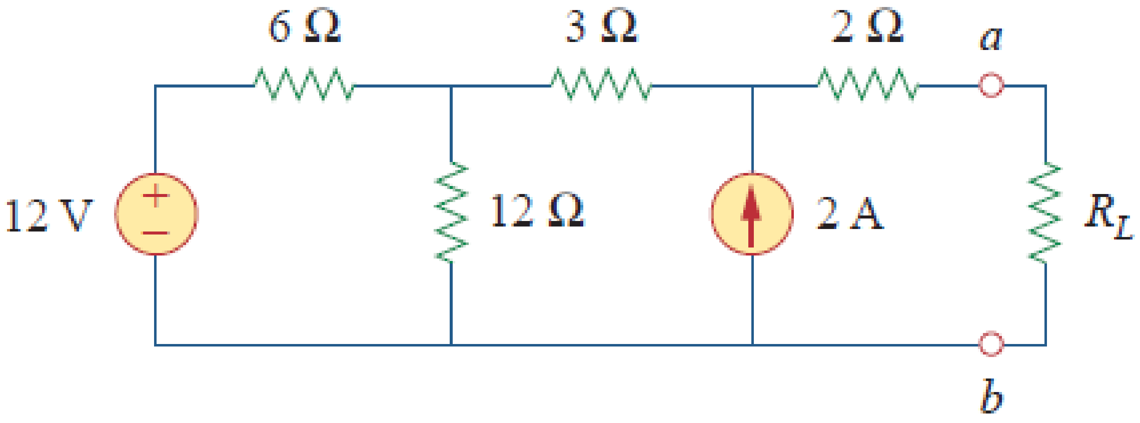
$$\frac{d^2p}{dR_L^2} == V_{Th}^2 \frac{d}{dR_L}\left[\frac{R_{Th} - R_L}{(R_{Th} + R_L)^3}\right] < 0 \leftarrow maximum\ power$$

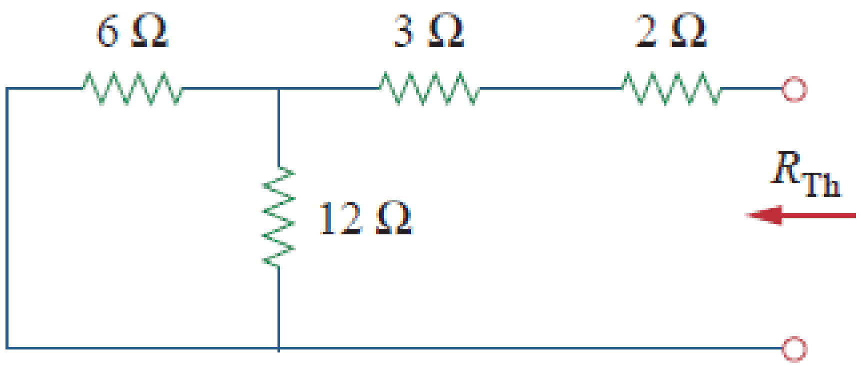
$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

$$\eta = \frac{p_{max}}{p_0} = \frac{V_{Th}^2 / 4R_{Th}}{V_{Th}^2 / 2R_{Th}} = 0.5 = 50\%$$

Maximum Power Transfer Example

Find R_L for maximum power transfer. Find the maximum power.

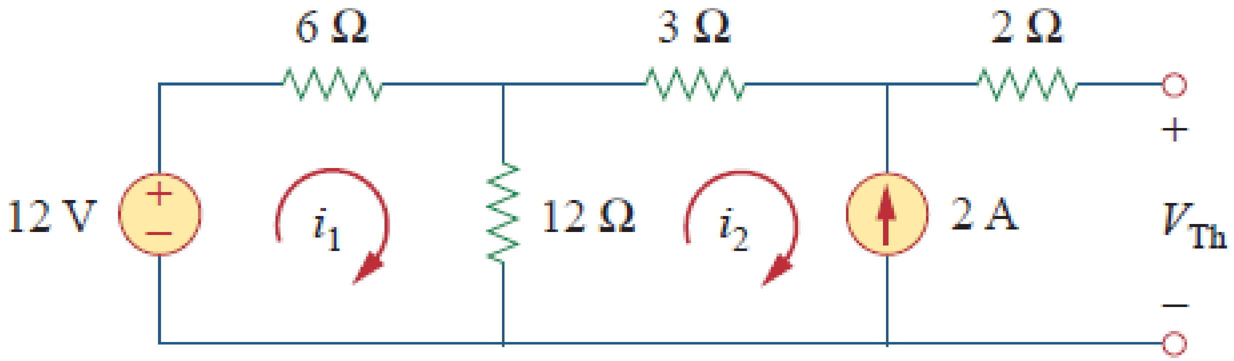




$$R_{Th} = 2 + 3 + 6 \times 12 = 5 + \frac{6 \cdot 12}{18} = 9\ \Omega$$

$$-12 + 18i_1 - 12i_2 = 0\,, \qquad i_2 = -2\ A$$

$$i_1 = -\frac{2}{3}\ A$$



$$-12 + 6i_1 + 3i_2 + 2 \cdot 0 + V_{Th} = 0 \rightarrow V_{Th} = 22\ V$$

$$R_L = R_{Th} = 9\ \Omega\,, \qquad p_{max} = \frac{22^2}{4 \cdot 9} = 13.44\ W$$

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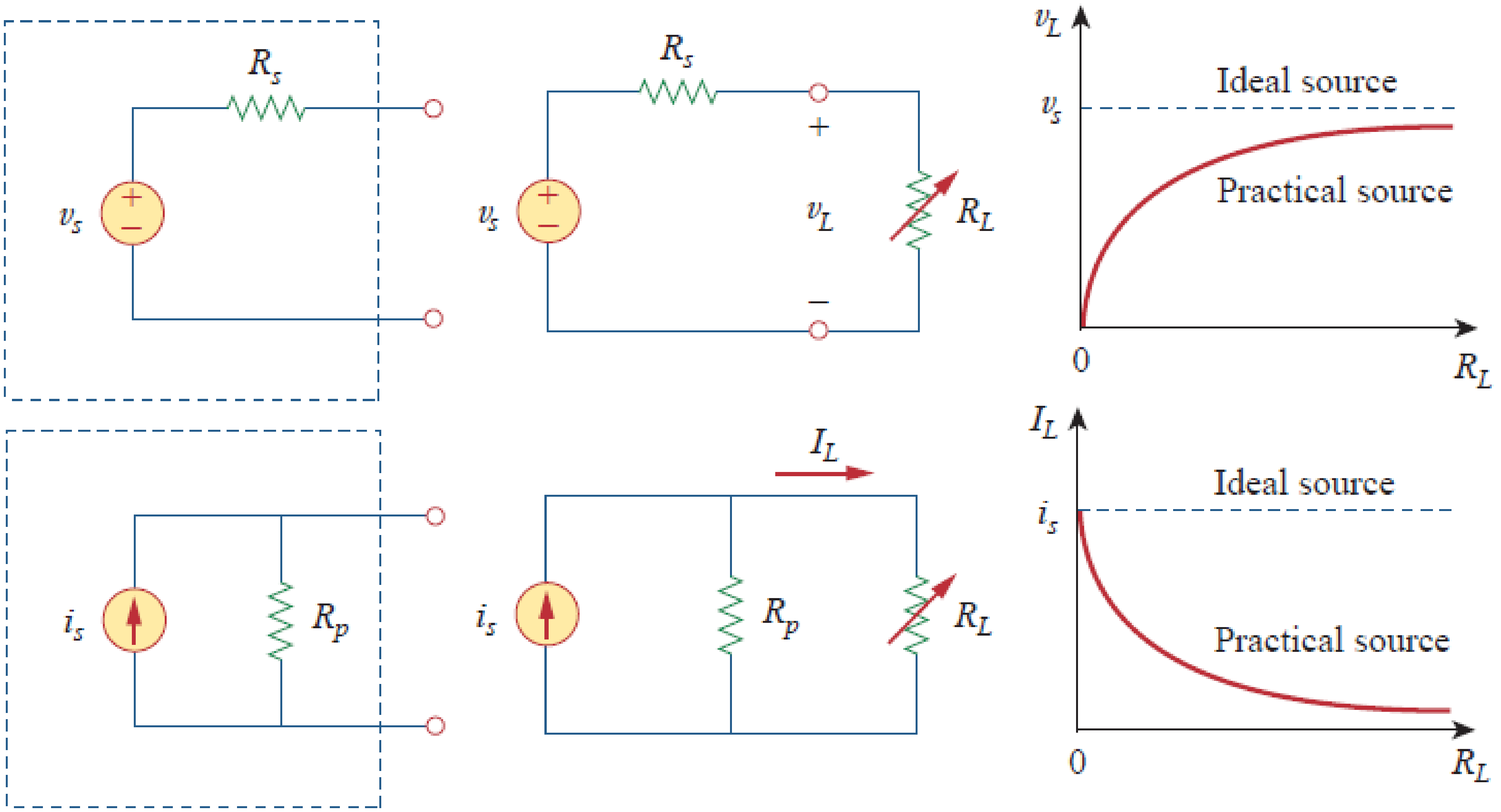
Applications – Practical Voltage and Current Sources



$$v_L = v_S \frac{R_L}{R_S + R_L}$$

$$i_L = i_S \frac{R_p}{R_p + R_L}$$

Questions