DR. GYURCSEK ISTVÁN

Resonance Circuits

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*
- *http://www.electronics-tutorials.ws/accircuits*
- *http://hyperphysics.phy-astr.gsu.edu/hbase/electric/serres.html*
- *https://en.wikipedia.org/wiki/RLC_circuit#Damping*

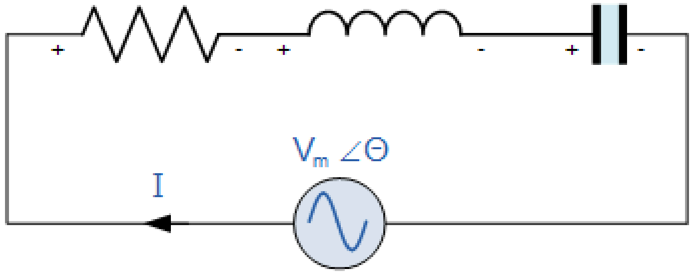
Progress in Content

- **Series Resonance Circuits**
- Parallel Resonance Circuits
- Free Resonance

Series Resonance Circuit

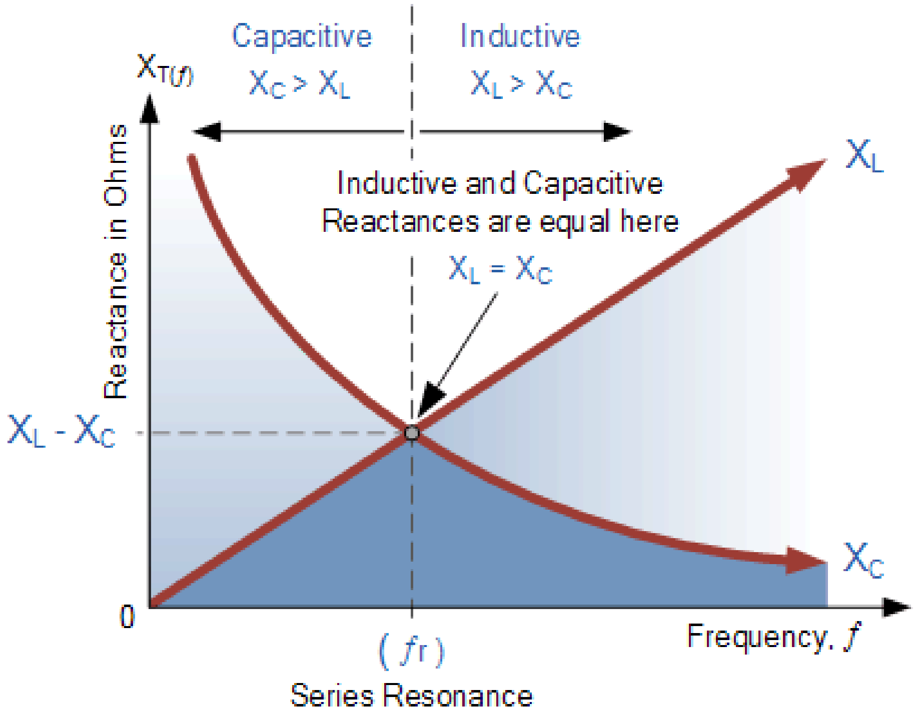
$$Z_{eq} = R + j\omega L - j\frac{1}{\omega C}$$

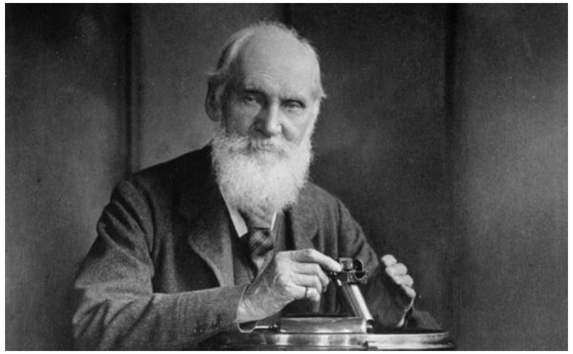
$$Resonance \rightarrow Im\{Z_{eq}\} = 0$$



$$X_L = X_C \rightarrow \omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r = \frac{1}{\sqrt{LC}} \; or \; f_r = \frac{1}{2\pi\sqrt{LC}}$$



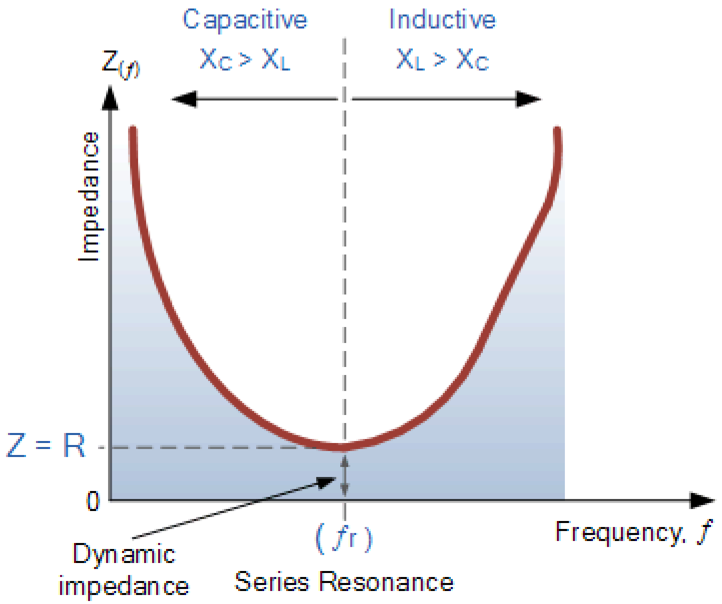


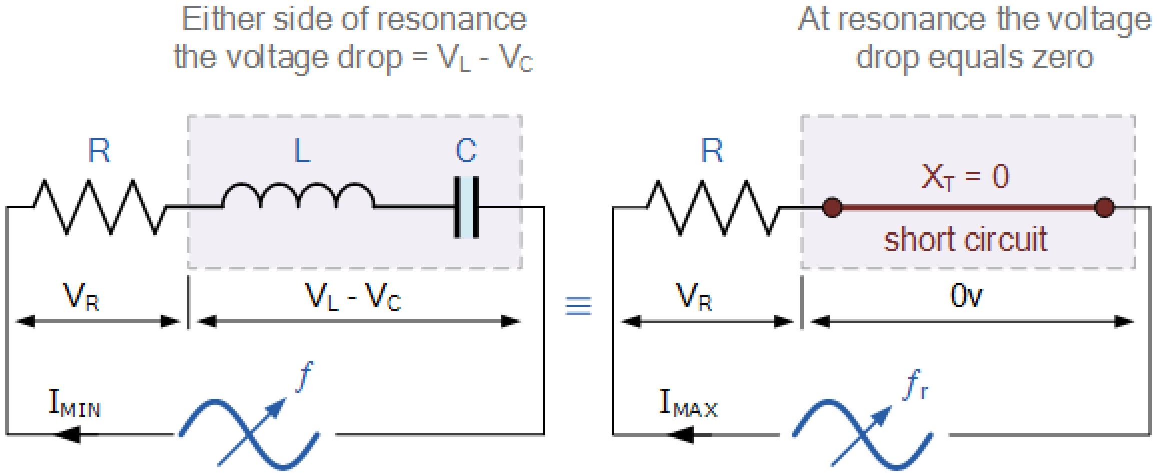
William Thomson
(Lord Kelvin - 1824-1907)

gyurcsek.istvan@mik.pte.hu

Series RLC Circuit at Resonance 1

$$\boldsymbol{Z} = R + jX_L - jX_C = R + j(X_L - X_C) \rightarrow Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$





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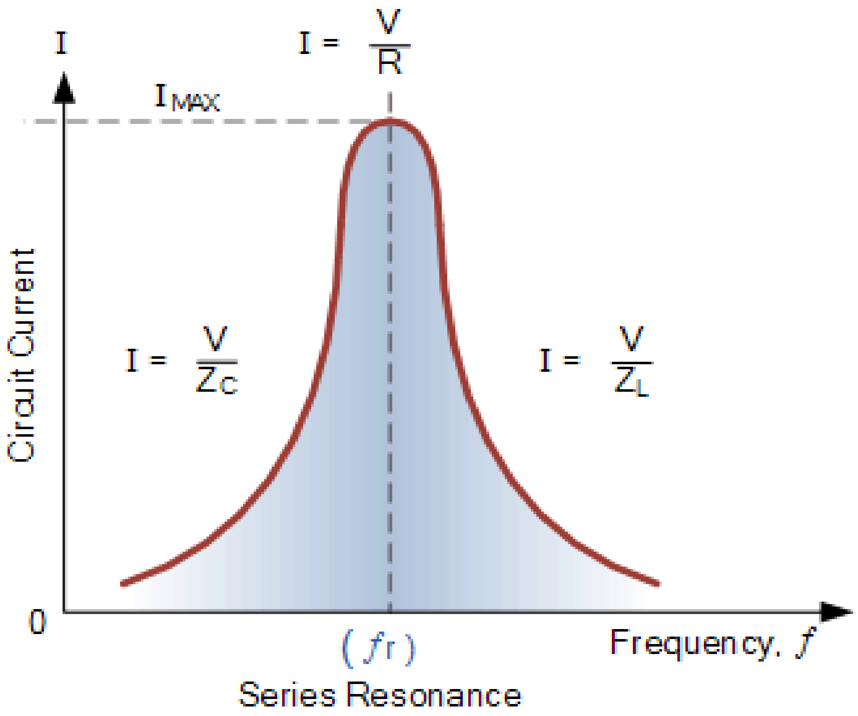
Series RLC Circuit at Resonance 2

$$\boldsymbol{I} = \frac{V}{\boldsymbol{Z}} = \frac{V}{R + j(X_L - X_C)}$$

$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



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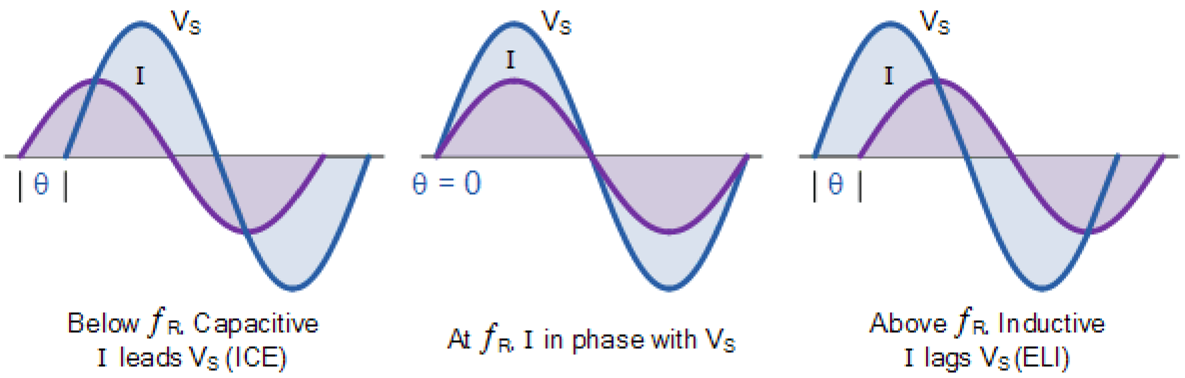
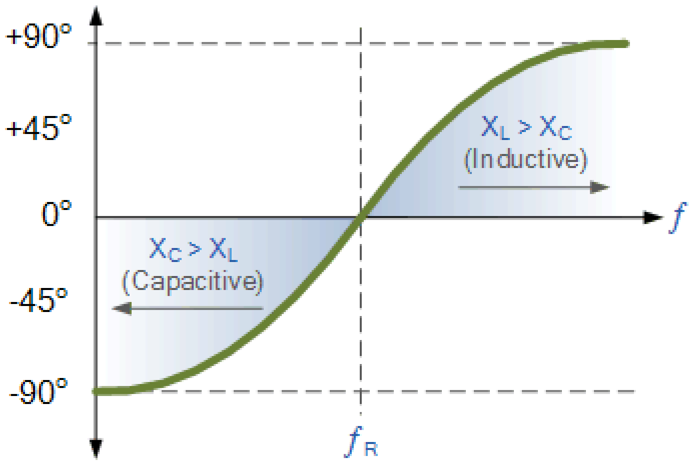
Phase Angle of a Series Resonance Circuit





$$\boldsymbol{Z} = R + j(X_L - X_C)$$

$$\rightarrow Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\rightarrow \varphi_Z = tan^{-1}\frac{X_L - X_C}{R} \quad (can\ be\ 0°\ !)$$

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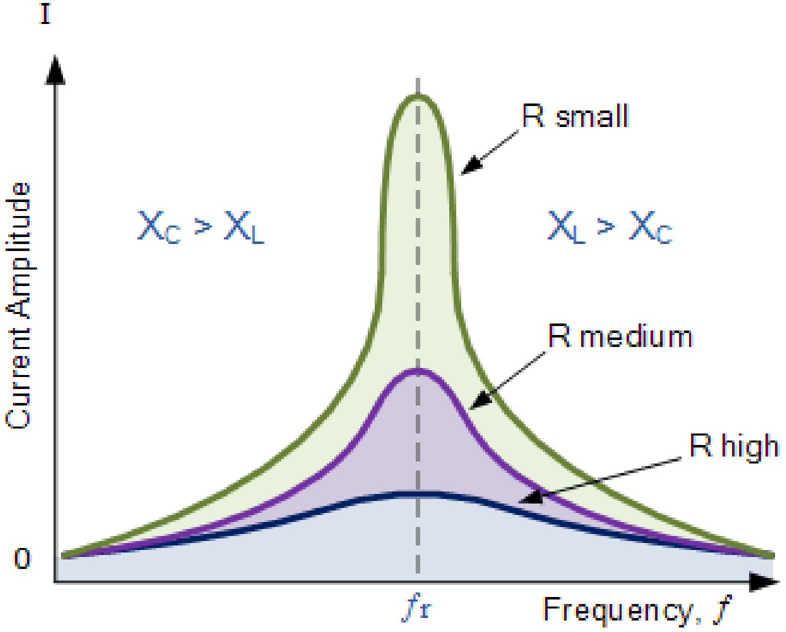
$$Q \triangleq 2\pi \cdot \frac{\text{'peak energy stored in the circuit'}}{\text{'dissipated energy in a period at resonance'}} = 2\pi \frac{{}^{1}\!/_{2}\, I_p^2 L}{{}^{1}\!/_{2}\, I_p^2 R T} = \frac{\omega_r L}{R} = \frac{X_r}{R} = \frac{1}{\omega_r R C}$$



$$Q = \frac{\omega L}{R} \rightarrow \omega = \frac{RQ}{L} \qquad Q = \frac{1}{\omega C R} \rightarrow \omega = \frac{1}{QCR}$$

$$\frac{RQ}{L} = \frac{1}{QCR} \rightarrow Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{Z_0}{R} \qquad Z_0 = \sqrt{\frac{L}{C}}$$

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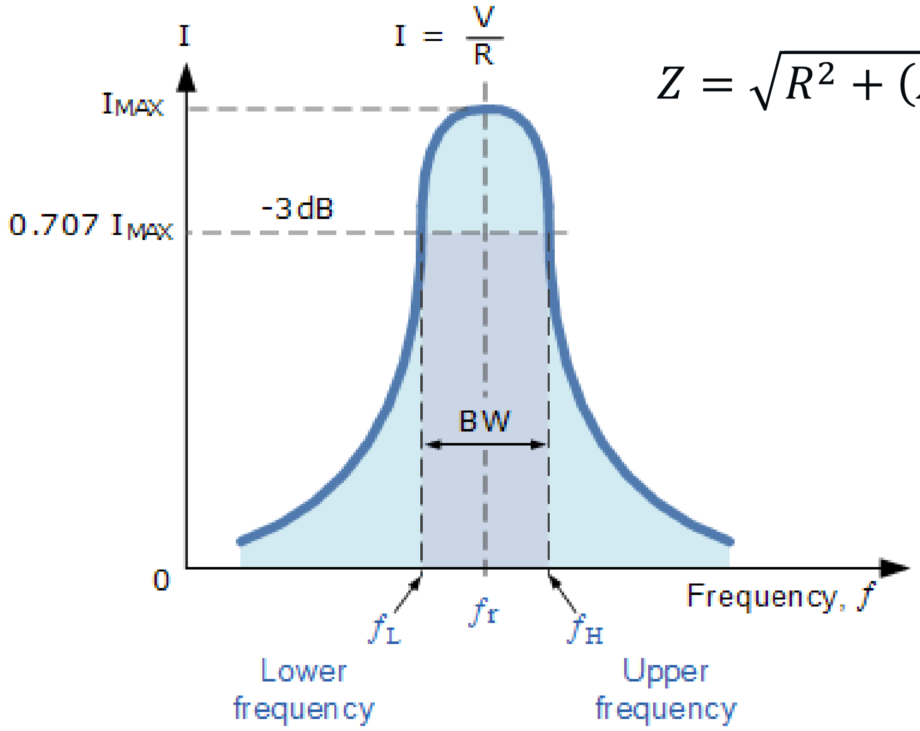
Bandwidth of a Series Resonance Circuit

$$BW = f_H - f_L \qquad \omega_r = \frac{1}{\sqrt{LC}} \rightarrow Z = min;\ I_S = max \qquad \frac{P_{max}}{2} \rightarrow \frac{I_{max}}{\sqrt{2}} \rightarrow Z = R \cdot \sqrt{2}$$

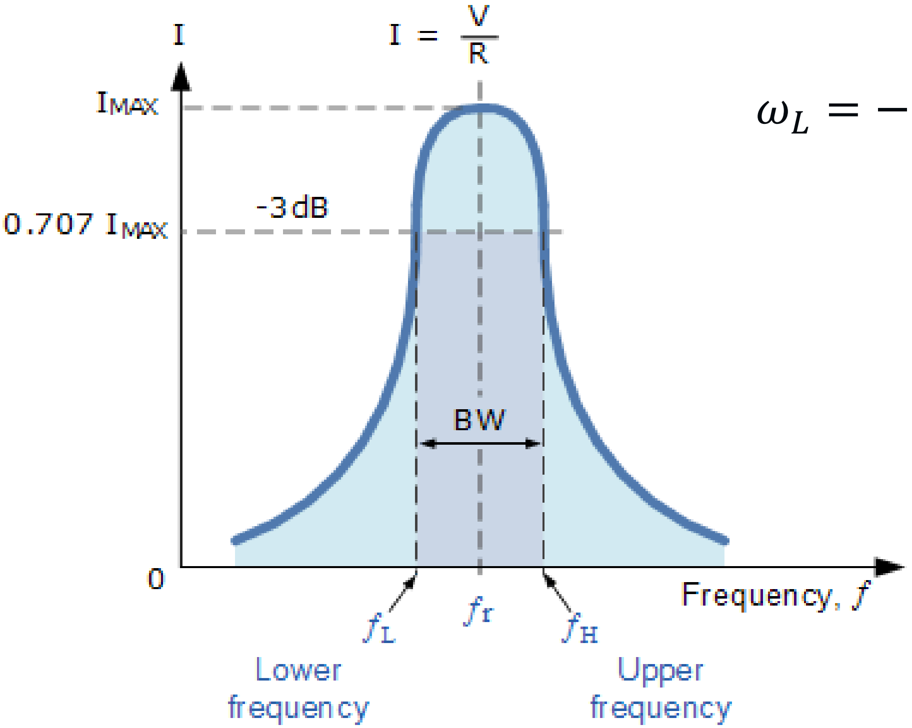


$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R \cdot \sqrt{2} \rightarrow \begin{cases} X_L - X_C = R \\ X_C - X_L = R \end{cases}$$

$$\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_H = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

BW and Q-factor of Series Resonance Circuit



$$\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad \omega_H = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\left.\begin{aligned} (a+b)(a-b) &= a^2 - b^2 \\ \omega_r &= \frac{1}{\sqrt{LC}} \end{aligned}\right\} \rightarrow \omega_r = \sqrt{\omega_L \omega_H}$$

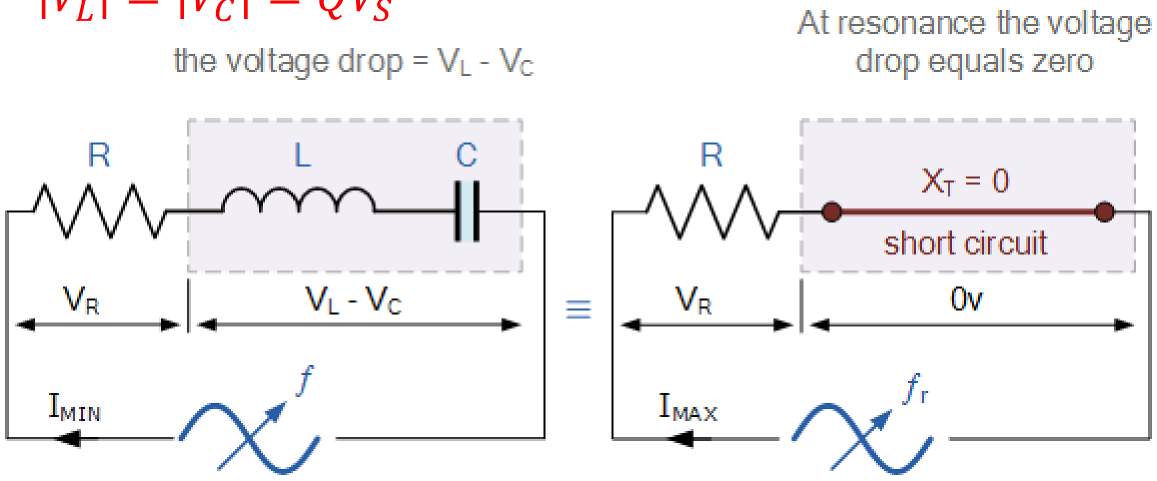
$$Q = \frac{\omega_r L}{R} \rightarrow \frac{R}{L} = \frac{\omega_r}{Q} \rightarrow \mathrm{BW} = f_H - f_L = \frac{1}{2\pi}\frac{R}{L} = \frac{f_r}{Q}$$

gyurcsek.istvan@mik.pte.hu

Voltage Resonance

Series resonance → voltage resonance

$$|V_L| = |V_C| = QV_S$$



$$V_L = IX_L = \frac{V_S}{R}X_L = \frac{\omega L}{R}V_S = QV_S \qquad V_C = IX_C = \frac{V_S}{R}X_C = \frac{1}{\omega CR}V_S = QV_S$$

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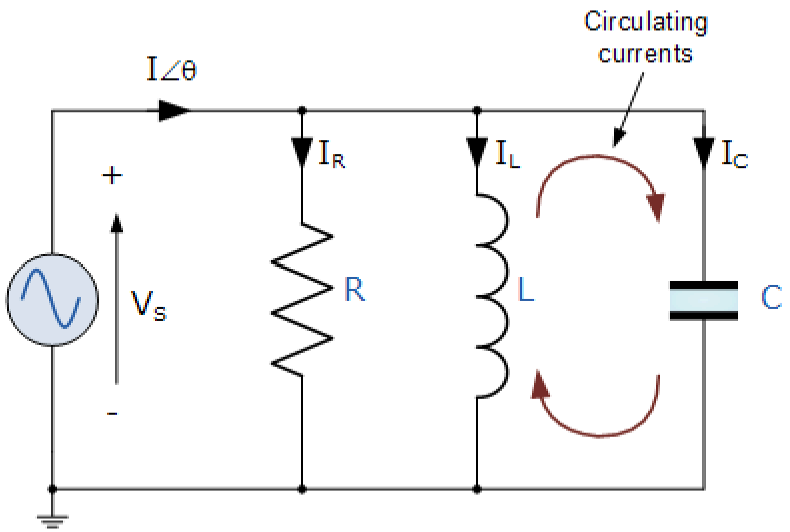
- **Parallel Resonance Circuits**
- Free Resonance

gyurcsek.istvan@mik.pte.hu

Parallel Resonance Circuit



$$Resonance \rightarrow Im\{\boldsymbol{Y}_{eq}\} = 0$$

$$\boldsymbol{Y} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

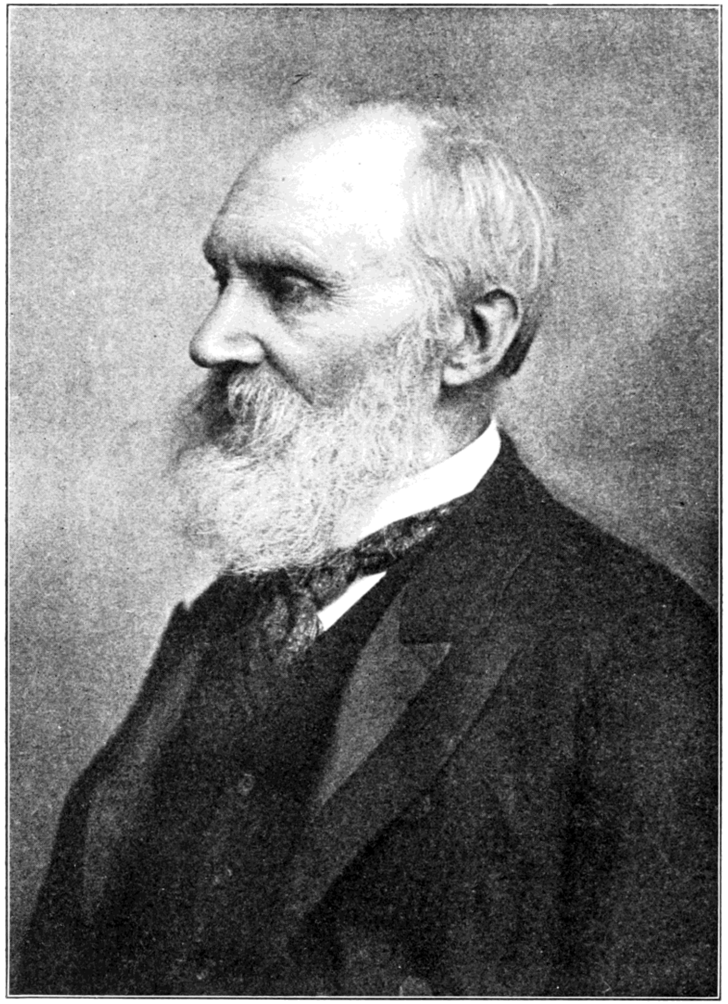
$$\boldsymbol{Y} = G - jB_L + jB_C$$

$$Admittance, Y = \frac{1}{Z} = \sqrt{G^2 + B^2}$$

$$Conductance, G = \frac{1}{R}$$

$$Inductive\ Susceptance, B_L = \frac{1}{2\pi f L}$$

$$Capacitive\ Susceptance, B_C = 2\pi f C$$

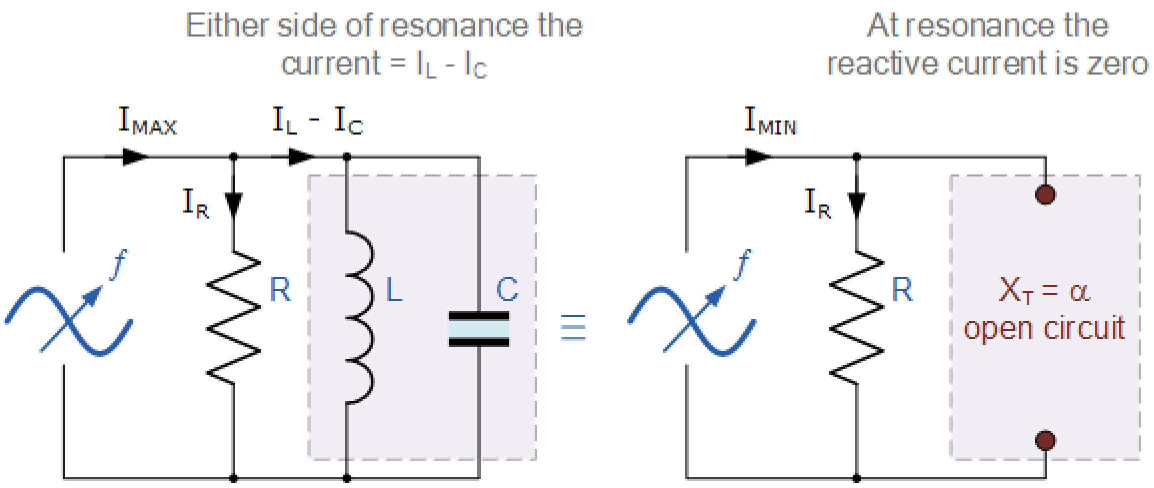


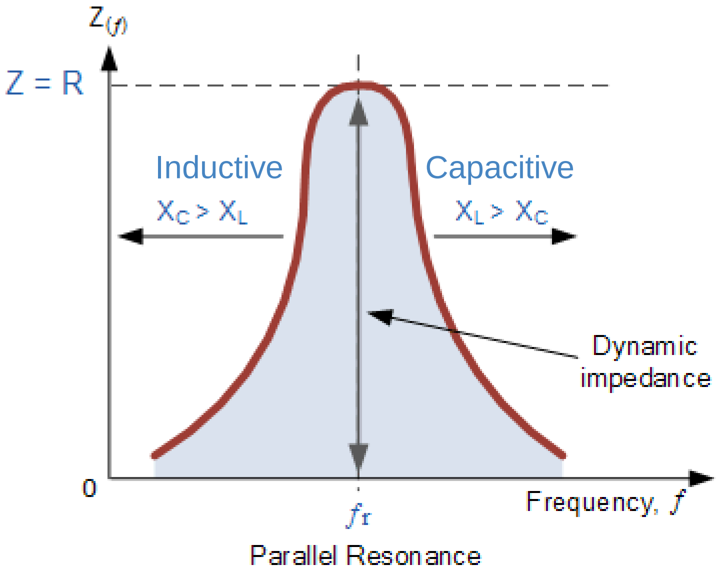
Lord William Thomson Kelvin
(1824-1907)

$$X_L = X_C \rightarrow \omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r = \frac{1}{\sqrt{LC}}\ or\ f_r = \frac{1}{2\pi\sqrt{LC}}$$

Impedance in Parallel Resonance Circuit





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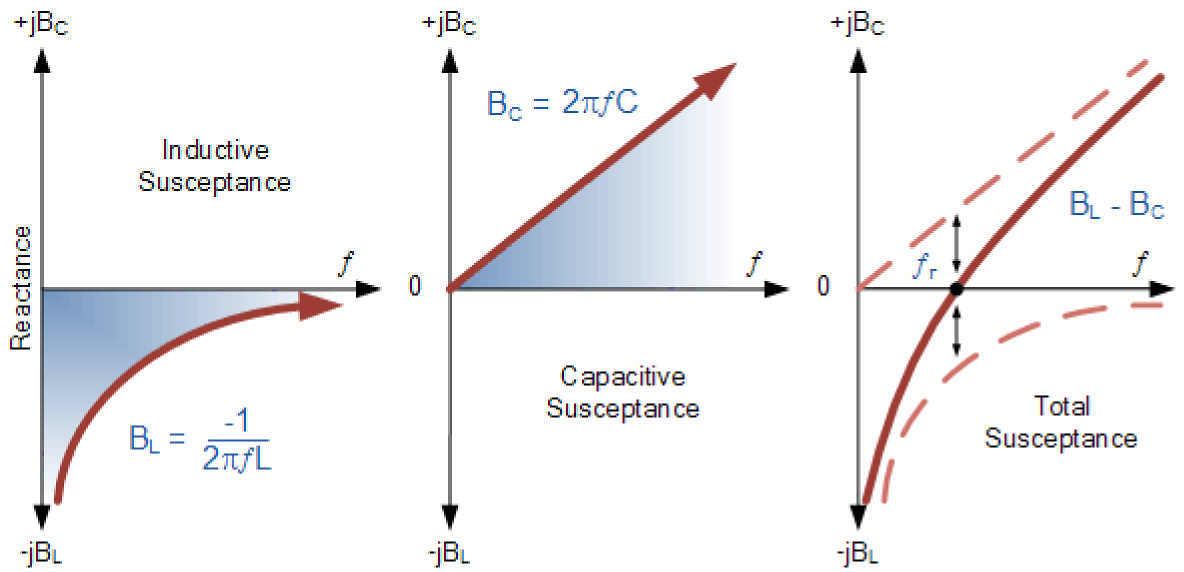
Susceptance in Parallel Resonance Circuit

$$\boldsymbol{Y} = G + jB_C - jB_L = G + j(B_C - B_L) \rightarrow Y = \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$



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Current in a Parallel Resonance Circuit

$$I_R = \frac{V}{R}$$

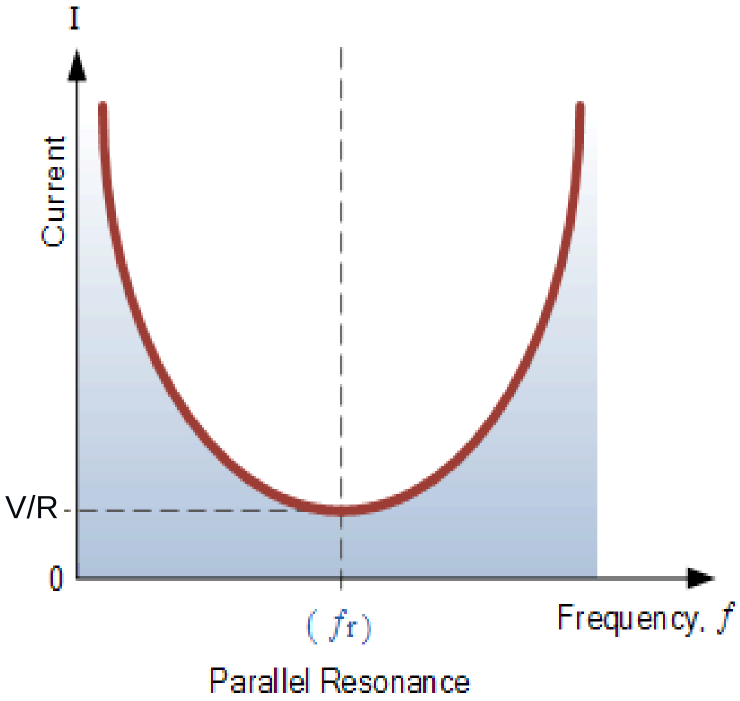
$$I_L = \frac{V}{X_L} = \frac{V}{\omega L} = \frac{V}{2\pi f L}$$

$$I_C = \frac{V}{X_C} = V 2\pi f C$$

$$\overline{I_T} = \overline{I_R} + \overline{I_L} + \overline{I_C}$$

$$I_T = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$I_T = \sqrt{I_R^2 + (0)^2} = I_R$$



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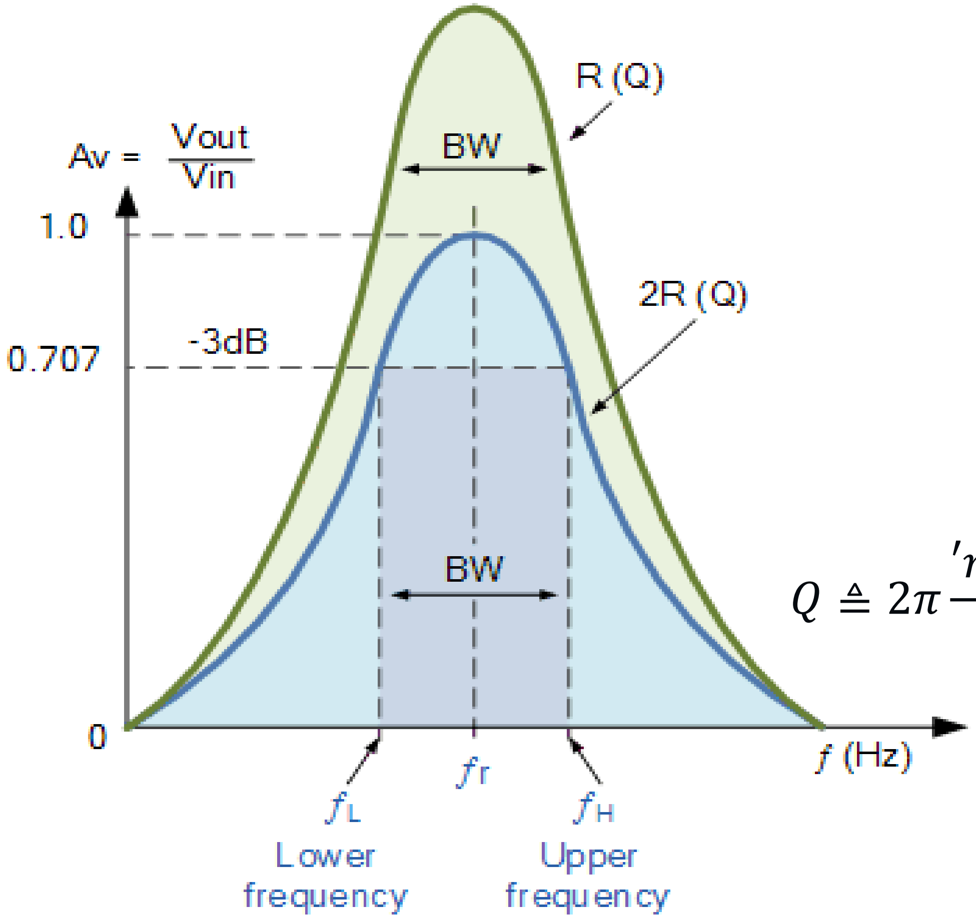
Bandwidth & Selectivity (Q-factor) of a Parallel Resonance Circuit



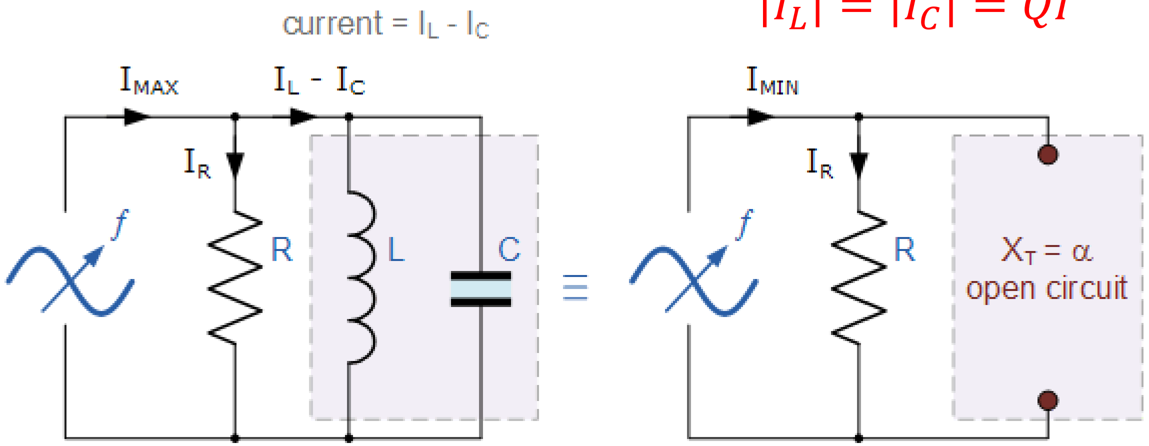
$$|B_C - B_L| = G \rightarrow \begin{cases} \omega_L = -\dfrac{G}{2C} + \sqrt{\left(\dfrac{G}{2C}\right)^2 + \dfrac{1}{LC}} \\ \omega_H = +\dfrac{G}{2C} + \sqrt{\left(\dfrac{G}{2C}\right)^2 + \dfrac{1}{LC}} \end{cases}$$

$$Q \triangleq 2\pi \frac{'max.energy\ stored'}{'power\ loss'} = \frac{V^2\,B_C}{V^2\,G} = \frac{\omega_r\,C}{G} = \frac{1}{\omega_r\,L\,G} = \frac{1}{G}\sqrt{\frac{C}{L}} = \frac{Y_0}{G}$$

$$Q = \frac{\omega_r\,C}{G} \rightarrow \frac{G}{C} = \frac{\omega_r}{Q} \rightarrow \mathrm{BW} = f_H - f_L = \frac{1}{2\pi}\frac{G}{C} = \frac{f_r}{Q}$$

Parallel resonance → current resonance

$|I_L| = |I_C| = QI$



$$I_L = \frac{V_S}{X_L} = \frac{IR}{X_L} = \frac{R}{\omega L} I = QI$$

$$I_C = \frac{V_S}{X_C} = \frac{IR}{X_C} = \omega CRI = QI$$

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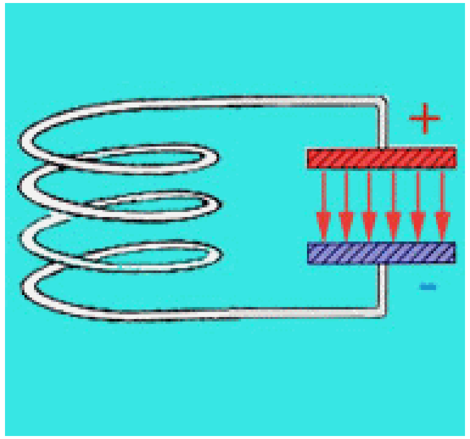
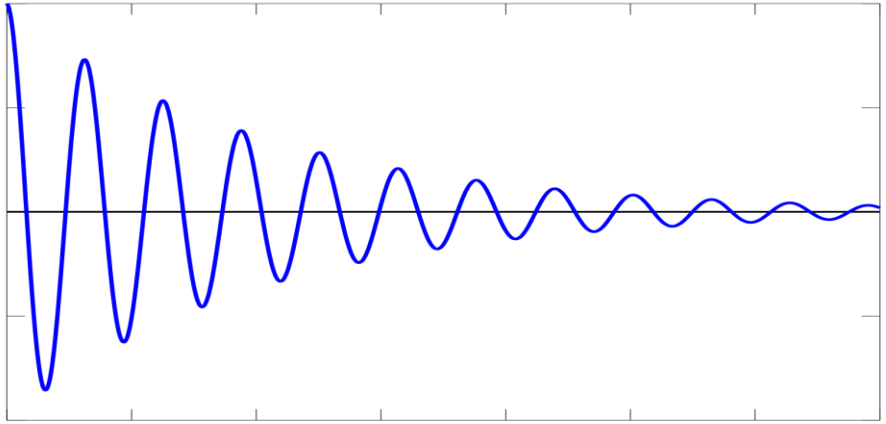
- Parallel Resonance Circuits
- **Free Resonance**

Free resonance (LC, RLC)



$$W_L = \frac{1}{2} L \cdot I^2, \qquad W_C = \frac{1}{2} C \cdot V^2$$

$$\frac{1}{2} L \cdot I^2 = \frac{1}{2} C \cdot V^2 \rightarrow \frac{V}{I} = \sqrt{\frac{L}{C}} = Z_0$$



$$i(t) = I_0\, e^{-\delta t} \sin \omega t \quad (\delta{:}\, damping\ factor)$$

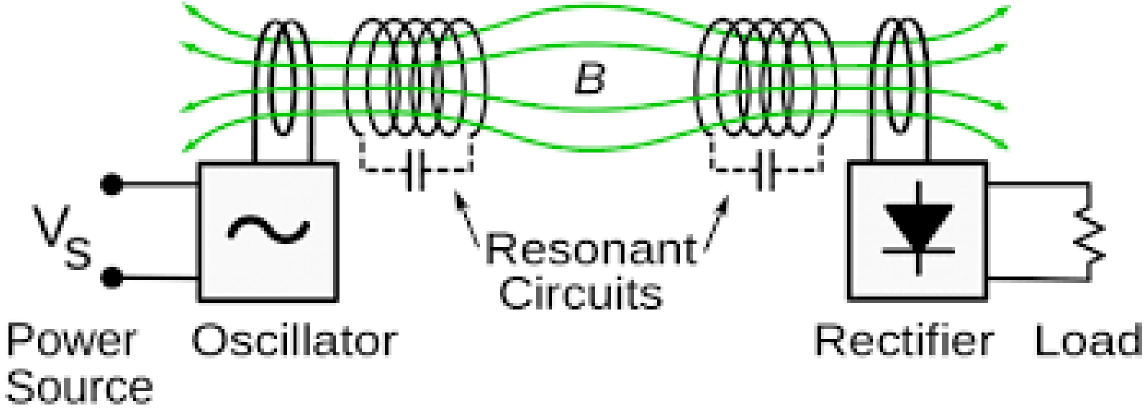
$$\omega = \sqrt{\omega_0^2 - \delta^2}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}, \qquad \delta = \frac{R}{2L}$$

undamped *(free oscillation)* $\rightarrow \delta = 0$

underdamped *(slowly dampening osc.)* $\rightarrow \delta < \omega_0$

critical damping *(no oscillation)* $\rightarrow \delta = \omega_0$

overdamped *(no oscillation)* $\rightarrow \delta > \omega_0$



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Resonance Frequency using Impure Components

$$Resonance \rightarrow Im\{\mathbf{Z}_{eq}\} = 0 \qquad \left.\begin{array}{l} \omega_r = \sqrt{\omega_L \omega_H} \\ \omega_0 \triangleq \dfrac{\omega_L + \omega_H}{2} \end{array}\right\} \rightarrow \omega_r \leq \omega_0$$

$$\omega_r = \sqrt{\left(\omega_0 - \frac{2\pi \cdot BW}{2}\right)\left(\omega_0 + \frac{2\pi \cdot BW}{2}\right)} = \sqrt{\omega_0^2 - \frac{\omega_0^2}{4Q^2}} = \omega_0 \sqrt{1 - \frac{1}{4Q^2}} \qquad \left.\begin{array}{l} Q \gg 1 \\ R \approx 0, \delta \approx 0 \end{array}\right\} \rightarrow \omega_r \approx \omega_0$$

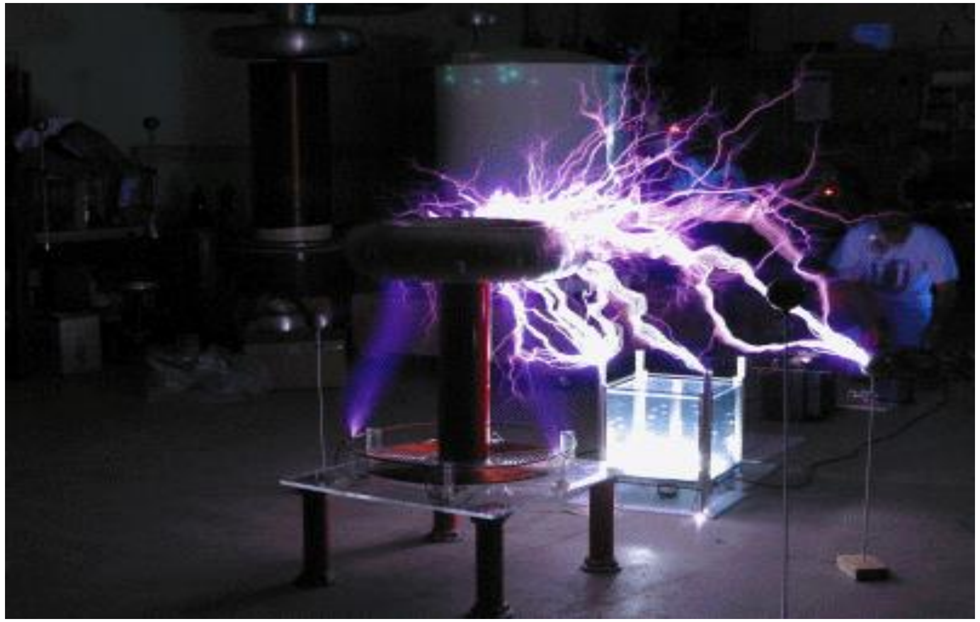
$$\omega_r = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\omega_0^2 - \delta^2}, \qquad \omega_0^2 = \frac{1}{LC}, \qquad \delta = \frac{R}{2L} \qquad \omega_r \approx \frac{\omega_L + \omega_H}{2}$$

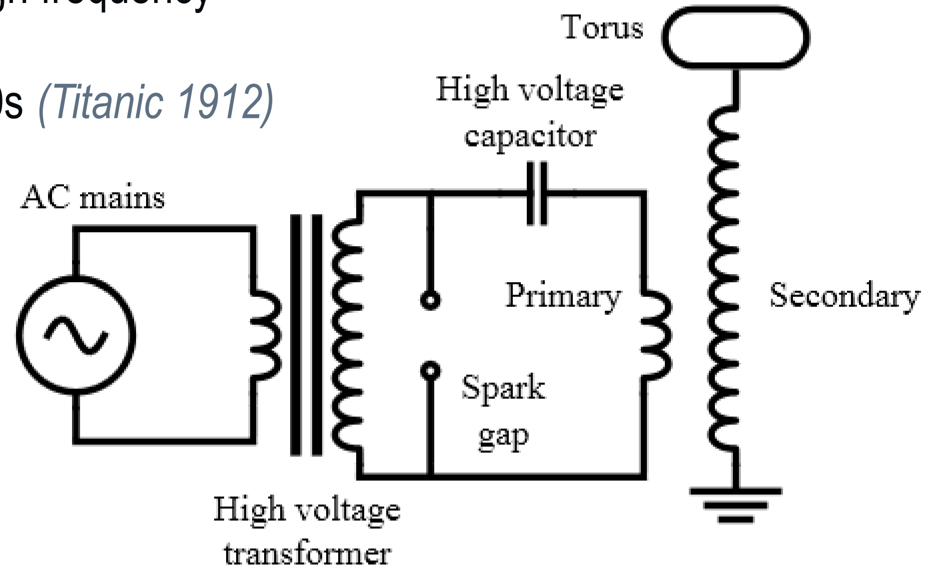
IN PRACTICE ... *coil and capacitor contains some resistance.*

- ❑ *Series RLC pure / impure components → no affect the resonance frequency*
- ❑ *Parallel RLC with impure components → affect the resonance frequency*

Application – Tesla Coil

- Air-core resonant transformer (N. Tesla 1891) → high-voltage, high frequency
- Transmission of electrical energy without wires
- Spark gap radio transmitters for wireless telegraphy until the 1920s *(Titanic 1912)*
- Today: transistor / thyristor instead of spark gap
- Primer and seconder circuits → tuned for same resonance freq





$$\omega_{P0} = \omega_{S0} \rightarrow L_P C_P = L_S C_S$$

$$\frac{1}{2} C_P U_P^2 = \frac{1}{2} C_S U_S^2 \rightarrow U_S = U_P \sqrt{\frac{C_P}{C_S}} = U_P \sqrt{\frac{L_S}{L_P}}$$

Questions