



DR. GYURCSEK ISTVÁN

Capacitors and Inductors

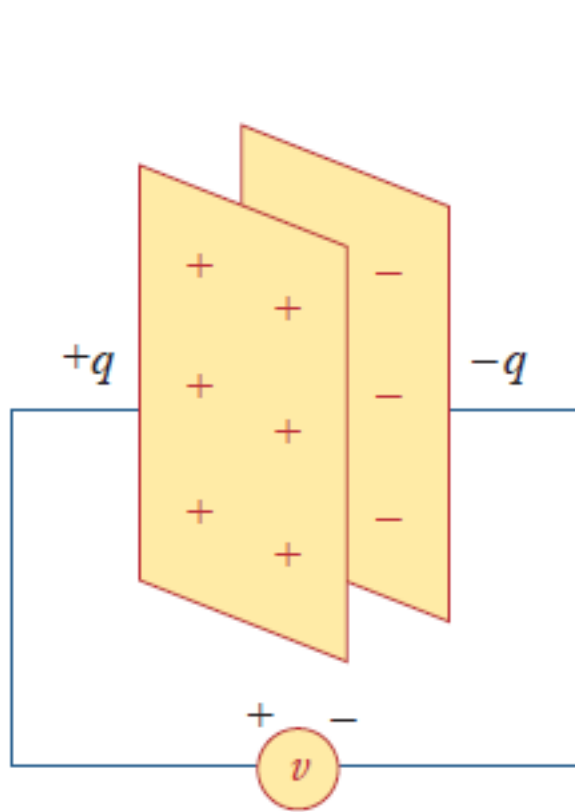
Sources and additional materials (recommended)

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- ❑ *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*

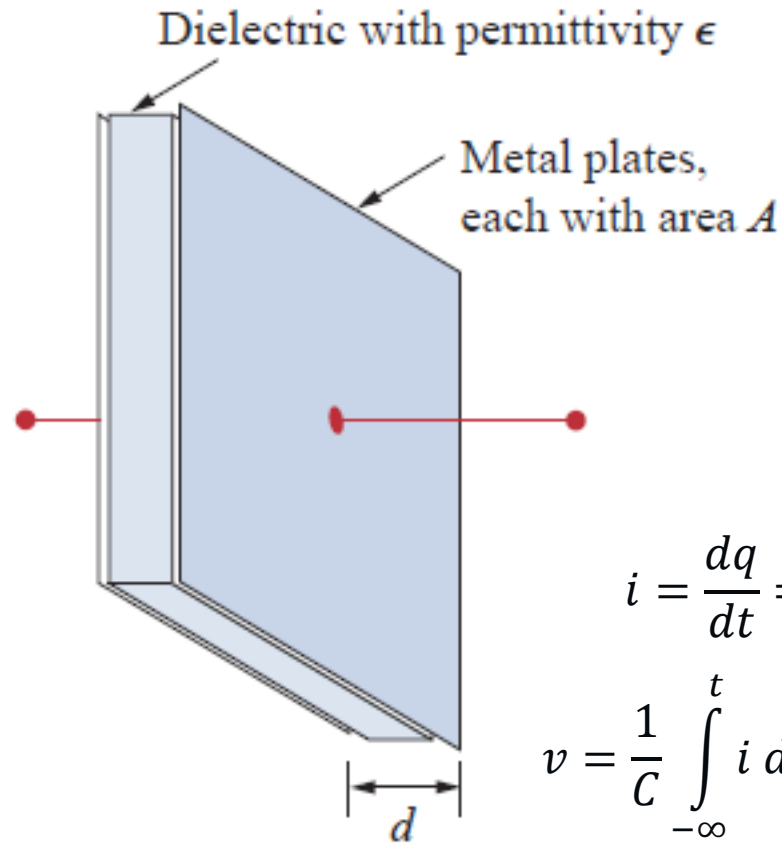


- Capacitance (Properties, Series and Parallel Connections)**
- Inductance (Properties, Series and Parallel Connections)
- Applications (Integrator, Differentiator)

Capacitors

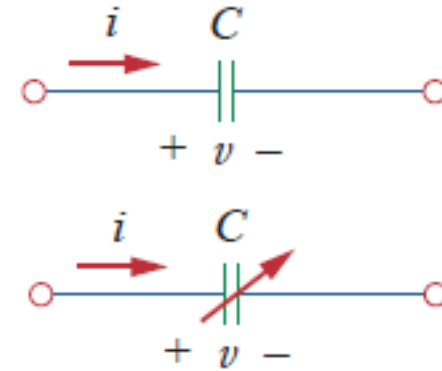


$$C = \frac{q}{v}, \quad [F] = \left[\frac{C}{V} \right]$$



$$C = \epsilon \frac{A}{d}$$

Symbols



$$i = \frac{dq}{dt} = C \frac{dv}{dt} \leftarrow \text{linear element}$$

$$v = \frac{1}{C} \int_{-\infty}^t i dt \rightarrow v = \frac{1}{C} \int_{t_0}^t i dt + v(t_0)$$

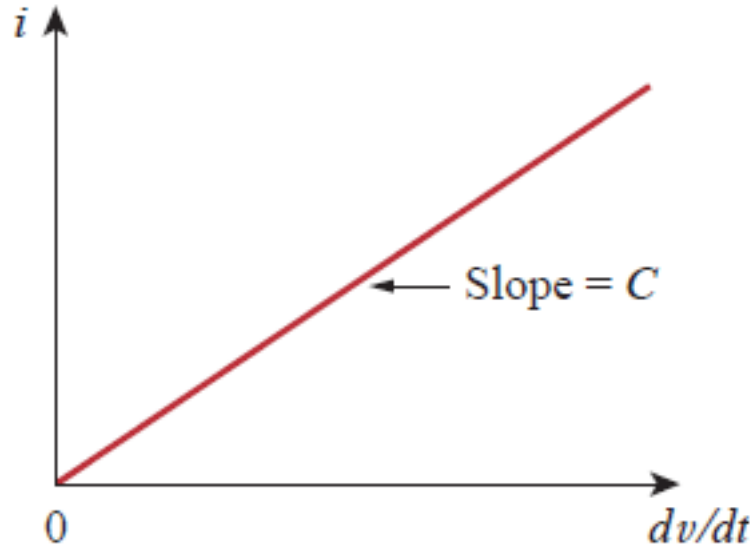
Memory element (v depends on past history of i .)

Properties of Capacitors



Property of linearity

$$i = C \frac{dv}{dt} \rightarrow$$



Energy stored in EF

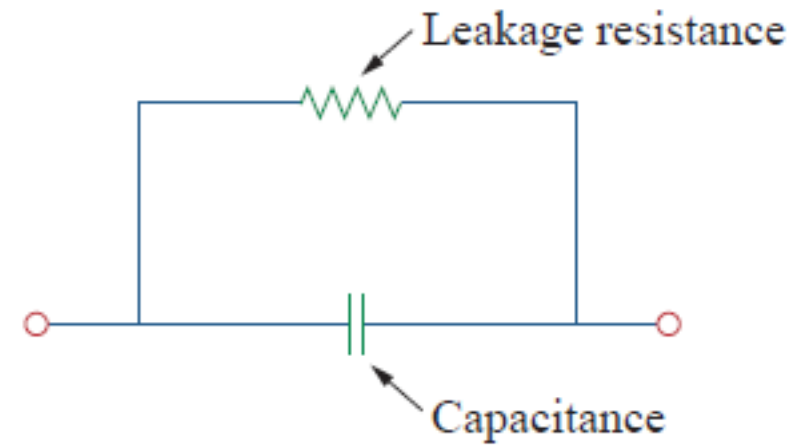
$$i = C \frac{dv}{dt} \rightarrow p = vi = vC \frac{dv}{dt}$$

$$w = \int_{-\infty}^t p dt = C \int_{-\infty}^t v \frac{dv}{dt} dt = C \int_{v(-\infty)}^{v(t)} v dv = \frac{1}{2} C v^2 \Big|_{v(-\infty)}^{v(t)}$$

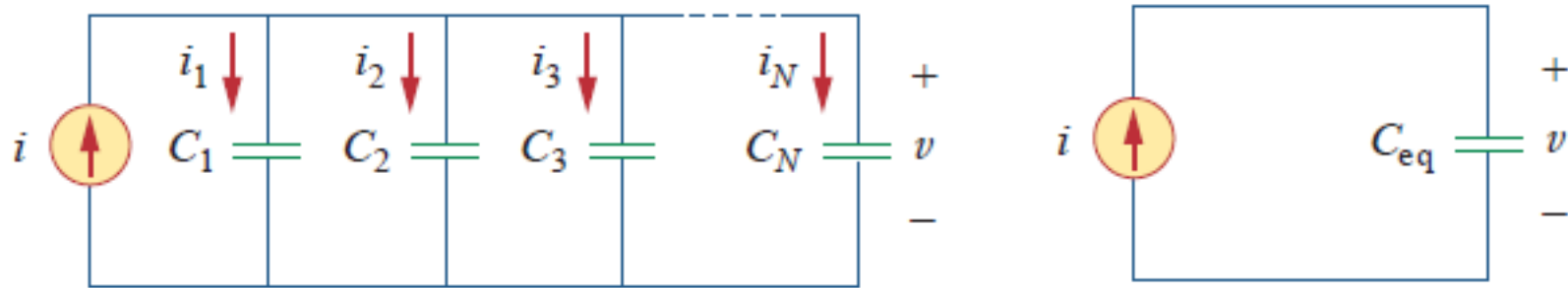
$$v(-\infty) = 0 \rightarrow w = \frac{1}{2} C v^2 \quad q = Cv \rightarrow w = \frac{1}{2} C v^2 = \frac{q^2}{2C}$$

Properties

- Linear element
- Energy storage (memory) element
- Open circuit on DC
- $v(-0) = v(+0)$
- Ideal C \rightarrow no dissipated energy
- Real capacitor model \rightarrow



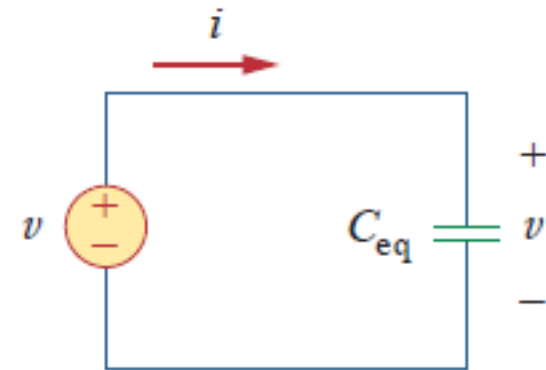
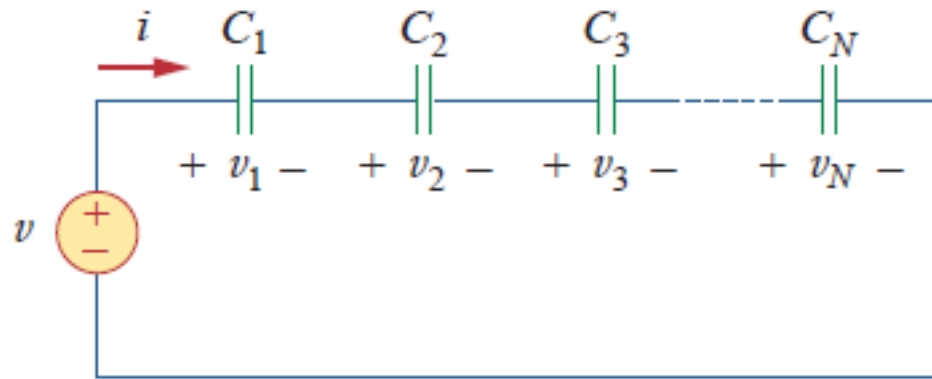
Parallel Capacitors



$$i = i_1 + i_2 + i_3 + \dots + i_N$$

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \dots + C_N \frac{dv}{dt} \quad i = \left(\sum_{k=1}^N C_k \right) \frac{dv}{dt} \rightarrow C_{eq} = \sum_{k=1}^N C_k$$

Series Capacitors



$$v = v_1 + v_2 + v_3 + \dots + v_N$$

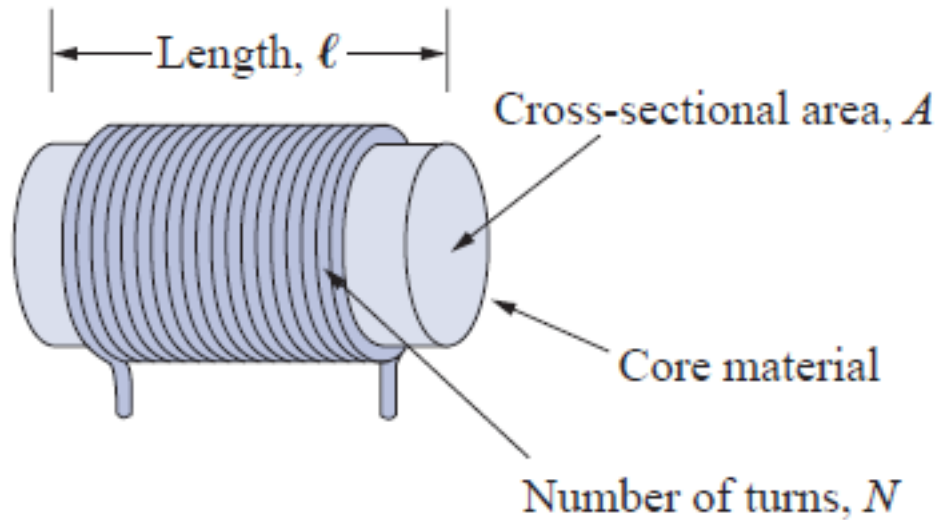
$$v = \frac{1}{C_1} \int_{t_0}^t i(t) dt + v_1(t_0) + \frac{1}{C_2} \int_{t_0}^t i(t) dt + v_2(t_0) + \dots + \frac{1}{C_N} \int_{t_0}^t i(t) dt + v_N(t_0)$$

$$v = \left(\sum_{k=1}^N \frac{1}{C_k} \right) \int_{t_0}^t i(t) dt + v(t_0) \rightarrow \frac{1}{C_{eq}} = \sum_{k=1}^N \frac{1}{C_k} \quad \left(\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow C_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2} \right)$$

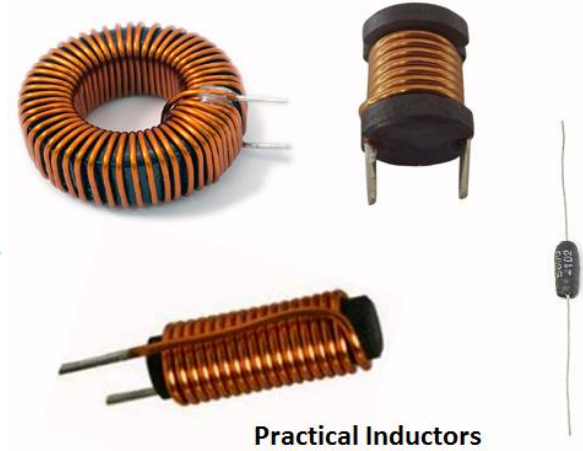
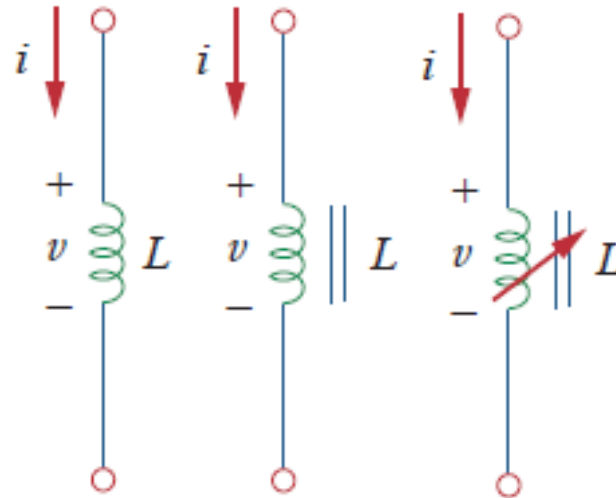


- Capacitance (Properties, Series and Parallel Connections)
- **Inductance (Properties, Series and Parallel Connections)**
- Applications (Integrator, Differentiator)

Inductors



Symbols



$$\left. \begin{array}{l} \text{(coil flux)} \rightarrow \Psi(t) = L \cdot i(t) \\ \text{(Faraday)} \rightarrow v(t) = \frac{d\Psi(t)}{dt} \end{array} \right\} \rightarrow v(t) = L \frac{di(t)}{dt} \quad di = \frac{1}{L} v dt \rightarrow i = \frac{1}{L} \int_{-\infty}^t v(t) dt$$

$$L \rightarrow [H] = \left[\frac{Vs}{A} \right] \quad \left(L = \mu \frac{N^2 A}{l} \right) \quad i = \frac{1}{L} \int_{t_0}^t v(t) dt + i(t_0)$$

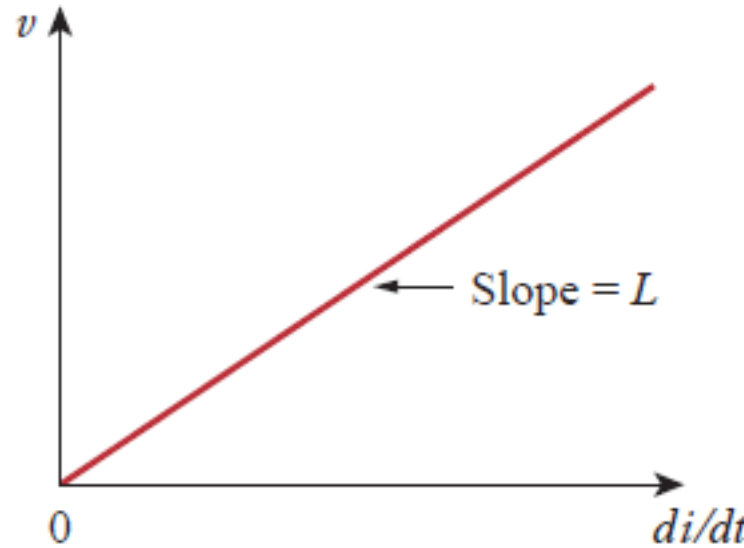
Memory element
(i depends on past history of v .)

Properties of Inductors



Property of linearity

$$v = L \frac{di}{dt} \rightarrow$$



Energy stored in MF

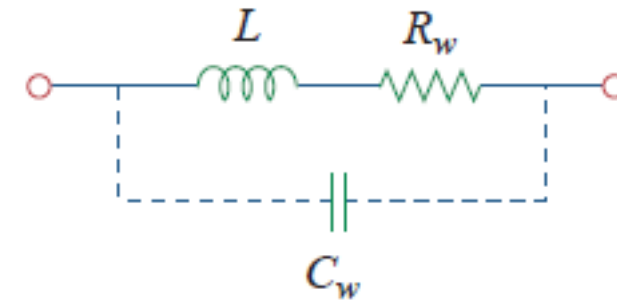
$$v = L \frac{di}{dt} \rightarrow p = vi = \left(L \frac{di}{dt} \right) i$$

$$w = \int_{-\infty}^t p dt = L \int_{-\infty}^t i \frac{di}{dt} dt = L \int_{i(-\infty)}^{i(t)} i di = \frac{1}{2} Li^2 \Big|_{i(-\infty)}^{i(t)}$$

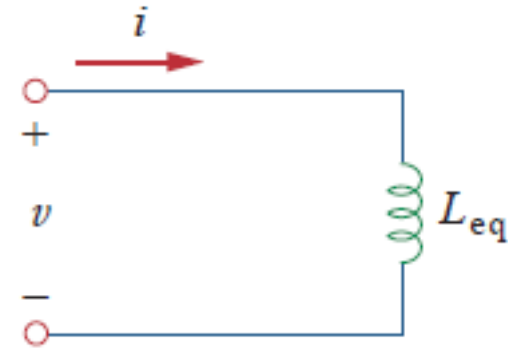
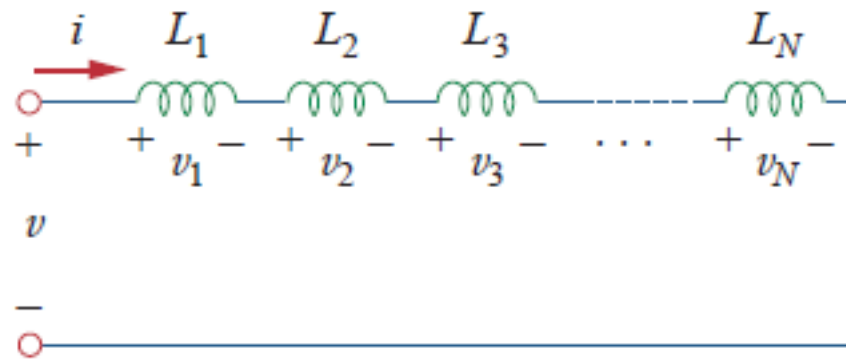
$$i(-\infty) = 0 \rightarrow w = \frac{1}{2} Li^2$$

Properties

- Linear element
- Energy storage (memory) element
- Short circuit on DC
- $i(-0) = i(+0)$
- Ideal L \rightarrow no dissipated energy
- Real inductor model \rightarrow



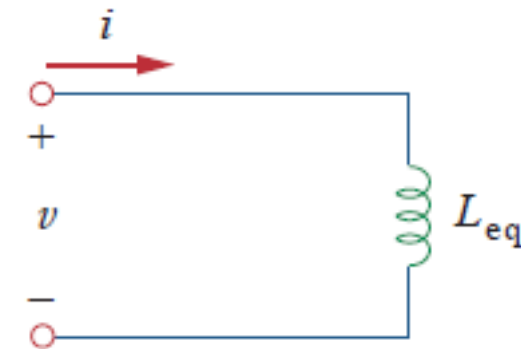
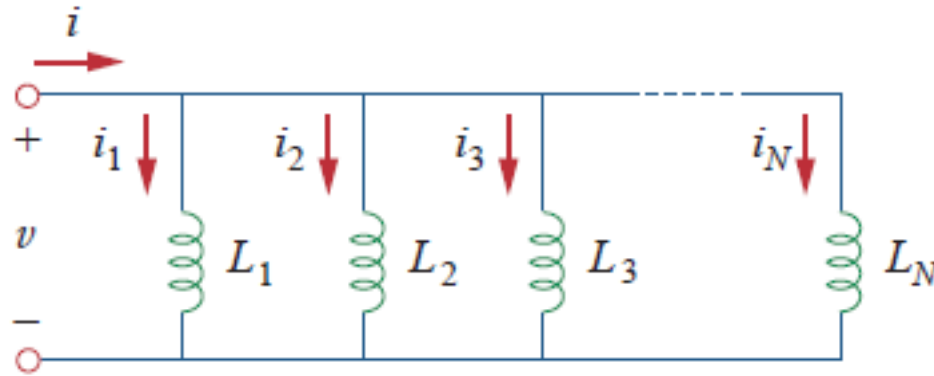
Series Inductors



$$v = v_1 + v_2 + v_3 + \dots + v_N$$

$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \dots + L_N \frac{di}{dt} \quad v = \left(\sum_{k=1}^N L_k \right) \frac{di}{dt} \rightarrow L_{eq} = \sum_{k=1}^N L_k$$

Parallel Inductors



$$i = i_1 + i_2 + i_3 + \dots + i_N$$

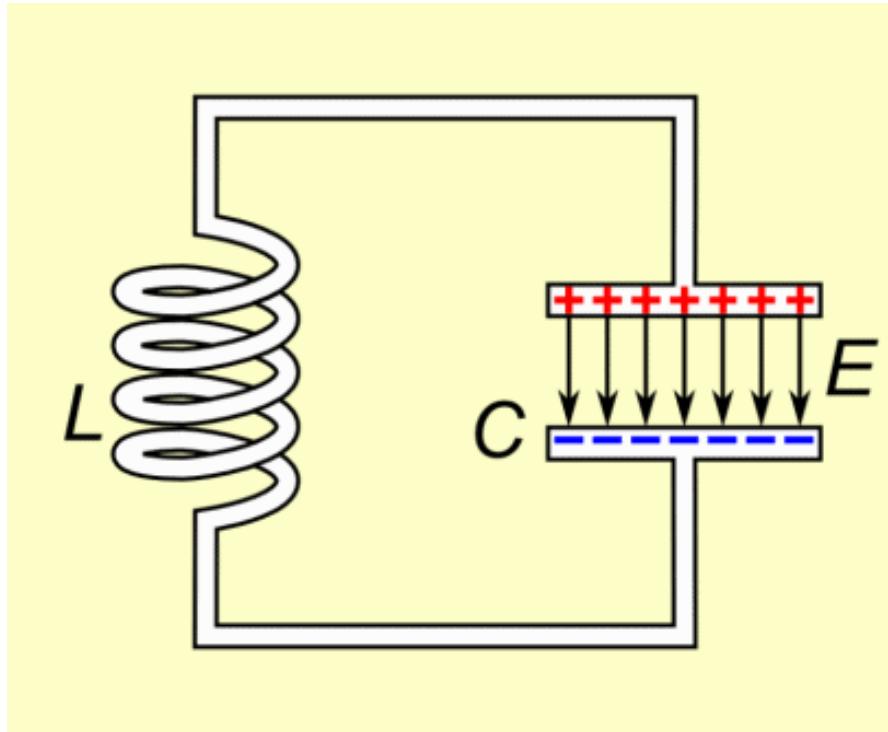
$$i = \frac{1}{L_1} \int_{t_0}^t v(t) dt + i_1(t_0) + \frac{1}{L_2} \int_{t_0}^t v(t) dt + i_2(t_0) + \dots + \frac{1}{L_N} \int_{t_0}^t v(t) dt + i_N(t_0)$$

$$i = \left(\sum_{k=1}^N \frac{1}{L_k} \right) \int_{t_0}^t v(t) dt + i(t_0) \rightarrow \frac{1}{L_{eq}} = \sum_{k=1}^N \frac{1}{L_k} \quad \left(\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \rightarrow L_{eq} = \frac{L_1 \cdot L_2}{L_1 + L_2} \right)$$



- ❑ Capacitance (Properties, Series and Parallel Connections)
- ❑ Inductance (Properties, Series and Parallel Connections)
- ❑ **Applications (Integrator, Differentiator)**

RLC Applications



LC special properties

- Temporary energy storage (*dc app*)
- C → no abrupt change in voltage (*dc app*)
- L → no abrupt change in current (*dc app*)
- Frequency sensitive behavior (*ac app*)

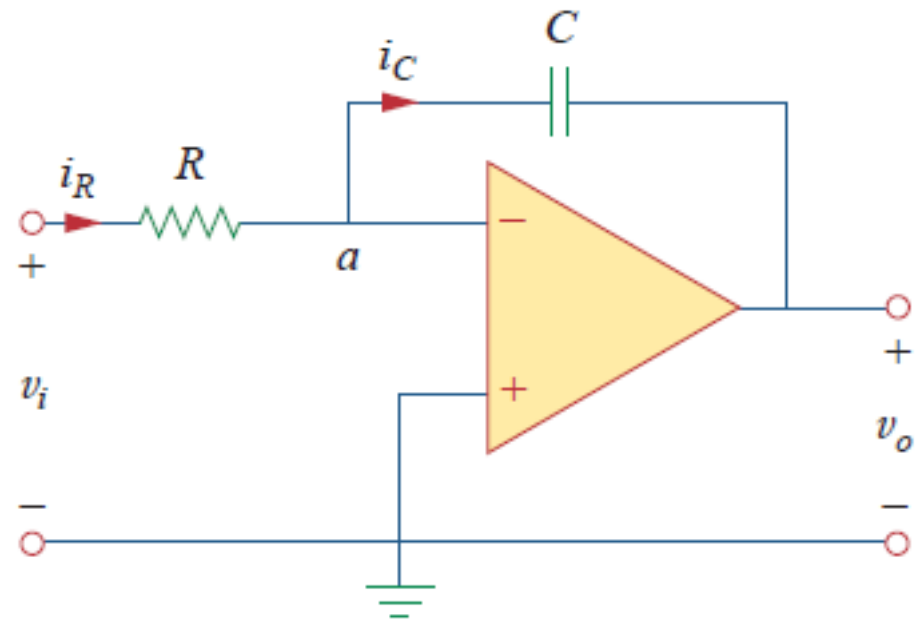
Resistors & Capacitors

- discrete form
- IC form

Inductors (coils)

- discrete form *only*
- limited in applications (*sensors, motors, etc.*)

App. – Integrator



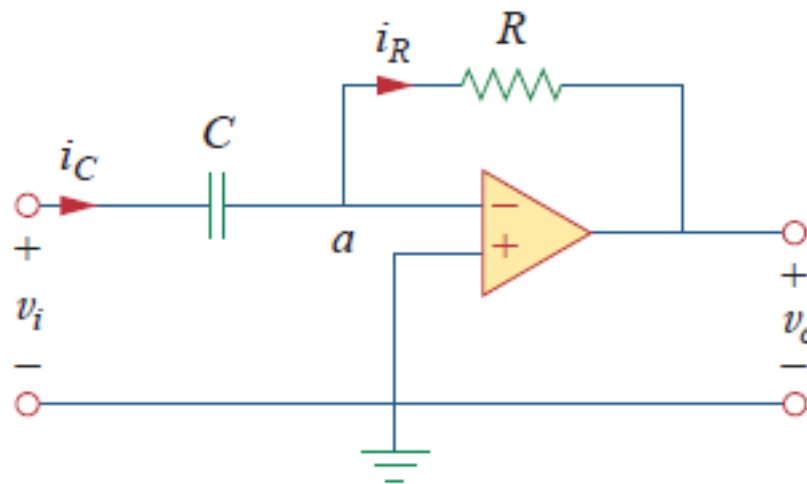
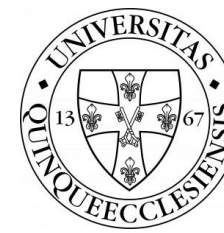
$$i_R = i_C \rightarrow \frac{v_i}{R} = -C \frac{dv_o}{dt}$$

$$dv_o = -\frac{1}{RC} v_i dt$$

$$v_o(t) - v_o(0) = -\frac{1}{RC} \int_0^t v_i dt$$

discharge integrator $\rightarrow v_o(0) = 0 \rightarrow$
$$v_o(t) = -\frac{1}{RC} \int_0^t v_i(t) dt$$

App. – Differentiator



$$i_R = i_C \rightarrow -\frac{v_o}{R} = C \frac{dv_i}{dt}$$

$$v_o = -RC \frac{dv_i}{dt}$$

Comments (*opamp application*)

- Not as popular as integrator
- Electronically unstable circuit
(*noise sensitive opamp circuit*)

Questions

