

DR. GYURCSEK ISTVÁN

# Resonance Circuits

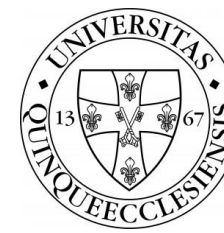
## *Sources and additional materials (recommended)*

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- ❑ *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*
- ❑ *<http://www.electronics-tutorials.ws/accircuits>*
- ❑ *<http://hyperphysics.phy-astr.gsu.edu/hbase/electric/serres.html>*
- ❑ *[https://en.wikipedia.org/wiki/RLC\\_circuit#Damping](https://en.wikipedia.org/wiki/RLC_circuit#Damping)*



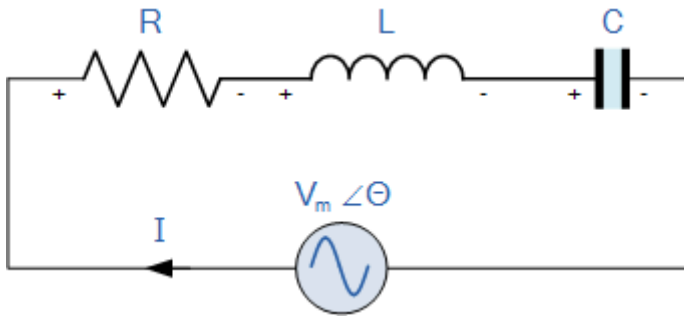
- Series Resonance Circuits**
- Parallel Resonance Circuits
- Free Resonance

# Series Resonance Circuit



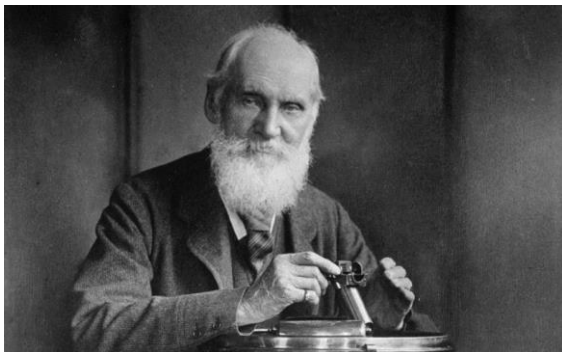
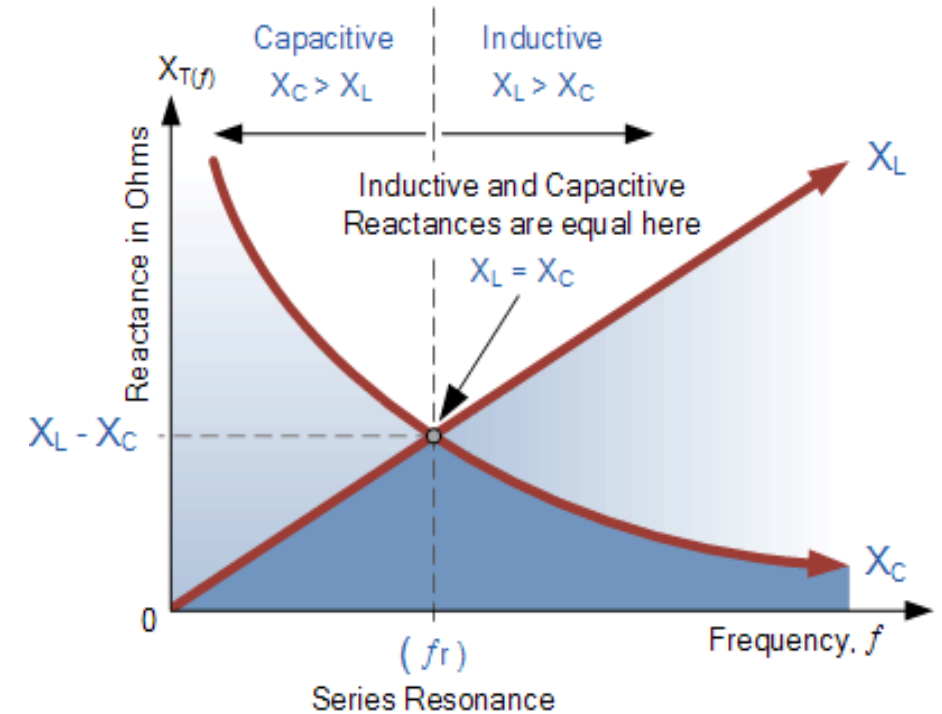
$$Z_{eq} = R + j\omega L - j\frac{1}{\omega C}$$

$$\text{Resonance} \rightarrow \text{Im}\{Z_{eq}\} = 0$$



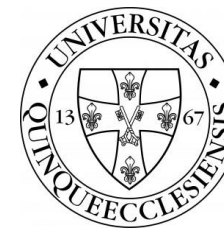
$$X_L = X_C \rightarrow \omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r = \frac{1}{\sqrt{LC}} \text{ or } f_r = \frac{1}{2\pi\sqrt{LC}}$$

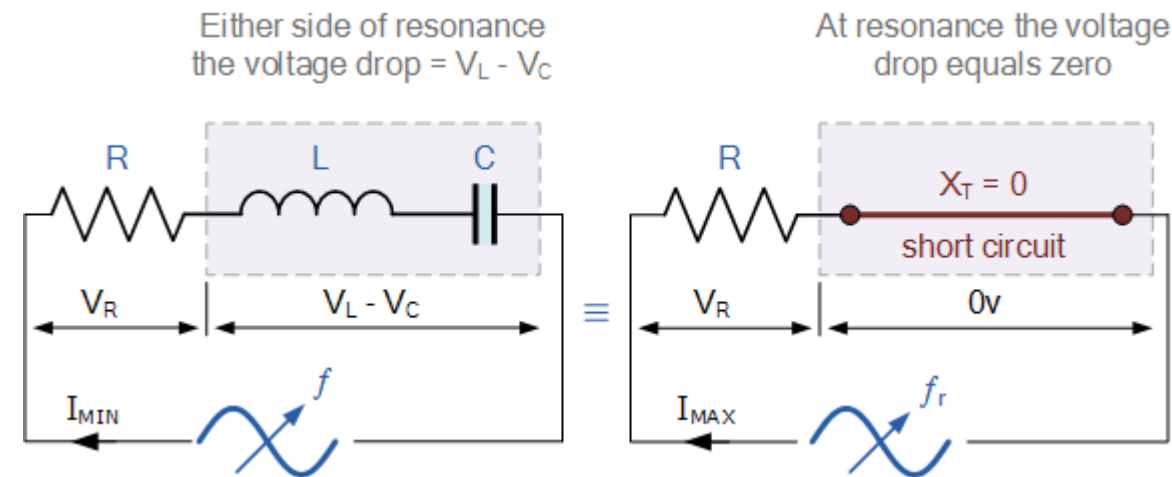
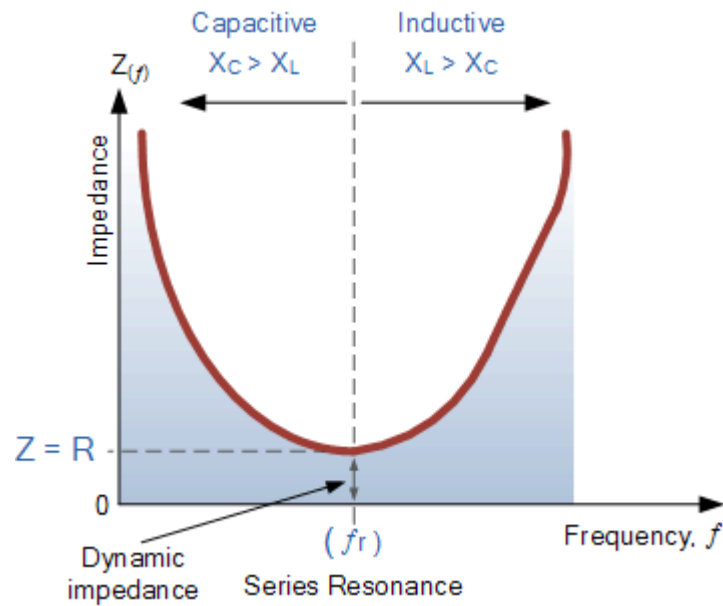


William Thomson  
(Lord Kelvin - 1824-1907)

# Series RLC Circuit at Resonance



$$\mathbf{Z} = R + jX_L - jX_C = R + j(X_L - X_C) \rightarrow Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

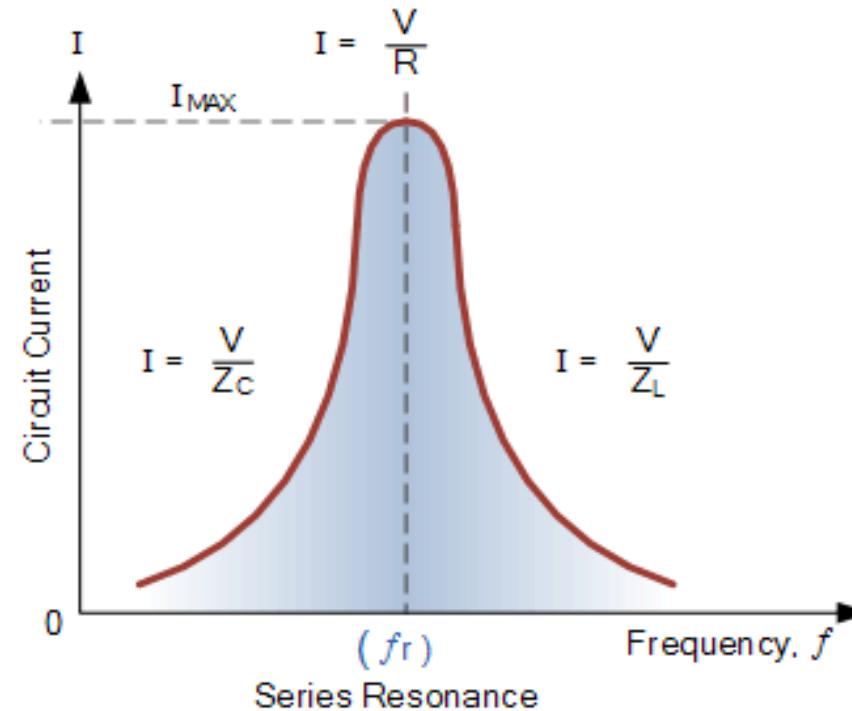


# Series RLC Circuit at Resonance

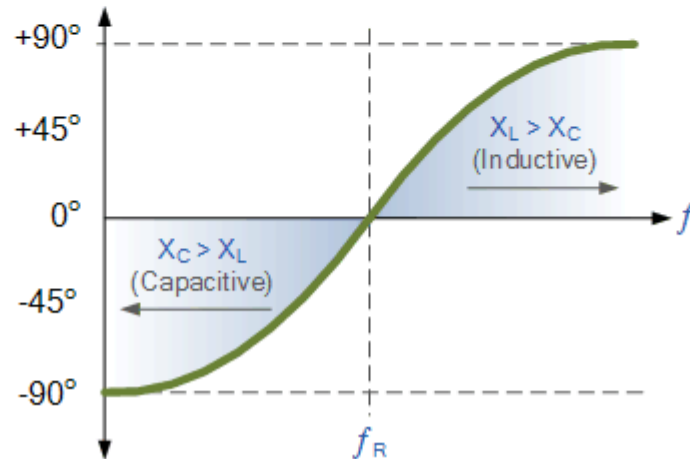
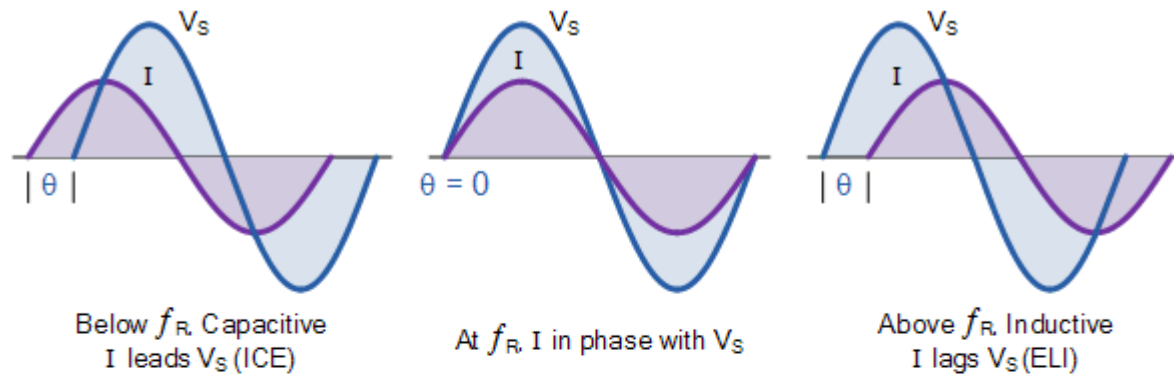
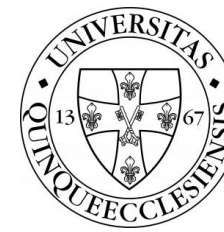


$$I = \frac{V}{Z} = \frac{V}{R + j(X_L - X_C)}$$

$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



# Phase of Series Resonance Circuit



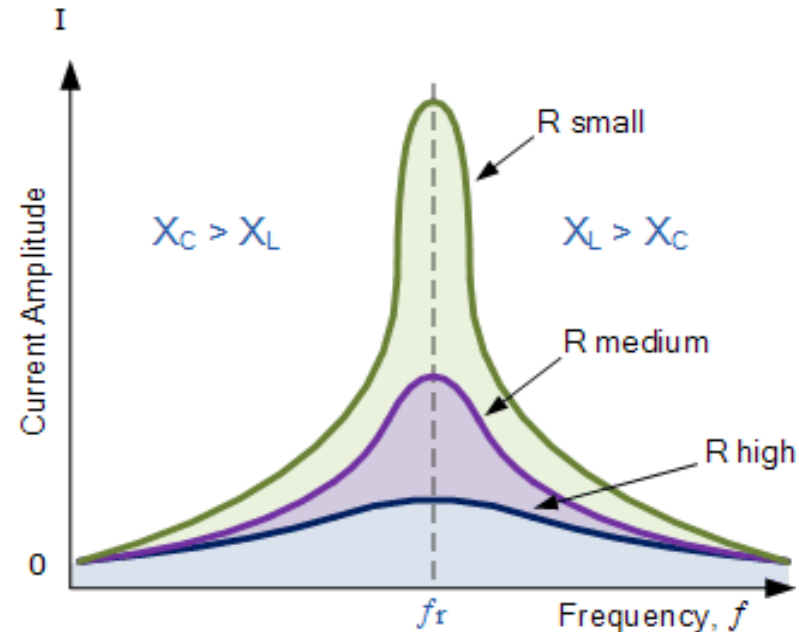
$$Z = R + j(X_L - X_C)$$

$$\rightarrow Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\rightarrow \varphi_Z = \tan^{-1} \frac{X_L - X_C}{R} \text{ (can be } 0^\circ \text{ !)}$$

# Q-factor and Wave Impedance at Series Resonance

$$Q \triangleq 2\pi \cdot \frac{\text{'peak energy stored in the circuit'}}{\text{'dissipated energy in a period at resonance'}} = 2\pi \frac{\frac{1}{2} I_p^2 L}{\frac{1}{2} I_p^2 R T} = \frac{\omega_r L}{R} = \frac{X_r}{R} = \frac{1}{\omega_r R C}$$



$$Q = \frac{\omega L}{R} \rightarrow \omega = \frac{RQ}{L}$$

$$Q = \frac{1}{\omega C R} \rightarrow \omega = \frac{1}{Q C R}$$

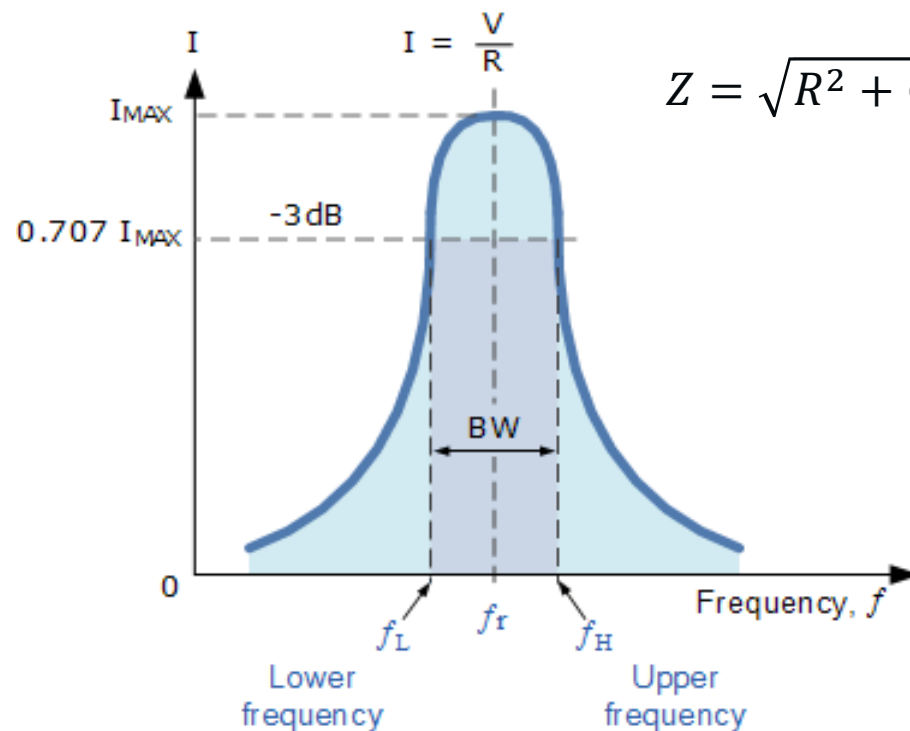
$$\frac{RQ}{L} = \frac{1}{Q C R} \rightarrow Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{Z_0}{R}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

# Bandwidth at Series Resonance



$$BW = f_H - f_L \quad \omega_r = \frac{1}{\sqrt{LC}} \rightarrow Z = \min; I_S = \max \quad \frac{P_{max}}{2} \rightarrow \frac{I_{max}}{\sqrt{2}} \rightarrow Z = R \cdot \sqrt{2}$$



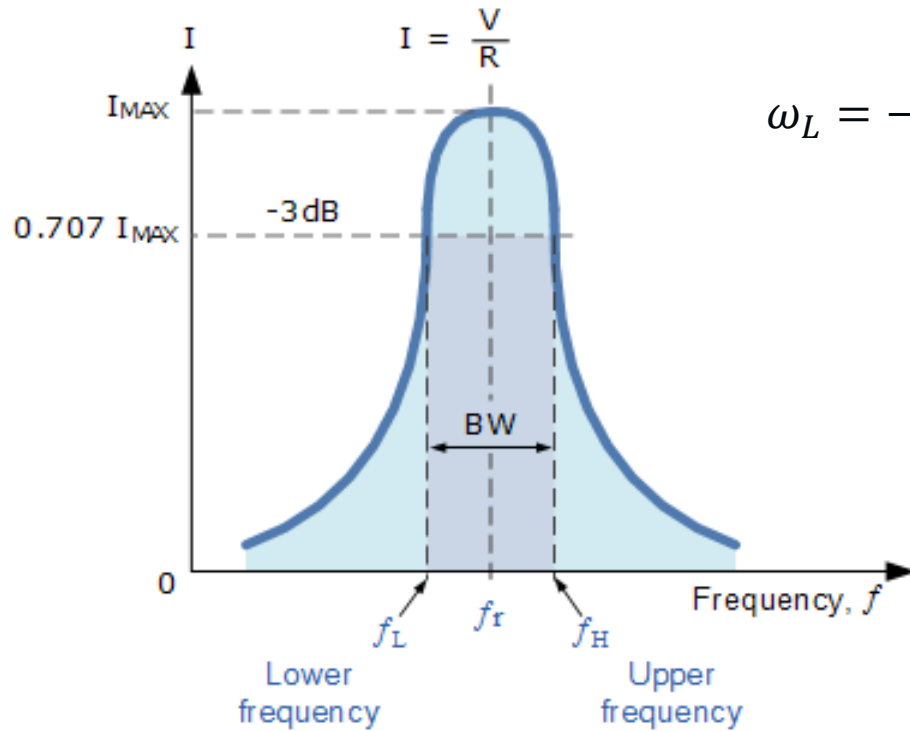
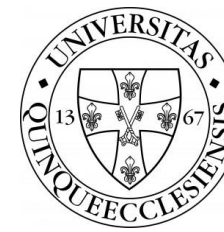
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R \cdot \sqrt{2} \rightarrow \begin{cases} X_L - X_C = R \\ X_C - X_L = R \end{cases}$$

$$\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_H = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$



# BW and Q-factor at Series Resonance



$$\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \quad \omega_H = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\left. \begin{aligned} (a+b)(a-b) &= a^2 - b^2 \\ \omega_r &= \frac{1}{\sqrt{LC}} \end{aligned} \right\} \rightarrow \omega_r = \sqrt{\omega_L \omega_H}$$

$$Q = \frac{\omega_r L}{R} \rightarrow \frac{R}{L} = \frac{\omega_r}{Q} \rightarrow BW = f_H - f_L = \frac{1}{2\pi} \frac{R}{L} = \frac{f_r}{Q}$$

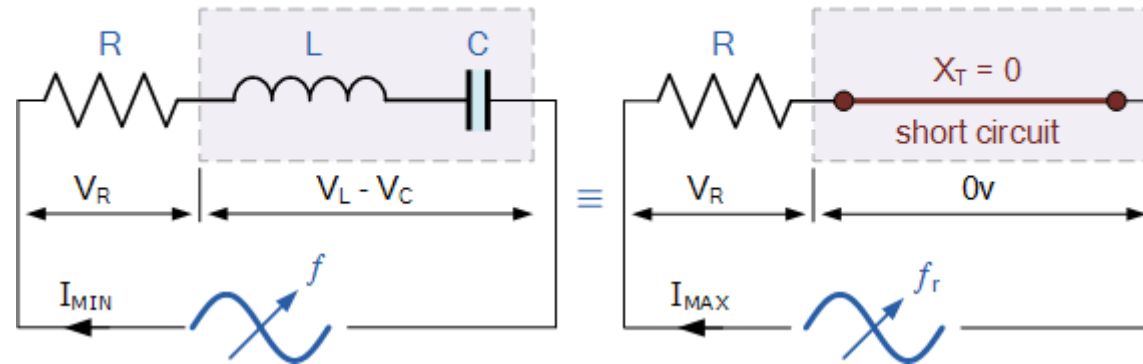
# Voltage Resonance



Series resonance  $\rightarrow$  voltage resonance

$$|V_L| = |V_C| = QV_S$$

the voltage drop =  $V_L - V_C$



At resonance the voltage drop equals zero

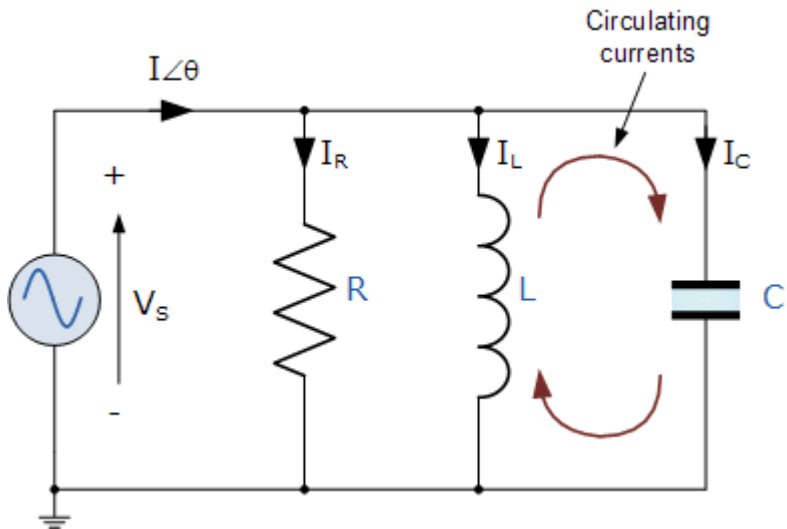
$$V_L = IX_L = \frac{V_S}{R} X_L = \frac{\omega L}{R} V_S = QV_S$$

$$V_C = IX_C = \frac{V_S}{R} X_C = \frac{1}{\omega CR} V_S = QV_S$$



- Series Resonance Circuits
- Parallel Resonance Circuits**
- Free Resonance

# Parallel Resonance Circuit



*Resonance*  $\rightarrow \text{Im}\{Y_{eq}\} = 0$

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$Y = G - jB_L + jB_C$$

Admittance,  $Y = \frac{1}{Z} = \sqrt{G^2 + B^2}$

Conductance,  $G = \frac{1}{R}$

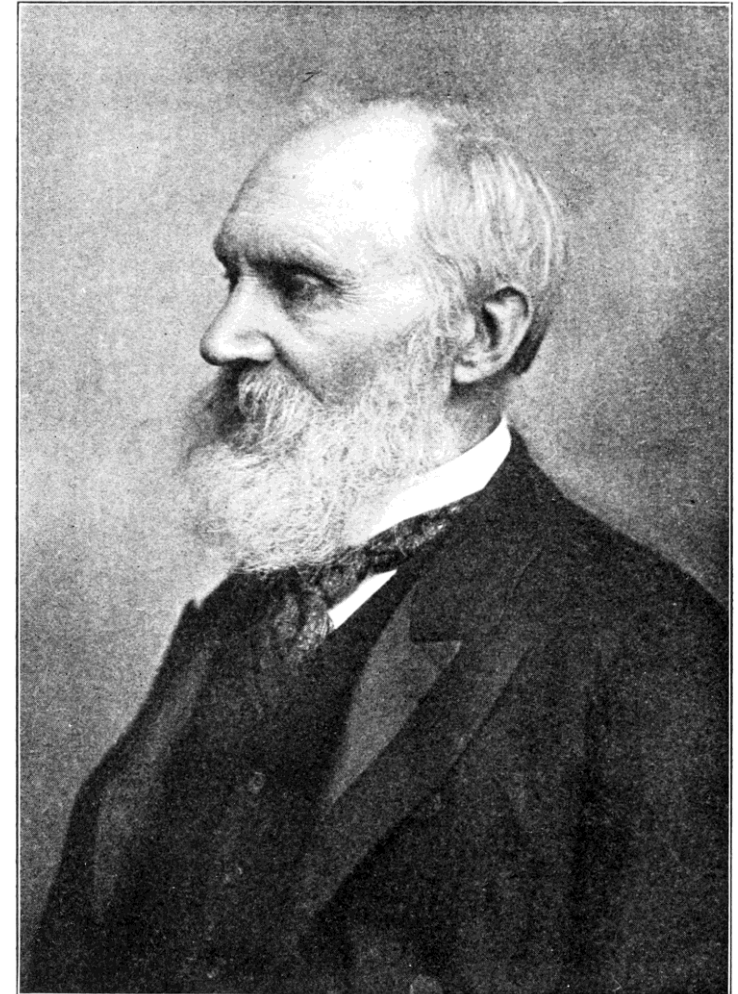
Inductive Susceptance,  $B_L = \frac{1}{2\pi fL}$

Capacitive Susceptance,  $B_C = 2\pi fC$

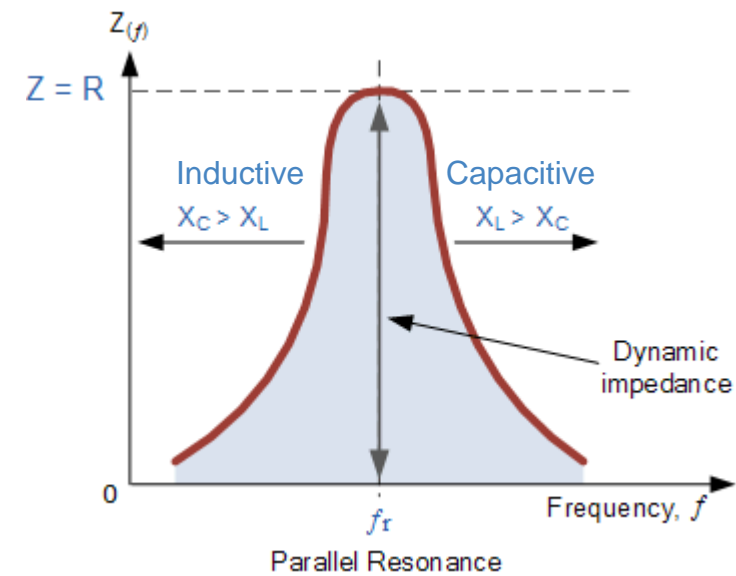
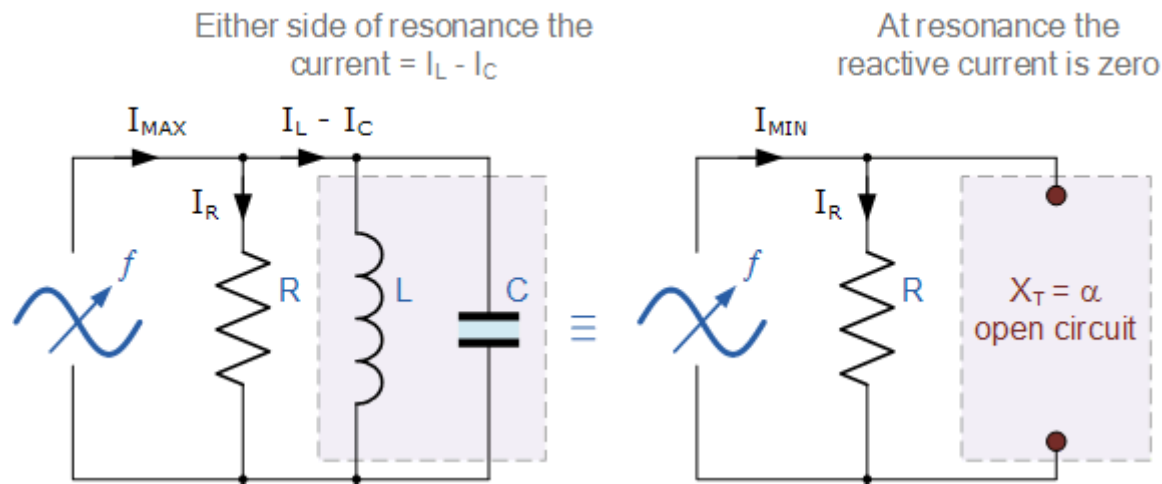
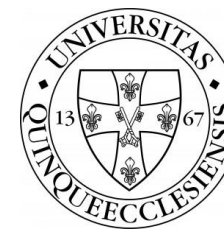
Lord William Thomson Kelvin  
(1824-1907)

$$X_L = X_C \rightarrow \omega_r L = \frac{1}{\omega_r C}$$

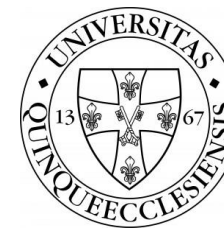
$$\omega_r = \frac{1}{\sqrt{LC}} \text{ or } f_r = \frac{1}{2\pi\sqrt{LC}}$$



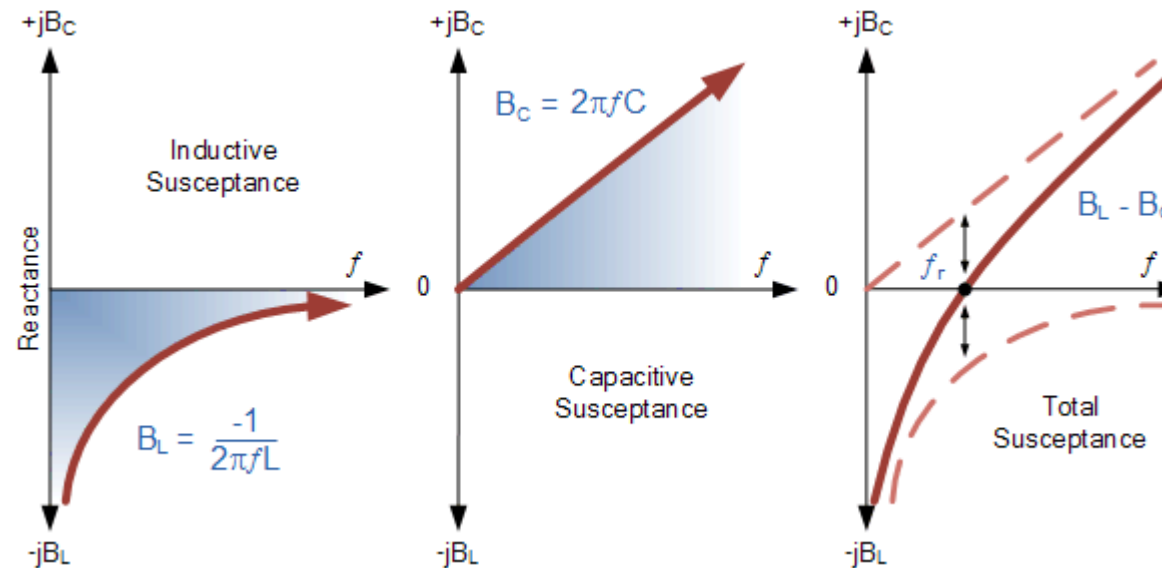
# Impedance at Parallel Resonance



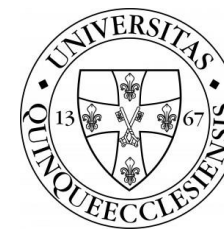
# Susceptance at Parallel Resonance



$$Y = G + jB_C - jB_L = G + j(B_C - B_L) \rightarrow Y = \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$



# Current in a Parallel Resonance Circuit



$$I_R = \frac{V}{R}$$

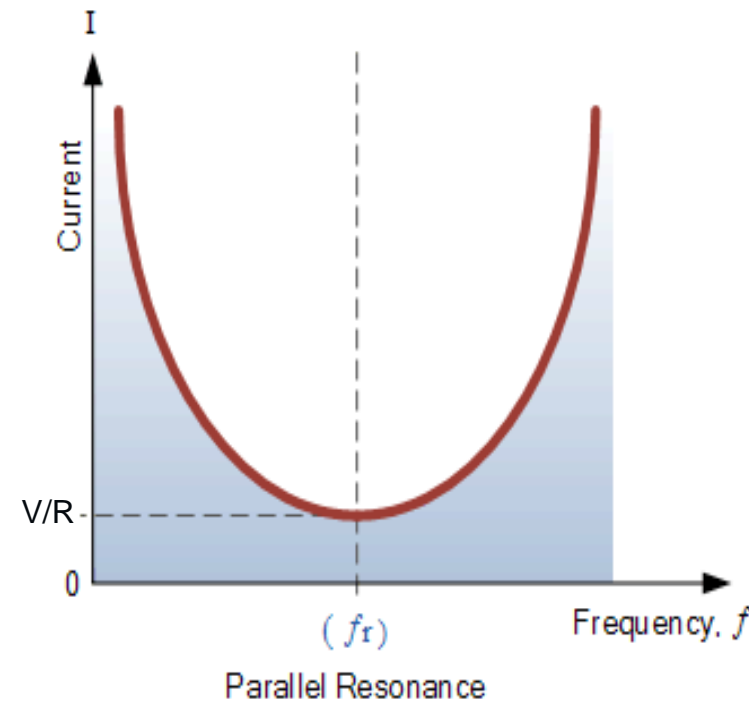
$$I_L = \frac{V}{X_L} = \frac{V}{\omega L} = \frac{V}{2\pi f L}$$

$$I_C = \frac{V}{X_C} = V 2\pi f C$$

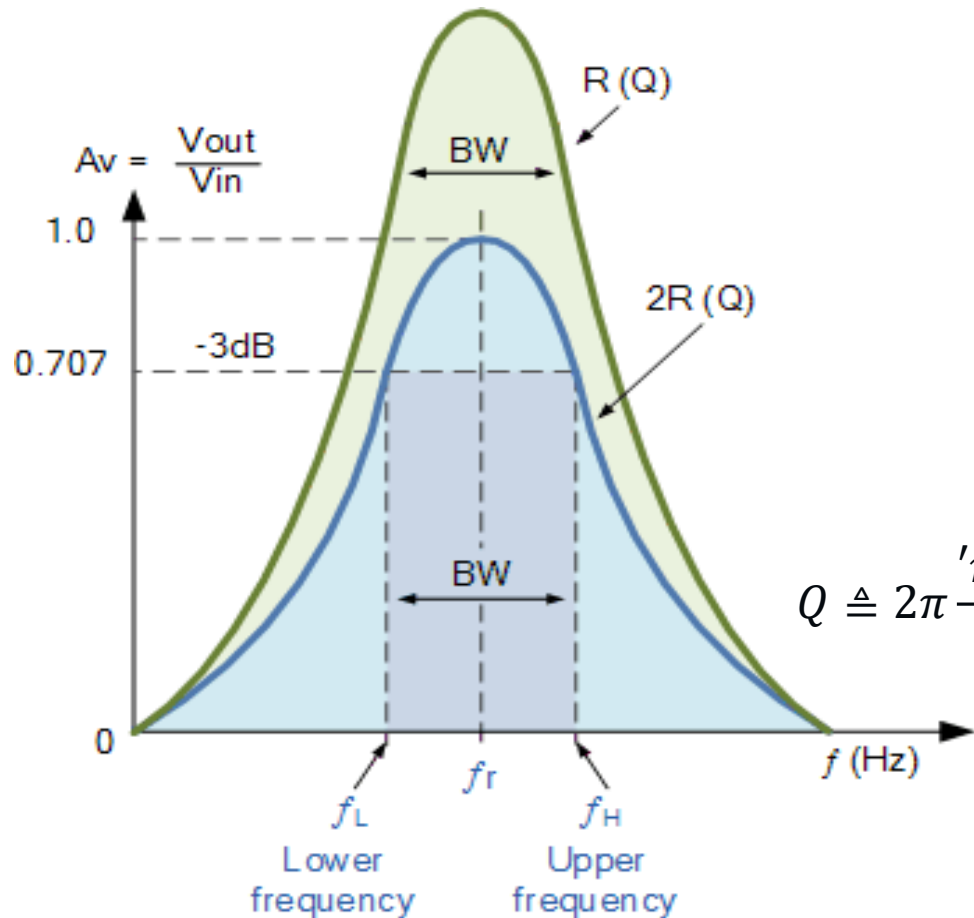
$$\bar{I}_T = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

$$I_T = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$I_T = \sqrt{I_R^2 + (0)^2} = I_R$$



# Bandwidth & Selectivity (Q-factor) of a Parallel Resonance Circuit



$$|B_C - B_L| = G \rightarrow \begin{cases} \omega_L = -\frac{G}{2C} + \sqrt{\left(\frac{G}{2C}\right)^2 + \frac{1}{LC}} \\ \omega_H = +\frac{G}{2C} + \sqrt{\left(\frac{G}{2C}\right)^2 + \frac{1}{LC}} \end{cases}$$

$$Q \triangleq 2\pi \frac{\text{'max. energy stored'}}{\text{'power loss'}} = \frac{V^2 B_C}{V^2 G} = \frac{\omega_r C}{G} = \frac{1}{\omega_r L G} = \frac{1}{G} \sqrt{\frac{C}{L}} = \frac{Y_0}{G}$$

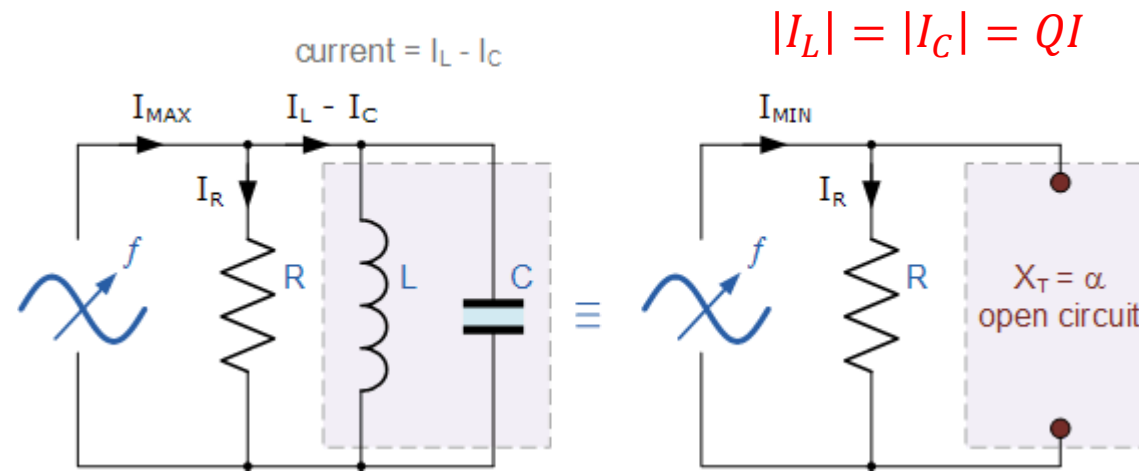
$$Q = \frac{\omega_r C}{G} \rightarrow \frac{G}{C} = \frac{\omega_r}{Q} \rightarrow BW = f_H - f_L = \frac{1}{2\pi C} \frac{G}{Q} = \frac{f_r}{Q}$$



# Current Resonance



Parallel resonance → current resonance



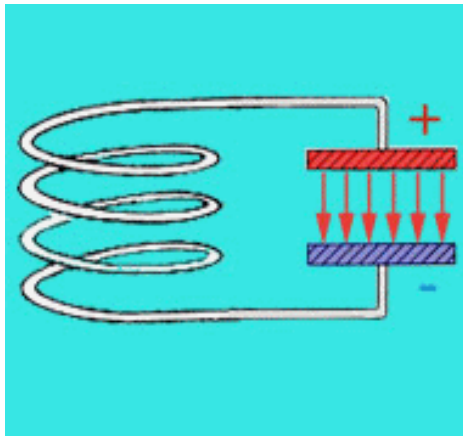
$$I_L = \frac{V_S}{X_L} = \frac{IR}{X_L} = \frac{R}{\omega L} I = QI$$

$$I_C = \frac{V_S}{X_C} = \frac{IR}{X_C} = \omega C R I = QI$$



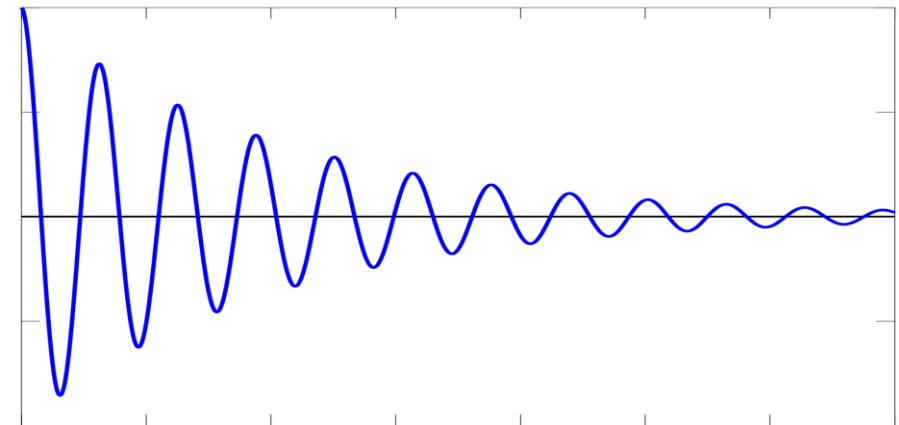
- Series Resonance Circuits
- Parallel Resonance Circuits
- Free Resonance**

# Free resonance (LC, RLC)



$$W_L = \frac{1}{2} L \cdot I^2, \quad W_C = \frac{1}{2} C \cdot V^2$$

$$\frac{1}{2} L \cdot I^2 = \frac{1}{2} C \cdot V^2 \rightarrow \frac{V}{I} = \sqrt{\frac{L}{C}} = Z_0$$



$$i(t) = I_0 e^{-\delta t} \sin \omega t \quad (\delta: \text{damping factor})$$

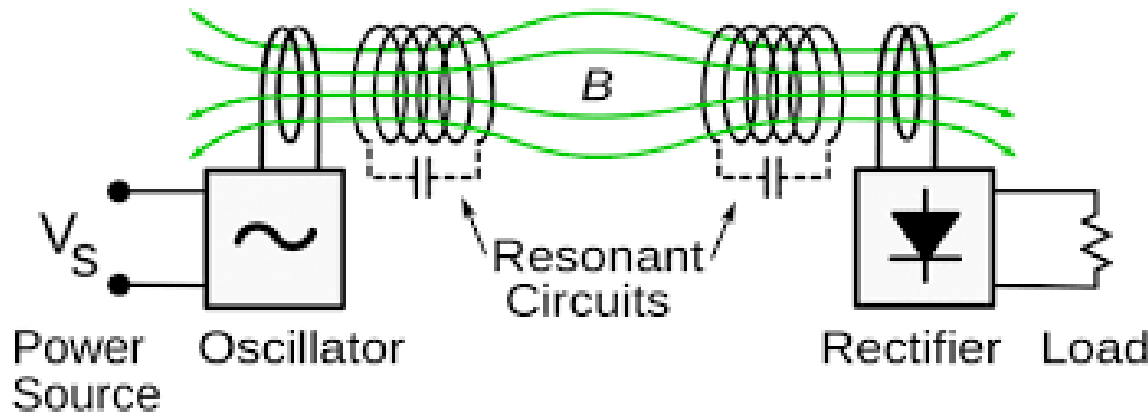
$$\omega = \sqrt{\omega_0^2 - \delta^2}, \quad \omega_0 = \frac{1}{\sqrt{LC}}, \quad \delta = \frac{R}{2L}$$

undamped (free oscillation)  $\rightarrow \delta = 0$

underdamped (slowly dampening osc.)  $\rightarrow \delta < \omega_0$

critical damping (no oscillation)  $\rightarrow \delta = \omega_0$

overdamped (no oscillation)  $\rightarrow \delta > \omega_0$



# Effect of Impure Components



$$\text{Resonance} \rightarrow \text{Im}\{\mathbf{Z}_{eq}\} = 0 \quad \left. \begin{array}{l} \omega_r = \sqrt{\omega_L \omega_H} \\ \omega_0 \triangleq \frac{\omega_L + \omega_H}{2} \end{array} \right\} \rightarrow \omega_r \leq \omega_0$$

$$\omega_r = \sqrt{\left(\omega_0 - \frac{2\pi \cdot BW}{2}\right) \left(\omega_0 + \frac{2\pi \cdot BW}{2}\right)} = \sqrt{\omega_0^2 - \frac{\omega_0^2}{4Q^2}} = \omega_0 \sqrt{1 - \frac{1}{4Q^2}}$$

$$\left. \begin{array}{l} Q \gg 1 \\ R \approx 0, \delta \approx 0 \end{array} \right\} \rightarrow \omega_r \approx \omega_0$$

$$\omega_r = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\omega_0^2 - \delta^2}, \quad \omega_0^2 = \frac{1}{LC}, \quad \delta = \frac{R}{2L}$$

$$\omega_r \approx \frac{\omega_L + \omega_H}{2}$$

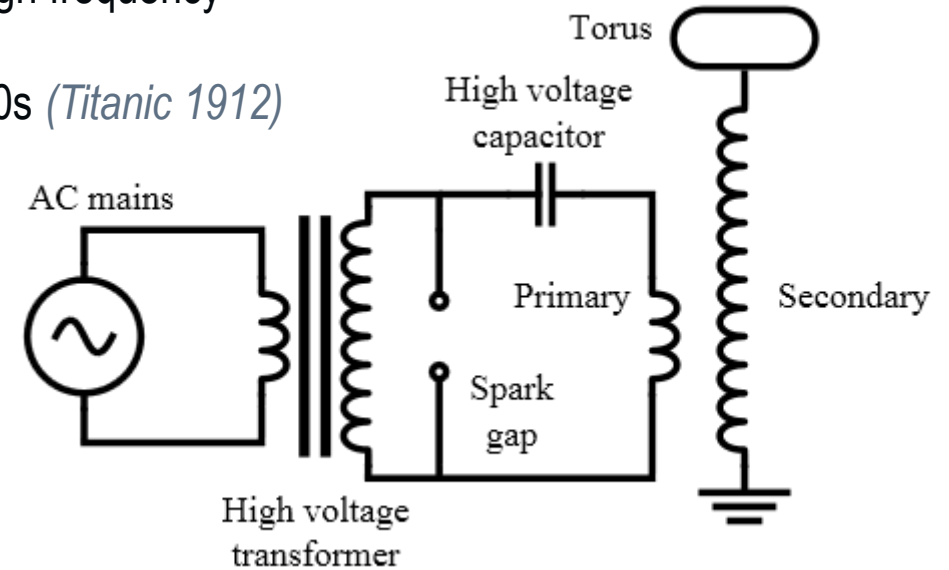
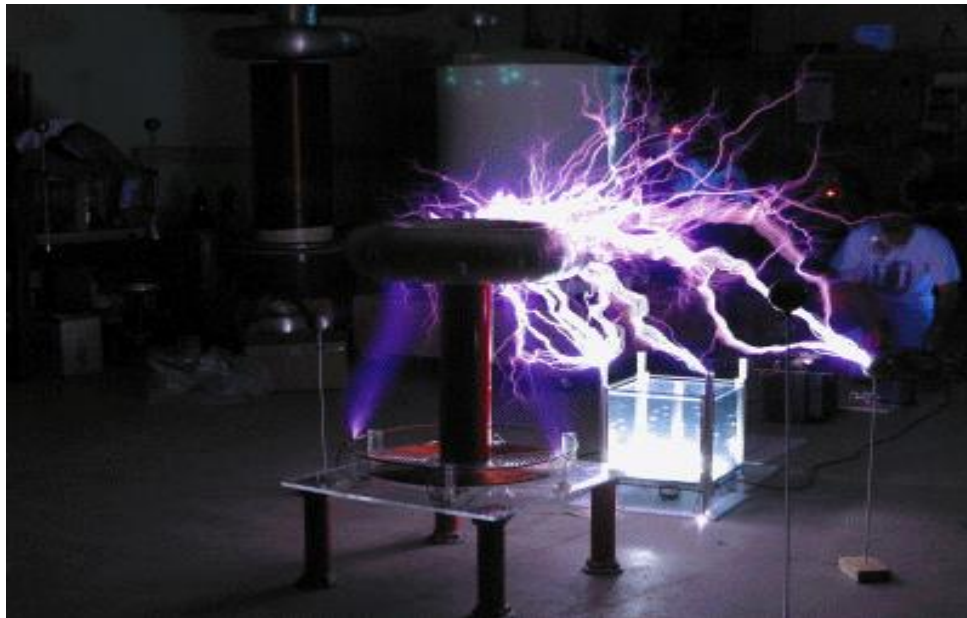
**IN PRACTICE ...** coil and capacitor contains some resistance.

- ❑ Series RLC pure / impure components  $\rightarrow$  no affect the resonance frequency
- ❑ Parallel RLC with impure components  $\rightarrow$  affect the resonance frequency

# App. – Tesla Coil



- ❑ Air-core resonant transformer (N. Tesla 1891) → high-voltage, high frequency
- ❑ Transmission of electrical energy without wires
- ❑ Spark gap radio transmitters for wireless telegraphy until the 1920s (*Titanic 1912*)
- ❑ Today: transistor / thyristor instead of spark gap
- ❑ Primer and seconder circuits → tuned for same resonance freq



$$\omega_{P0} = \omega_{S0} \rightarrow L_P C_P = L_S C_S$$

$$\frac{1}{2} C_P U_P^2 = \frac{1}{2} C_S U_S^2 \rightarrow U_S = U_P \sqrt{\frac{C_P}{C_S}} = U_P \sqrt{\frac{L_S}{L_P}}$$

# Questions

