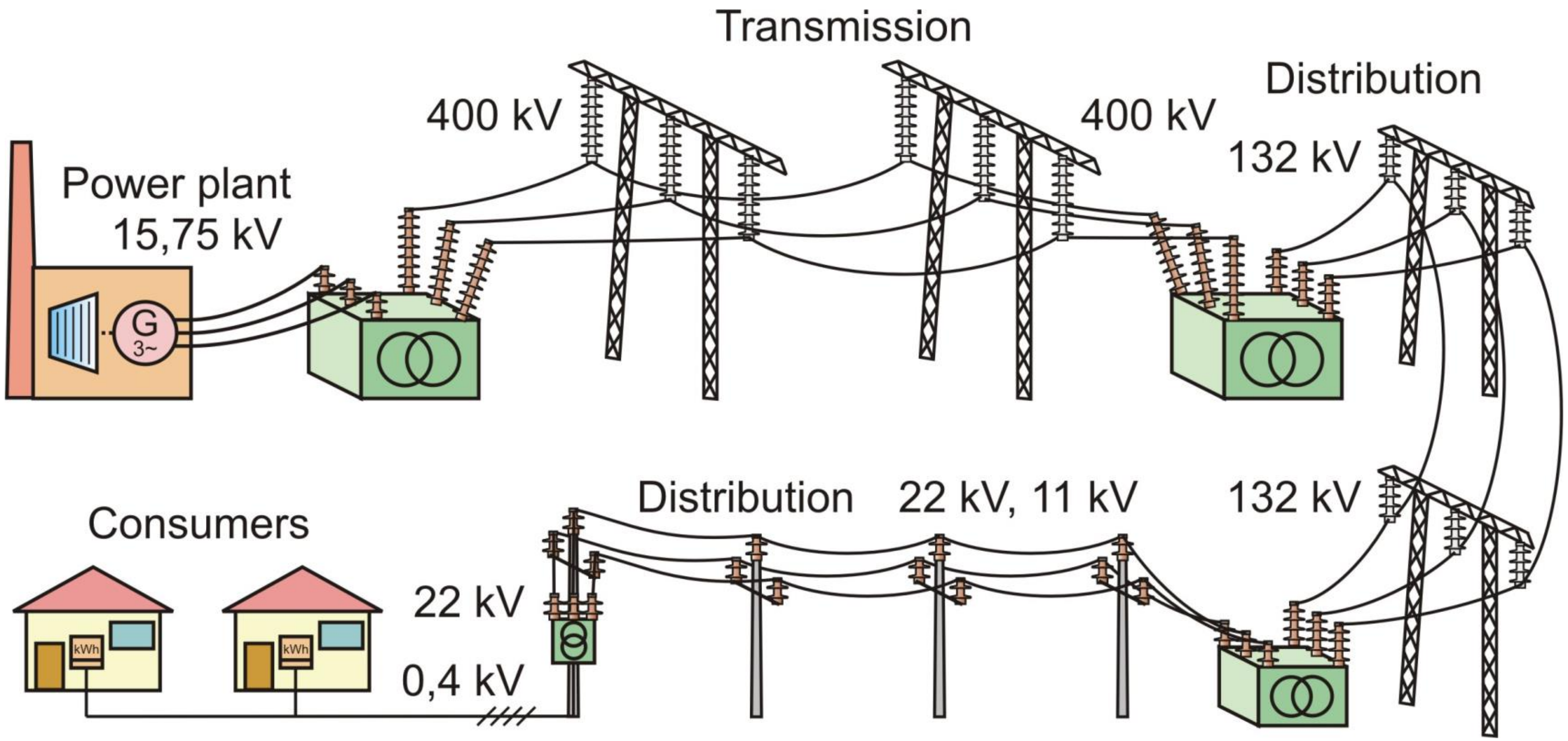
DR. GYURCSEK ISTVÁN

Three-Phase Circuits

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Fleckenstein: Three-Phase Electrical Power (ISBN-13: 978-1498737777)*
- *Delmar: 3-phase Circuits and Electrical Machines (ISBN-13: 978-1439059821)*
- *Mayergoyz - Lawson: Basic Electric Circuit Theory (ISBN13: 978-0124808652)*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*

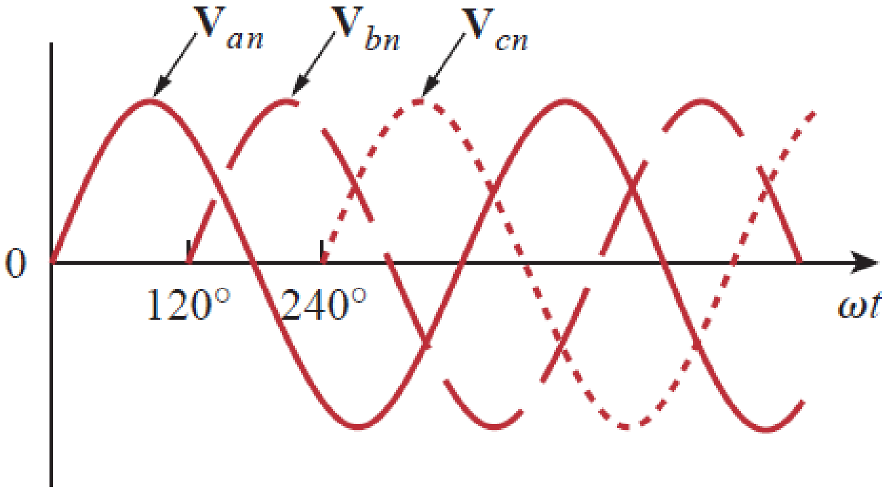
Generation, Transmission, Distribution



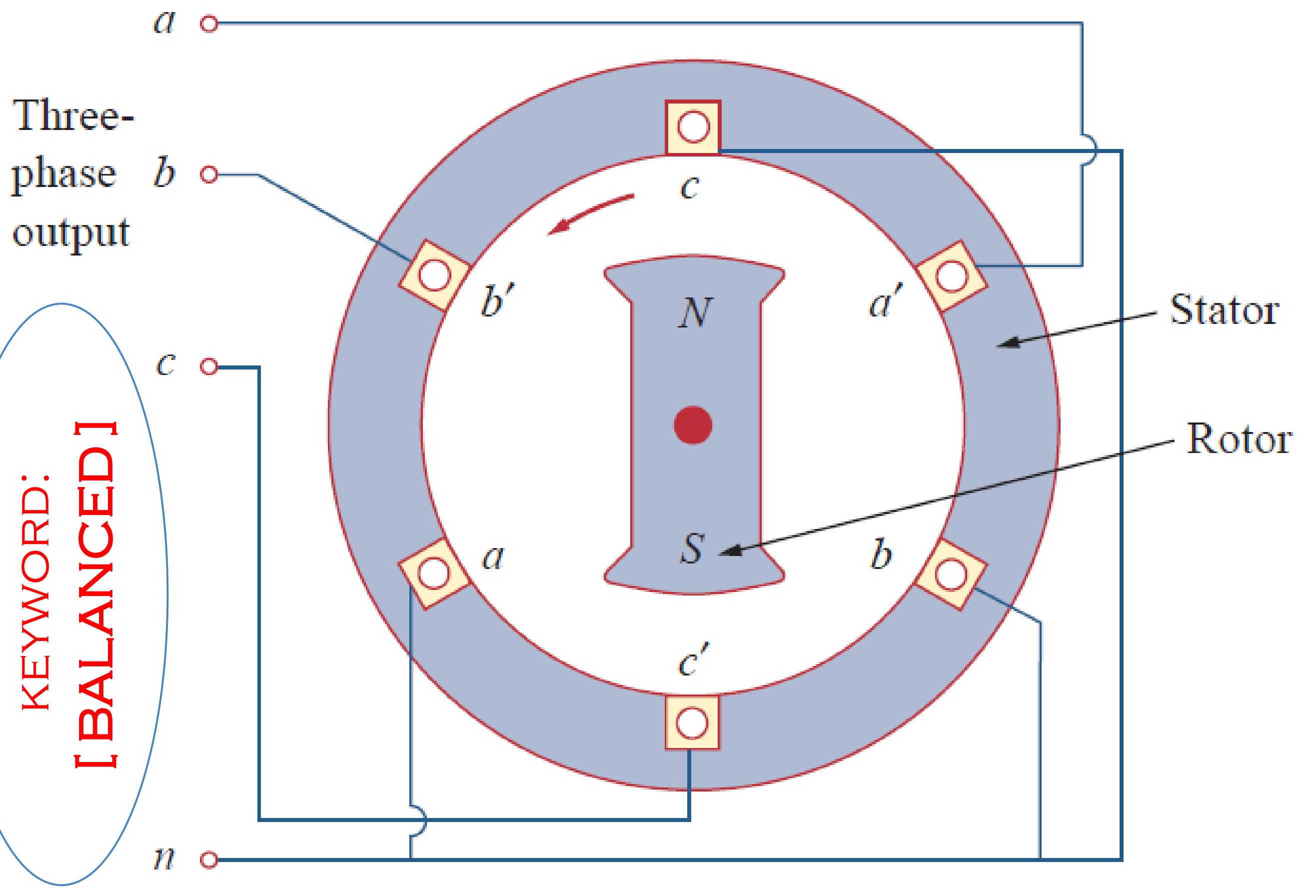
Why Three-Phase... ?

‚Three-phase power is the most efficient way that electricity could be produced, transmitted, consumed.'

- ❑ Single-phase power → falls to zero three times
- ❑ Three-phase power → never falls to zero
- ❑ Delivered power is the same at any instant
- ❑ Motors / transformers → 150% operating charact.
- ❑ 33% save in mass of conductors *(see later)*



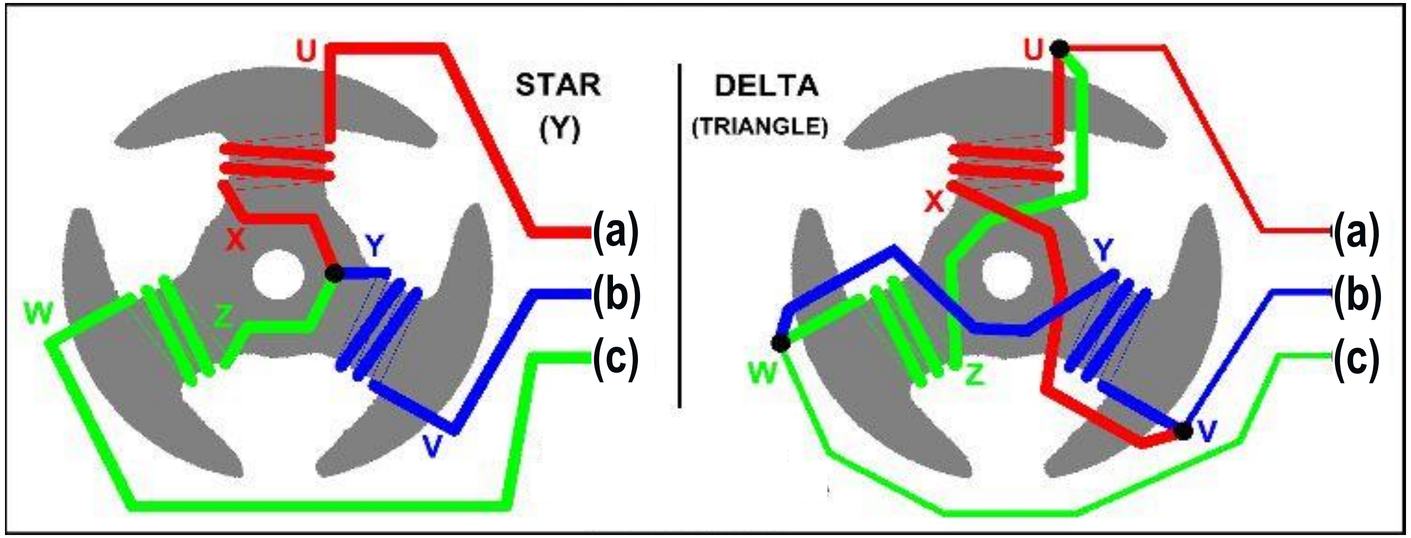
KEYWORD:
[BALANCED]

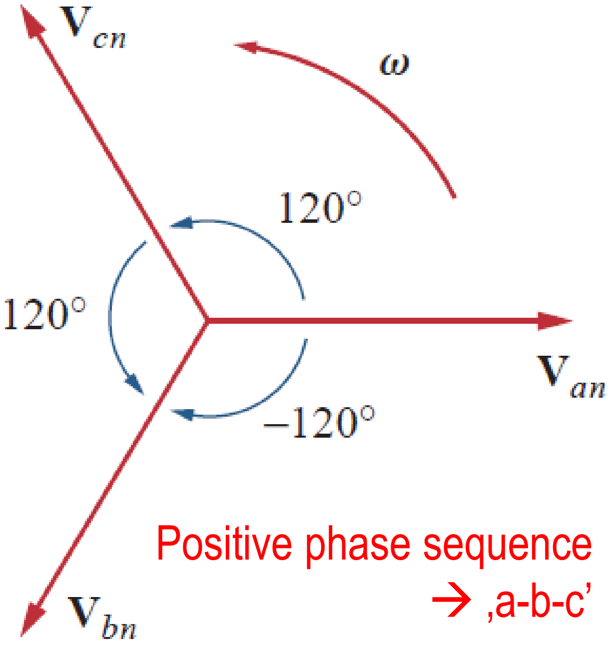


gyurcsek.istvan@mik.pte.hu

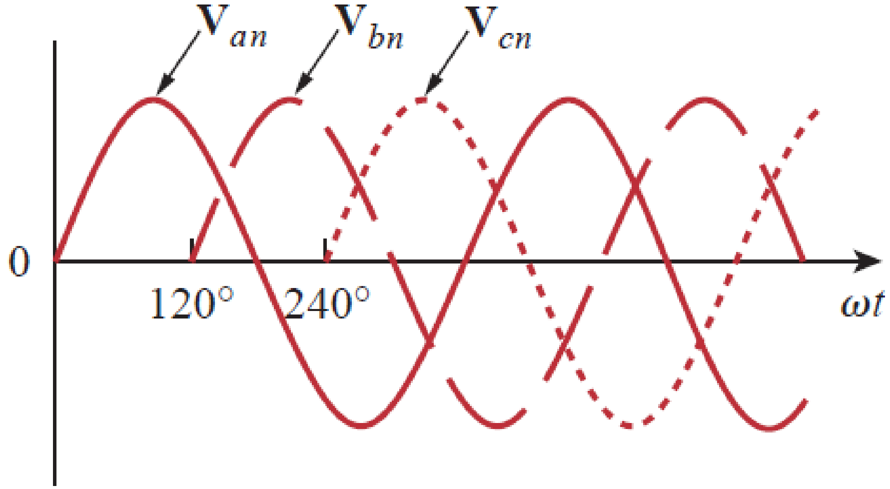
- **Balanced Three-Phase Voltages**
- Balanced Wye-Wye Connection
- Balanced Wye-Delta Connection
- Balanced Delta-Delta Connection
- Balanced Delta-Wye Connection
- Power in Balanced Systems
- Unbalanced Three-Phase Systems
- Power in Unbalanced Systems

Three-Phase Voltages





Positive phase sequence
→ ‚a-b-c'

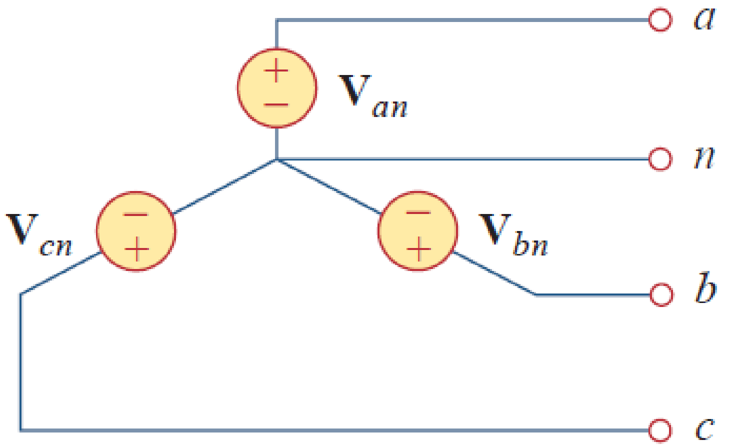


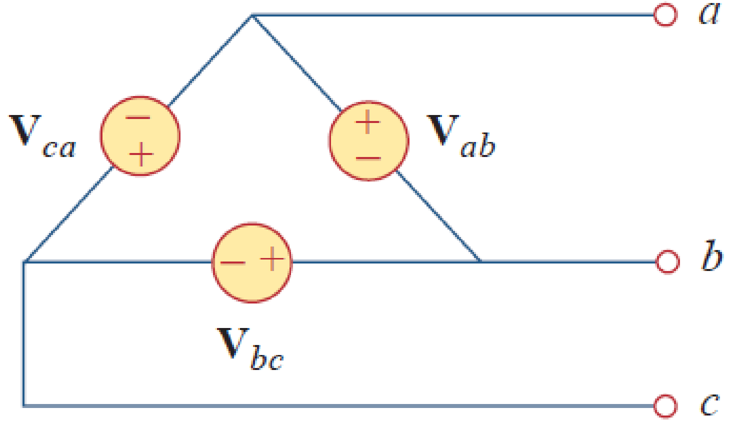
$$v_{an}(t) = V_p \sin \omega t \rightarrow \boldsymbol{V}_{an} = V_p e^{j0°}$$

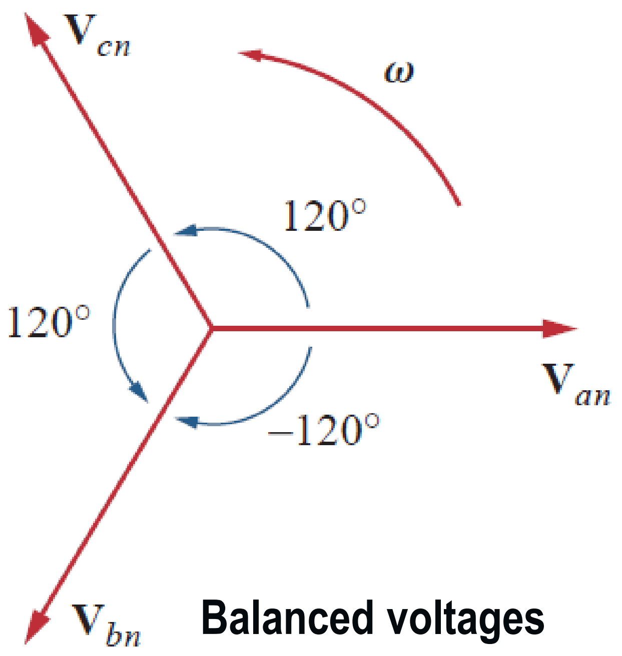
$$v_{bn}(t) = V_p \sin(\omega t - 120°) \rightarrow \boldsymbol{V}_{bn} = V_p e^{-j120°}$$

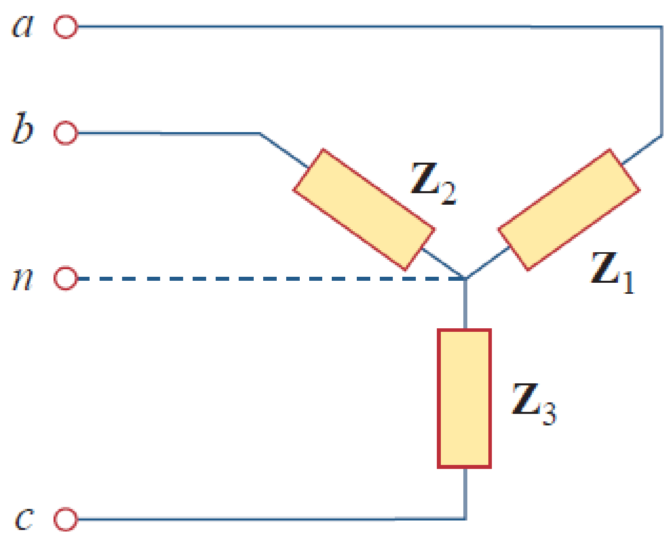
$$v_{cn}(t) = V_p \sin(\omega t + 120°) \rightarrow \boldsymbol{V}_{cn} = V_p e^{j120°}$$

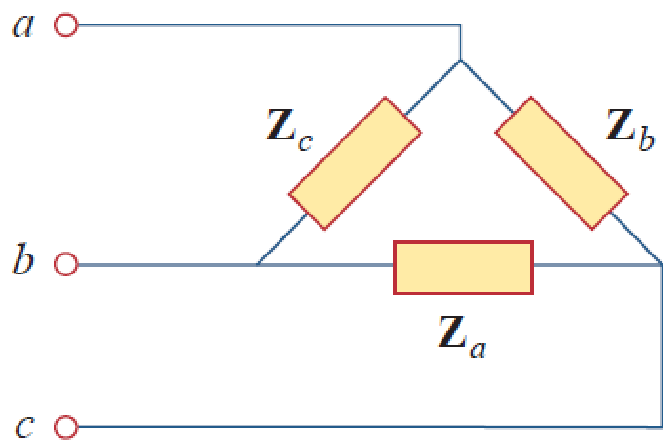
Source to Load Connections











Possible source - load connections

(1) $Y - Y$. (2) $Y - \Delta$, (3) $\Delta - \Delta$, (4) $\Delta - Y$

Balanced voltages

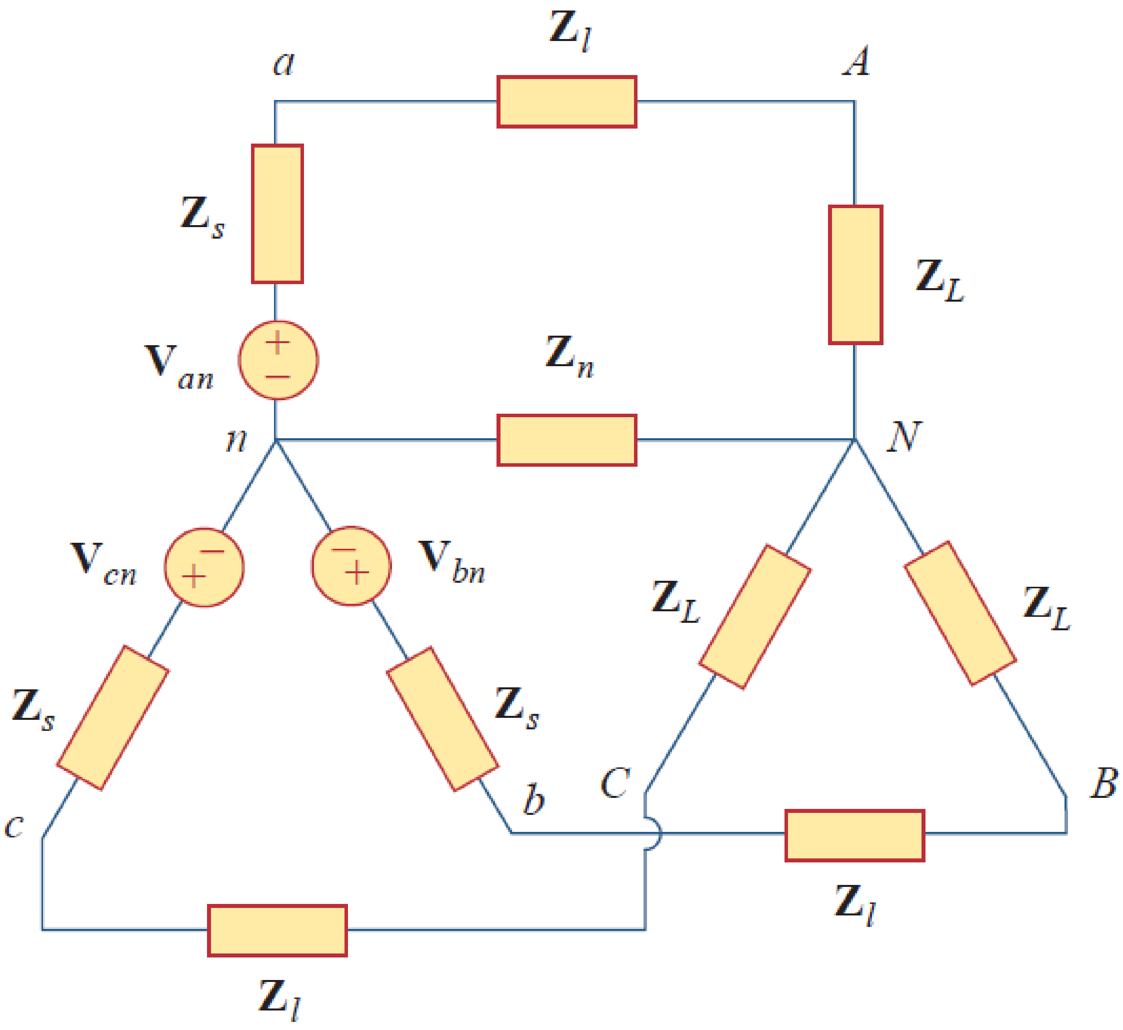
- ❑ $\boldsymbol{V}_{an} + \boldsymbol{V}_{bn} + \boldsymbol{V}_{cn} = 0$
- ❑ $V_{an} = V_{bn} = V_{cn} = V$

Balanced load

- ❑ $\boldsymbol{Z}_1 = \boldsymbol{Z}_2 = \boldsymbol{Z}_3 = \boldsymbol{Z}_Y$
- ❑ $\boldsymbol{Z}_a = \boldsymbol{Z}_b = \boldsymbol{Z}_c = \boldsymbol{Z}_\Delta$

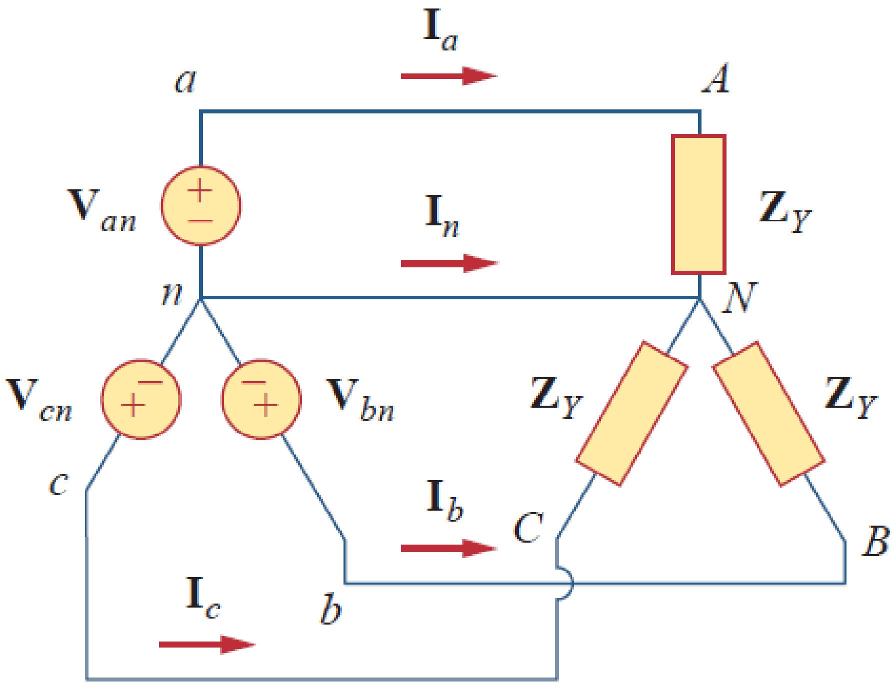
- Balanced Three-Phase Voltages
- **Balanced Wye-Wye Connection**
- Balanced Wye-Delta Connection
- Balanced Delta-Delta Connection
- Balanced Delta-Wye Connection
- Power in Balanced Systems
- Unbalanced Three-Phase Systems
- Power in Unbalanced Systems

Balanced Wye-Wye...



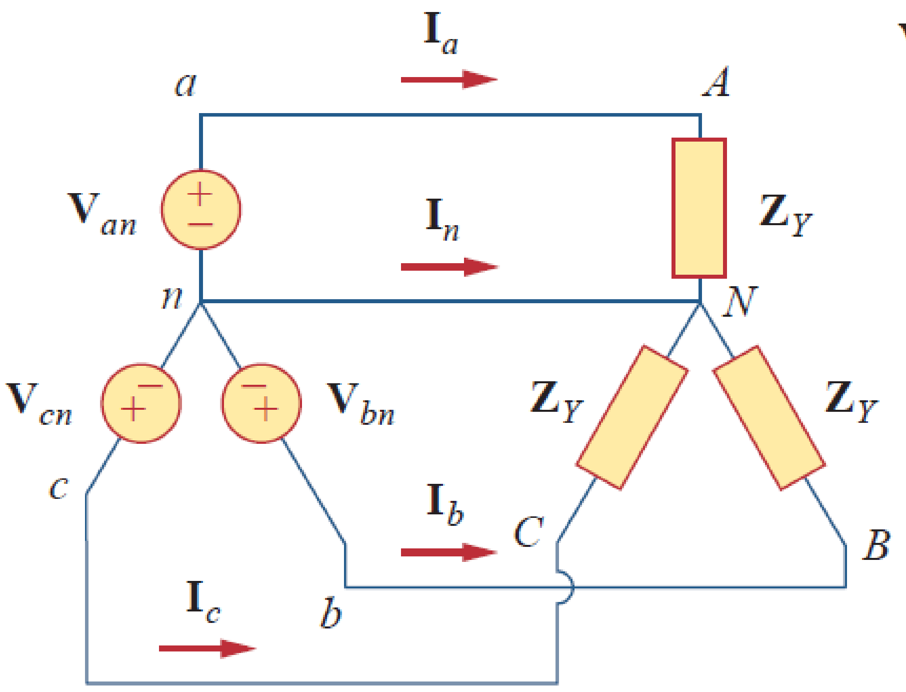
$$Z_Y = Z_S + Z_l + Z_L$$

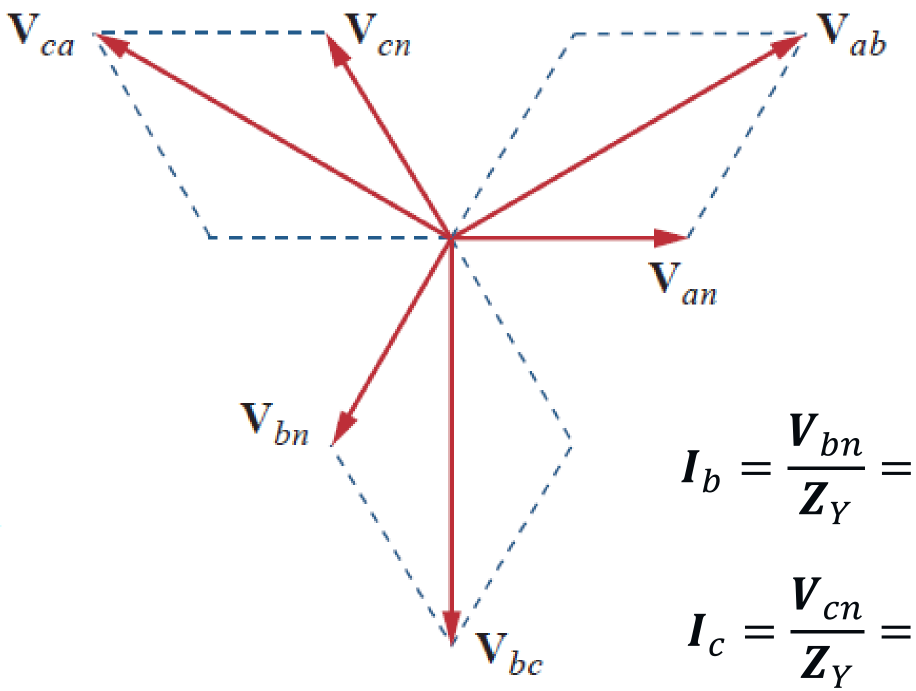
$$Z_n \approx 0 \, , Z_S \approx 0 \, , Z_l \approx 0 \rightarrow Z_Y = Z_L$$



gyurcsek.istvan@mik.pte.hu

Balanced Wye-Wye...





$$\boldsymbol{I}_a = \frac{\boldsymbol{V}_{an}}{\boldsymbol{Z}_Y}$$

$$\boldsymbol{I}_b = \frac{\boldsymbol{V}_{bn}}{\boldsymbol{Z}_Y} = \frac{\boldsymbol{V}_{an}e^{-j120°}}{\boldsymbol{Z}_Y} = \boldsymbol{I}_a e^{-j120°}$$

$$\boldsymbol{I}_c = \frac{\boldsymbol{V}_{cn}}{\boldsymbol{Z}_Y} = \frac{\boldsymbol{V}_{an}e^{-j240°}}{\boldsymbol{Z}_Y} = \boldsymbol{I}_a e^{-j240°}$$

$\boldsymbol{V}_{an} = V_p e^{j0°}$, $\boldsymbol{V}_{bn} = V_p e^{-j120°}$, $\boldsymbol{V}_{cn} = V_p e^{j120°}$ $\quad V_L = \sqrt{3}V_p$, $\quad I_L = I_p$ $\quad \boldsymbol{I}_a + \boldsymbol{I}_b + \boldsymbol{I}_c = 0$

$$\boldsymbol{V}_{ab} = \boldsymbol{V}_{an} - \boldsymbol{V}_{bn} = V_p e^{j0°} - V_p e^{-j120°} = V_p\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V_p e^{j30°}$$

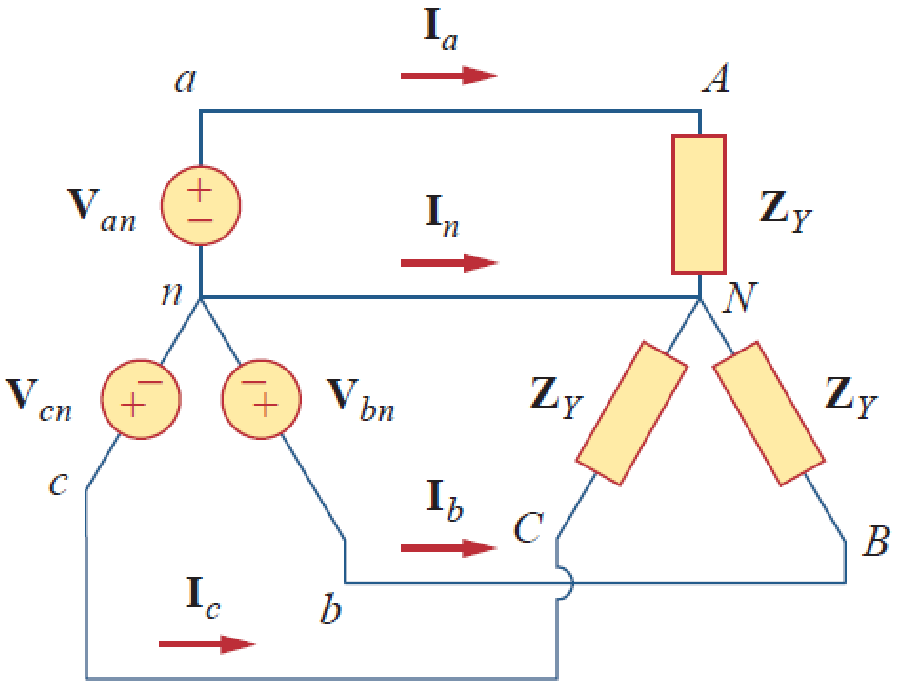
$$\boldsymbol{I}_n = -(\boldsymbol{I}_a + \boldsymbol{I}_b + \boldsymbol{I}_c) = 0$$

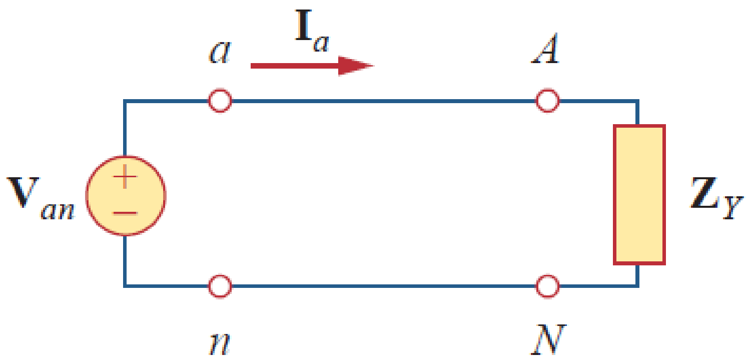
$$\boldsymbol{V}_{bc} = \boldsymbol{V}_{bn} - \boldsymbol{V}_{cn} = \cdots = \sqrt{3}V_p e^{-j90°} \qquad \boldsymbol{V}_{ca} = \boldsymbol{V}_{cn} - \boldsymbol{V}_{an} = \cdots = \sqrt{3}V_p e^{-j210°}$$

$$\boldsymbol{V}_{nN} = \boldsymbol{Z}_n \boldsymbol{I}_n = 0$$

Balanced Wye-Wye...

Single phase equivalent





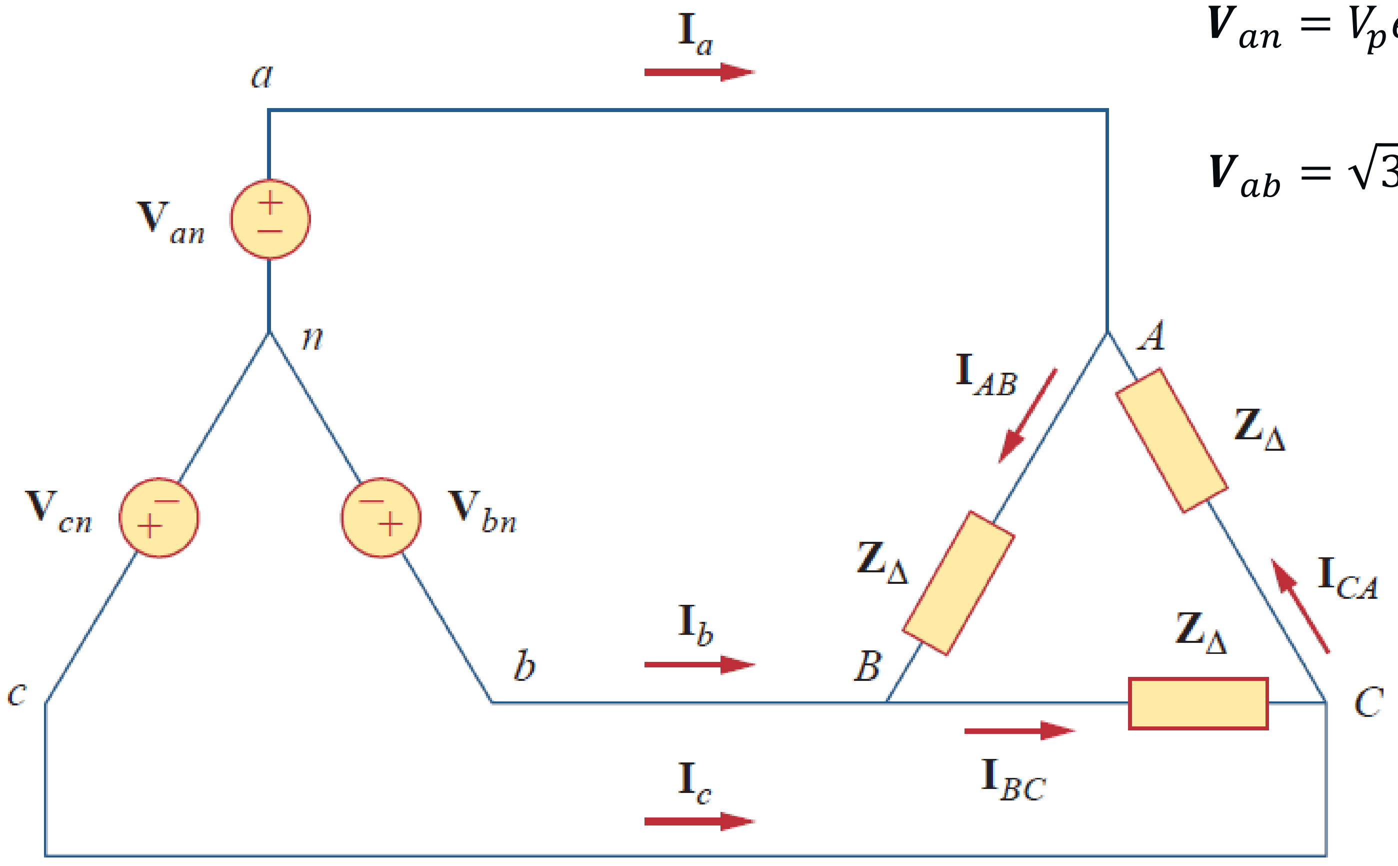
$$I_a = \frac{V_{an}}{Z_Y}$$

$$I_b = I_a e^{-j120°}, \qquad I_c = I_a e^{-j240°}$$

- Balanced Three-Phase Voltages
- Balanced Wye-Wye Connection
- **Balanced Wye-Delta Connection**
- Balanced Delta-Delta Connection
- Balanced Delta-Wye Connection
- Power in Balanced Systems
- Unbalanced Three-Phase Systems
- Power in Unbalanced Systems

Balanced Wye-Delta...

The most practical and most commonly used three-phase system.



$$\boldsymbol{V}_{an} = V_p e^{j0°}, \qquad \boldsymbol{V}_{bn} = V_p e^{-j120°}, \qquad \boldsymbol{V}_{cn} = V_p e^{j120°}$$

$$\boldsymbol{V}_{ab} = \sqrt{3}V_p e^{j30°} = \boldsymbol{V}_{AB}, \qquad \boldsymbol{V}_{bc} = \sqrt{3}V_p e^{-j90°} = \boldsymbol{V}_{BC}$$

$$\boldsymbol{V}_{ca} = \sqrt{3}V_p e^{j150°} = \boldsymbol{V}_{CA}$$

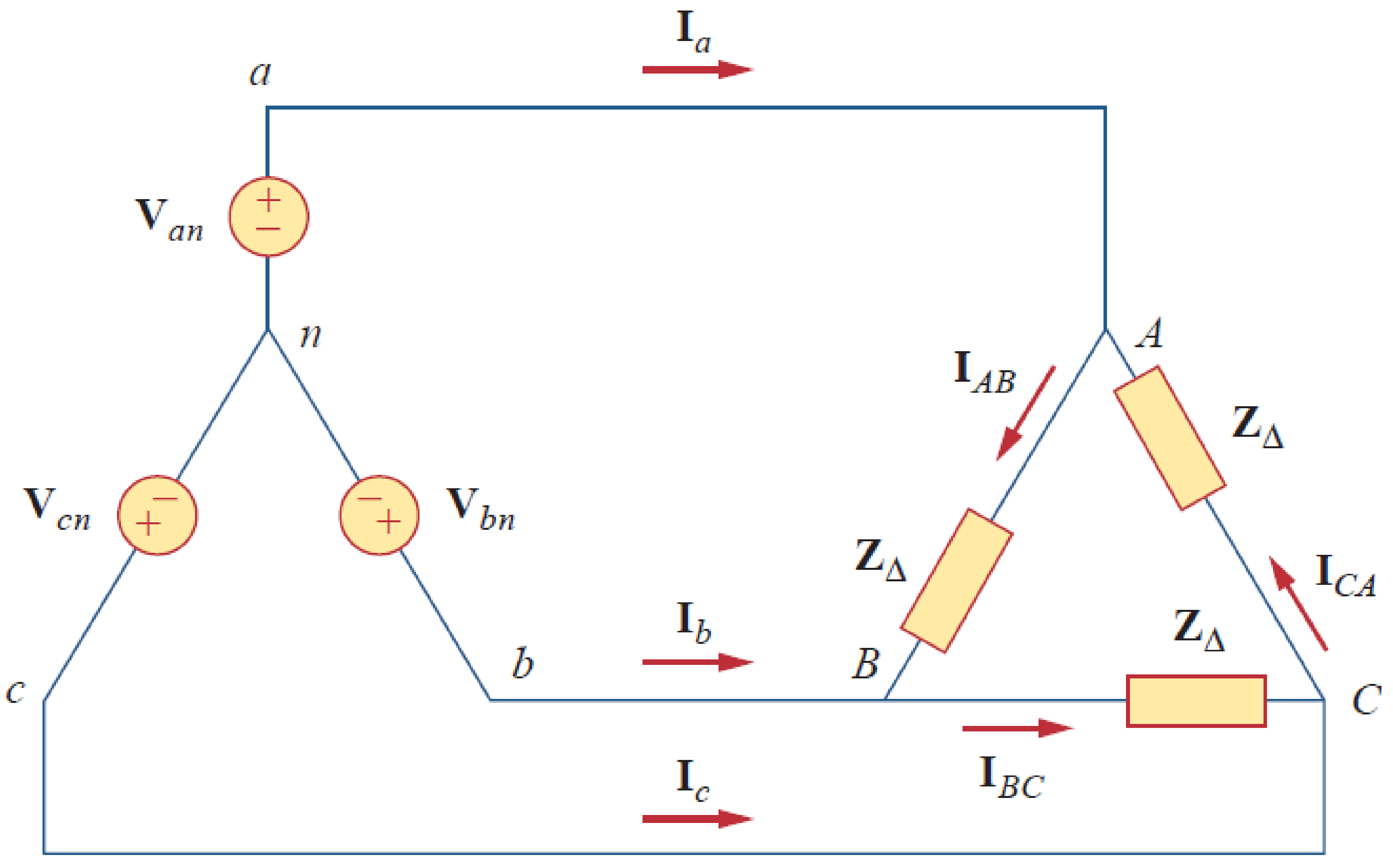
$$\boldsymbol{I}_{AB} = \frac{\boldsymbol{V}_{AB}}{\boldsymbol{Z}_{\Delta}}, \qquad \boldsymbol{I}_{BC} = \frac{\boldsymbol{V}_{BC}}{\boldsymbol{Z}_{\Delta}}, \qquad \boldsymbol{I}_{CA} = \frac{\boldsymbol{V}_{CA}}{\boldsymbol{Z}_{\Delta}}$$

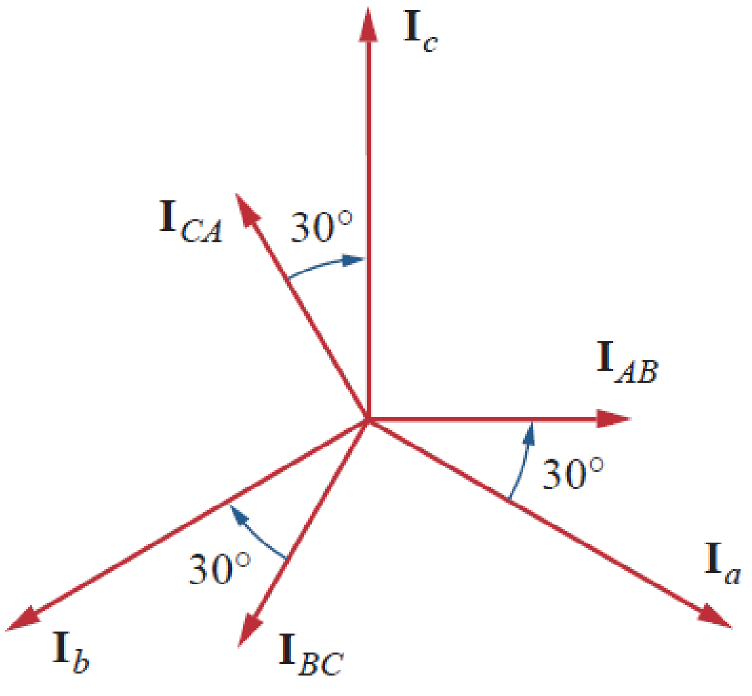
Another way… $-\boldsymbol{V}_{an} + \boldsymbol{Z}_{\Delta}\boldsymbol{I}_{AB} + \boldsymbol{V}_{bn} = 0$

$$\boldsymbol{I}_{AB} = \frac{\boldsymbol{V}_{an} - \boldsymbol{V}_{bn}}{\boldsymbol{Z}_{\Delta}} = \frac{\boldsymbol{V}_{ab}}{\boldsymbol{Z}_{\Delta}} = \frac{\boldsymbol{V}_{AB}}{\boldsymbol{Z}_{\Delta}}$$

gyurcsek.istvan@mik.pte.hu

Balanced Wye-Delta…





$I_L = \sqrt{3} I_P$

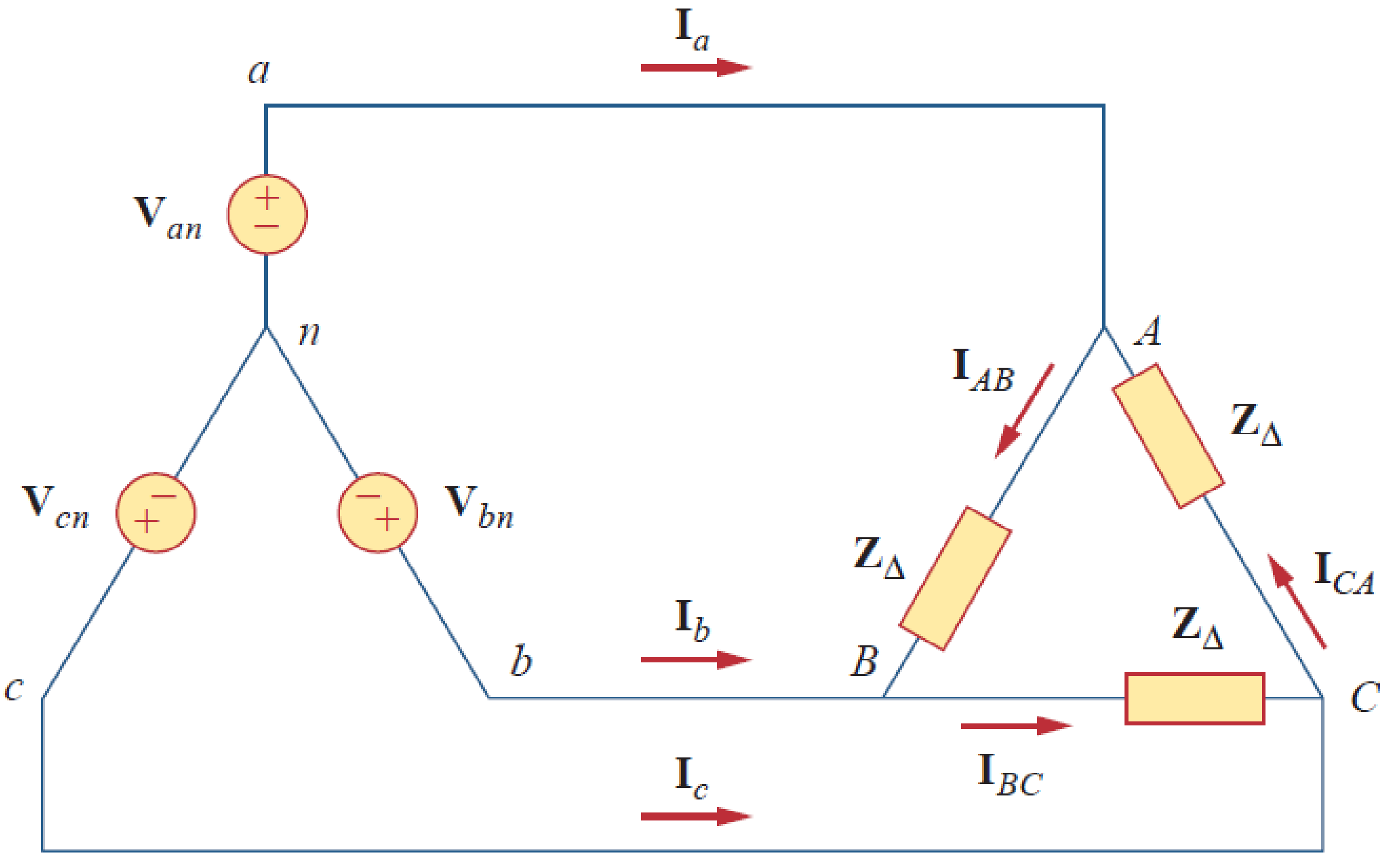
$V_L = V_P$

$\boldsymbol{I_a = I_{AB} - I_{CA}}$, $\boldsymbol{I_b = I_{BC} - I_{AB}}$, $\boldsymbol{I_c = I_{CA} - I_{BC}}$

$$\boldsymbol{I_{CA}} = \boldsymbol{I_{AB}} e^{-j240°} \rightarrow \boldsymbol{I_a} = \boldsymbol{I_{AB}} - \boldsymbol{I_{CA}} = \boldsymbol{I_{AB}}\left(1 - e^{-j240°}\right) = \boldsymbol{I_{AB}}(1 + 0.5 - j0.866) = \boldsymbol{I_{AB}}\sqrt{\boldsymbol{3}} e^{-j30°}$$

gyurcsek.istvan@mik.pte.hu

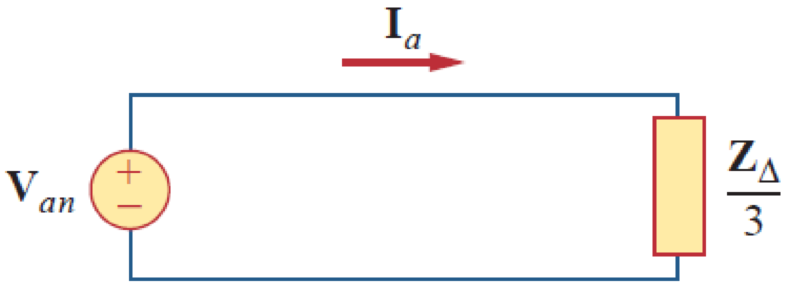
Balanced Wye-Delta...



- Wye – delta equivalent transform

$$Z_Y = \frac{Z_\Delta}{3}$$

- Single phase equivalent

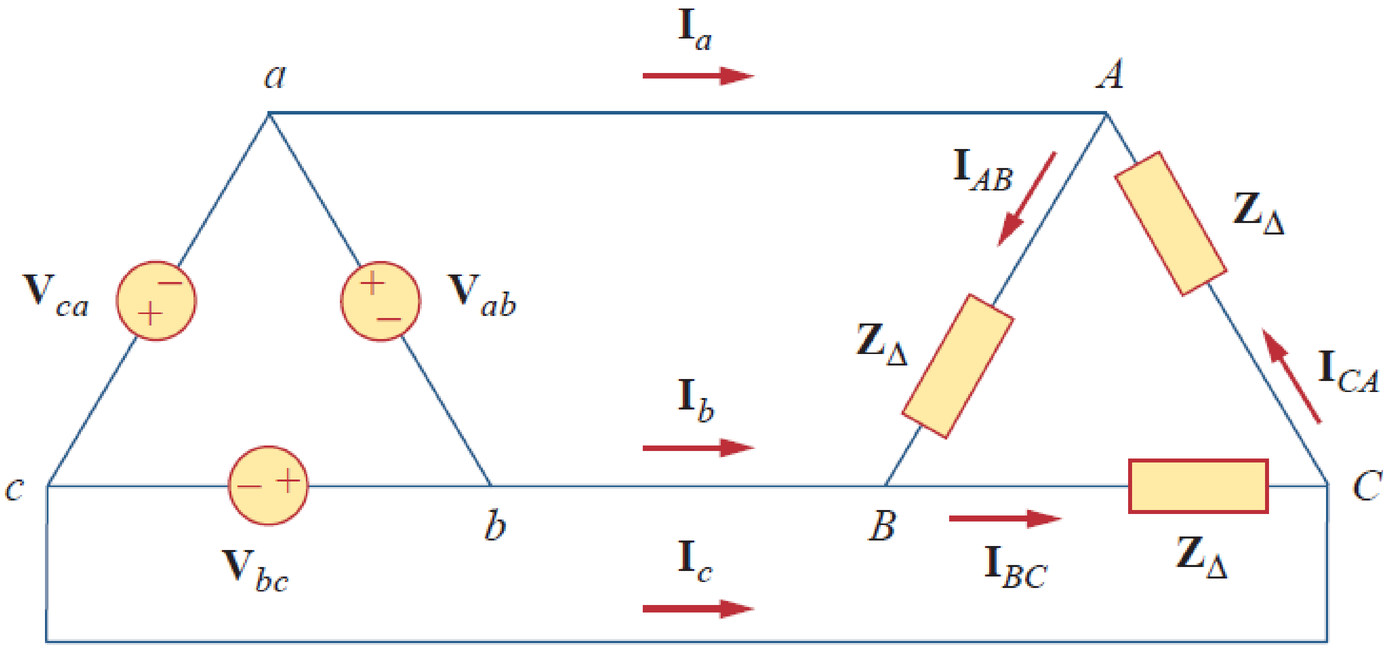


$$I_a = \frac{V_{an}}{Z_Y}$$

 gyurcsek.istvan@mik.pte.hu

- ❑ Balanced Three-Phase Voltages
- ❑ Balanced Wye-Wye Connection
- ❑ Balanced Wye-Delta Connection
- ❑ **Balanced Delta-Delta Connection**
- ❑ Balanced Delta-Wye Connection
- ❑ Power in Balanced Systems
- ❑ Unbalanced Three-Phase Systems
- ❑ Power in Unbalanced Systems

Balanced Delta-Delta...



$$\boldsymbol{I}_{AB} = \frac{\boldsymbol{V}_{AB}}{\boldsymbol{Z}_\Delta} = \frac{\boldsymbol{V}_{ab}}{\boldsymbol{Z}_\Delta} \qquad \boldsymbol{I}_a = \boldsymbol{I}_{AB} - \boldsymbol{I}_{CA}$$

$$\boldsymbol{I}_{BC} = \frac{\boldsymbol{V}_{BC}}{\boldsymbol{Z}_\Delta} = \frac{\boldsymbol{V}_{bc}}{\boldsymbol{Z}_\Delta} \qquad \boldsymbol{I}_b = \boldsymbol{I}_{BC} - \boldsymbol{I}_{AB}$$

$$\boldsymbol{I}_{CA} = \frac{\boldsymbol{V}_{CA}}{\boldsymbol{Z}_\Delta} = \frac{\boldsymbol{V}_{ca}}{\boldsymbol{Z}_\Delta} \qquad \boldsymbol{I}_c = \boldsymbol{I}_{CA} - \boldsymbol{I}_{BC}$$

$$\boldsymbol{V}_{ab} = V_p e^{j0°}, \qquad \boldsymbol{V}_{bc} = V_p e^{-j120°}, \qquad \boldsymbol{V}_{ca} = V_p e^{j120°}$$

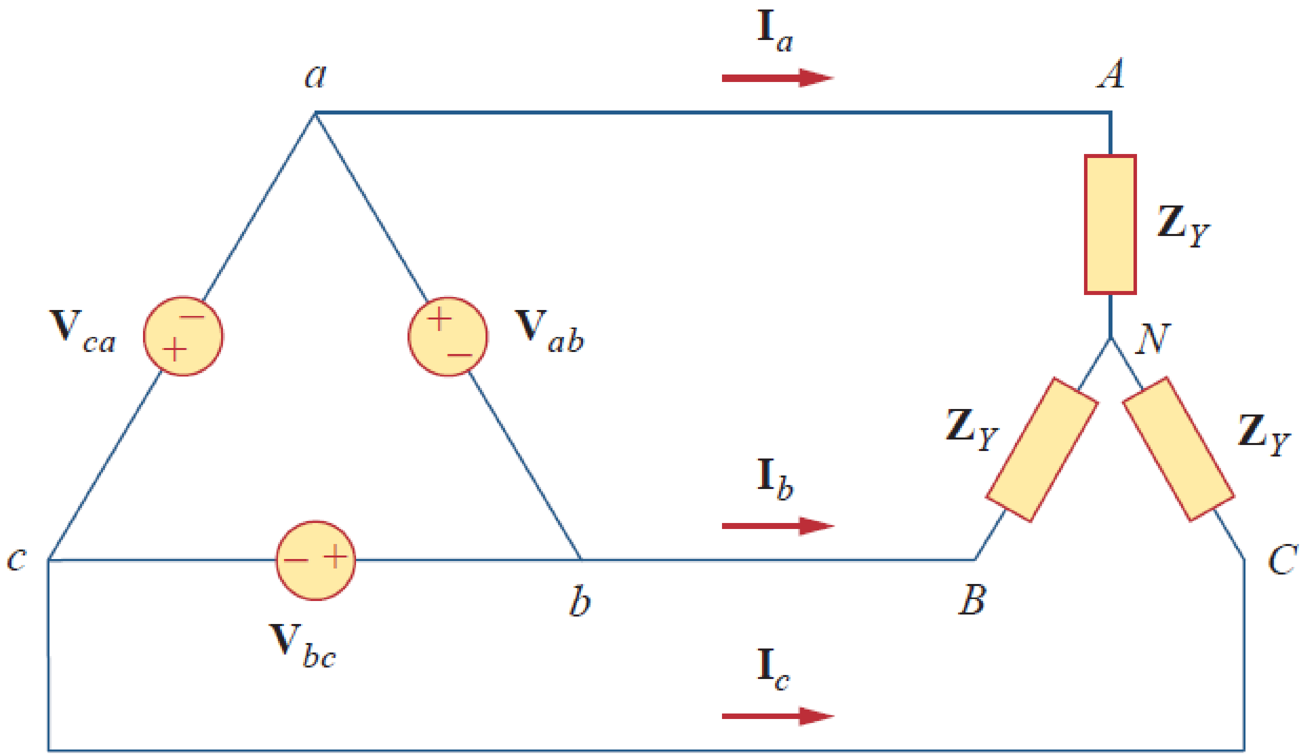
$$\boldsymbol{V}_{ab} = \boldsymbol{V}_{AB}, \qquad \boldsymbol{V}_{bc} = \boldsymbol{V}_{BC}, \qquad \boldsymbol{V}_{ca} = \boldsymbol{V}_{CA}$$

$$I_L = \sqrt{3} I_P$$

$$V_L = V_P$$

- ❑ Balanced Three-Phase Voltages
- ❑ Balanced Wye-Wye Connection
- ❑ Balanced Wye-Delta Connection
- ❑ Balanced Delta-Delta Connection
- ❑ **Balanced Delta-Wye Connection**
- ❑ Power in Balanced Systems
- ❑ Unbalanced Three-Phase Systems
- ❑ Power in Unbalanced Systems

Balanced Delta-Wye...



$$\boldsymbol{V}_{ab} = V_p e^{j0^\circ}, \qquad \boldsymbol{V}_{bc} = V_p e^{-j120^\circ}, \qquad \boldsymbol{V}_{ca} = V_p e^{j120^\circ}$$

$$-\boldsymbol{V}_{ab} + \boldsymbol{Z}_Y \boldsymbol{I}_a - \boldsymbol{Z}_Y \boldsymbol{I}_b = 0 \rightarrow \boldsymbol{Z}_Y(\boldsymbol{I}_a - \boldsymbol{I}_b) = \boldsymbol{V}_{ab} = V_p e^{j0^\circ}$$

$$\boldsymbol{I}_a - \boldsymbol{I}_b = \frac{V_p e^{j0^\circ}}{\boldsymbol{Z}_Y}$$

$$\boldsymbol{I}_b = \boldsymbol{I}_a e^{-j120^\circ}$$

$$\rightarrow \boldsymbol{I}_a - \boldsymbol{I}_b = \boldsymbol{I}_a(1 + 0.5 + j0.866)$$

$$= \boldsymbol{I}_a \sqrt{3} e^{j30^\circ}$$

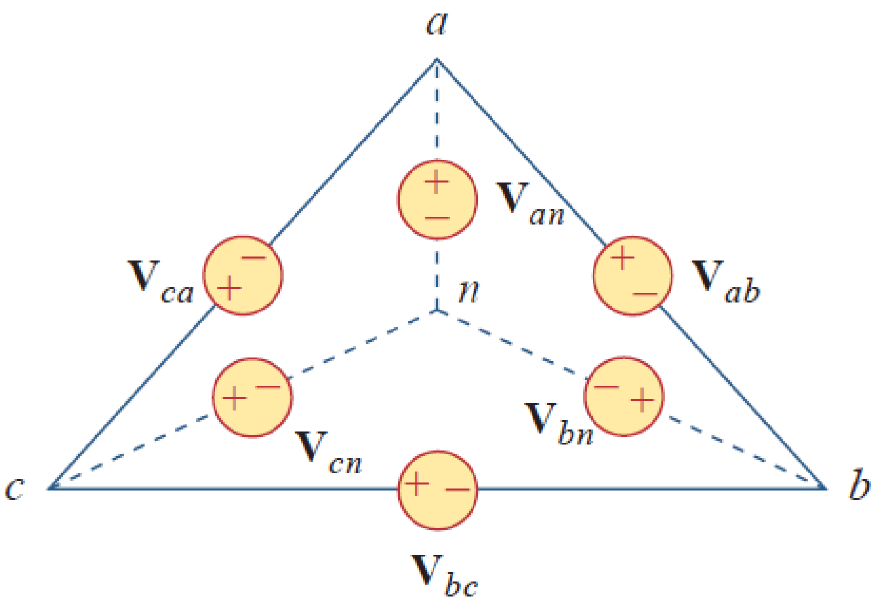
$$\boldsymbol{I}_a = \frac{V_p / \sqrt{3}\, e^{-j30^\circ}}{\boldsymbol{Z}_Y}$$

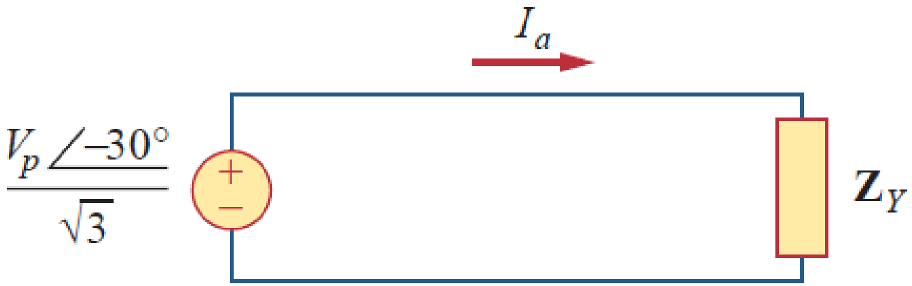
$$\boldsymbol{I}_b = \boldsymbol{I}_a e^{-j120^\circ}$$

$$\boldsymbol{I}_c = \boldsymbol{I}_a e^{j120^\circ}$$

Balanced Delta-Wye...

Delta – Wye – source transform $\boldsymbol{V}_{an} = \frac{V_p}{\sqrt{3}} e^{-j30°}$, $\boldsymbol{V}_{bn} = \frac{V_p}{\sqrt{3}} e^{-j150°}$, $\boldsymbol{V}_{cn} = \frac{V_p}{\sqrt{3}} e^{j90°}$





$$\boldsymbol{I}_a = \frac{V_p/\sqrt{3}\, e^{-j30°}}{\boldsymbol{Z}_Y}$$

$$\boldsymbol{V}_{AN} = \boldsymbol{I}_a \boldsymbol{Z}_Y = \frac{V_p}{\sqrt{3}} e^{-j30°}$$

$$\boldsymbol{V}_{BN} = \boldsymbol{V}_{AN} e^{-j120°}$$

$$\boldsymbol{V}_{CN} = \boldsymbol{V}_{AN} e^{j120°}$$

gyurcsek.istvan@mik.pte.hu

- ❑ Balanced Three-Phase Voltages
- ❑ Balanced Wye-Wye Connection
- ❑ Balanced Wye-Delta Connection
- ❑ Balanced Delta-Delta Connection
- ❑ Balanced Delta-Wye Connection
- ❑ **Power in Balanced Systems**
- ❑ Unbalanced Three-Phase Systems
- ❑ Power in Unbalanced Systems

Power in a Balanced System

Instantaneous power for Y connected load

$v_{AN} = V_p\sqrt{2}\cos\omega t\,, \qquad v_{BN} = V_p\sqrt{2}\cos(\omega t - 120°)\,, \qquad v_{CN} = V_p\sqrt{2}\cos(\omega t + 120°)$

$i_a = I_p\sqrt{2}\cos(\omega t - \Theta)\,, \qquad i_b = I_p\sqrt{2}\cos(\omega t - \Theta - 120°)\,, \qquad i_c = I_p\sqrt{2}\cos(\omega t - \Theta + 120°)$

$$p = p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c =$$

$$= 2V_pI_p[\cos\omega t\cos(\omega t - \Theta) + \cos(\omega t - 120°)\cos(\omega t - \Theta - 120°) + \cos(\omega t + 120°)\cos(\omega t - \Theta + 120°)]$$

$$\cos A\cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

$$p = V_pI_p[3\cos\Theta + \cos(2\omega t - \Theta) + \cos(2\omega t - \Theta - 240°) + \cos(2\omega t - \Theta + 240°)]$$

$\cos(A \mp B) = \cos A\cos B \pm \sin A\sin B\,, \qquad \cos(2\omega t - \Theta) = \cos\alpha$

$$p = V_pI_p[3\cos\Theta + \cos(\alpha) + \cos(\alpha - 240°) + \cos(\alpha + 240°)] = V_pI_p\left[3\cos\Theta + \cos(\alpha) + 2\left(-\frac{1}{2}\right)\cos(\alpha)\right]$$

$$p = 3V_pI_p\cos\Theta$$

Power in a Balanced System

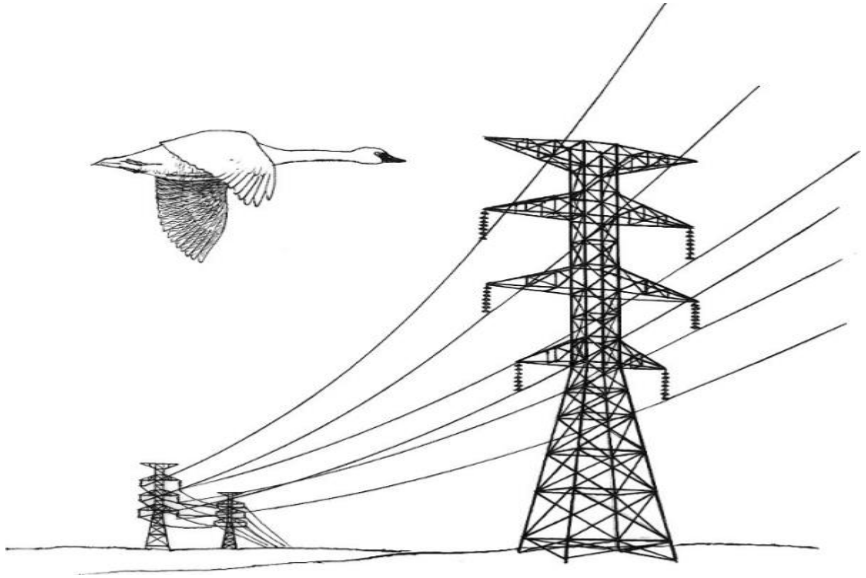
$p = 3V_p I_p \cos\Theta$

$average\ PWR\ per\ phase \rightarrow P_P = \frac{p}{3} = V_P I_P \cos\Theta$

$reactive\ PWR\ per\ phase \rightarrow Q_P = V_P I_P \sin\Theta$

$apparent\ PWR\ per\ phase \rightarrow S_P = V_P I_P$

$complex\ PWR\ per\ phase \rightarrow \boldsymbol{S}_P = P_P + jQ_P = \boldsymbol{V}_P \boldsymbol{I}_P^*$



$total\ average\ PWR \rightarrow P = P_a + P_b + P_c = 3P_P = 3V_P I_P \cos\Theta = \sqrt{3} V_L I_L \cos\Theta \leftarrow next\ page$

$total\ reactive\ PWR \rightarrow Q = Q_a + Q_b + Q_c = 3Q_P = 3V_P I_P \sin\Theta = \sqrt{3} V_L I_L \sin\Theta \leftarrow next\ page$

$total\ complex\ PWR \rightarrow \boldsymbol{S} = 3\boldsymbol{S}_P = 3\boldsymbol{V}_P \boldsymbol{I}_P^* = 3I_P^2 \boldsymbol{Z}_P = \frac{3V_P^2}{\boldsymbol{Z}_P^*} = P + jQ = \sqrt{3} V_L I_L e^{j\Theta} \leftarrow next\ page$

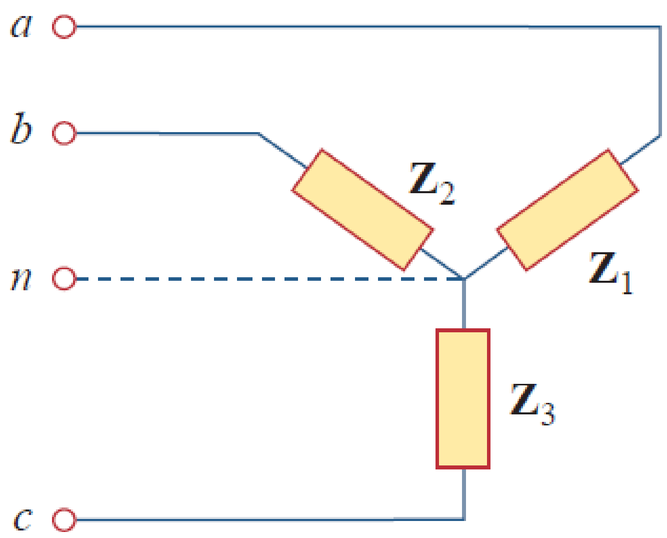
Note - Wye vs. Delta Loads

Wye connected load

$$\left(V_L = \sqrt{3}V_P\,, I_L = I_P\right) \rightarrow \; S = 3V_P I_P = 3\frac{V_L}{\sqrt{3}} I_L = \sqrt{3} V_L I_L$$

$$P = 3V_P I_P \cos\Theta = 3\frac{V_L}{\sqrt{3}} I_L \cos\Theta = \sqrt{3} V_L I_L \cos\Theta$$

$$Q = 3V_P I_P \sin\Theta = 3\frac{V_L}{\sqrt{3}} I_L \sin\Theta = \sqrt{3} V_L I_L \sin\Theta$$

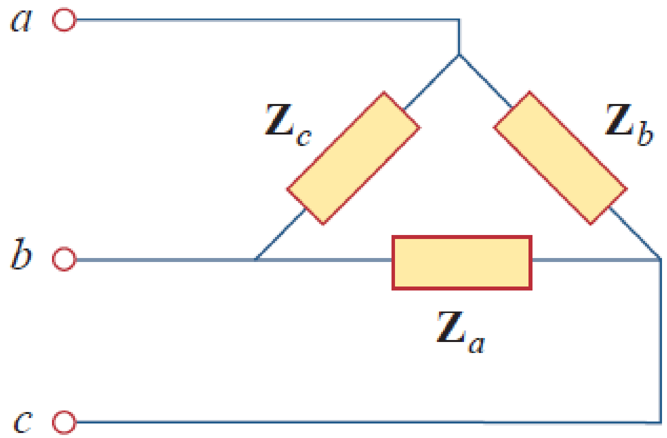


Delta connected load

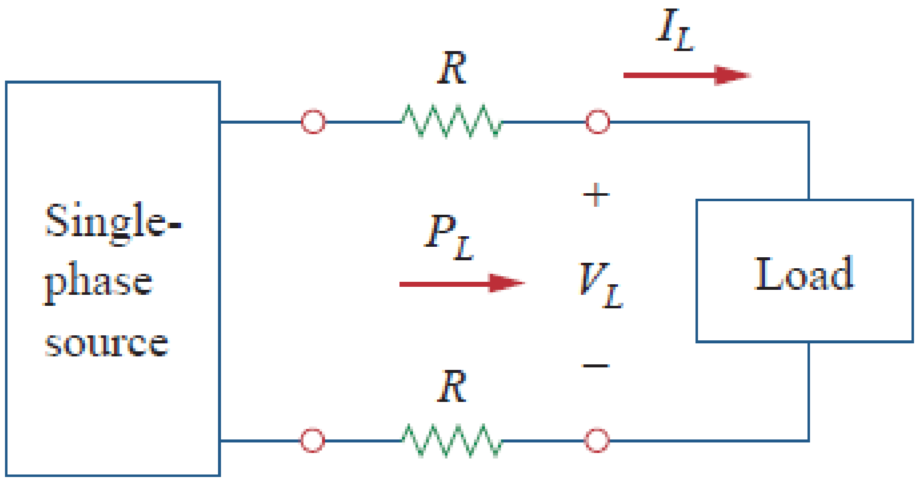
$$\left(V_L = V_P, I_L = \sqrt{3}I_P\right) \rightarrow \quad S = 3V_P I_P = 3V_L \frac{I_L}{\sqrt{3}} = \sqrt{3} V_L I_L$$

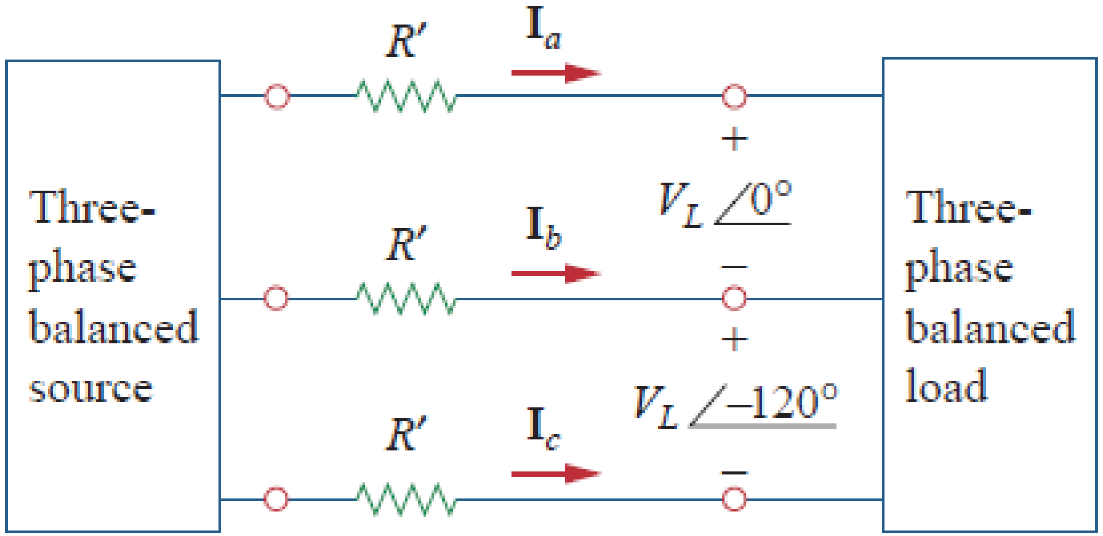
$$P = 3V_P I_P \cos\Theta = 3V_L \frac{I_L}{\sqrt{3}} \cos\Theta = \sqrt{3} V_L I_L \cos\Theta$$

$$Q = 3V_P I_P \sin\Theta = 3V_L \frac{I_L}{\sqrt{3}} \sin\Theta = \sqrt{3} V_L I_L \sin\Theta$$



Power Loss...





$$I_L = \frac{P_L}{V_L} \rightarrow P_{loss} = 2I_L^2 R = 2R\frac{P_L^2}{V_L^2}$$

$$I_L' = I_a = I_b = I_c = \frac{P_L}{\sqrt{3}V_L} \rightarrow P_{loss}' = 3(I_L')^2 R' = 3R'\frac{P_L^2}{3V_L^2} = R'\frac{P_L^2}{V_L^2}$$

$$\frac{P_{loss}}{P_{loss}'} = \frac{2R}{R'} = \frac{2\ {}^{\rho l}/_{\pi r^2}}{{}^{\rho l}/_{\pi r'^2}} = \frac{2r'^2}{r^2} \rightarrow same\ PWR\ loss \rightarrow 2r'^2 = r^2$$

$$\frac{material\ for\ single - phase}{material\ for\ three - phase} = \frac{2(r^2\pi l)}{3(r'^2\pi l)} = \frac{2r^2}{3r'^2} = \frac{2}{3}\cdot 2 = 1.33 \rightarrow$$ 33% more material for single-phase transmission!

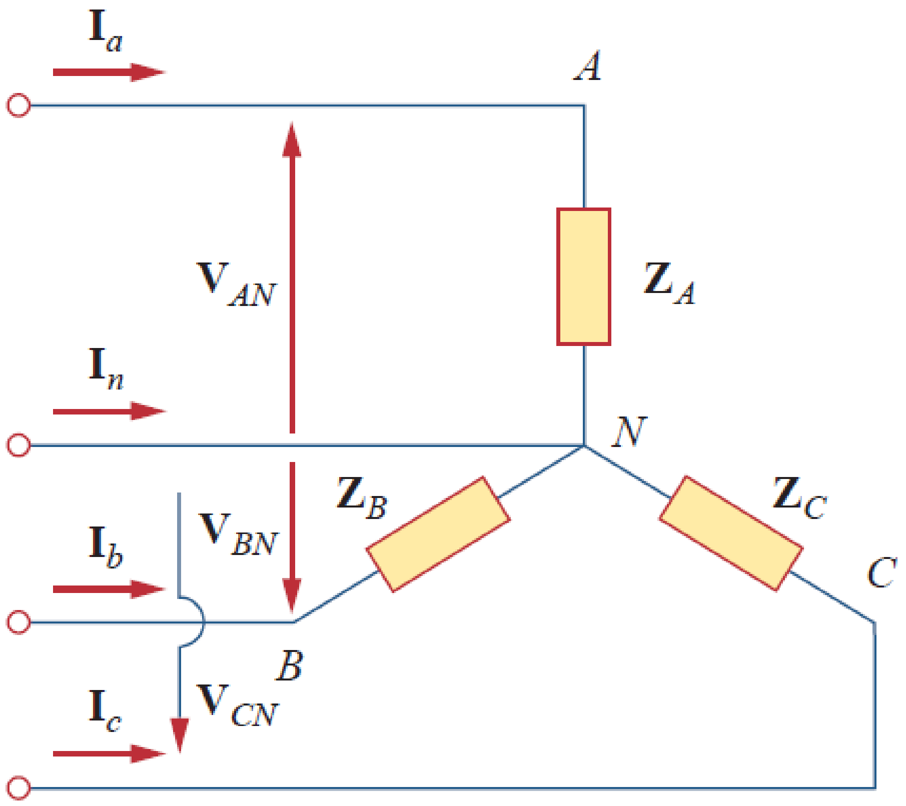
- ❑ Balanced Three-Phase Voltages
- ❑ Balanced Wye-Wye Connection
- ❑ Balanced Wye-Delta Connection
- ❑ Balanced Delta-Delta Connection
- ❑ Balanced Delta-Wye Connection
- ❑ Power in Balanced Systems
- ❑ **Unbalanced Three-Phase Systems**
- ❑ Power in Unbalanced Systems

gyurcsek.istvan@mik.pte.hu

Unbalanced Systems

Unbalanced system is due to

- ❑ *(unbalanced voltage sources)*
- ❑ unbalanced load.



$$\boldsymbol{I}_a = \frac{\boldsymbol{V}_{AN}}{\boldsymbol{Z}_A}, \qquad \boldsymbol{I}_b = \frac{\boldsymbol{V}_{BN}}{\boldsymbol{Z}_B}, \qquad \boldsymbol{I}_c = \frac{\boldsymbol{V}_{CN}}{\boldsymbol{Z}_C}$$

$$\boldsymbol{I}_n = -(\boldsymbol{I}_a + \boldsymbol{I}_b + \boldsymbol{I}_c)$$

Three-wire system → $\boldsymbol{I}_a + \boldsymbol{I}_b + \boldsymbol{I}_c = 0$

The same could be used (accordingly) for…

- ❑ *wye-connected source; wye-connected load*
- ❑ *delta-connected source; wye-connected load*
- ❑ *wye-connected source; delta-connected load*
- ❑ *delta-connected source; delta-connected load*

gyurcsek.istvan@mik.pte.hu

Non-Ideal Cables...

KVL + KCL + Characteristics…

$$Z_A = Z'_A + Z_l + Z_S \qquad V_{an} - V_{AN} + V_{Nn} = 0$$

$$Z_B = Z'_B + Z_l + Z_S \qquad V_{bn} - V_{BN} + V_{Nn} = 0$$

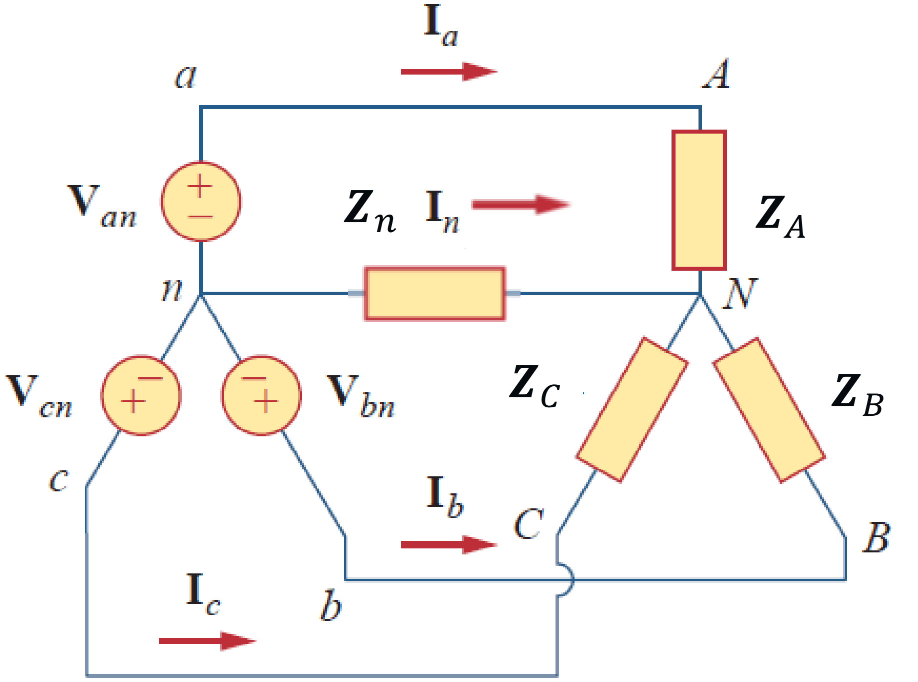
$$Z_C = Z'_C + Z_l + Z_S \qquad V_{cn} - V_{CN} + V_{Nn} = 0$$

$$I_n = V_{Nn} \cdot Y_n \qquad I_n = -(I_a + I_b + I_c)$$

$$I_a = V_{AN} \cdot Y_A \qquad I_a = (V_{an} + V_{Nn}) \cdot Y_A$$

$$I_b = V_{BN} \cdot Y_B \qquad I_b = (V_{bn} + V_{Nn}) \cdot Y_B$$

$$I_c = V_{CN} \cdot Y_C \qquad I_c = (V_{cn} + V_{Nn}) \cdot Y_C$$



(Millman's Theorem)

$$V_{Nn} = \frac{V_{an} \cdot Y_A + V_{bn} \cdot Y_B + V_{cn} \cdot Y_C}{Y_A + Y_B + Y_C + Y_n}$$

Non-Ideal Cables...

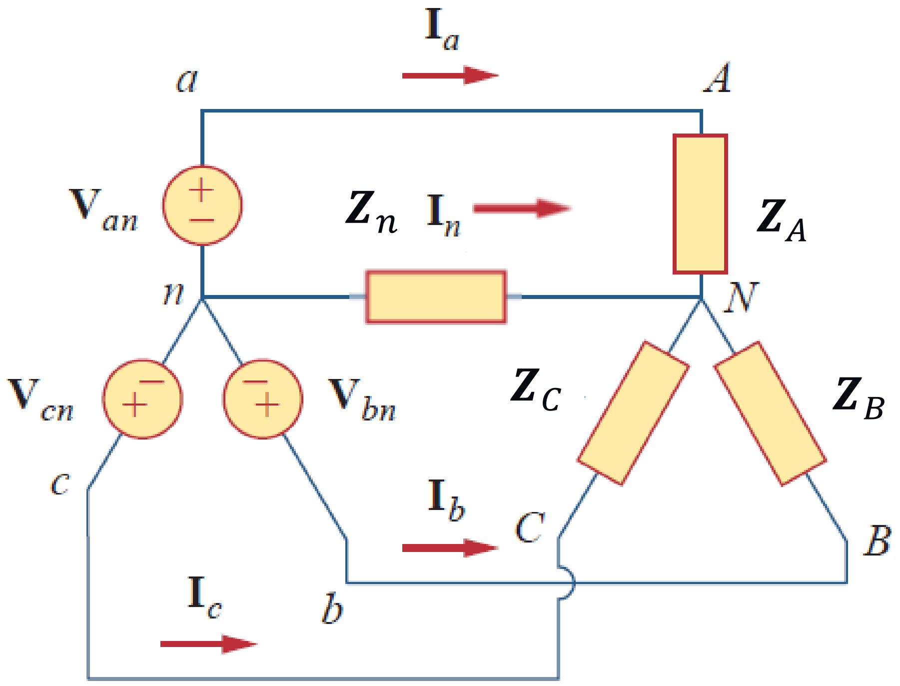
$$\boldsymbol{V}_{Nn} = \frac{\boldsymbol{V}_{an} \cdot \boldsymbol{Y}_A + \boldsymbol{V}_{bn} \cdot \boldsymbol{Y}_B + \boldsymbol{V}_{cn} \cdot \boldsymbol{Y}_C}{\boldsymbol{Y}_A + \boldsymbol{Y}_B + \boldsymbol{Y}_C + \boldsymbol{Y}_n}$$

❑ In case of balanced load

$$\boldsymbol{Y}_A = \boldsymbol{Y}_B = \boldsymbol{Y}_C \rightarrow \boldsymbol{V}_{Nn} = \frac{\boldsymbol{Y}_A \cdot (\boldsymbol{V}_{an} + \boldsymbol{V}_{bn} + \boldsymbol{V}_{cn})}{3 \cdot \boldsymbol{Y}_A + \boldsymbol{Y}_n} = 0$$

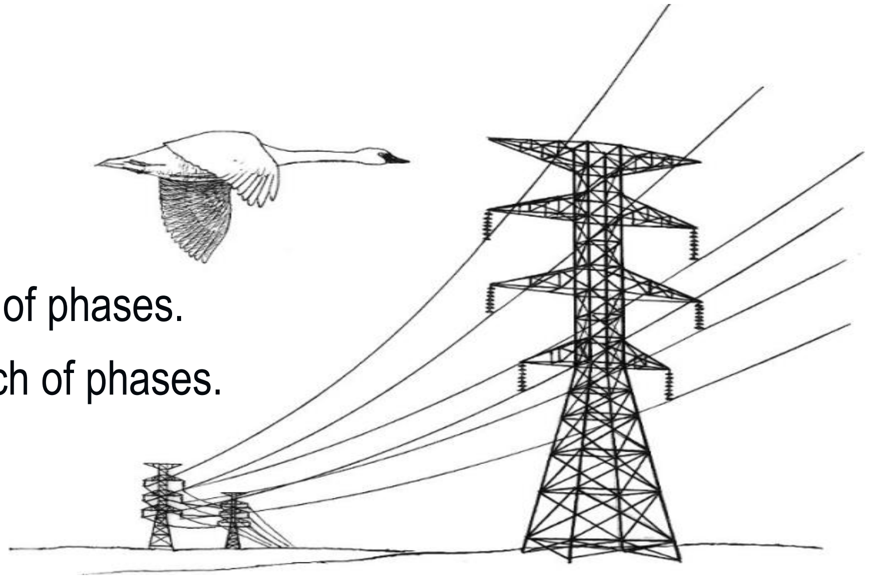
❑ In case of unbalanced load + ideal neutral wire

$$\boldsymbol{Y}_N = \infty \rightarrow \boldsymbol{V}_{Nn} = 0\,, \qquad \boldsymbol{I}_n > 0$$



- Balanced Three-Phase Voltages
- Balanced Wye-Wye Connection
- Balanced Wye-Delta Connection
- Balanced Delta-Delta Connection
- Balanced Delta-Wye Connection
- Power in Balanced Systems
- Unbalanced Three-Phase Systems
- **Power in Unbalanced Systems**

Power in Unbalanced Systems

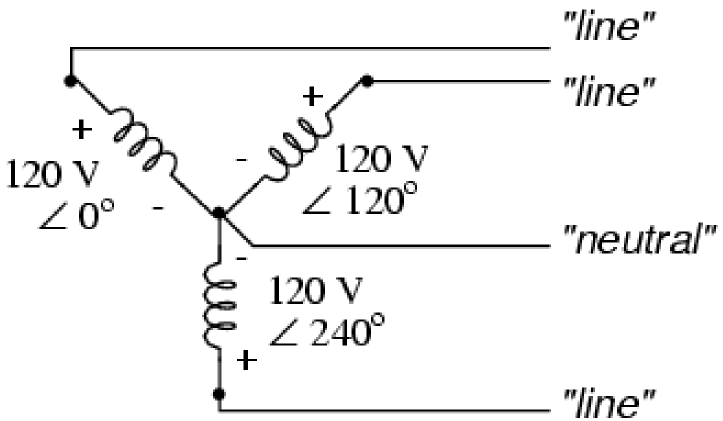


- The total PWR is not simply three times the PWR in any of phases.
- The total PWR is the sum of the unbalanced PWR in each of phases.
- Reactive PWR has sign (CAP → (-), IND → (+)
- *(Refer to Tellegen's theorem)*

- Total average PWR $\rightarrow P = P_a + P_b + P_c\,,$ $\quad P \neq 3P_P, P \neq 3V_P I_P \cos\Theta\,, P \neq \sqrt{3}V_L I_L \cos\Theta$
- Total reactive PWR $\rightarrow Q = Q_a + Q_b + Q_c\,,$ $\quad Q \neq 3Q_P, Q \neq 3V_P I_P \sin\Theta\,, Q \neq \sqrt{3}V_L I_L \sin\Theta$
- Total complex PWR $\rightarrow \boldsymbol{S} = \boldsymbol{S}_a + \boldsymbol{S}_b + \boldsymbol{S}_c = P + jQ\,,$ $\quad \boldsymbol{S} \neq 3\boldsymbol{S}_P, S \neq 3\boldsymbol{V}_P\boldsymbol{I}_P^*, S \neq \sqrt{3}V_L I_L e^{j\Theta}$

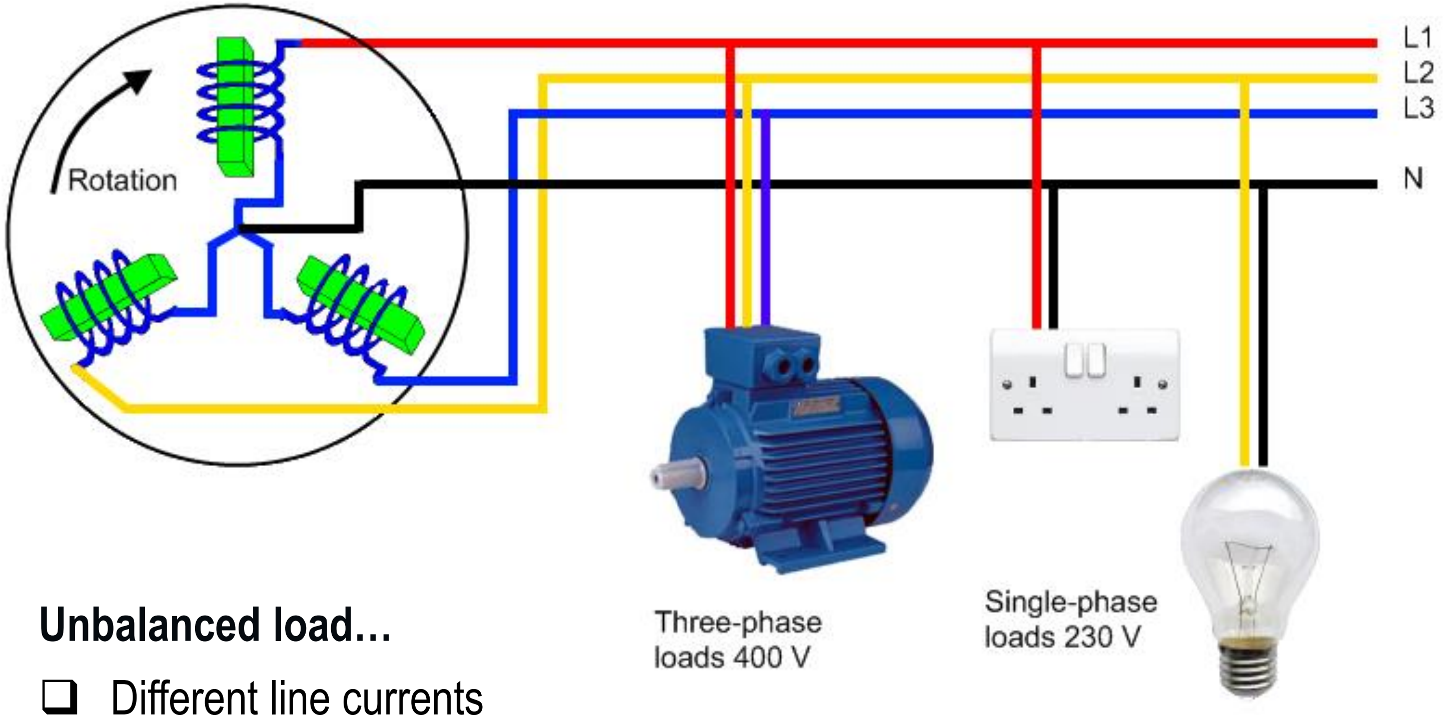
gyurcsek.istvan@mik.pte.hu

Summary



Balanced load…

- ❑ Voltages are 120° out of phase with each other
- ❑ Wye connection
  - ❑ $V_L = \sqrt{3}\, V_P$ $\qquad I_L = I_P$
  - ❑ Common (neutral) nodes → same potential
  - ❑ No neutral current (3 wire/4 wire systems)
- ❑ Delta connection
  - ❑ $V_L = V_P$ $\qquad I_L = \sqrt{3}\, I_P$



Unbalanced load…

- ❑ Different line currents
- ❑ Wye connection
  - ❑ Common (neutral) nodes → different potential
  - ❑ Neutral current occurs (4 wire system)
- ❑ Total power → sum of different powers in phases

gyurcsek.istvan@mik.pte.hu

Questions