

DR. GYURCSEK ISTVÁN

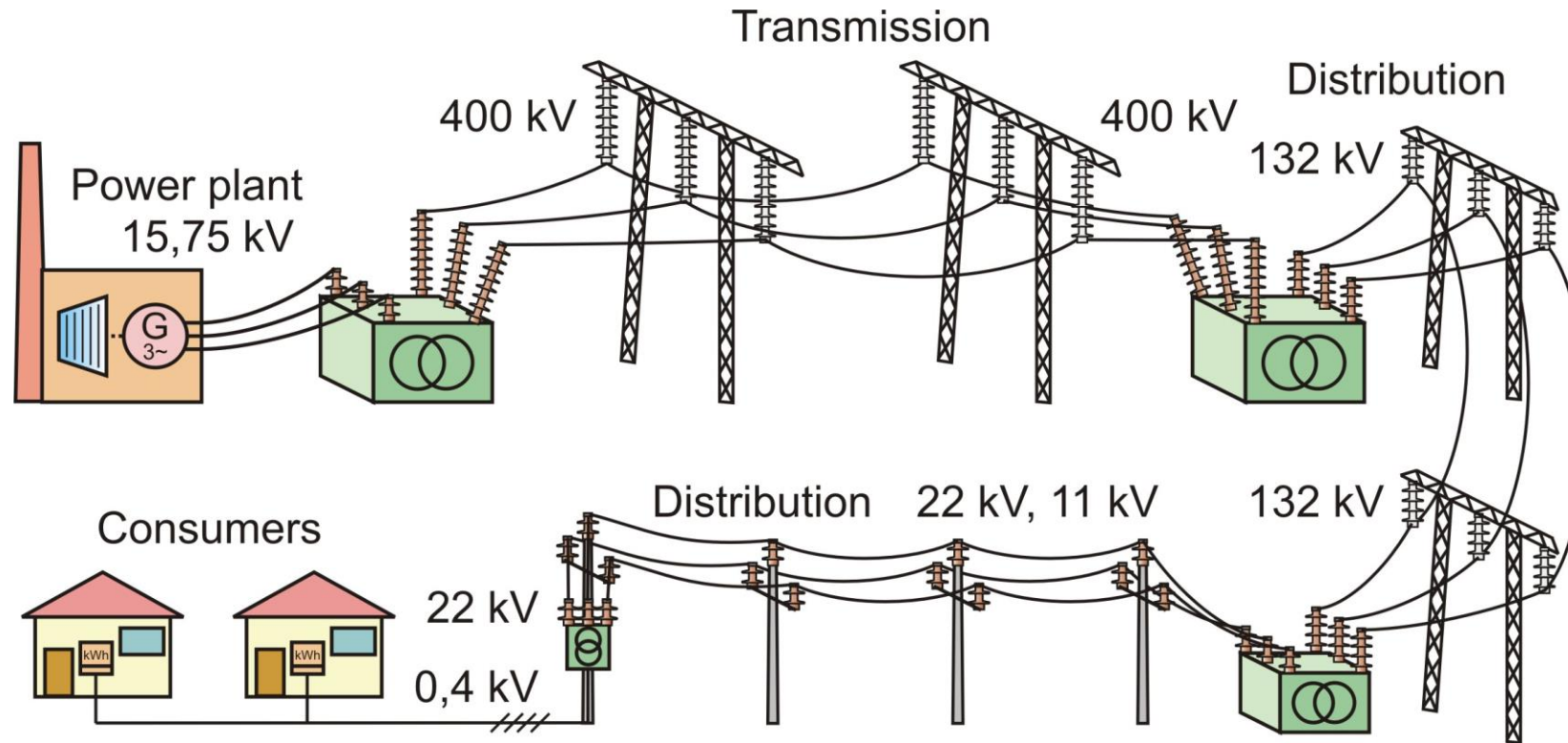
Three-Phase Circuits

Sources and additional materials (recommended)

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Fleckenstein: Three-Phase Electrical Power (ISBN-13: 978-1498737777)*
- ❑ *Delmar: 3-phase Circuits and Electrical Machines (ISBN-13: 978-1439059821)*
- ❑ *Mayergoyz - Lawson: Basic Electric Circuit Theory (ISBN13: 978-0124808652)*
- ❑ *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*

EXAMPLE (schematic diagram)

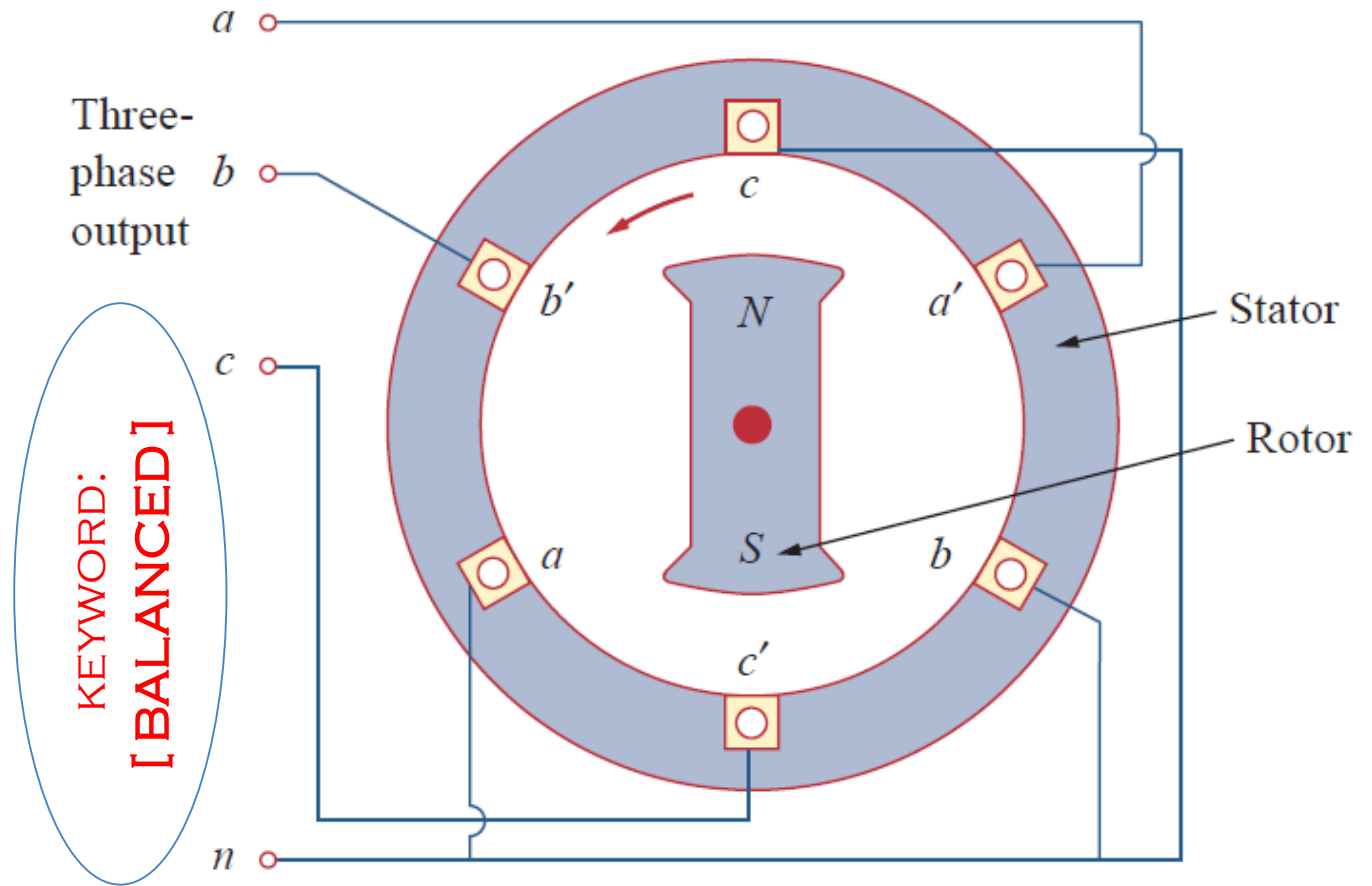
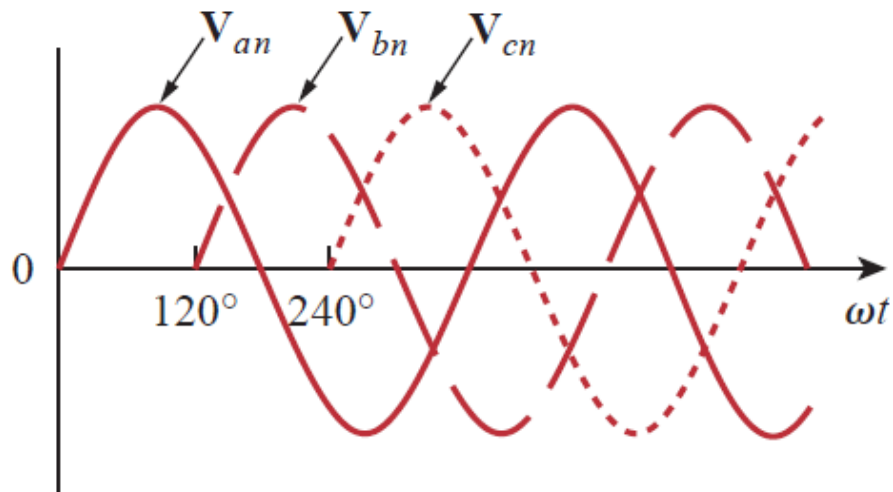
Generation, Transmission and Distribution of Electrical Power System



Why Three-Phase... ?

‘Three-phase power is the most efficient way that electricity could be produced, transmitted, consumed.’

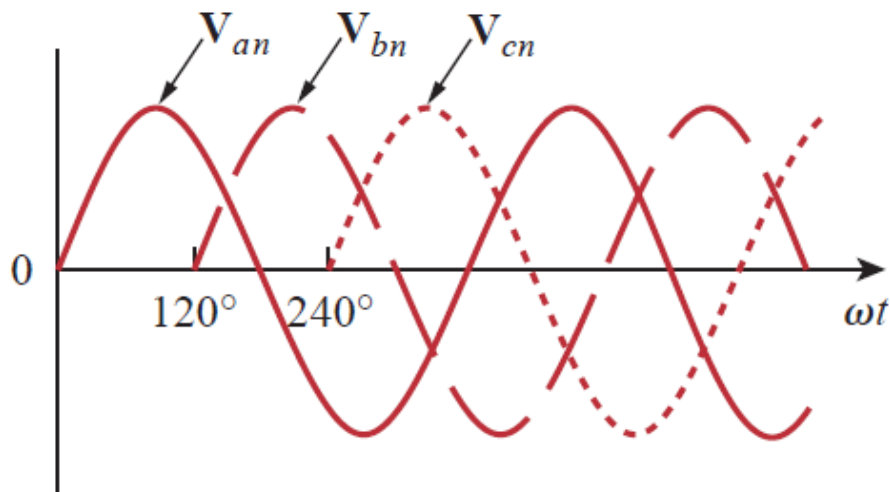
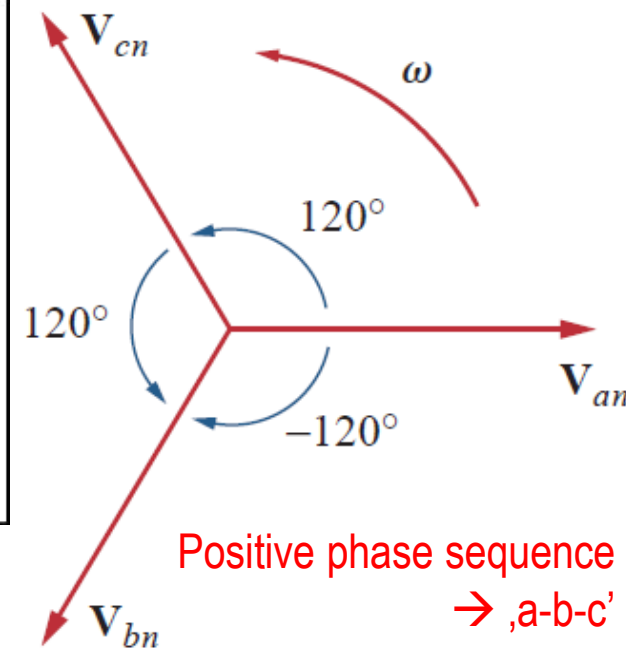
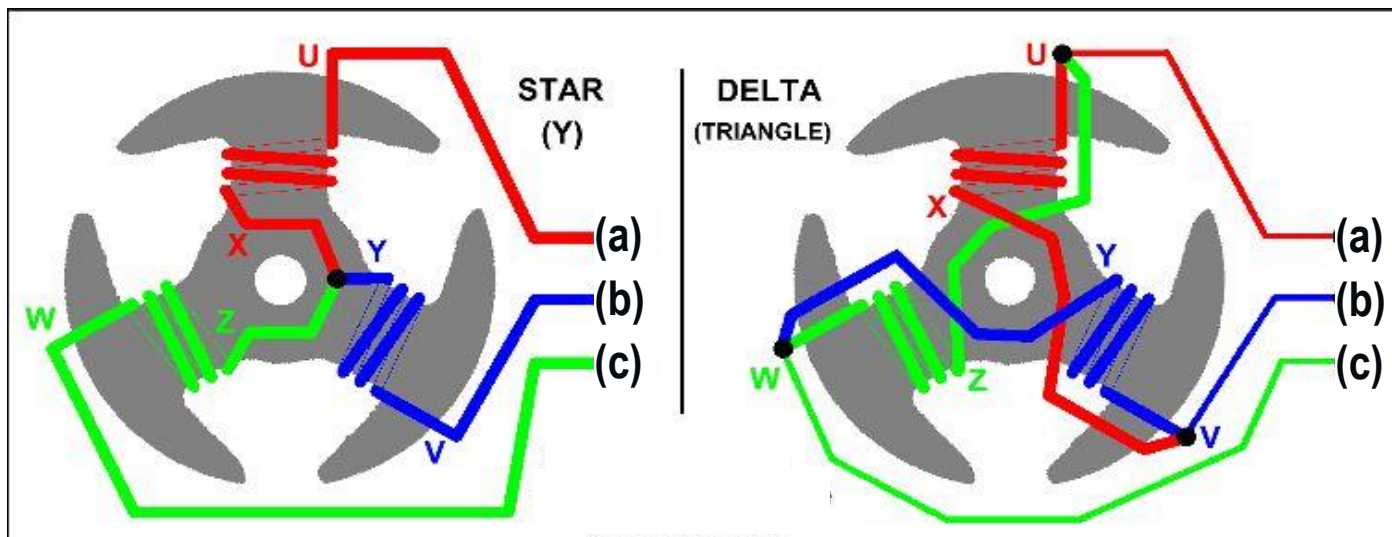
- ❑ Single-phase power → falls to zero three times
- ❑ Three-phase power → never falls to zero
- ❑ Delivered power is the same at any instant
- ❑ Motors / transformers → 150% operating charact.
- ❑ 33% save in mass of conductors (*see later*)





- Balanced Three-Phase Voltages**
- Balanced Wye-Wye Connection
- Balanced Wye-Delta Connection
- Balanced Delta-Delta Connection
- Balanced Delta-Wye Connection
- Power in Balanced Systems
- Unbalanced Three-Phase Systems
- Power in Unbalanced Systems

Three-Phase Voltages

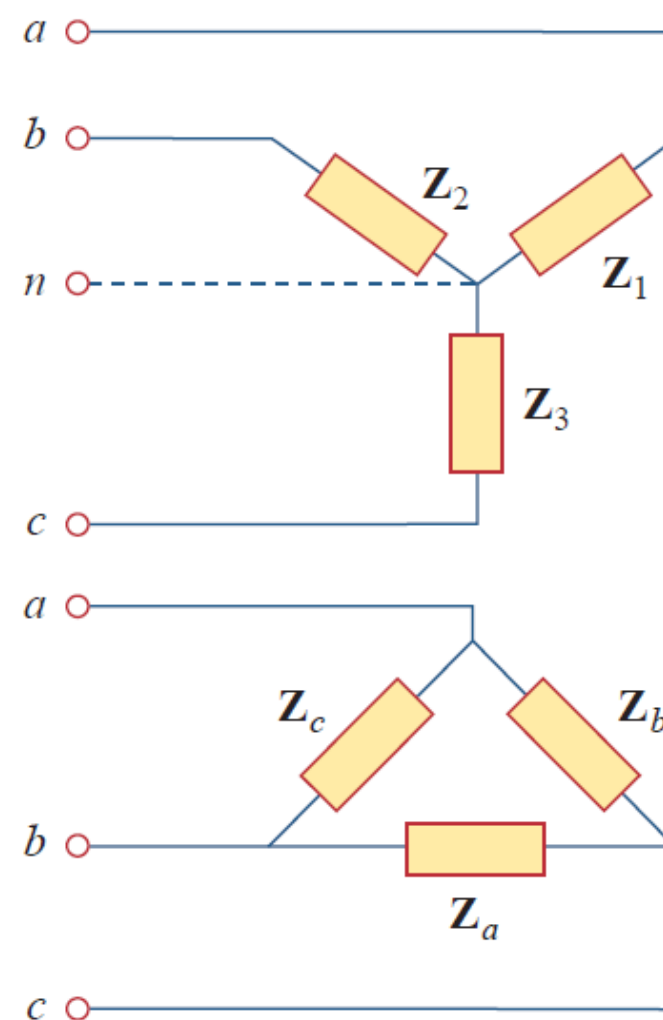
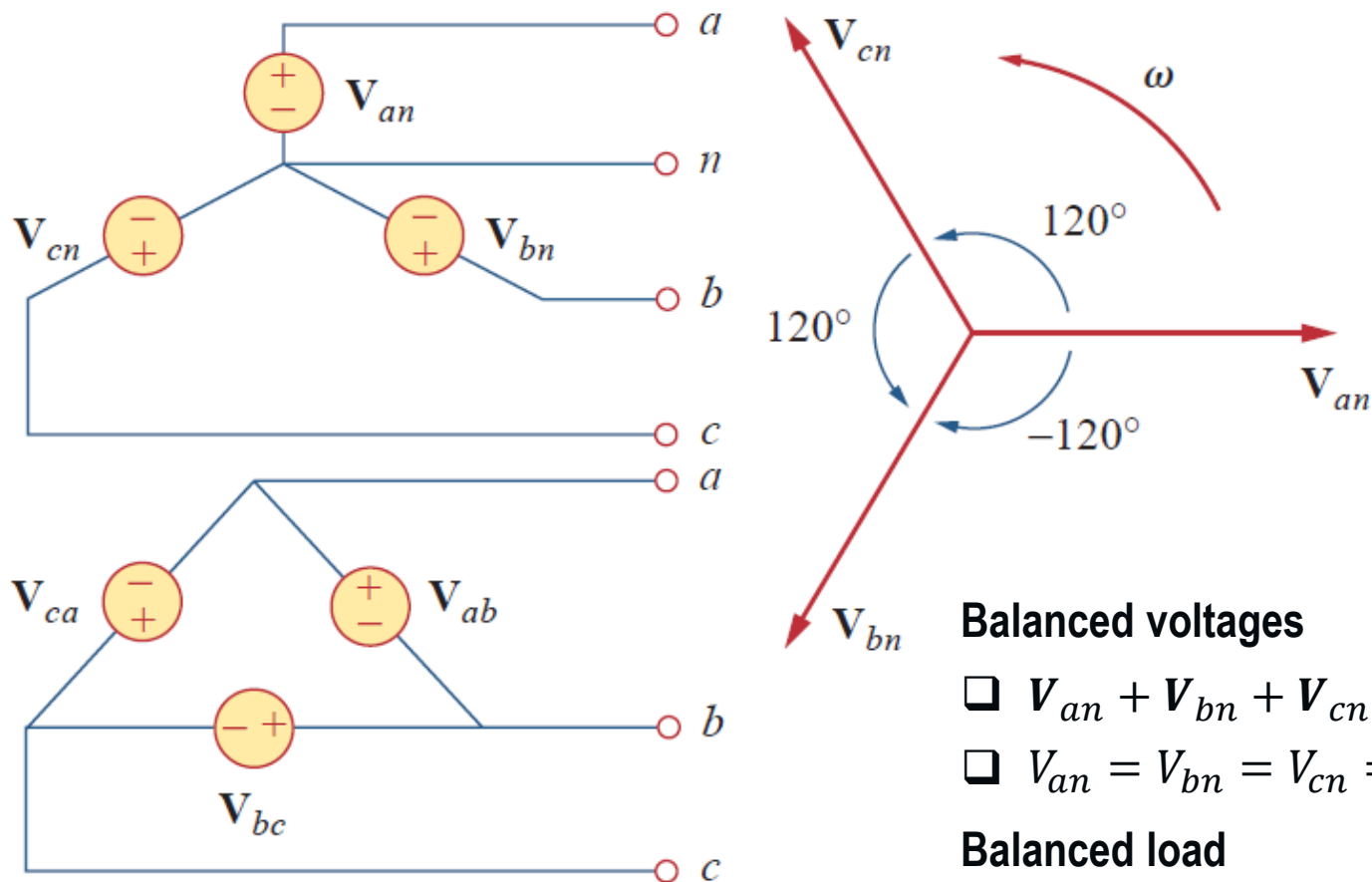


$$v_{an}(t) = V_p \sin \omega t \rightarrow \mathbf{V}_{an} = V_p e^{j0^\circ}$$

$$v_{bn}(t) = V_p \sin(\omega t - 120^\circ) \rightarrow \mathbf{V}_{bn} = V_p e^{-j120^\circ}$$

$$v_{cn}(t) = V_p \sin(\omega t + 120^\circ) \rightarrow \mathbf{V}_{cn} = V_p e^{j120^\circ}$$

Possible Source to Load Connections



Balanced voltages

- $V_{an} + V_{bn} + V_{cn} = 0$
- $V_{an} = V_{bn} = V_{cn} = V$

Balanced load

- $Z_1 = Z_2 = Z_3 = Z_Y$
- $Z_a = Z_b = Z_c = Z_{\Delta}$

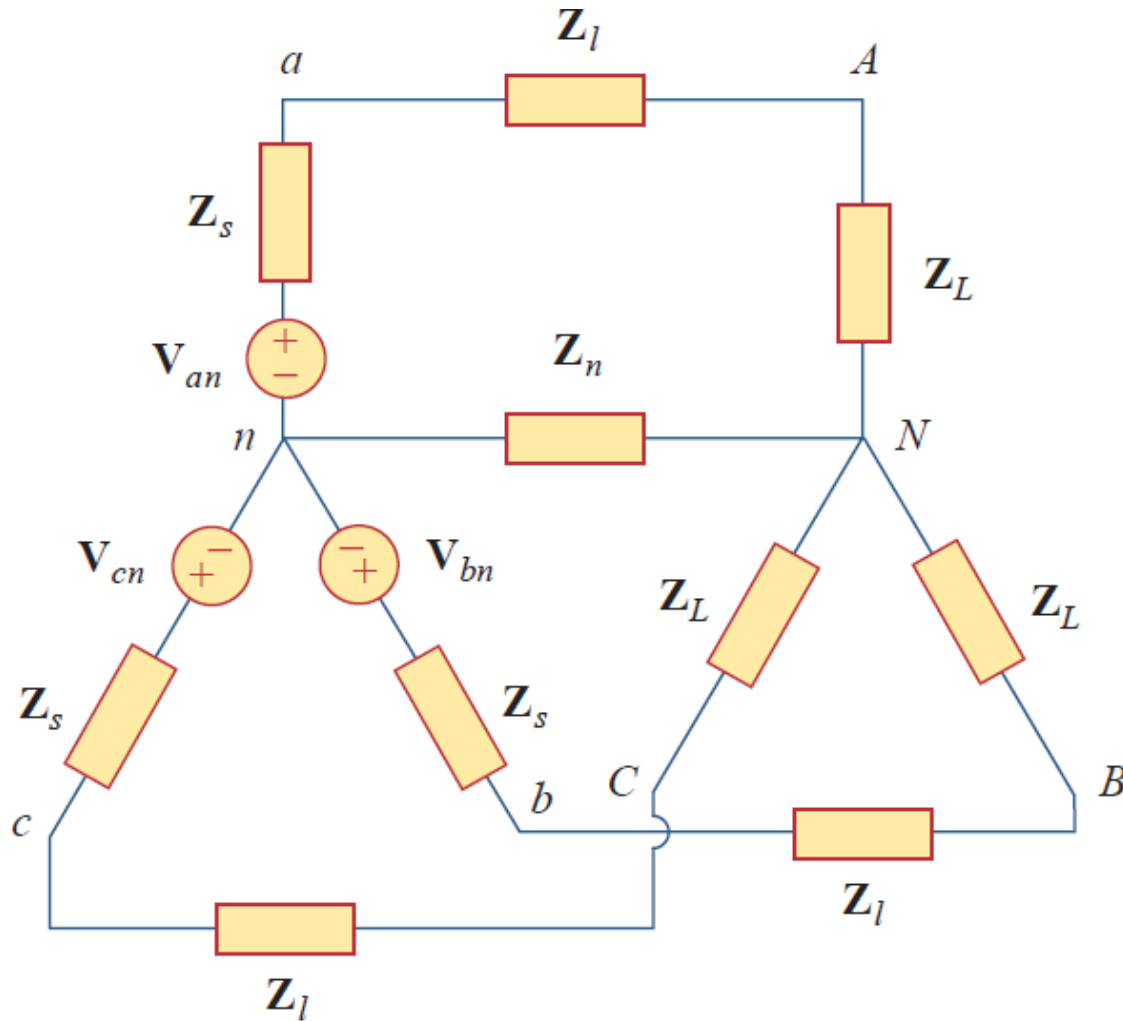
Possible source - load connections

- (1) Y - Y. (2) Y - Δ, (3) Δ - Δ, (4) Δ - Y



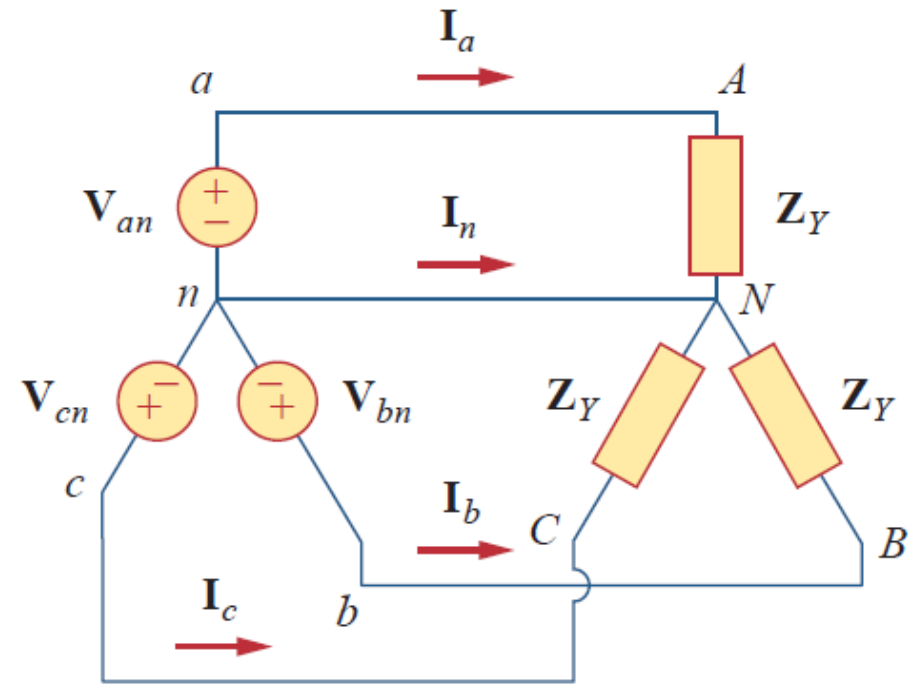
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Balanced Wye-Wye Connection 1

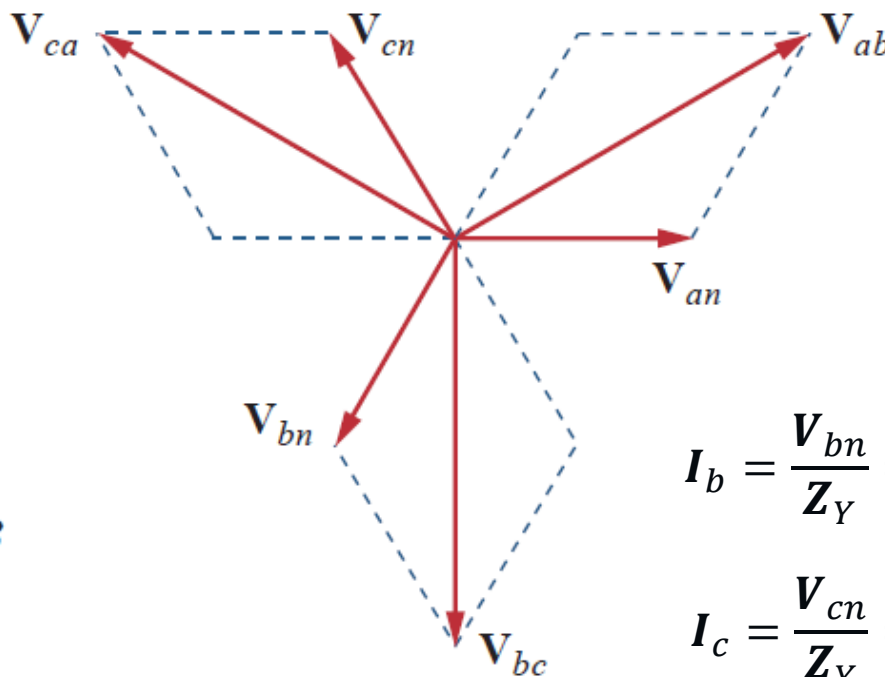
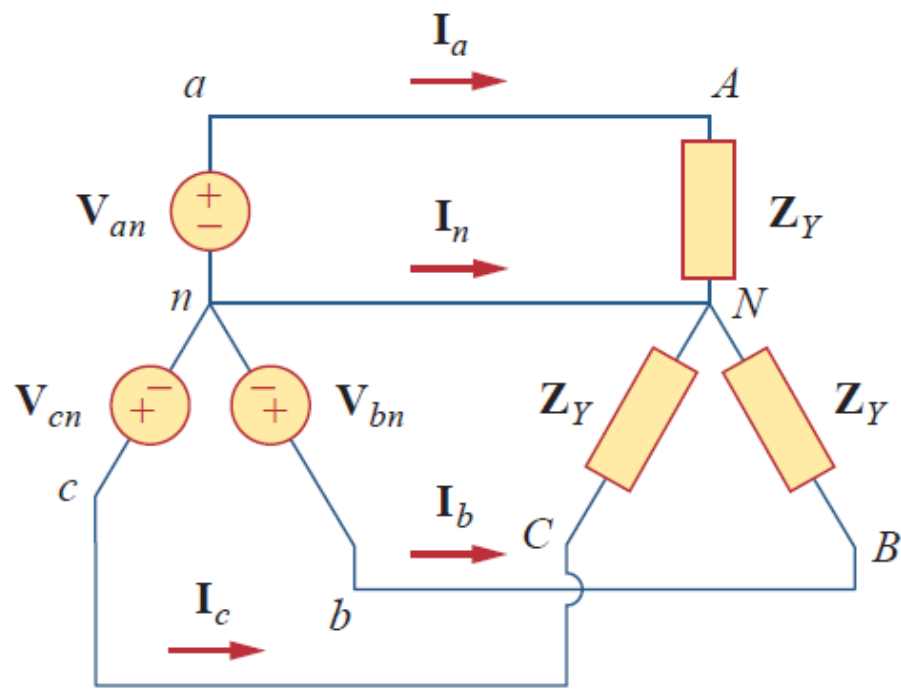


$$Z_Y = Z_S + Z_l + Z_L$$

$$Z_n \approx 0, Z_S \approx 0, Z_l \approx 0 \rightarrow Z_Y = Z_L$$



Balanced Wye-Wye Connection 2



$$I_a = \frac{V_{an}}{Z_Y}$$

$$I_b = \frac{V_{bn}}{Z_Y} = \frac{V_{an}e^{-j120^\circ}}{Z_Y} = I_a e^{-j120^\circ}$$

$$I_c = \frac{V_{cn}}{Z_Y} = \frac{V_{an}e^{-j240^\circ}}{Z_Y} = I_a e^{-j240^\circ}$$

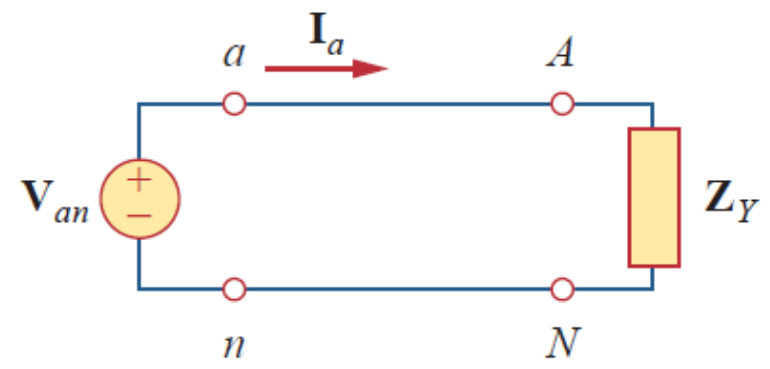
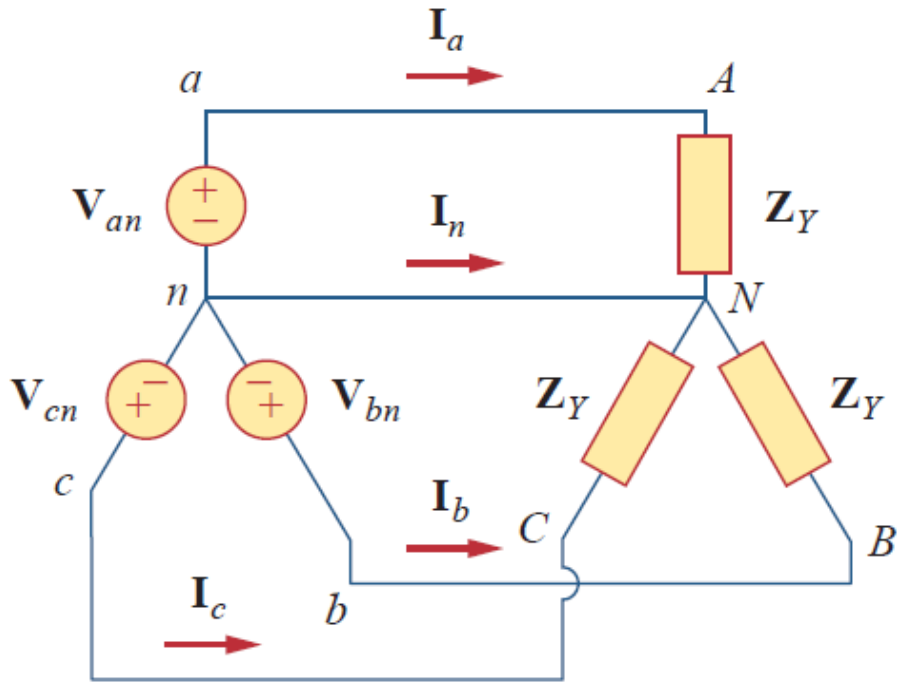
$$V_{an} = V_p e^{j0^\circ}, \quad V_{bn} = V_p e^{-j120^\circ}, \quad V_{cn} = V_p e^{j120^\circ} \quad V_L = \sqrt{3}V_p, \quad I_L = I_p \quad I_a + I_b + I_c = 0$$

$$V_{ab} = V_{an} - V_{bn} = V_p e^{j0^\circ} - V_p e^{-j120^\circ} = V_p \left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2} \right) = \sqrt{3}V_p e^{j30^\circ} \quad I_n = -(I_a + I_b + I_c) = 0$$

$$V_{bc} = V_{bn} - V_{cn} = \dots = \sqrt{3}V_p e^{-j90^\circ} \quad V_{ca} = V_{cn} - V_{an} = \dots = \sqrt{3}V_p e^{-j210^\circ} \quad V_{nN} = Z_n I_n = 0$$

Balanced Wye-Wye Connection 3

Single phase equivalent



$$I_a = \frac{V_{an}}{Z_Y}$$

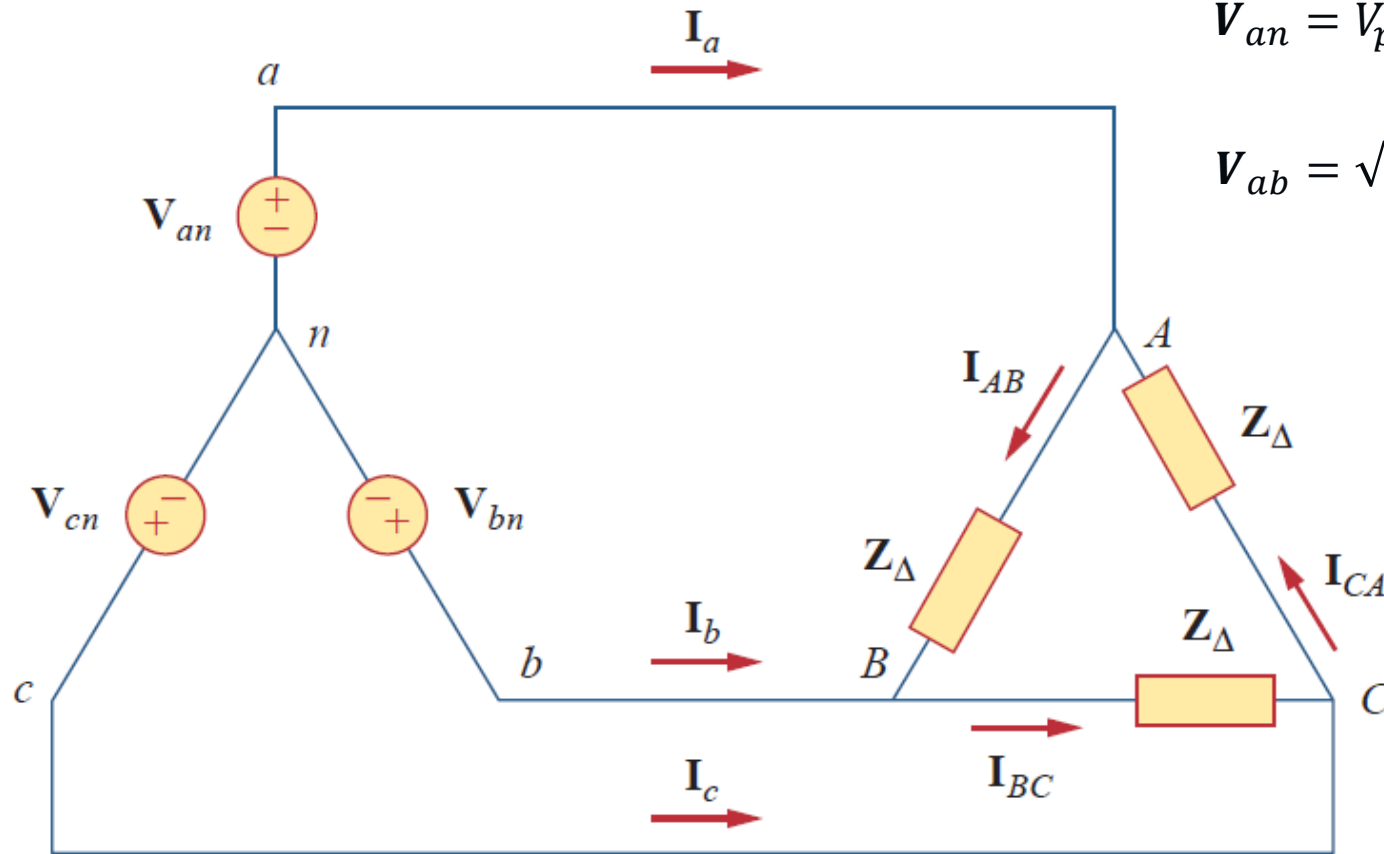
$$I_b = I_a e^{-j120^\circ}, \quad I_c = I_a e^{-j240^\circ}$$



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Balanced Wye-Delta Connection 1

The most practical and most commonly used three-phase system.



$$V_{an} = V_p e^{j0^\circ}, \quad V_{bn} = V_p e^{-j120^\circ}, \quad V_{cn} = V_p e^{j120^\circ}$$

$$V_{ab} = \sqrt{3}V_p e^{j30^\circ} = V_{AB}, \quad V_{bc} = \sqrt{3}V_p e^{-j90^\circ} = V_{BC}$$

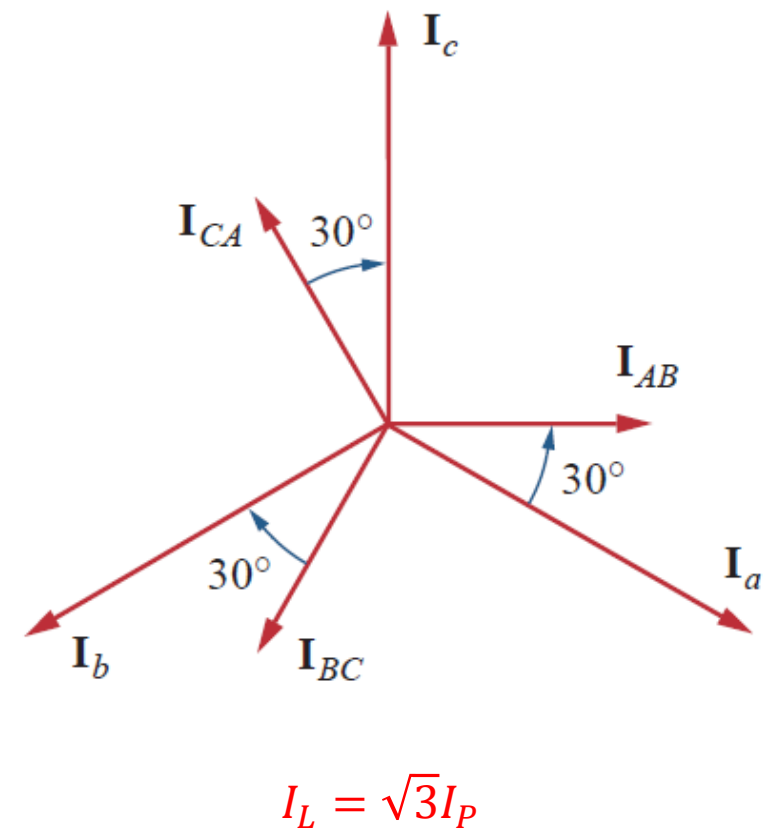
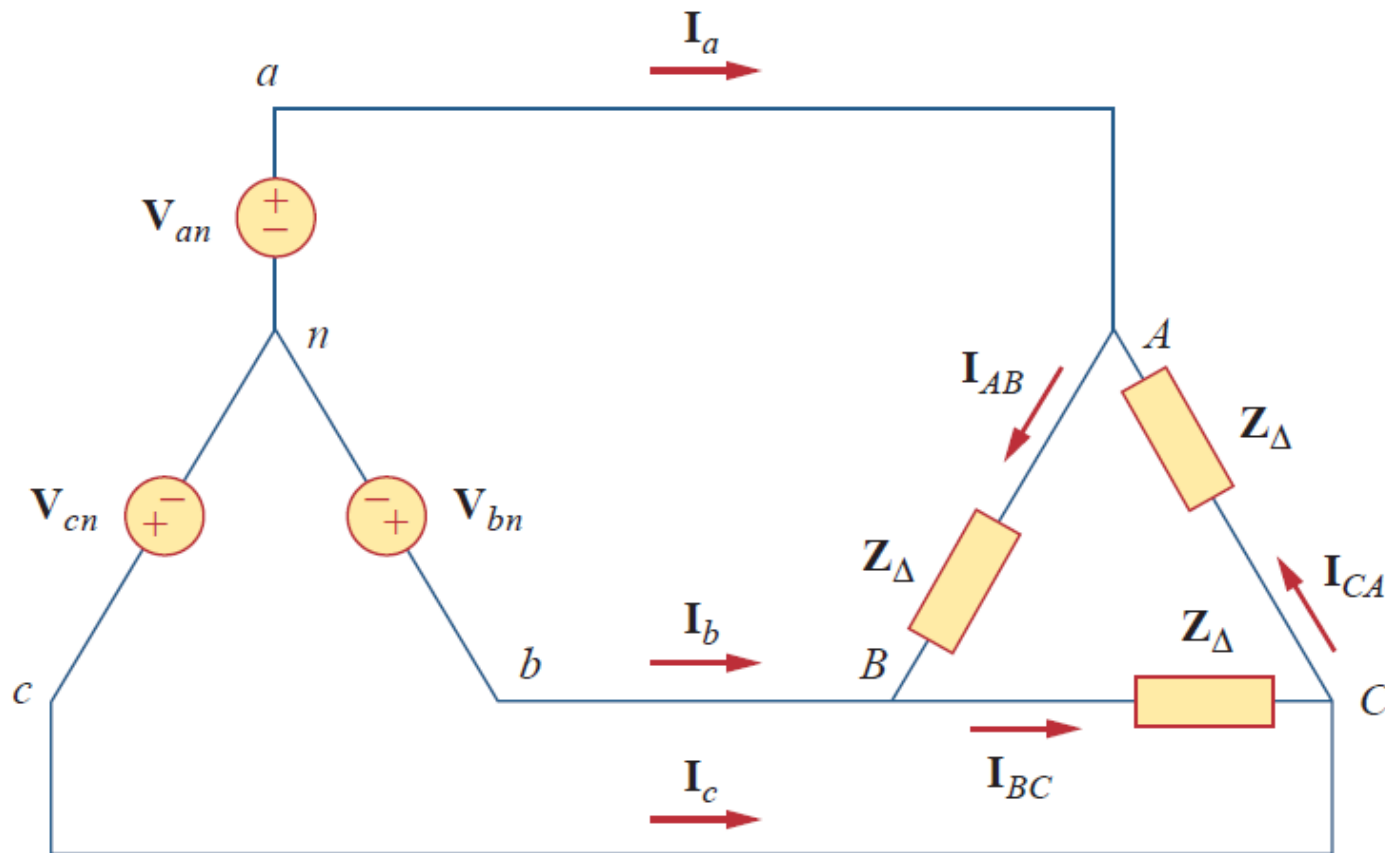
$$V_{ca} = \sqrt{3}V_p e^{j150^\circ} = V_{CA}$$

$$I_{AB} = \frac{V_{AB}}{Z_\Delta}, \quad I_{BC} = \frac{V_{BC}}{Z_\Delta}, \quad I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

Another way... $-V_{an} + Z_\Delta I_{AB} + V_{bn} = 0$

$$I_{AB} = \frac{V_{bn} - V_{an}}{Z_\Delta} = \frac{V_{ab}}{Z_\Delta} = \frac{V_{AB}}{Z_\Delta}$$

Balanced Wye-Delta Connection 2



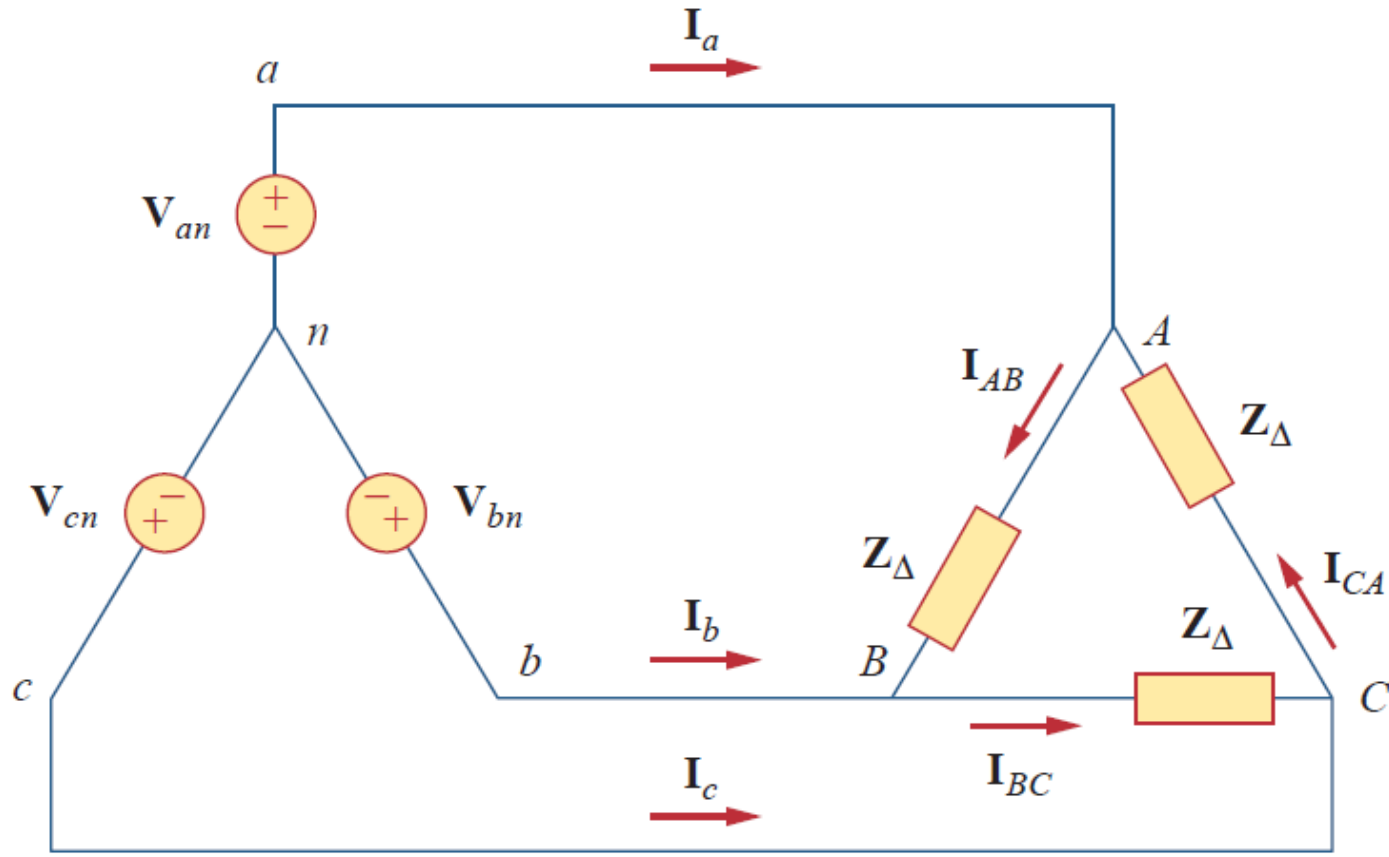
$$I_L = \sqrt{3} I_P$$

$$V_L = V_P$$

$$I_a = I_{AB} - I_{CA}, \quad I_b = I_{BC} - I_{AB}, \quad I_c = I_{CA} - I_{BC}$$

$$I_{CA} = I_{AB} e^{-j240^\circ} \rightarrow I_a = I_{AB} - I_{CA} = I_{AB} (1 - e^{-j240^\circ}) = I_{AB} (1 + 0.5 - j0.866) = I_{AB} \sqrt{3} e^{-j30^\circ}$$

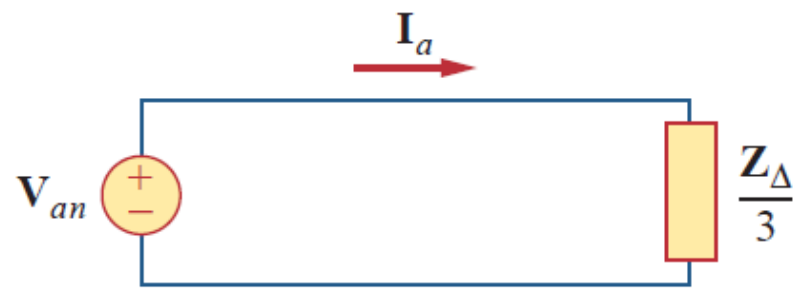
Balanced Wye-Delta Connection 3



□ Wye – delta equivalent transform

$$Z_Y = \frac{Z_{\Delta}}{3}$$

□ Single phase equivalent

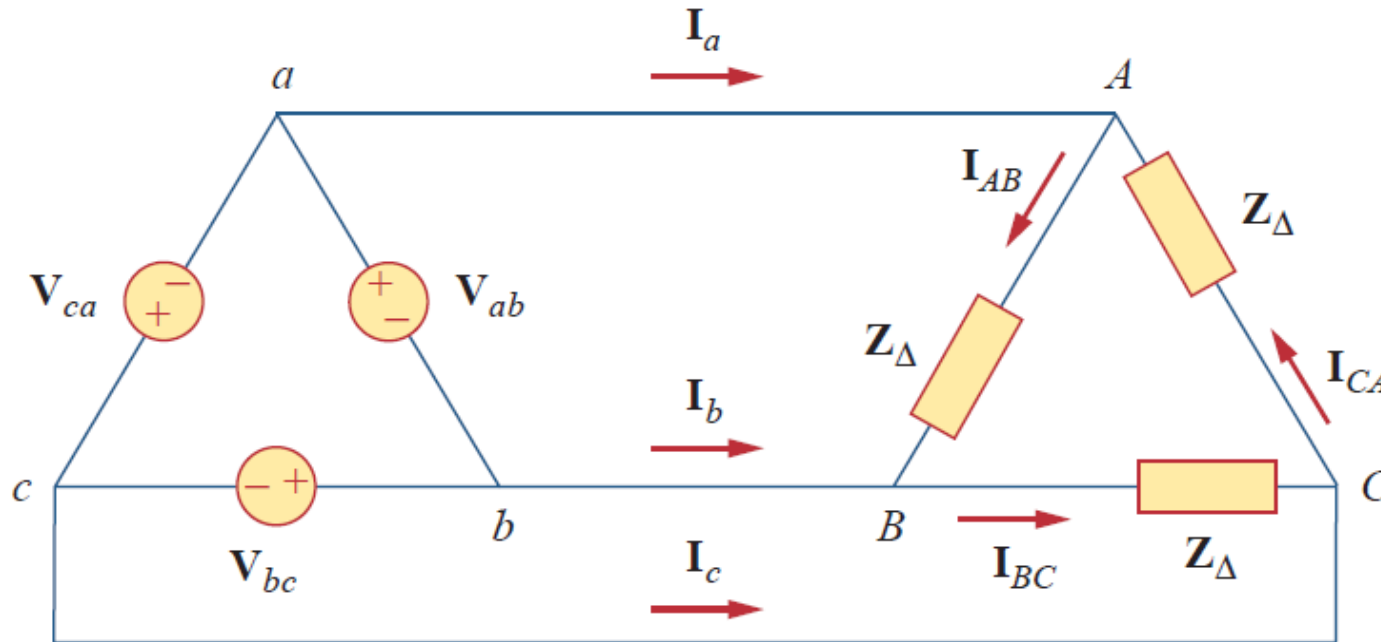


$$I_a = \frac{V_{an}}{Z_Y}$$



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Balanced Delta-Delta Connection



$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{V_{ab}}{Z_{\Delta}} \quad I_a = I_{AB} - I_{CA}$$

$$I_{BC} = \frac{V_{BC}}{Z_{\Delta}} = \frac{V_{bc}}{Z_{\Delta}} \quad I_b = I_{BC} - I_{AB}$$

$$I_{CA} = \frac{V_{CA}}{Z_{\Delta}} = \frac{V_{ca}}{Z_{\Delta}} \quad I_c = I_{CA} - I_{BC}$$

$$V_{ab} = V_p e^{j0^\circ}, \quad V_{bc} = V_p e^{-j120^\circ}, \quad V_{ca} = V_p e^{j120^\circ}$$

$$V_{ab} = V_{AB}, \quad V_{bc} = V_{BC}, \quad V_{ca} = V_{CA}$$

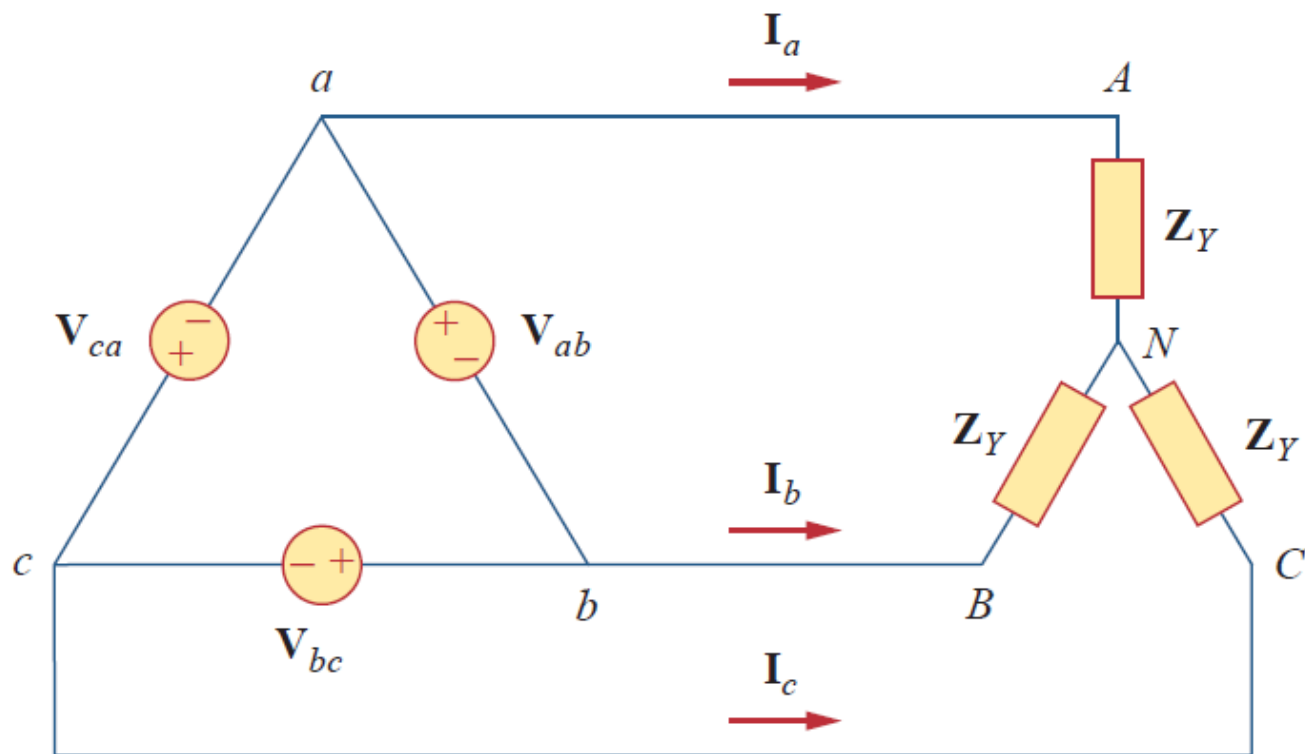
$$I_L = \sqrt{3} I_P$$

$$V_L = V_P$$



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Balanced Delta-Wye Connection 1



$$V_{ab} = V_p e^{j0^\circ}, \quad V_{bc} = V_p e^{-j120^\circ}, \quad V_{ca} = V_p e^{j120^\circ}$$

$$-V_{ab} + Z_Y I_a - Z_Y I_b = 0 \rightarrow Z_Y (I_a - I_b) = V_{ab} = V_p e^{j0^\circ}$$

$$I_a - I_b = \frac{V_p e^{j0^\circ}}{Z_Y}$$

$$I_b = I_a e^{-j120^\circ}$$

$$\rightarrow I_a - I_b = I_a (1 + 0.5 + j0.866)$$

$$= I_a \sqrt{3} e^{j30^\circ}$$

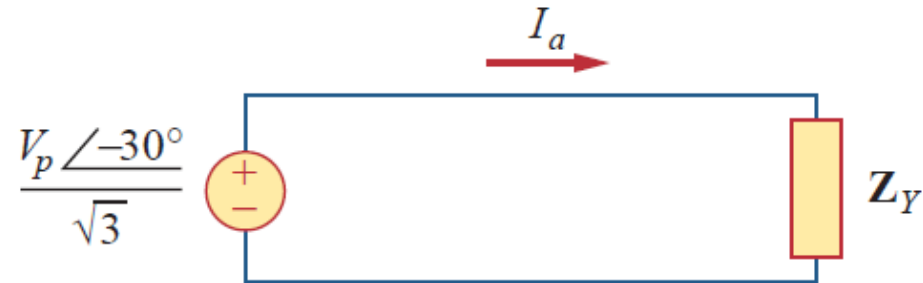
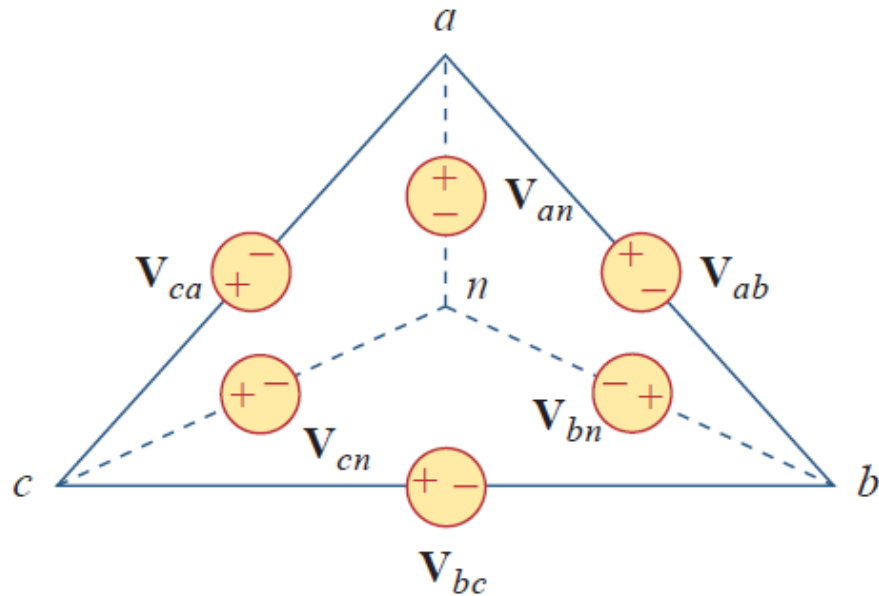
$$I_a = \frac{V_p / \sqrt{3} e^{-j30^\circ}}{Z_Y}$$

$$I_b = I_a e^{-j120^\circ}$$

$$I_c = I_a e^{j120^\circ}$$

Balanced Delta-Wye Connection 2

Delta – Wye – source transform $V_{an} = \frac{V_p}{\sqrt{3}} e^{-j30^\circ}$, $V_{bn} = \frac{V_p}{\sqrt{3}} e^{-j150^\circ}$, $V_{cn} = \frac{V_p}{\sqrt{3}} e^{j90^\circ}$



$$I_a = \frac{V_p / \sqrt{3} e^{-j30^\circ}}{Z_Y}$$

$$V_{BN} = V_{AN} e^{-j120^\circ}$$

$$V_{AN} = I_a Z_Y = \frac{V_p}{\sqrt{3}} e^{-j30^\circ}$$

$$V_{CN} = V_{AN} e^{j120^\circ}$$



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Power in a Balanced System 1

Instantaneous power for Y connected load

$$v_{AN} = V_p \sqrt{2} \cos \omega t, \quad v_{BN} = V_p \sqrt{2} \cos(\omega t - 120^\circ), \quad v_{CN} = V_p \sqrt{2} \cos(\omega t + 120^\circ)$$

$$i_a = I_p \sqrt{2} \cos(\omega t - \theta), \quad i_b = I_p \sqrt{2} \cos(\omega t - \theta - 120^\circ), \quad i_c = I_p \sqrt{2} \cos(\omega t - \theta + 120^\circ)$$

$$p = p_a + p_b + p_c = v_{AN} i_a + v_{BN} i_b + v_{CN} i_c =$$

$$= 2V_p I_p [\cos \omega t \cos(\omega t - \theta) + \cos(\omega t - 120^\circ) \cos(\omega t - \theta - 120^\circ) + \cos(\omega t + 120^\circ) \cos(\omega t - \theta + 120^\circ)]$$

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

$$p = V_p I_p [3 \cos \theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ) + \cos(2\omega t - \theta + 240^\circ)]$$

$$\cos(A \mp B) = \cos A \cos B \pm \sin A \sin B, \quad \cos(2\omega t - \theta) = \cos \alpha$$

$$p = V_p I_p [3 \cos \theta + \cos(\alpha) + \cos(\alpha - 240^\circ) + \cos(\alpha + 240^\circ)] = V_p I_p \left[3 \cos \theta + \cos(\alpha) + 2 \left(-\frac{1}{2} \right) \cos(\alpha) \right]$$

$$p = 3V_p I_p \cos \theta$$

Power in a Balanced System 2

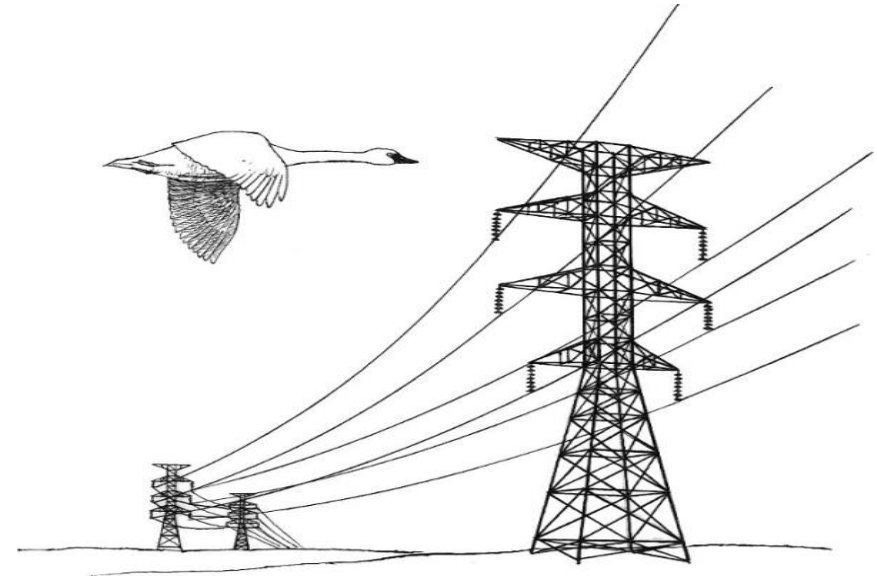
$$p = 3V_p I_p \cos \theta$$

$$\text{average PWR per phase} \rightarrow P_p = \frac{p}{3} = V_p I_p \cos \theta$$

$$\text{reactive PWR per phase} \rightarrow Q_p = V_p I_p \sin \theta$$

$$\text{apparent PWR per phase} \rightarrow S_p = V_p I_p$$

$$\text{complex PWR per phase} \rightarrow \mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^*$$



$$\text{total average PWR} \rightarrow P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos \theta = \sqrt{3}V_L I_L \cos \theta \leftarrow \text{next page}$$

$$\text{total reactive PWR} \rightarrow Q = Q_a + Q_b + Q_c = 3Q_p = 3V_p I_p \sin \theta = \sqrt{3}V_L I_L \sin \theta \leftarrow \text{next page}$$

$$\text{total complex PWR} \rightarrow \mathbf{S} = 3\mathbf{S}_p = 3\mathbf{V}_p \mathbf{I}_p^* = 3I_p^2 \mathbf{Z}_p = \frac{3V_p^2}{\mathbf{Z}_p^*} = P + jQ = \sqrt{3}V_L I_L e^{j\theta} \leftarrow \text{next page}$$

Note – Wye vs. Delta Connected Loads

Wye connected load

$$(V_L = \sqrt{3}V_P, I_L = I_P) \rightarrow S = 3V_P I_P = 3 \frac{V_L}{\sqrt{3}} I_L = \sqrt{3}V_L I_L$$

$$P = 3V_P I_P \cos \theta = 3 \frac{V_L}{\sqrt{3}} I_L \cos \theta = \sqrt{3}V_L I_L \cos \theta$$

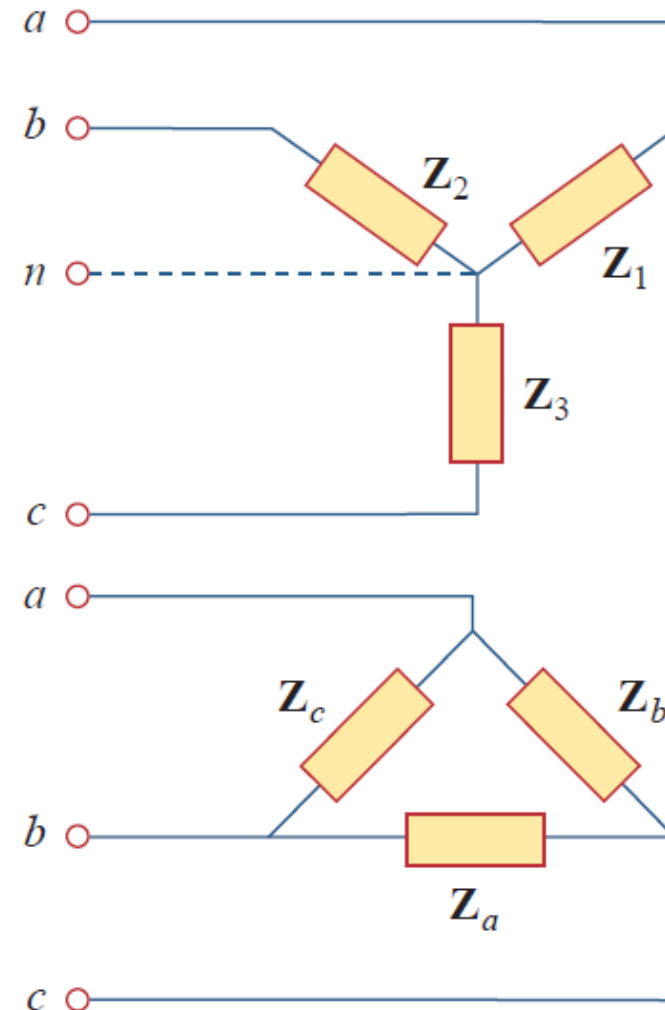
$$Q = 3V_P I_P \sin \theta = 3 \frac{V_L}{\sqrt{3}} I_L \sin \theta = \sqrt{3}V_L I_L \sin \theta$$

Delta connected load

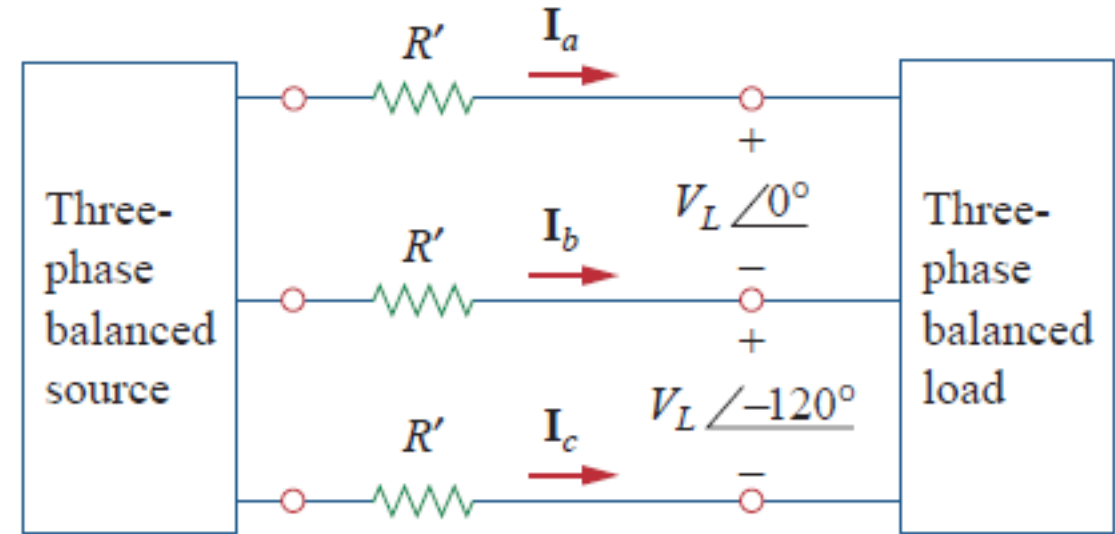
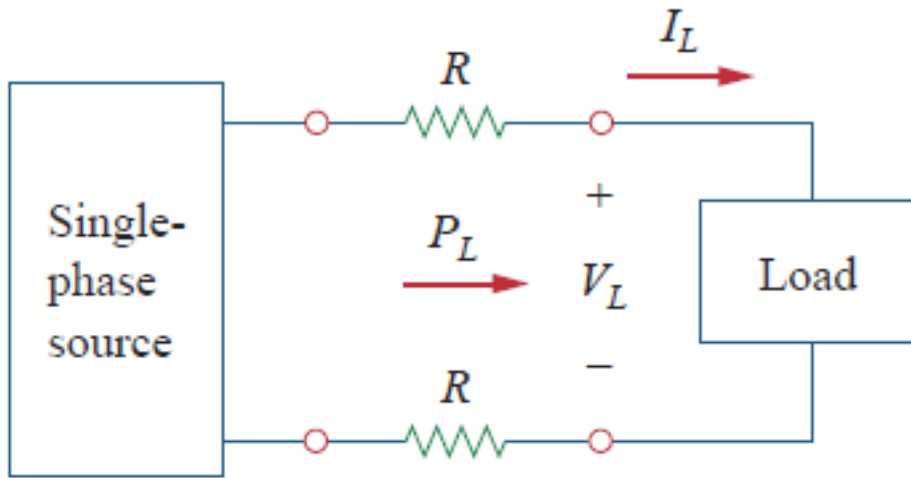
$$(V_L = V_P, I_L = \sqrt{3}I_P) \rightarrow S = 3V_P I_P = 3V_L \frac{I_L}{\sqrt{3}} = \sqrt{3}V_L I_L$$

$$P = 3V_P I_P \cos \theta = 3V_L \frac{I_L}{\sqrt{3}} \cos \theta = \sqrt{3}V_L I_L \cos \theta$$

$$Q = 3V_P I_P \sin \theta = 3V_L \frac{I_L}{\sqrt{3}} \sin \theta = \sqrt{3}V_L I_L \sin \theta$$



Power Loss in Single-Phase vs. Three-Phase Transmission Systems



$$I_L = \frac{P_L}{V_L} \rightarrow P_{loss} = 2I_L^2 R = 2R \frac{P_L^2}{V_L^2}$$

$$I'_L = I_a = I_b = I_c = \frac{P_L}{\sqrt{3}V_L} \rightarrow P'_{loss} = 3(I'_L)^2 R' = 3R' \frac{P_L^2}{3V_L^2} = R' \frac{P_L^2}{V_L^2}$$

$$\frac{P_{loss}}{P'_{loss}} = \frac{2R}{R'} = \frac{2 \frac{\rho l}{\pi r^2}}{\frac{\rho l}{\pi r'^2}} = \frac{2r'^2}{r^2} \rightarrow \text{same PWR loss} \rightarrow 2r'^2 = r^2$$

$$\frac{\text{material for single - phase}}{\text{material for three - phase}} = \frac{2(r^2 \pi l)}{3(r'^2 \pi l)} = \frac{2r^2}{3r'^2} = \frac{2}{3} \cdot 2 = 1.33 \rightarrow \text{33\% more material for single-phase transmission!}$$

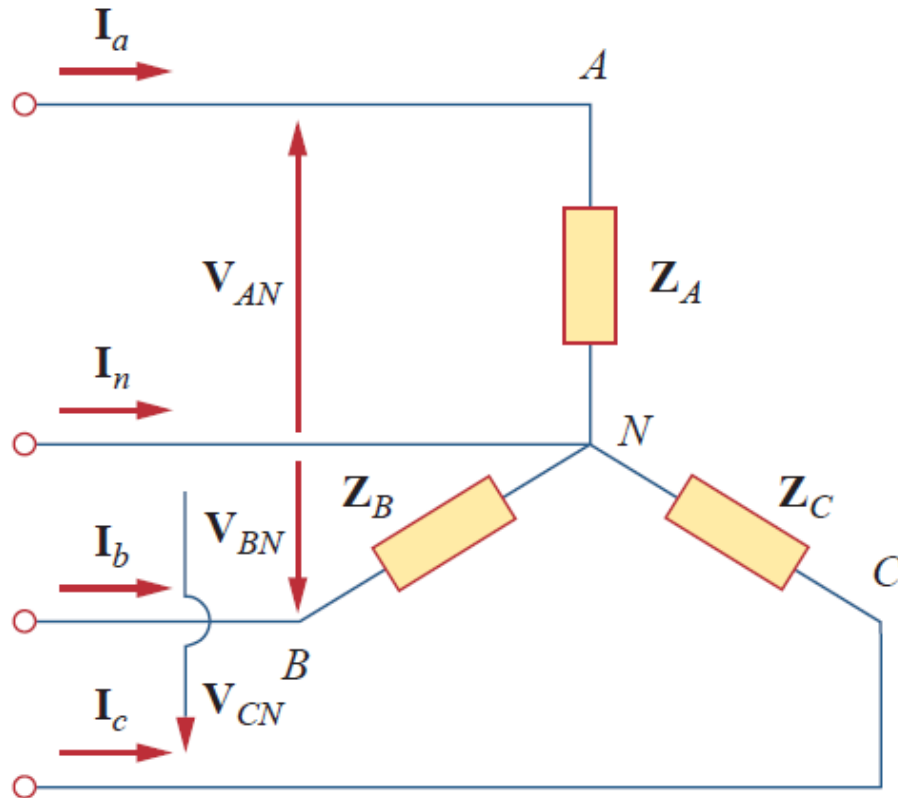


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Unbalanced Three-Phase Systems

Unbalanced system is due to

- (unbalanced voltage sources)
- unbalanced load.



$$I_a = \frac{V_{AN}}{Z_A}, \quad I_b = \frac{V_{BN}}{Z_B}, \quad I_c = \frac{V_{CN}}{Z_C}$$

$$I_n = -(I_a + I_b + I_c)$$

Three-wire system $\rightarrow I_a + I_b + I_c = 0$

The same could be used (accordingly) for...

- wye-connected source; wye-connected load
- delta-connected source; wye-connected load
- wye-connected source; delta-connected load
- delta-connected source; delta-connected load

Influence of Non-Ideal Cables 1

KVL + KCL + Characteristics...

$$Z_A = Z'_A + Z_l + Z_S$$

$$Z_B = Z'_B + Z_l + Z_S$$

$$Z_C = Z'_C + Z_l + Z_S$$

$$I_n = V_{Nn} \cdot Y_n$$

$$I_a = V_{AN} \cdot Y_A$$

$$I_b = V_{BN} \cdot Y_B$$

$$I_c = V_{CN} \cdot Y_C$$

$$V_{an} - V_{AN} + V_{Nn} = 0$$

$$V_{bn} - V_{BN} + V_{Nn} = 0$$

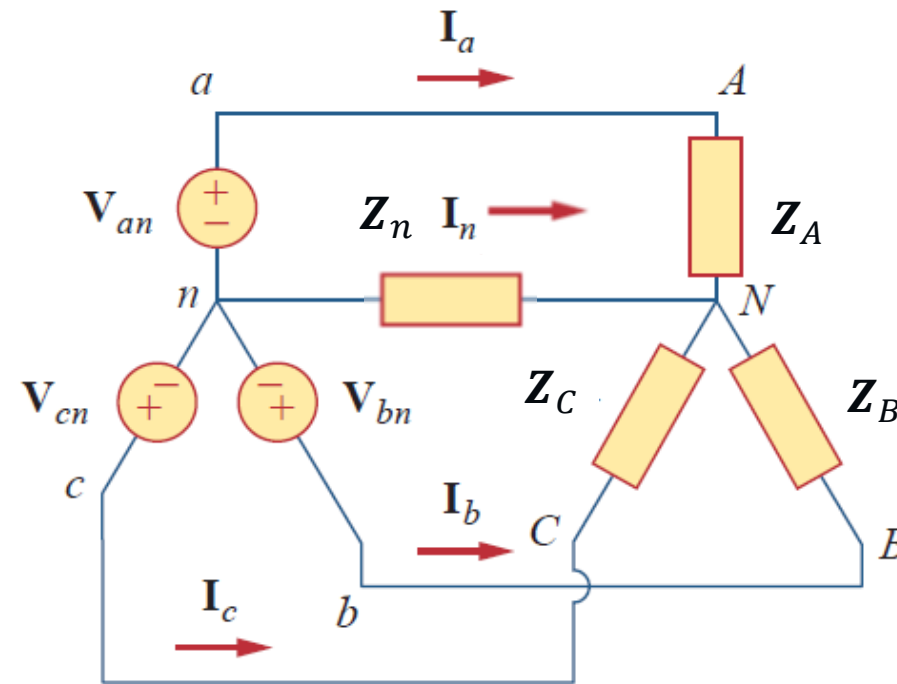
$$V_{cn} - V_{CN} + V_{Nn} = 0$$

$$I_n = -(I_a + I_b + I_c)$$

$$I_a = (V_{an} + V_{Nn}) \cdot Y_A$$

$$I_b = (V_{bn} + V_{Nn}) \cdot Y_B$$

$$I_c = (V_{cn} + V_{Nn}) \cdot Y_C$$



(Millman's Theorem)

$$V_{Nn} = \frac{V_{an} \cdot Y_A + V_{bn} \cdot Y_B + V_{cn} \cdot Y_C}{Y_A + Y_B + Y_C + Y_n}$$

Influence of Non-Ideal Cables 2

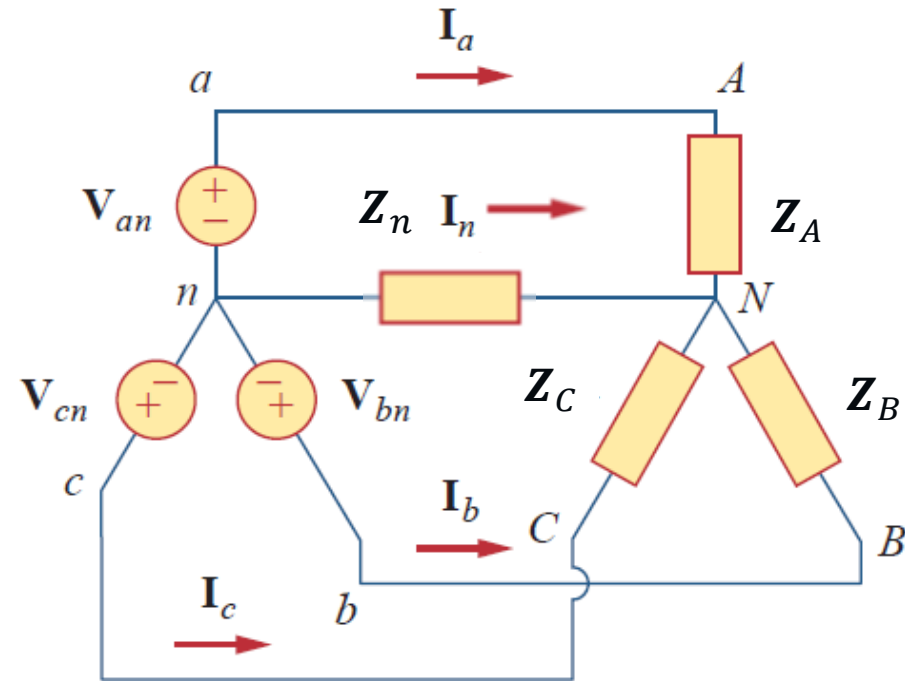
$$V_{Nn} = \frac{V_{an} \cdot Y_A + V_{bn} \cdot Y_B + V_{cn} \cdot Y_C}{Y_A + Y_B + Y_C + Y_n}$$

□ In case of balanced load

$$Y_A = Y_B = Y_C \rightarrow V_{Nn} = \frac{Y_A \cdot (V_{an} + V_{bn} + V_{cn})}{3 \cdot Y_A + Y_n} = 0$$

□ In case of unbalanced load + ideal neutral wire

$$Y_N = \infty \rightarrow V_{Nn} = 0, \quad I_n > 0$$



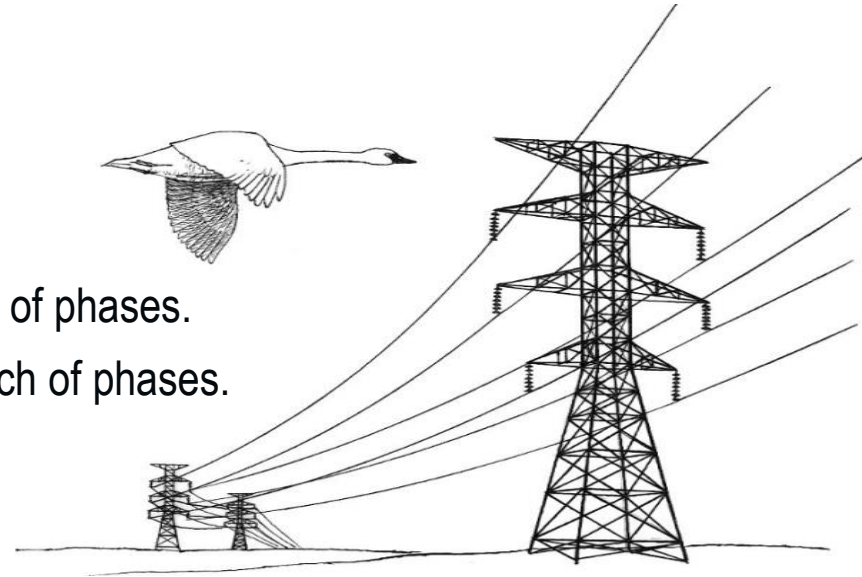


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Power in Unbalanced Systems

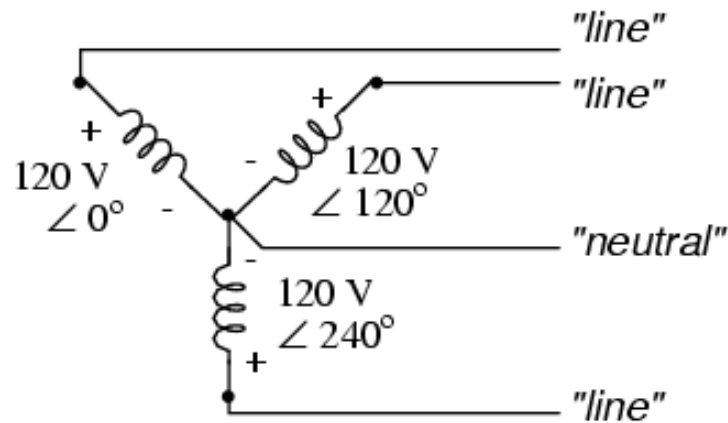


- ❑ The total PWR is not simply three times the PWR in any of phases.
- ❑ The total PWR is the sum of the unbalanced PWR in each of phases.
- ❑ Reactive PWR has sign (CAP \rightarrow (-), IND \rightarrow (+))
- ❑ (Refer to Tellegen's theorem)



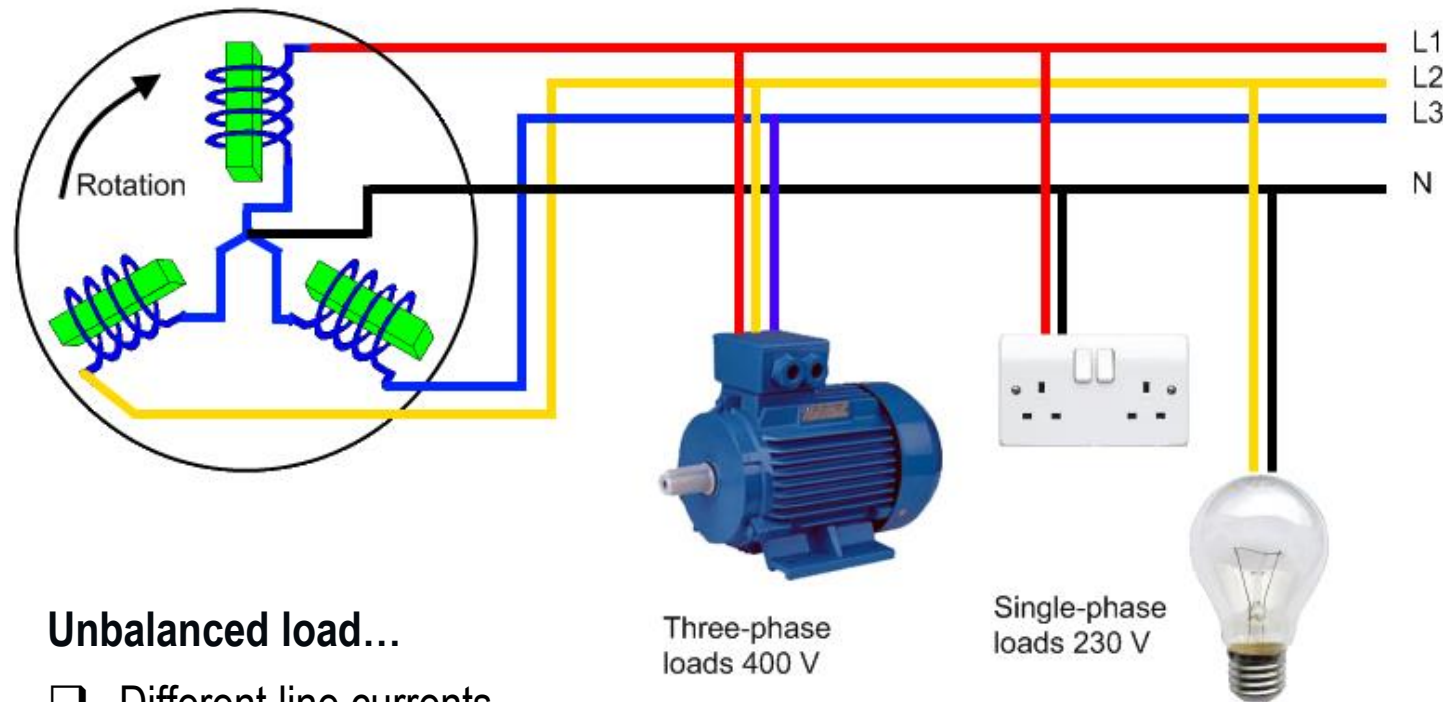
- ❑ Total average PWR $\rightarrow P = P_a + P_b + P_c$, $P \neq 3P_p, P \neq 3V_p I_p \cos \theta, P \neq \sqrt{3}V_L I_L \cos \theta$
- ❑ Total reactive PWR $\rightarrow Q = Q_a + Q_b + Q_c$, $Q \neq 3Q_p, Q \neq 3V_p I_p \sin \theta, Q \neq \sqrt{3}V_L I_L \sin \theta$
- ❑ Total complex PWR $\rightarrow \mathbf{S} = \mathbf{S}_a + \mathbf{S}_b + \mathbf{S}_c = P + jQ$, $\mathbf{S} \neq 3\mathbf{S}_p, \mathbf{S} \neq 3V_p \mathbf{I}_p^*, \mathbf{S} \neq \sqrt{3}V_L I_L e^{j\theta}$

Balanced and Unbalanced Loads (Summary)



Balanced load...

- Voltages are 120° out of phase with each other
- Wye connection
 - $V_L = \sqrt{3} V_P$ $I_L = I_P$
 - Common (neutral) nodes \rightarrow same potential
 - No neutral current (3 wire/4 wire systems)
- Delta connection
 - $V_L = V_P$ $I_L = \sqrt{3} I_P$



Unbalanced load...

- Different line currents
- Wye connection
 - Common (neutral) nodes \rightarrow different potential
 - Neutral current occurs (4 wire system)
- Total power \rightarrow sum of different powers in phases

