



DR. GYURCSEK ISTVÁN

Two-Port Networks

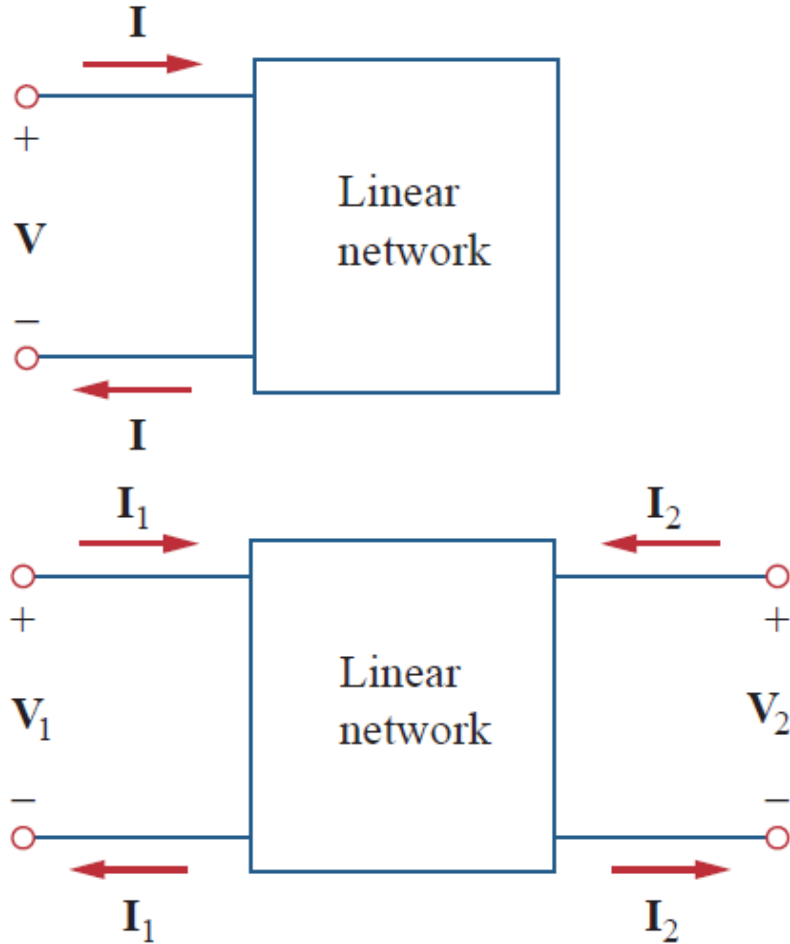
Sources and additional materials (recommended)

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- ❑ *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*

Introduction



Network analysis \rightarrow determ. of V, I at certain nodes (*terminals*)



Port

- ❑ Pair of terminals
- ❑ Same current enters / leaves

One-port

- ❑ Element / device
- ❑ i.e. $R, L, C,$

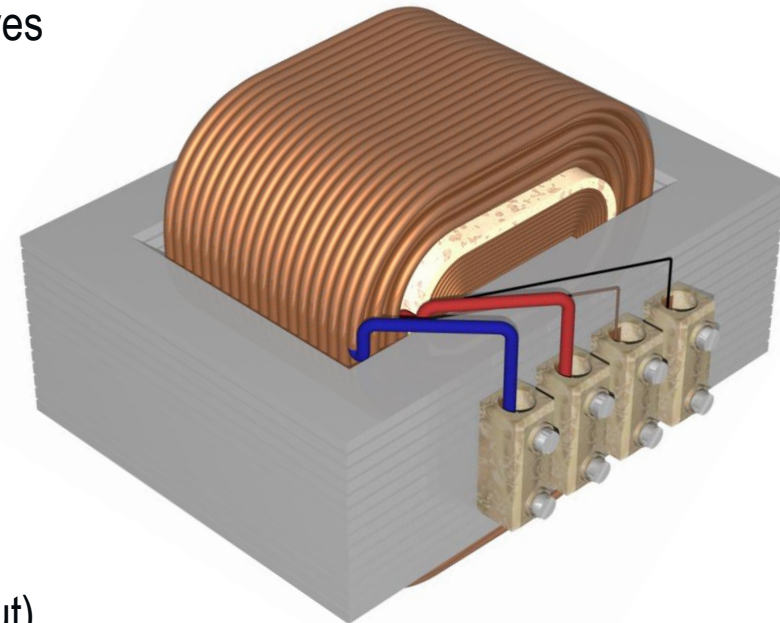
Two-port network

- ❑ Two separate ports
- ❑ Input & output

Two-port characteristic

- ❑ 4 params (2 input, 2 output)
- ❑ Only 2 of them are independent!
- ❑ Relationship b/w V_1, V_2, I_1, I_2

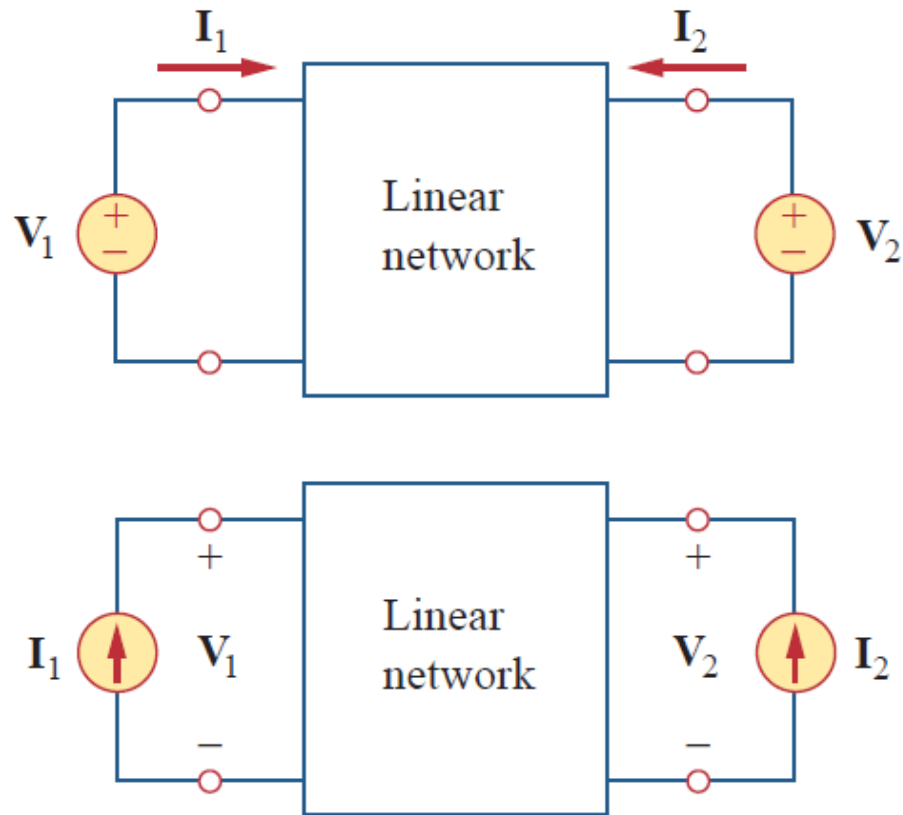
Example \rightarrow transformer as a two-port network





- ❑ **Two-Port Characteristics**
- ❑ Relations between Characteristics
- ❑ Two-Port Interconnections
- ❑ Bartlett's Bisection Theorem
- ❑ Two-Ports with Finite Terminations
- ❑ Applications

Two-Port Characteristics



Two-port variables

- V_1, V_2, I_1, I_2
- Two of them are independent (*only!*)
- Relationship b/w 4 variables

Network driven by

- voltage sources
- current sources
- (*mix of them*)

Two-port parameters

[input] [output] [transfer] → OC & SC params

- Six characteristics (*some of them might be missing*)
- impedance – admittance params
- hybrid – inverse hybrid params
- transmission – inverse transmission params

OC Impedance Parameters



$$\begin{aligned} V_1 &= z_{11} \cdot I_1 + z_{12} \cdot I_2 \\ V_2 &= z_{21} \cdot I_1 + z_{22} \cdot I_2 \end{aligned} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [z] \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

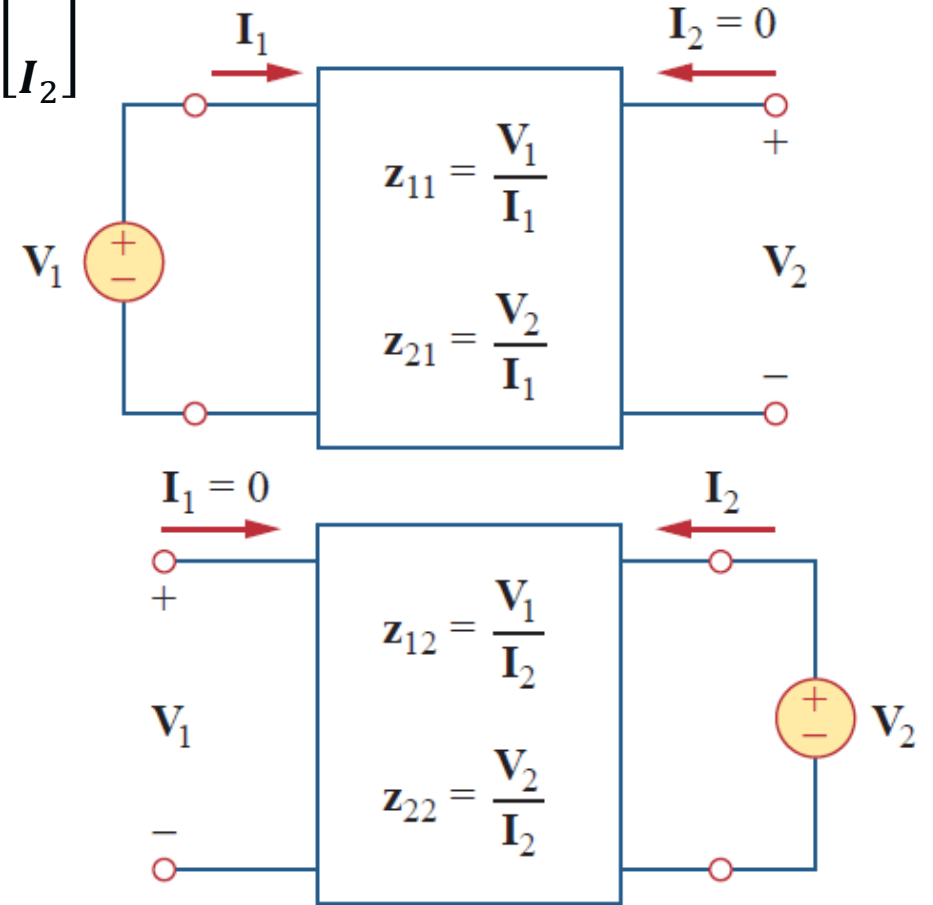
$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \leftarrow \text{OC input impedance}$$

$$z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} \leftarrow \text{OC transfer impedance (port 2} \rightarrow \text{port 1)}$$

$$z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} \leftarrow \text{OC transfer impedance (port 1} \rightarrow \text{port 2)}$$

$$z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} \leftarrow \text{OC output impedance}$$

Calculating / measuring circuits



OC Impedance Parameters



$$V_1 = z_{11} \cdot I_1 + z_{12} \cdot I_2$$

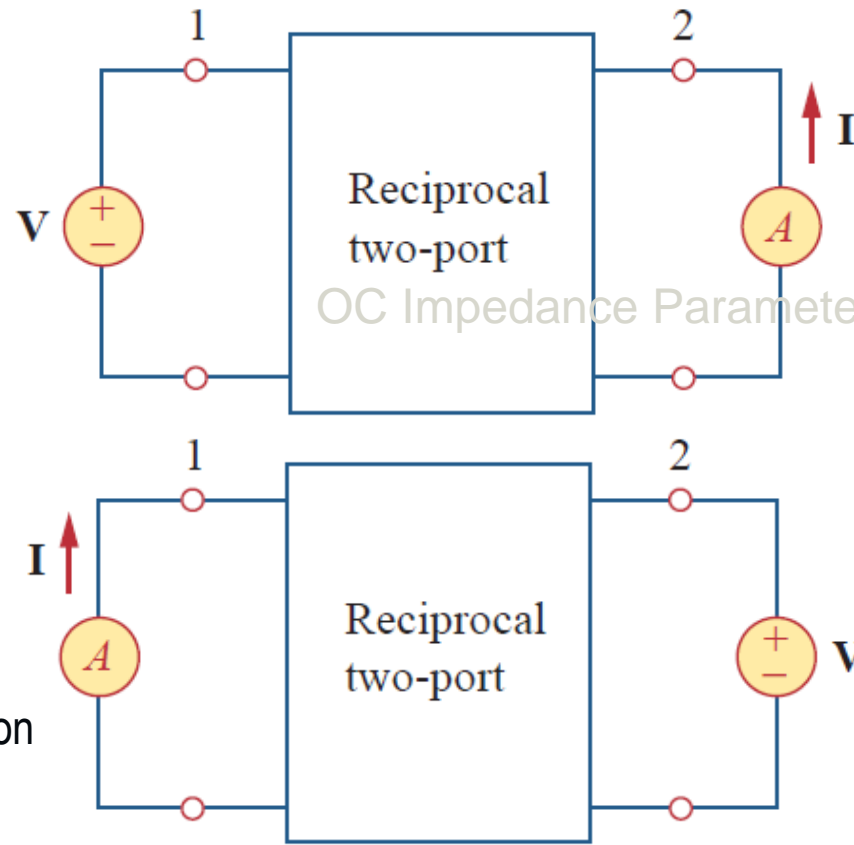
$$V_2 = z_{21} \cdot I_1 + z_{22} \cdot I_2$$

Symmetrical two-port network

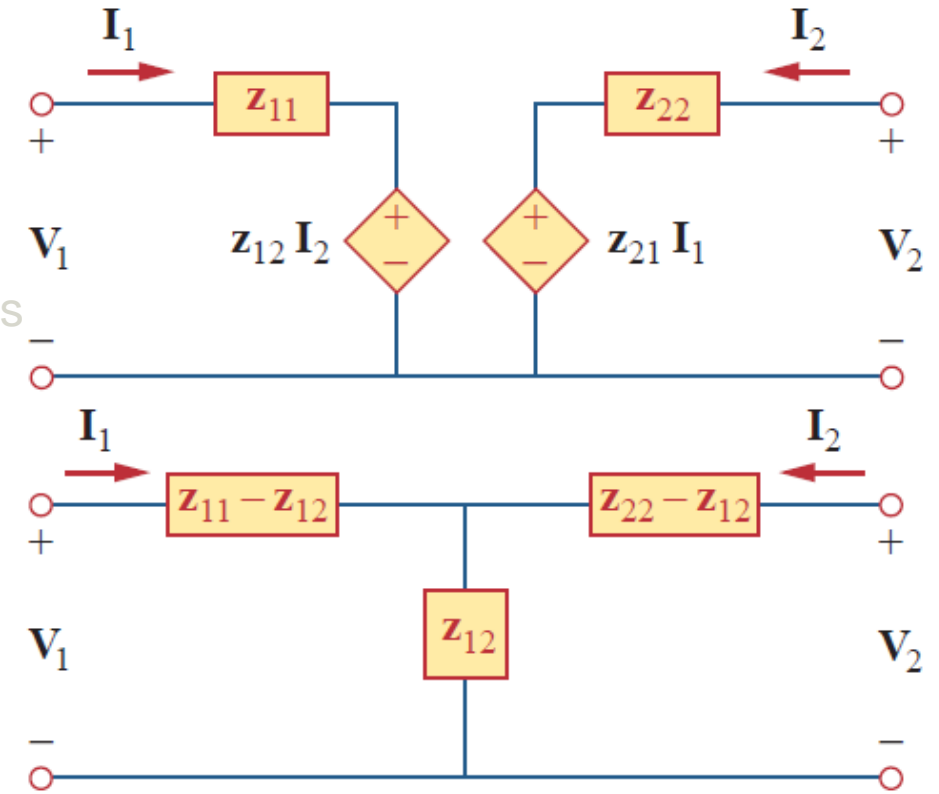
- ,mirrorlike' symmetry
- two similar halves
- $z_{11} = z_{22}$

Reciprocal two-port network

- linear network
- no dependent sources
- Interchanged ports of excitation & response $\rightarrow z_{12} = z_{21}$

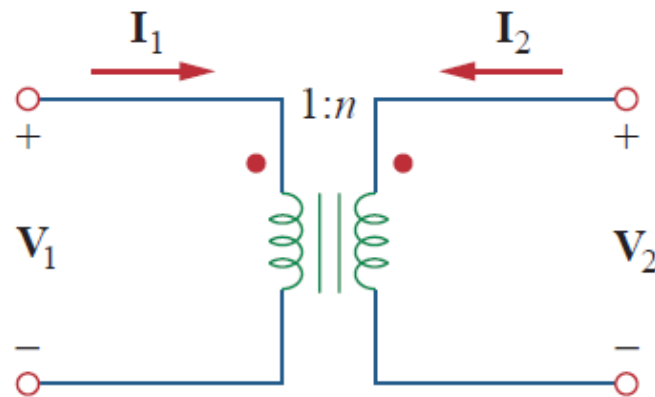


General equivalent circuit



Equivalent circuit for reciprocal network

Comment ...



No z parameters for some two-ports!
i.e. ideal transformer \rightarrow impossible to express V_1 and V_2 in terms of I_1 and I_2 .

$$V_1 = \frac{1}{n} V_2, \quad I_1 = -n I_2$$

SC Admittance Parameters



$$\begin{aligned} I_1 &= y_{11} \cdot V_1 + y_{12} \cdot V_2 \\ I_2 &= y_{21} \cdot V_1 + y_{22} \cdot V_2 \end{aligned} \rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [y] \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

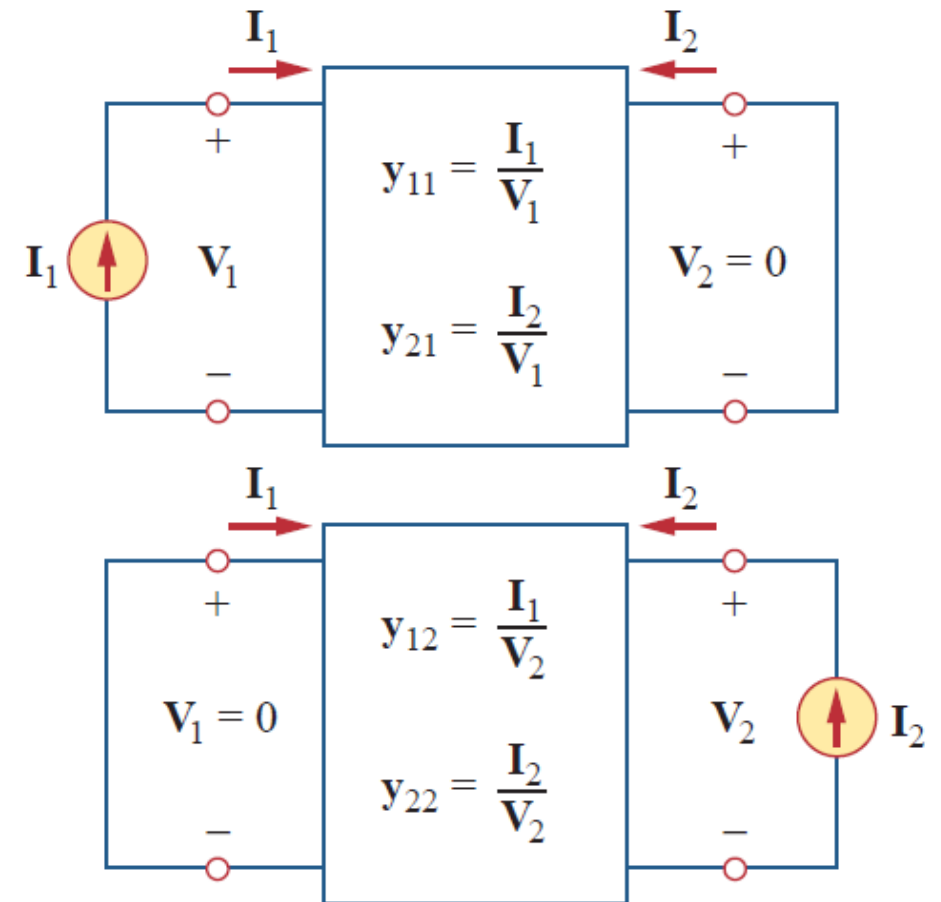
$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \leftarrow \text{SC input admittance}$$

$$y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} \leftarrow \text{SC transfer admittance (port 2} \rightarrow \text{port 1)}$$

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} \leftarrow \text{SC transfer admittance (port 1} \rightarrow \text{port 2)}$$

$$y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0} \leftarrow \text{SC output admittance}$$

Calculating / measuring circuits



SC Admittance Parameters



$$I_1 = y_{11} \cdot V_1 + y_{12} \cdot V_2$$

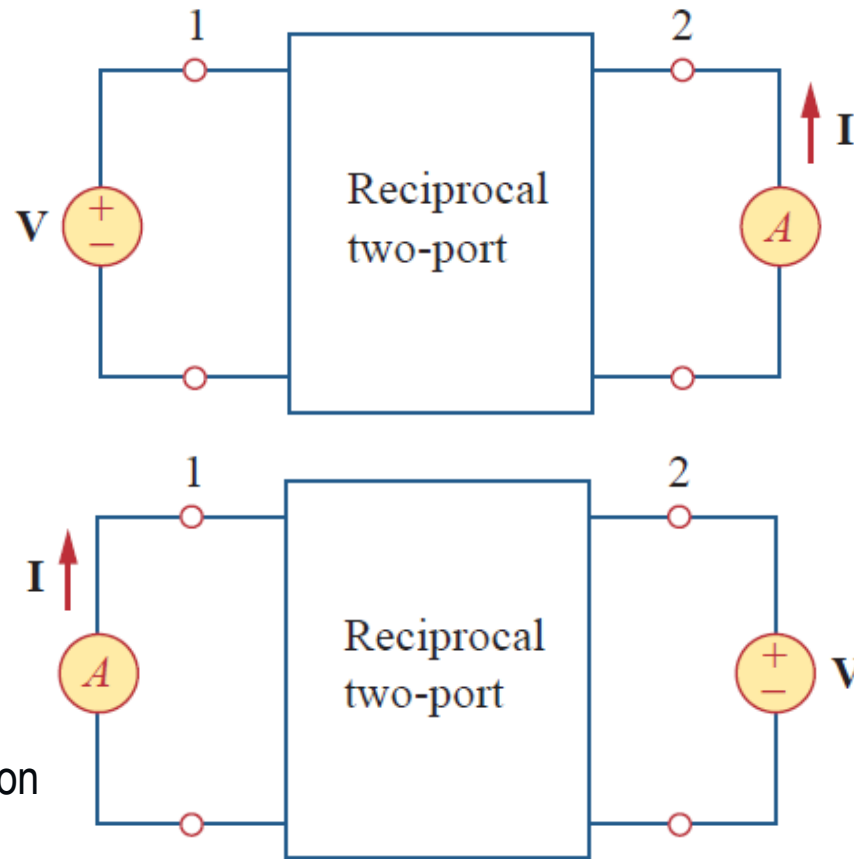
$$I_2 = y_{21} \cdot V_1 + y_{22} \cdot V_2$$

Symmetrical two-port network

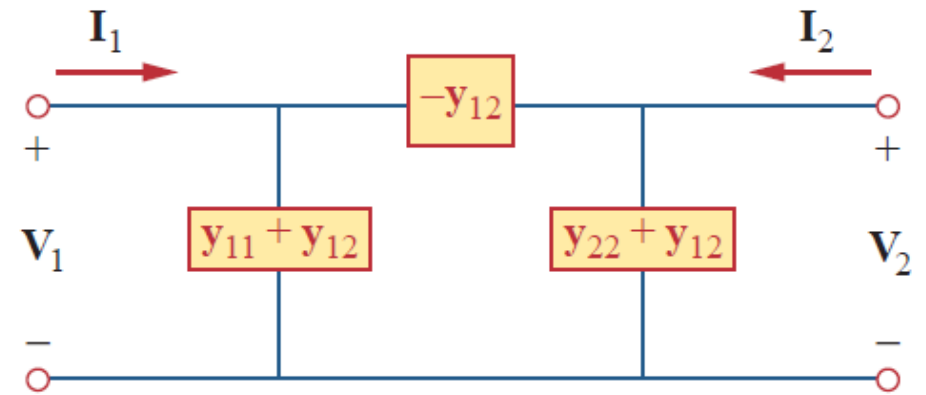
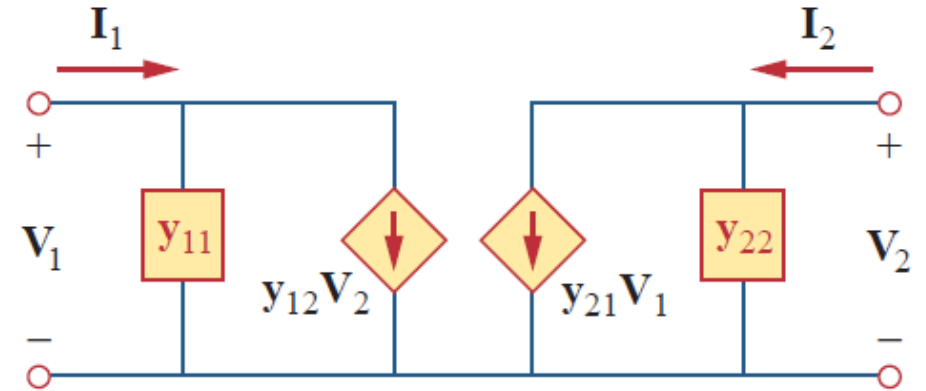
- ,mirrorlike' symmetry
- two similar halves
- $y_{11} = y_{22}$

Reciprocal two-port network

- linear network
- no dependent sources
- Interchanged ports of excitation & response $\rightarrow y_{12} = y_{21}$



General equivalent circuit



Equivalent circuit for reciprocal network

Hybrid Parameters



$$\begin{aligned} V_1 &= h_{11} \cdot I_1 + h_{12} \cdot V_2 \\ I_2 &= h_{21} \cdot I_1 + h_{22} \cdot V_2 \end{aligned} \rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = [h] \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} \leftarrow \text{SC input impedance}$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} \leftarrow \text{OC reverse voltage gain}$$

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0} \leftarrow \text{SC forward current gain}$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} \leftarrow \text{OC output admittance}$$

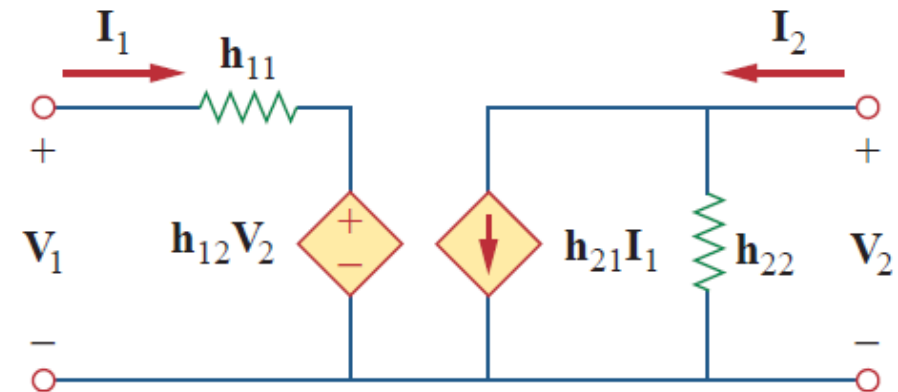
Calculating / measuring circuits

□ ...like it is for z and y params

Symmetrical if... $\Delta = \pm 1$

Reciprocal if... $h_{12} = -h_{21}$

Hybrid model of a two-port network (used for BJT)



Inverse Hybrid Parameters



$$\begin{aligned} I_1 &= g_{11} \cdot V_1 + g_{12} \cdot I_2 \\ V_2 &= g_{21} \cdot V_1 + g_{22} \cdot I_2 \end{aligned} \rightarrow \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \rightarrow \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [g] \cdot \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0} \leftarrow \text{OC input admittance}$$

$$g_{12} = \left. \frac{I_1}{I_2} \right|_{V_1=0} \leftarrow \text{SC reverse current gain}$$

$$g_{21} = \left. \frac{V_2}{V_1} \right|_{I_2=0} \leftarrow \text{OC forward voltage gain}$$

$$g_{22} = \left. \frac{V_2}{I_2} \right|_{V_1=0} \leftarrow \text{SC output impedance}$$

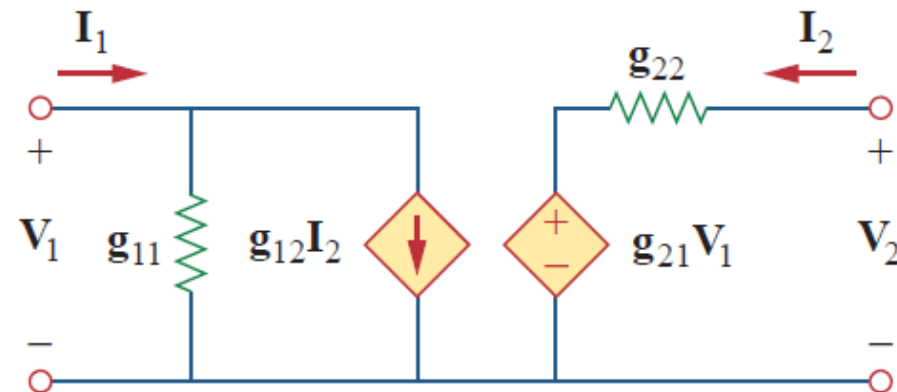
Calculating / measuring circuits

□ ...like it is for z and y params

Symmetrical if... $\Delta = \pm 1$

Reciprocal if... $g_{12} = -g_{21}$

Hybrid model of a two-port network (used for FET)



Transmission Parameters



$$\begin{aligned} V_1 &= A \cdot V_2 - B \cdot I_2 \\ I_1 &= C \cdot V_2 - D \cdot I_2 \end{aligned} \rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T] \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Calculating / measuring circuits

□ ...like it is for z and y params

Symmetrical if... $A = D$

Reciprocal if... $\Delta = 1$

$(AD - BC = 1)$

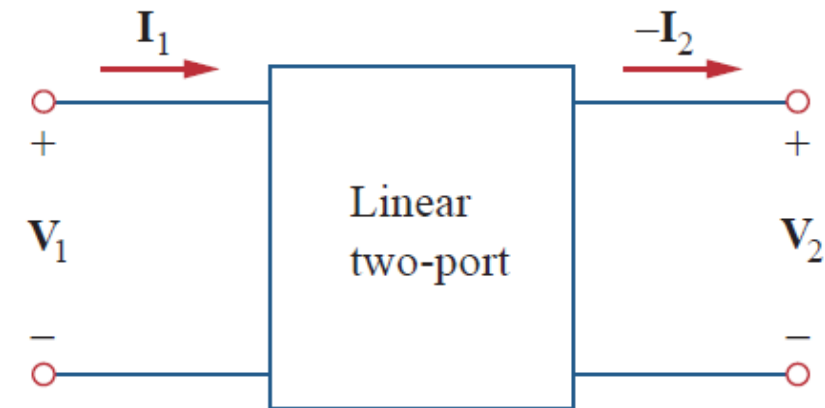
$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \leftarrow \text{OC voltage ratio}$$

$$B = - \left. \frac{V_1}{I_2} \right|_{V_2=0} \leftarrow \text{negative SC transfer impedance}$$

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0} \leftarrow \text{OC transfer admittance}$$

$$D = - \left. \frac{I_1}{I_2} \right|_{V_2=0} \leftarrow \text{negative SC current ratio}$$

Transmission model \rightarrow used for cascaded nw.



Inverse Transmission Parameters



$$\begin{aligned} V_2 &= a \cdot V_1 - b \cdot I_1 \\ I_2 &= c \cdot V_1 - d \cdot I_1 \end{aligned} \rightarrow \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} \rightarrow \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = [t] \cdot \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

Calculating / measuring circuits

□ ...like it is for z and y params

Symmetrical if... $a = d$

Reciprocal if... $\Delta = 1$

$(ad - bc = 1)$

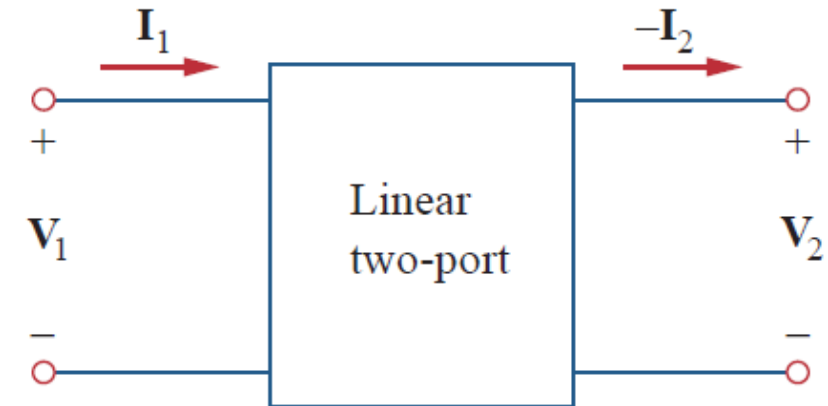
$$a = \left. \frac{V_2}{V_1} \right|_{I_1=0} \leftarrow \text{OC voltage gain}$$

$$b = - \left. \frac{V_2}{I_1} \right|_{V_1=0} \leftarrow \text{negative SC transfer impedance}$$

$$c = \left. \frac{I_2}{V_1} \right|_{I_1=0} \leftarrow \text{OC transfer admittance}$$

$$d = - \left. \frac{I_2}{I_1} \right|_{V_2=0} \leftarrow \text{negative SC current gain}$$

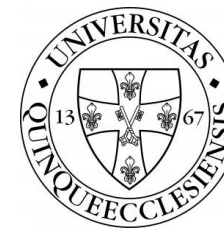
Transmission model \rightarrow used for cascaded nw.





- Two-Port Characteristics
- Relations between Characteristics**
- Two-Port Interconnections
- Bartlett's Bisection Theorem
- Two-Ports with Finite Terminations
- Applications

Relationship between Parameters



If 2 sets exist \rightarrow relation can be calculated

Example process 1 ,y' params from ,z' params

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [z] \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow A^{-1} = \frac{\text{adjoint } A}{\Delta} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc}$$

$$y = z^{-1} = \frac{\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}}{z_{11}z_{22} - z_{12}z_{21}} = \begin{bmatrix} \frac{z_{22}}{\Delta} & -\frac{z_{12}}{\Delta} \\ -\frac{z_{21}}{\Delta} & \frac{z_{11}}{\Delta} \end{bmatrix}$$

Example process 2 ,h' params from ,z' params

$$(1): V_1 = z_{11} \cdot I_1 + z_{12} \cdot I_2$$

$$(2): V_2 = z_{21} \cdot I_1 + z_{22} \cdot I_2$$

$$(2): \rightarrow I_2 = -\frac{z_{21}}{z_{22}} I_1 + \frac{1}{z_{22}} V_2$$

$$(I_2 \rightarrow 1): V_1 = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}} I_1 + \frac{z_{12}}{z_{22}} V_2$$

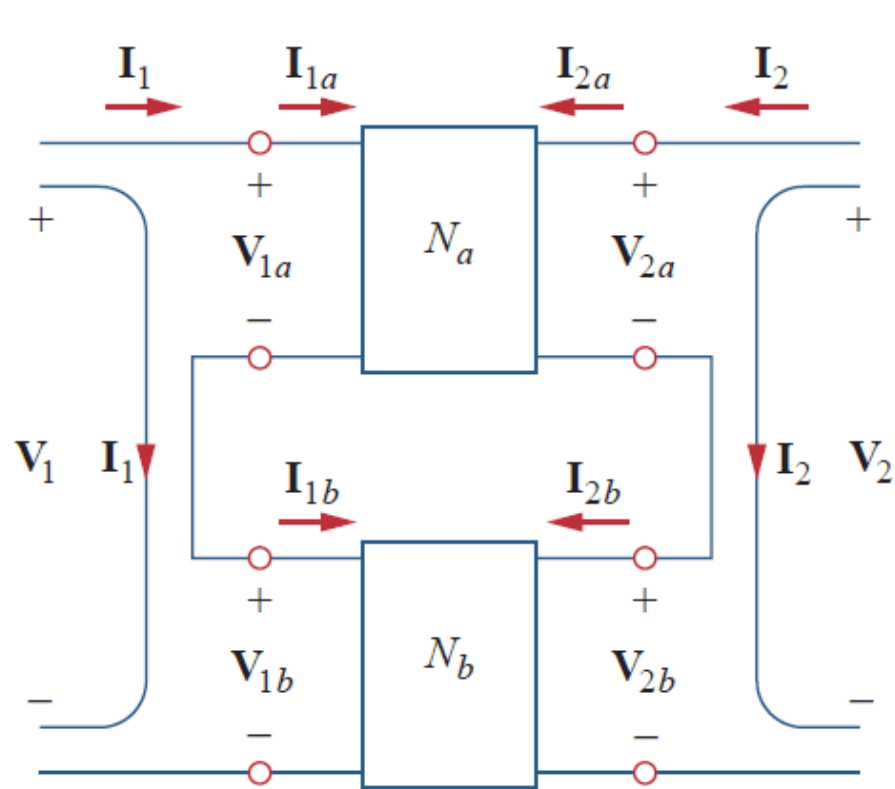
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{\Delta}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Other relationships... https://en.wikipedia.org/wiki/Two-port_network

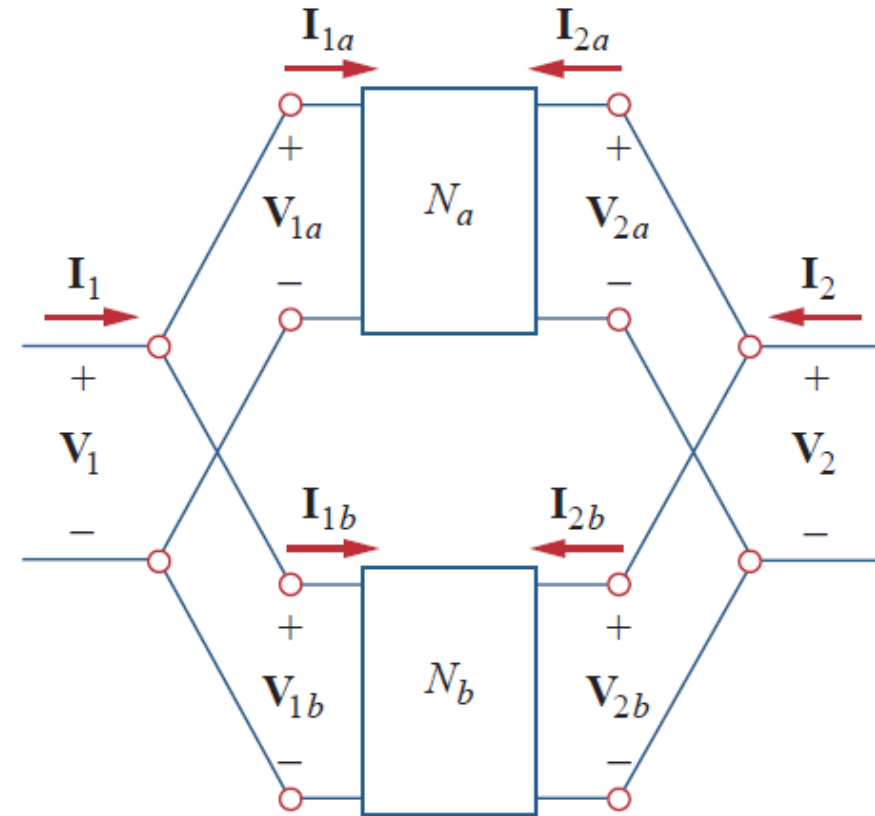


- Two-Port Characteristics
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Interconnection of Two-Ports

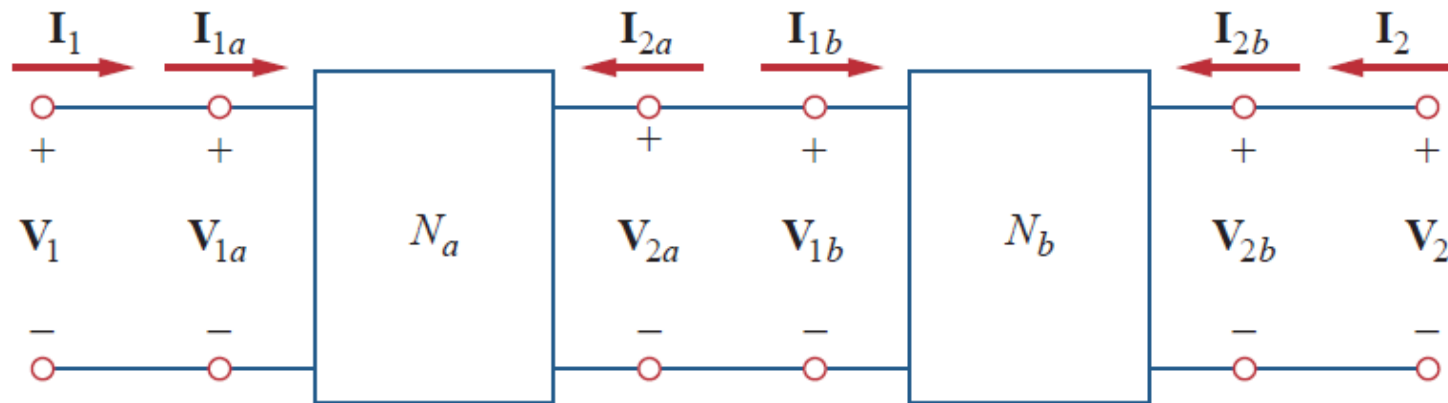


$$\dots \rightarrow [\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b]$$



$$\dots \rightarrow [\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b]$$

Cascaded Interconnection



$$\dots \rightarrow [T] = [T_a] \cdot [T_b]$$

$$\dots \rightarrow [t] = [t_a] \cdot [t_b]$$

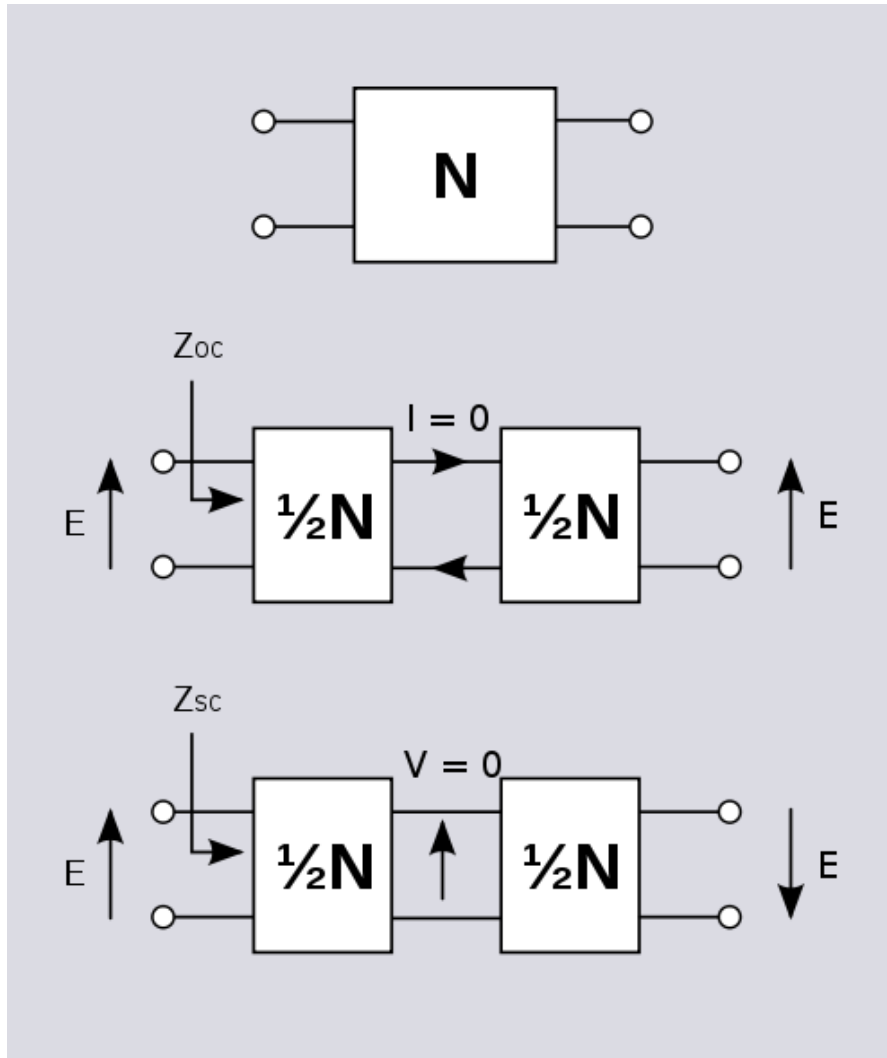
Questions





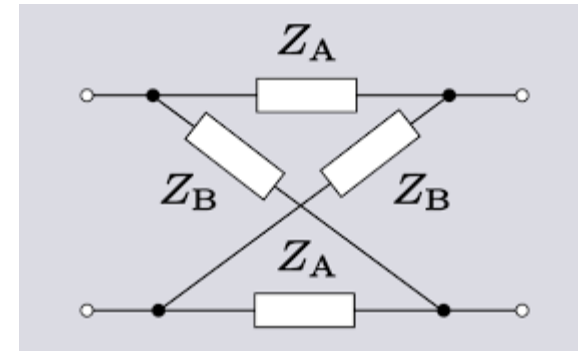
- Two-Port Characteristics
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Bartlett's Bisection Theorem



Symmetric network

- ❑ 2 nw. parameters only
- ❑ 1/2 circuit for calculation
- ❑ Lattice equivalent circuit



$$Z_A = Z_{sc(\text{of halfsection})} = Z_{11} - Z_{12}$$

$$Z_B = Z_{oc(\text{of halfsection})} = Z_{11} + Z_{12}$$

Proving Bartlett's Theorem



$$Z_A = Z_{11} - Z_{12}$$

$$Z_B = Z_{11} + Z_{12}$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = (Z_A + Z_B) \times (Z_A + Z_B) = \frac{Z_A + Z_B}{2}$$

$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2$$

$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2$$

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = (Z_A + Z_B) \times (Z_A + Z_B) = \frac{Z_A + Z_B}{2}$$

$$V_2 = V_B - V_A = \frac{I_1}{2} \cdot Z_B - \frac{I_1}{2} \cdot Z_A = \frac{I_1}{2} \cdot (Z_B - Z_A)$$

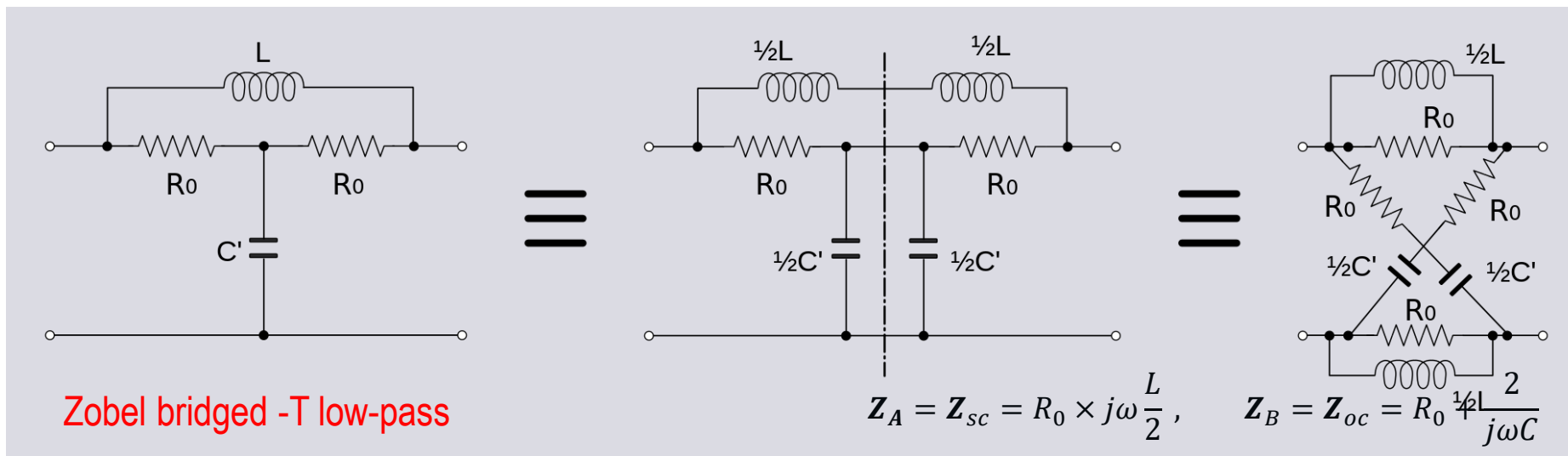
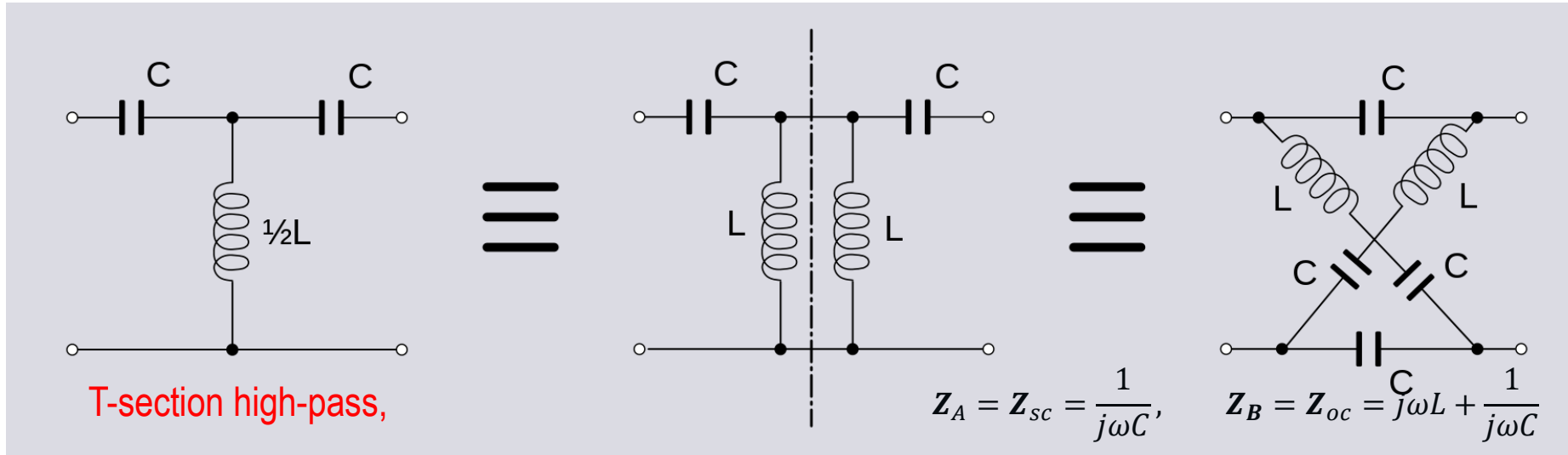
$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = (Z_A + Z_B) \times (Z_A + Z_B) = \frac{Z_A + Z_B}{2}$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = \frac{I_1 \cdot (Z_B - Z_A)}{2 \cdot I_1} = \frac{Z_B - Z_A}{2} = Z_{12}$$

$$V_A = \frac{I_1}{2} \cdot Z_A, \quad V_B = \frac{I_1}{2} \cdot Z_B$$

$$Z_{11} + Z_{12} = \frac{2 \cdot Z_B}{2} = Z_B, \quad Z_{11} - Z_{12} = \frac{2 \cdot Z_A}{2} = Z_A$$

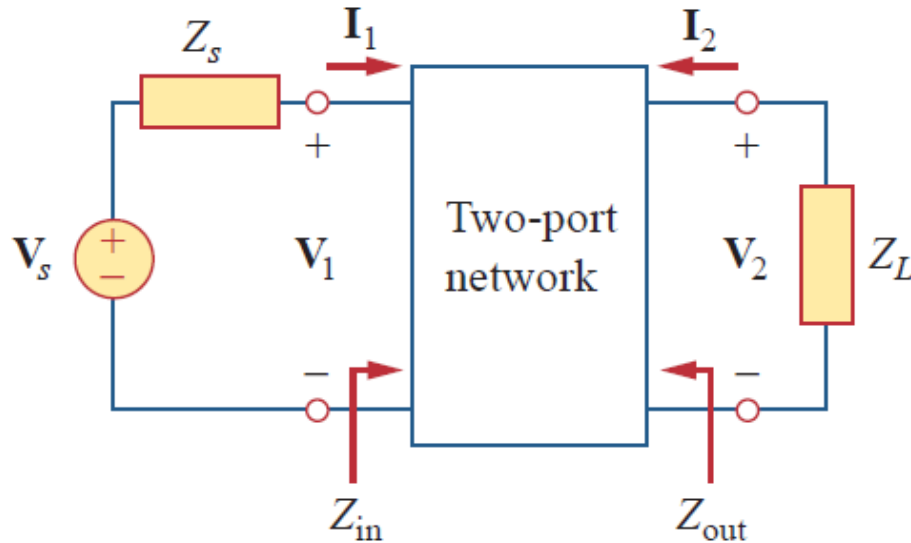
Examples for Lattice equivalents





- ❑ Two-Port Characteristics
- ❑ Relations between Characteristics
- ❑ Two-Port Interconnections
- ❑ Bartlett's Bisection Theorem
- ❑ **Two-Ports with Finite Terminations**
- ❑ Applications

Finite Termination



Any of the out of 6 params set can be used.
(\rightarrow *h* params ... most useful)

Most important parameters...

$$A_v = \frac{V_2(s)}{V_1(s)} \leftarrow \text{voltage gain}$$

$$A_i = \frac{I_2(s)}{I_1(s)} \leftarrow \text{current gain}$$

$$z_{in} = \frac{V_1(s)}{I_1(s)} \leftarrow \text{input imp.}$$

$$z_{out} = \frac{V_2(s)}{I_2(s)} \Big|_{V_s=0} \leftarrow \text{output imp.}$$

Terminated Transmission Line



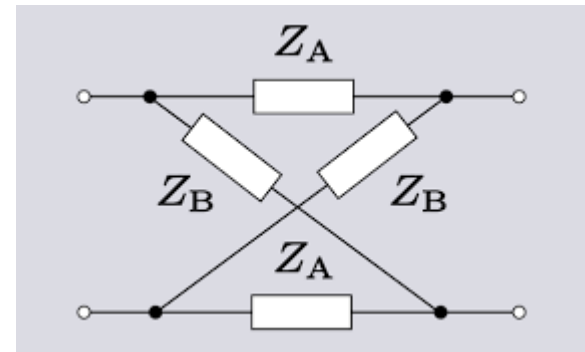
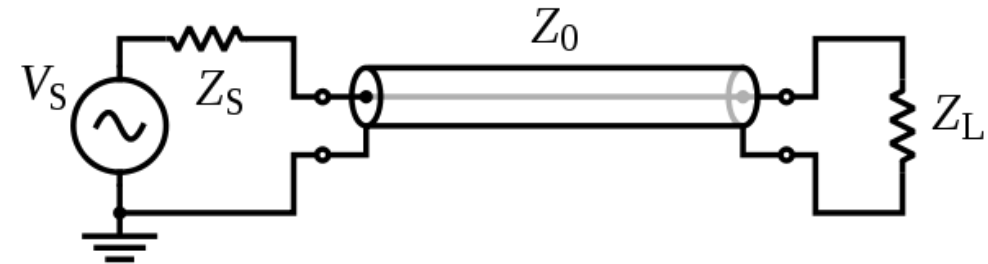
Symmetric network → 2 network parameters only → [wave impedance], [wave transmission coefficient]

Wave impedance (characteristic impedance)

$$Z_{01}(def) = \left. \frac{V_1}{I_1} \right|_{refl=0} = \sqrt{Z_{1oc} \cdot Z_{1sc}} = \sqrt{\frac{Z_{11}}{Y_{11}}}$$

$$Z_{02}(def) = \left. \frac{V_2}{I_2} \right|_{refl=0} = \sqrt{Z_{2oc} \cdot Z_{2sc}} = \sqrt{\frac{Z_{22}}{Y_{22}}}$$

(https://en.wikipedia.org/wiki/Characteristic_impedance)



Properties

- ❑ Ideal transmission line → $Z_0 = Z_0$ (real)
- ❑ Transparent network → $Z_L = Z_0 \rightarrow Z_{in} = Z_0$
- ❑ Refl. free connection → $Z_L = Z_S = Z_0$
- ❑ Lattice equivalent → $Z_0 = \sqrt{Z_A \cdot Z_B}$

$$Z_0 = \sqrt{Z_{OC} \cdot Z_{SC}} \leftarrow Z_{OC} = \frac{Z_A + Z_B}{2}, \quad Z_{SC} = 2(Z_A \times Z_B)$$

Terminated Transmission Line



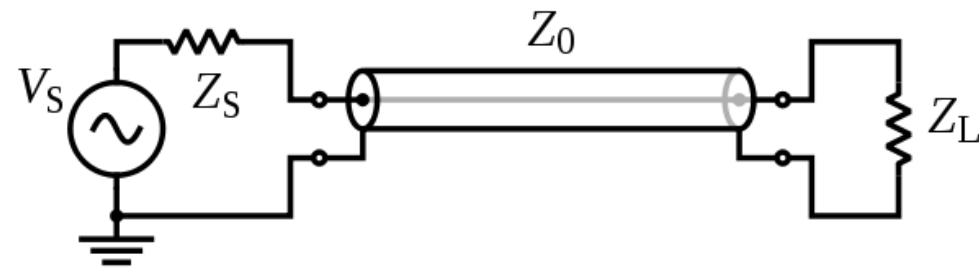
Wave propagation coefficient (*propagation constant, transmission coefficient*)

$$\Gamma = \sqrt{\frac{P_S}{P_2}}, \quad P_S = \frac{V_S^2}{4Z_S} \text{ (max. PWR transfer),} \quad P_2 = \frac{V_2^2}{Z_L}$$

$$\text{In general} \rightarrow \Gamma(\omega) = \frac{V_S}{2V_2} \sqrt{\frac{Z_L}{Z_S}}$$

$$Z_S = Z_L = Z_0 \rightarrow \Gamma(\omega) = \Gamma_0(\omega) = \frac{V_S}{2V_2} \sqrt{\frac{Z_0}{Z_0}}$$

$$\rightarrow \Gamma_0(\omega) = \frac{V_S}{2V_2} = \frac{V_1}{V_2} = \frac{1}{H(\omega)}$$



❑ Wave transmission (*hullámátvitel*) $\rightarrow g_0 = \ln \Gamma_0 = \ln(\Gamma_0 e^{jb_0}) = a_0 + jb_0$

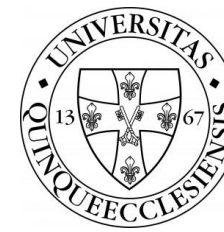
❑ Wave attenuation (*hullámcsillapítás*) $\rightarrow a_0 = \ln \Gamma_0$ [Np]

❑ Wave phase (*hullámforgatás*) $\rightarrow b_0$ [rad] https://en.wikipedia.org/wiki/Propagation_constant

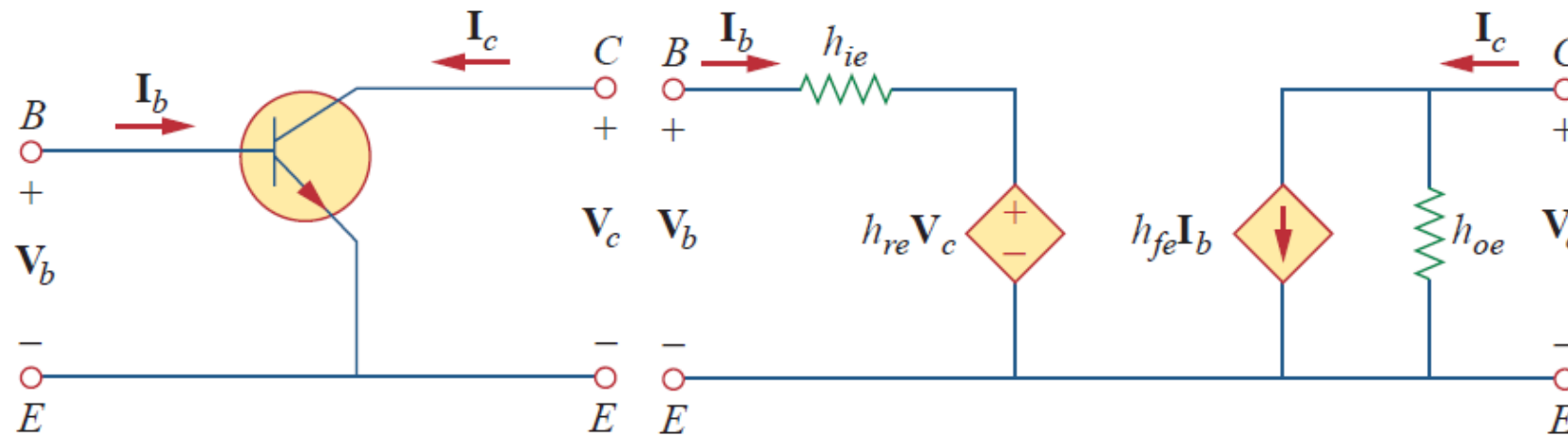


- Two-Port Characteristics
- Relations between Characteristics
- Two-Port Interconnections
- Bartlett's Bisection Theorem
- Two-Ports with Finite Terminations
- Applications**

App. – Isolating Amplifier



Goal is to find the 'most important params' voltage and current gains, input and output impedances.



$$V_b = h_{ie} \cdot I_b + h_{re} \cdot V_c$$

$$I_c = h_{fe} \cdot I_b + h_{oe} \cdot V_c$$

$h_{ie} (= h_{11}) \rightarrow$

Base input impedance

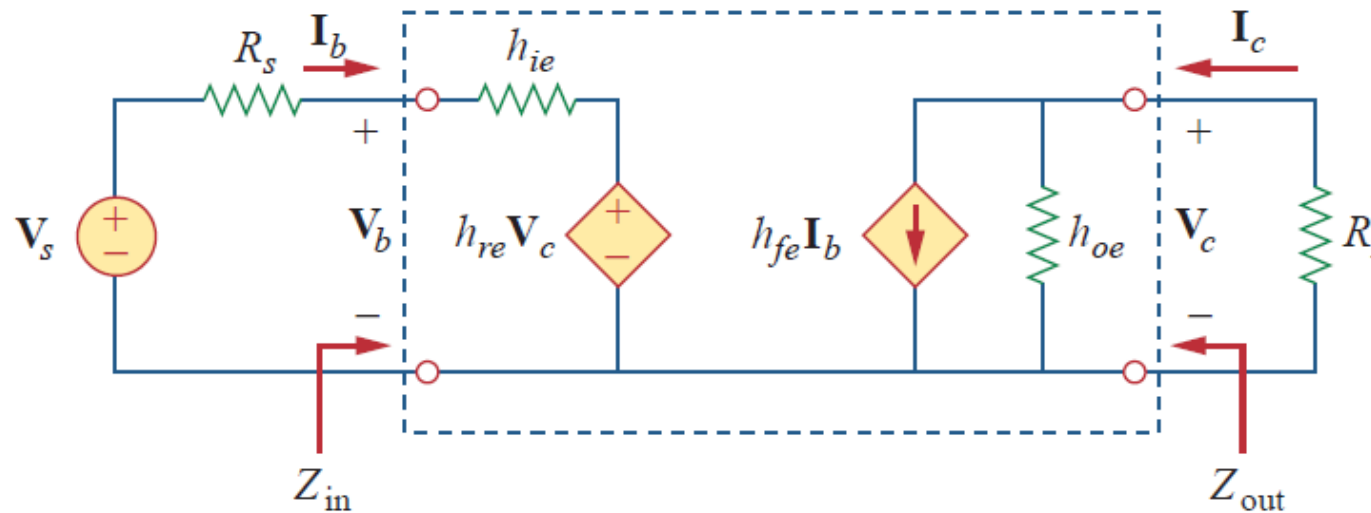
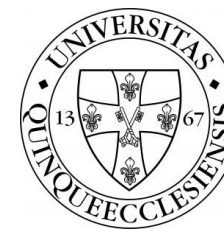
$h_{oe} (= h_{22}) \rightarrow$

Output admittance

$h_{re} (= h_{12}) \rightarrow$ Reverse voltage feedback ratio

$h_{fe} (= h_{21}) \rightarrow$ Base – collector current gain

App. – Isolating Amplifier



$$\dots \rightarrow A_v = \frac{V_c}{V_b} = \frac{-h_{fe}R_L}{h_{ie} + (h_{ie}h_{oe} - h_{re}h_{fe})R_L}$$

$$\dots \rightarrow z_{in} = \frac{V_b}{I_b} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L}$$

$$\dots \rightarrow A_i = \frac{I_c}{I_b} = \frac{h_{fe}}{1 + h_{oe}R_L}$$

$$\dots \rightarrow z_{out} = \left. \frac{V_C}{I_c} \right|_{V_S=0} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}}$$

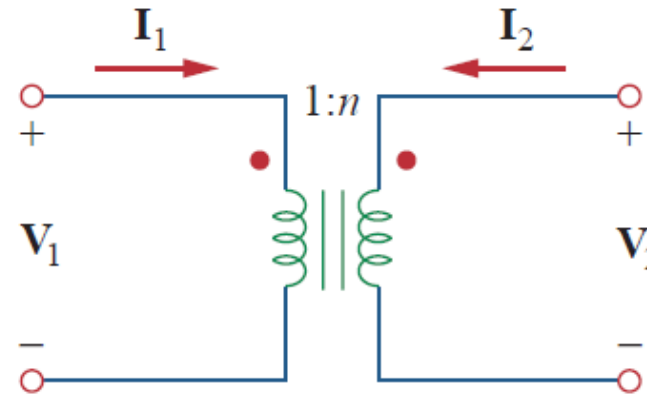
App. – Ideal Transformer



Ideal transformer → has no **z** params (4th supplementary LTI element supplementing R-L-C)

Transmission characteristic

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1/n & 0 \\ 0 & -n \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$



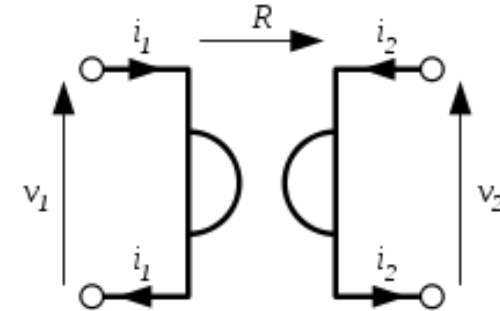
Energy-free device! $P = V_1 \cdot I_1 + V_2 \cdot I_2 = V_1 \cdot I_1 + n \cdot V_1 \left(-\frac{1}{n} I_1 \right) = 0$

Impedance transformer $Z_{in} = \frac{V_1}{I_1} = \frac{V_2/n}{n \cdot I_2} = \frac{1}{n^2} \cdot \frac{V_2}{I_2} \rightarrow Z_{in} = \frac{1}{n^2} \cdot Z_L$

App. – Gyrator



$$\begin{aligned} V_1 &= -R \cdot I_2 \\ V_2 &= R \cdot I_1 \end{aligned} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & -R \\ R & 0 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



☐ Energy-free device →

$$P = V_1 \cdot I_1 + V_2 \cdot I_2 = V_1 \cdot I_1 + R \cdot I_1 \left(-\frac{V_1}{R} \right) = V_1 \cdot I_1 - V_1 \cdot I_1 = 0$$

Bernard Tellegen (1900-1990)

- ☐ Dutch electrical engineer
- ☐ Philips Laboratory
- ☐ Pentode(1926)
- ☐ Gyrator (1948)
- ☐ Tellegen's theorem (1952)

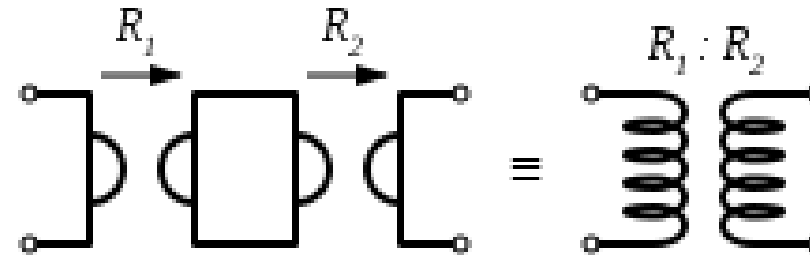
- ☐ Gyrator resistance R is the only parameter.
- ☐ Different z_{12} and z_{21} → non-reciprocal device
- ☐ Loaded gyrator

$$Z_{in} = \frac{V_1}{I_1} = \frac{-R \cdot I_2}{V_2/R} = -\frac{R^2}{V_2/I_2} = -\frac{R^2}{Z_L}$$

App. – Gyrator



2 gyrators → ideal transformer
(3rd candidate for 4th element?)



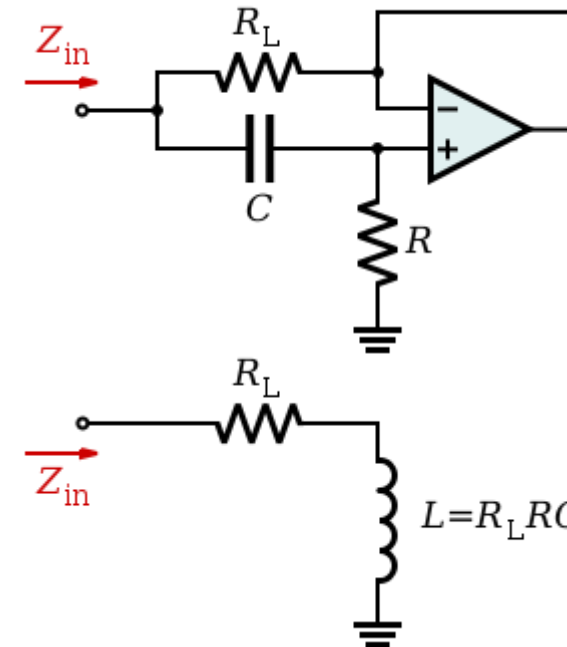
Impedance inversion

$$Z_{in} = (R_L + j\omega R_L RC) \times \left(R + \frac{1}{j\omega C} \right)$$

$\left(\begin{matrix} R \rightarrow \text{high} \\ \omega \rightarrow \text{low} \end{matrix} \right) \rightarrow RC \text{ negligible in parallel}$

$$Z_{in} = (R_L + j\omega R_L RC)$$

Small, ideal inductor at low frequency.



Questions

