DR. GYURCSEK ISTVÁN

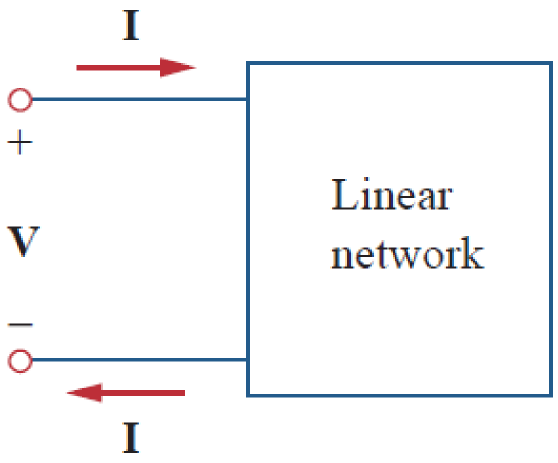
Two-Port Networks

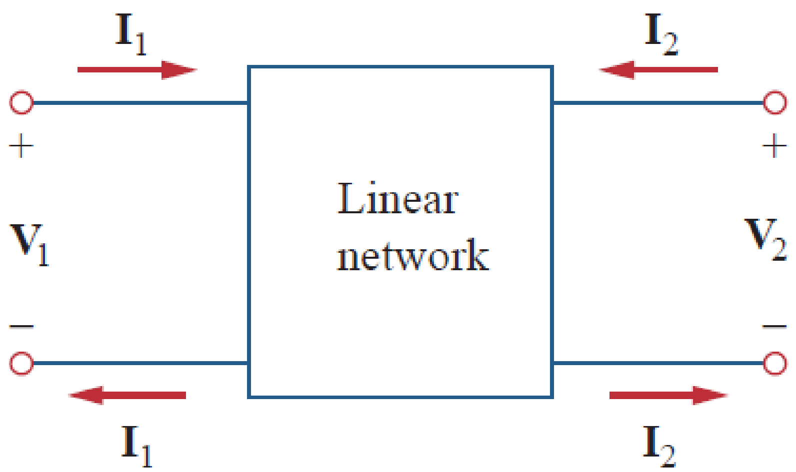
Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*

Introduction

Network analysis → determ. of *V, I* at certain nodes *(terminals)*





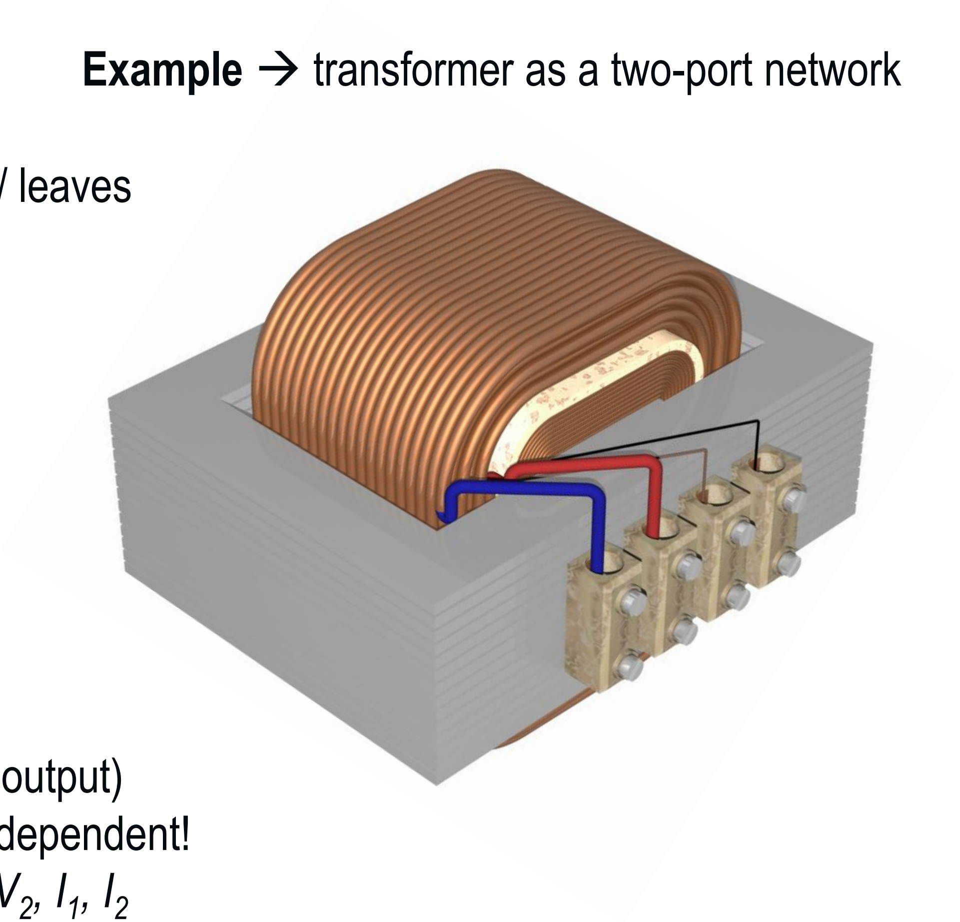
Port
- ❑ Pair of terminals
- ❑ Same current enters / leaves

One-port
- ❑ Element / device
- ❑ i.e. *R, L, C,*

Two-port network
- ❑ Two separate ports
- ❑ Input & output

Two-port characteristic
- ❑ 4 params (2 input, 2 output)
- ❑ Only 2 of them are independent!
- ❑ Relationship b/w V_1, V_2, I_1, I_2

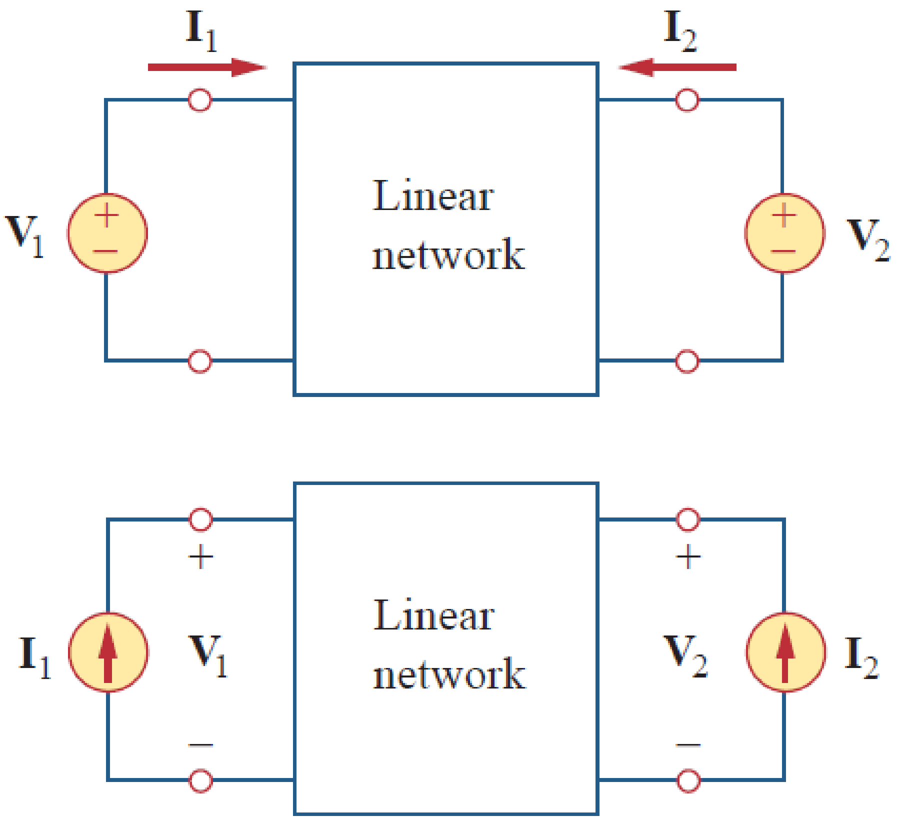
Example → transformer as a two-port network



Progress in Content

- **Two-Port Charactristics**
- Relations between Characteristics
- Two-Port Interconnections
- Bartlett’s Bisection Theorem
- Two-Ports with Finite Terminations
- Applications

Two-Port Characteristics



Two-port variables

- ❑ V_1, V_2, I_1, I_2
- ❑ Two of them are independent *(only!)*
- ❑ Relationship b/w 4 variables

Network driven by

- ❑ voltage sources
- ❑ current sources
- ❑ *(mix of them)*

Two-port parameters

[input] [output] [transfer] → OC & SC params

- ❑ Six characteristics *(some of them might be missing)*
- ❑ impedance – admittance params
- ❑ hybrid – inverse hybrid params
- ❑ transmission – inverse transmission params

Impedance Parameters *(Open-Circuit Impedance Params)* 1

$$\begin{matrix} V_1 = z_{11}\cdot I_1 + z_{12}\cdot I_2 \\ V_2 = z_{21}\cdot I_1 + z_{22}\cdot I_2 \end{matrix} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [z]\cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

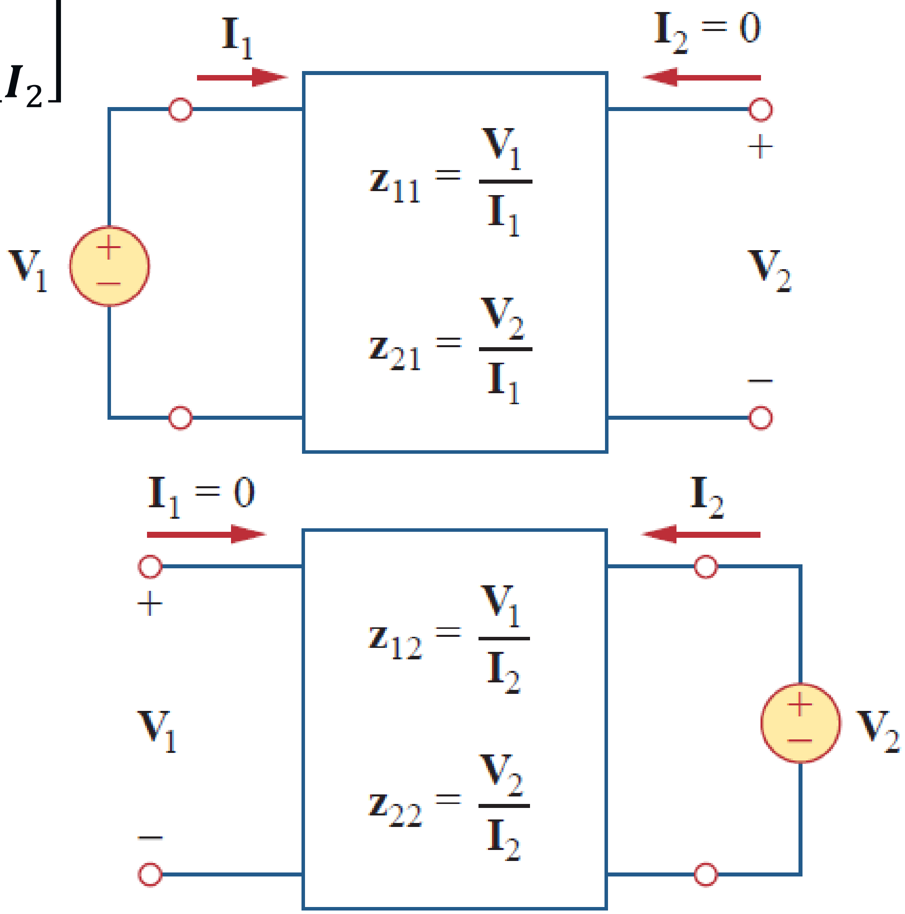
$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \leftarrow OC\ input\ impedance$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \leftarrow OC\ transfer\ impedance\ (port\ 2 \rightarrow port\ 1)$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \leftarrow OC\ transfer\ impedance (port\ 1 \rightarrow port\ 2)$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \leftarrow OC\ output\ impedance$$

Calculating / meauring circuits



Impedance Parameters *(Open-Circuit Impedance Params)* 2

$$V_1 = z_{11} \cdot I_1 + z_{12} \cdot I_2$$

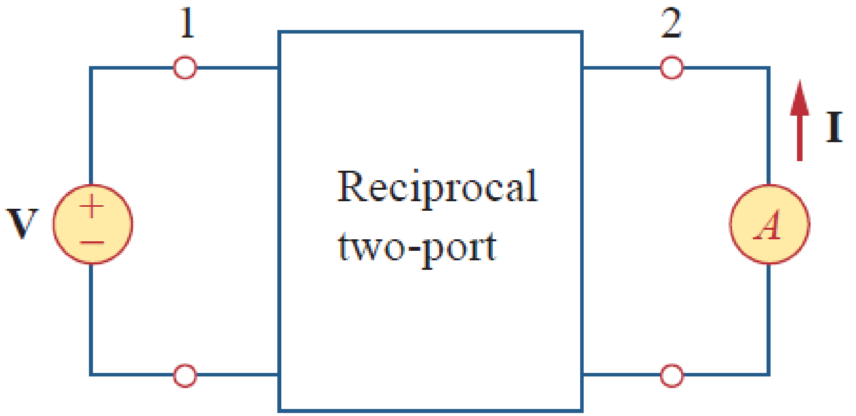
$$V_2 = z_{21} \cdot I_1 + z_{22} \cdot I_2$$

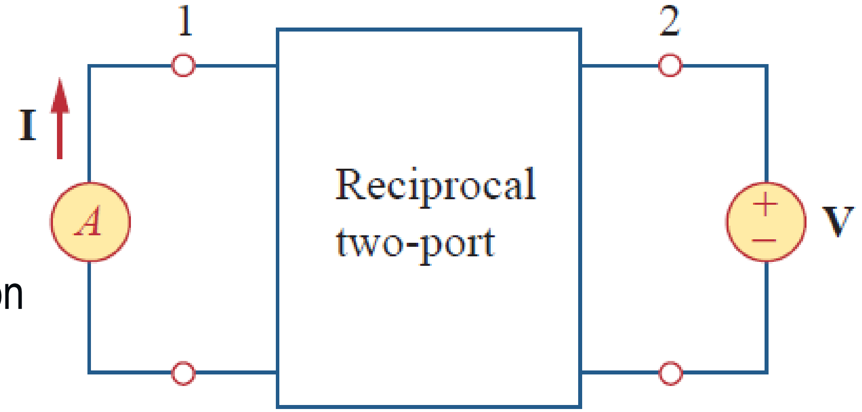
Symmetrical two-port network

- ❑ ‚mirrorlike' symmetry
- ❑ two similar halves
- ❑ $z_{11} = z_{22}$

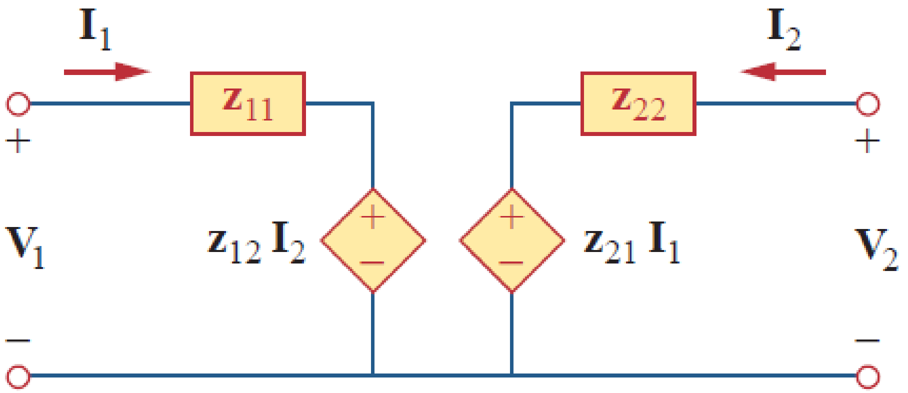
Reciprocal two-port network

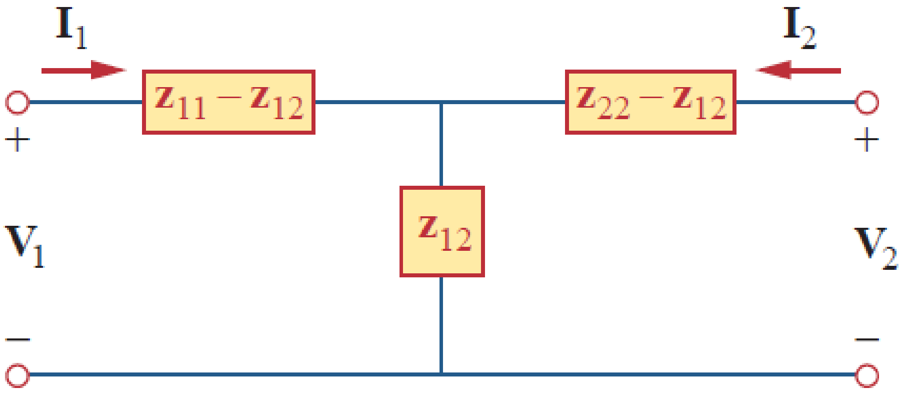
- ❑ linear network
- ❑ no dependent sources
- ❑ Interchanged ports of excitation & response → $z_{12} = z_{21}$



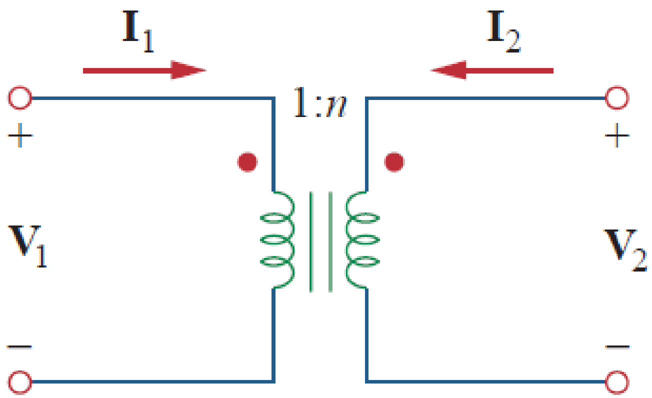


General equivalent circuit





Equivalent circuit for reciprocal network



No z parameters for some two-ports!
i.e. ideal transformer → impossible to express V_1 and V_2 in terms of I_1 and I_2.

$$\boldsymbol{V}_1 = \frac{1}{n}\boldsymbol{V}_2\,, \qquad \boldsymbol{I}_1 = -n\boldsymbol{I}_2$$

Admittance Parameters *(Short Circuit Admittance Params)* 1

$$\begin{aligned} I_1 &= y_{11}\cdot V_1 + y_{12}\cdot V_2 \\ I_2 &= y_{21}\cdot V_1 + y_{22}\cdot V_2 \end{aligned} \rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [y]\cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

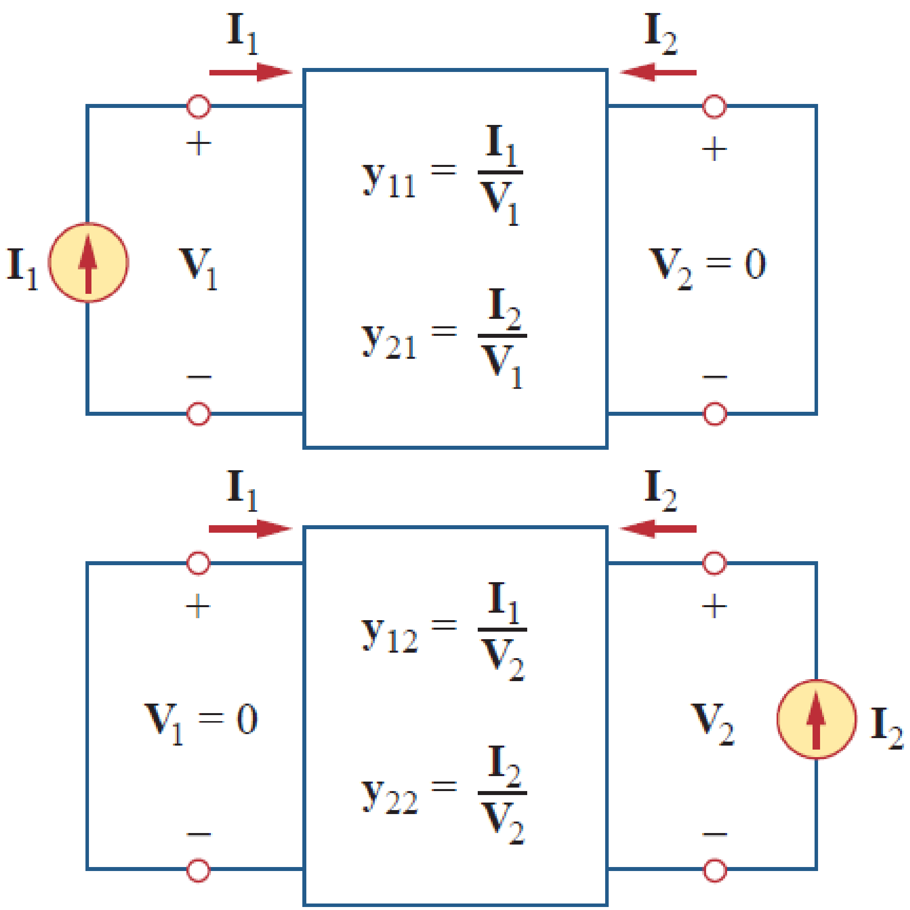
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \leftarrow SC\ input\ admittance$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \leftarrow SC\ transfer\ admittance\ (port\ 2 \rightarrow port\ 1)$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \leftarrow SC\ transfer\ admittance\ (port\ 1 \rightarrow port\ 2)$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \leftarrow SC\ output\ admittance$$

Calculating / meauring circuits



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Admittance Parameters *(Short Circuit Admittance Params)* 2

$$I_1 = y_{11} \cdot V_1 + y_{12} \cdot V_2$$

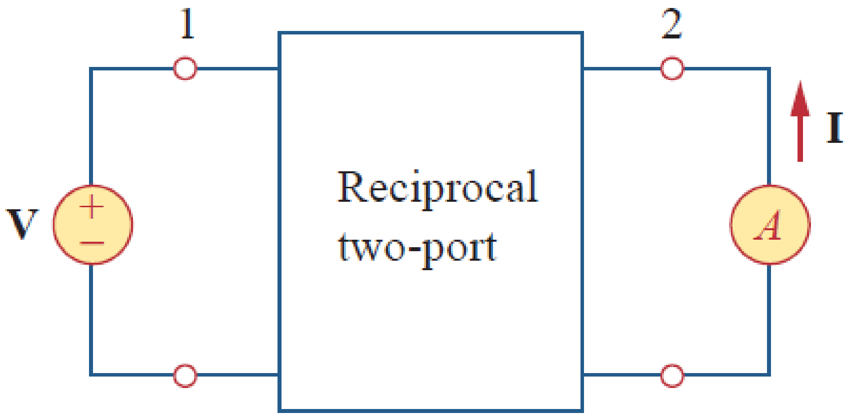
$$I_2 = y_{21} \cdot V_1 + y_{22} \cdot V_2$$

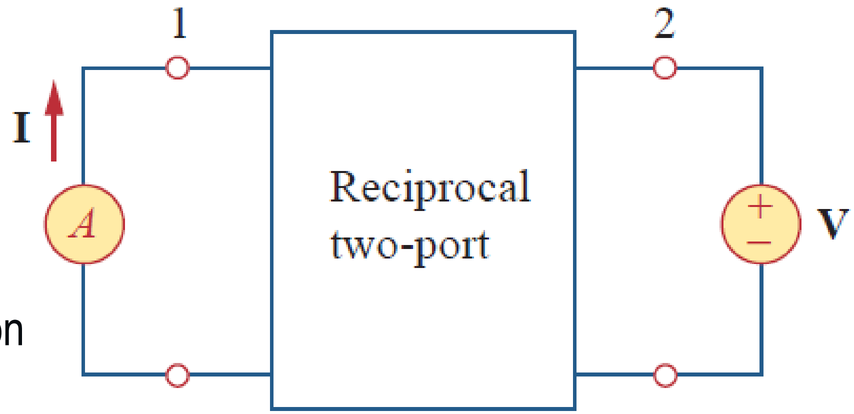
Symmetrical two-port network

- ‚mirrorlike' symmetry
- two similar halves
- $y_{11} = y_{22}$

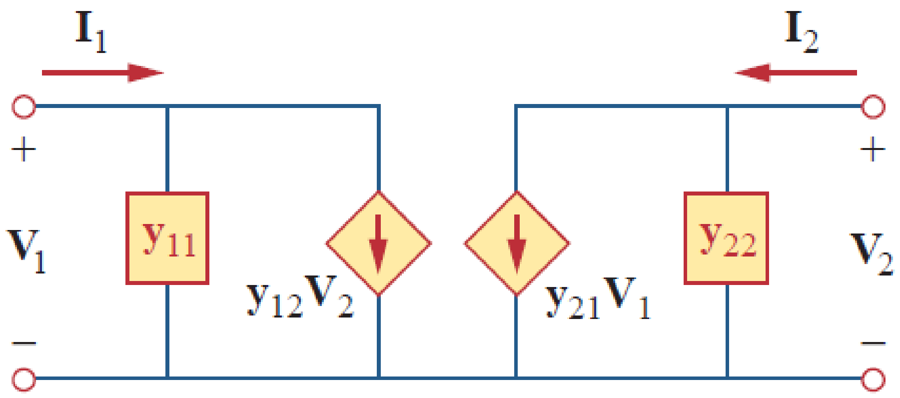
Reciprocal two-port network

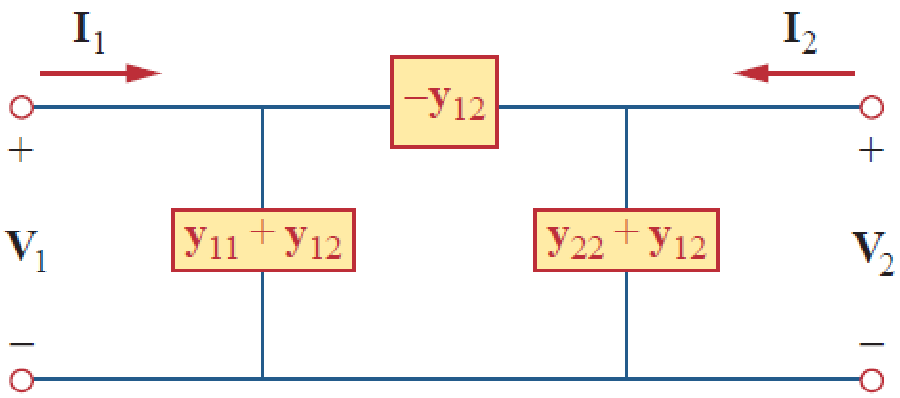
- linear network
- no dependent sources
- Interchanged ports of excitation & response → $y_{12} = y_{21}$





General equivalent circuit





Equivalent circuit for reciprocal network

Hybrid Parameters *(h Params)*

$$\begin{aligned} V_1 &= h_{11}\cdot I_1 + h_{12}\cdot V_2 \\ I_2 &= h_{21}\cdot I_1 + h_{22}\cdot V_2 \end{aligned} \rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = [h] \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \leftarrow SC\ imput\ impedance$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \leftarrow OC\ reverse\ voltage\ gain$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \leftarrow SC\ forward\ current\ gain$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \leftarrow OC\ output\ admittance$$

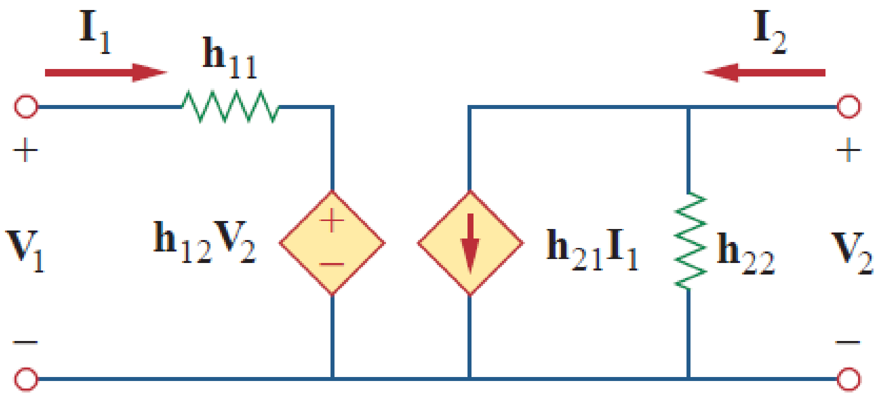
Calculating / meauring circuits

- ❑ *…like it is for z and y params*

Symmetrical if… $\Delta = \pm 1$

Reciprocal if… $h_{12} = -h_{21}$

Hybrid model of a two-port network *(used for BJT)*



Inverse Hybrid Parameters *(g Params)*

$$I_1 = g_{11} \cdot V_1 + g_{12} \cdot I_2$$
$$V_2 = g_{21} \cdot V_1 + g_{22} \cdot I_2 \rightarrow \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \rightarrow \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [g] \cdot \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \leftarrow OC\ imput\ admittance$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \leftarrow SC\ reverse\ current\ gain$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \leftarrow OC\ forward\ voltage\ gain$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \leftarrow SC\ output\ impedance$$

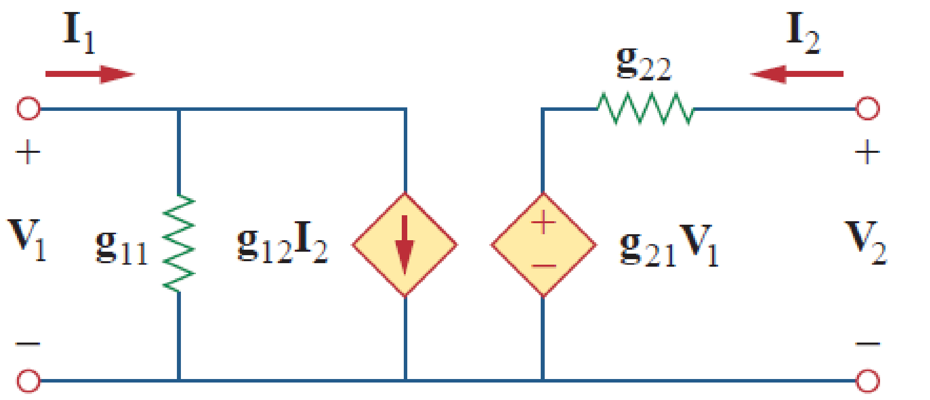
Calculating / meauring circuits

- ❑ *…like it is for z and y params*

Symmetrical if… $\Delta = \pm 1$

Reciprocal if… $g_{12} = -g_{21}$

Hybrid model of a two-port network *(used for FET)*



Transmission Parameters

$$V_1 = A \cdot V_2 - B \cdot I_2$$
$$I_1 = C \cdot V_2 - D \cdot I_2$$
$$\rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T] \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$A = \frac{V_1}{V_2}\bigg|_{I_2=0} \leftarrow OC\ voltage\ ratio$$

$$B = -\frac{V_1}{I_2}\bigg|_{V_2=0} \leftarrow negative\ SC\ transfer\ impedance$$

$$C = \frac{I_1}{V_2}\bigg|_{I_2=0} \leftarrow OC\ transfer\ admittance$$

$$D = -\frac{I_1}{I_2}\bigg|_{V_2=0} \leftarrow negative\ SC\ current\ ratio$$

Calculating / meauring circuits

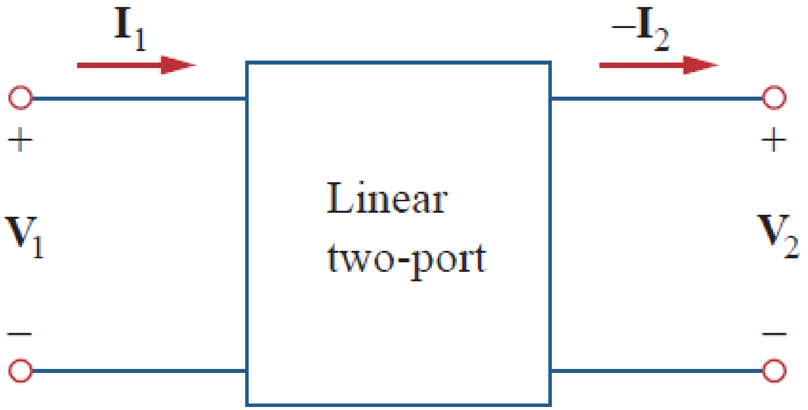
- ❑ *...like it is for z and y params*

Symmetrical if... $A = D$

Reciprocal if... $\Delta = 1$

$(AD - BC = 1)$

Transmission model → *used for cascaded nw.*



Inverse Transmission Parameters

$$\begin{aligned} V_2 &= a \cdot V_1 - b \cdot I_1 \\ I_2 &= c \cdot V_1 - d \cdot I_1 \end{aligned} \rightarrow \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} \rightarrow \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = [t] \cdot \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

Calculating / meauring circuits

- ❑ *...like it is for z and y params*

Symmetrical if... $a = d$

Reciprocal if... $\Delta = 1$

$(ad - bc = 1)$

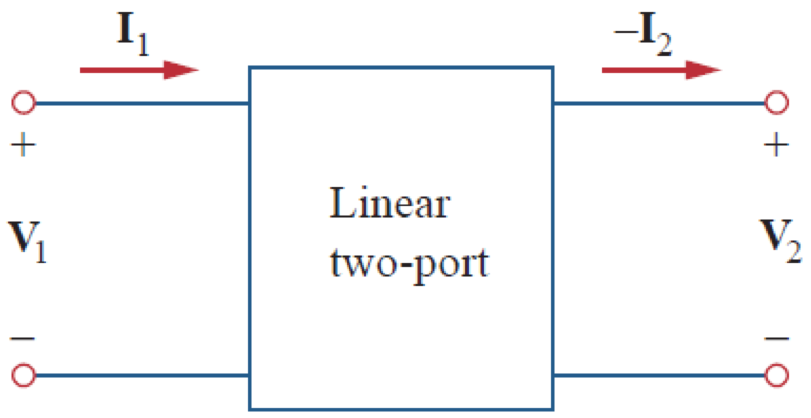
$$a = \left.\frac{V_2}{V_1}\right|_{I_1=0} \leftarrow OC\ voltage\ gain$$

$$b = -\left.\frac{V_2}{I_1}\right|_{V_1=0} \leftarrow negative\ SC\ transfer\ impedance$$

$$c = \left.\frac{I_2}{V_1}\right|_{I_1=0} \leftarrow OC\ transfer\ admittance$$

$$d = -\left.\frac{I_2}{I_1}\right|_{V_2=0} \leftarrow negative\ SC\ current\ gain$$

Transmission model → *used for cascaded nw.*



Progress in Content

- ❑ Two-Port Charactristics
- ❑ **Relations between Characteristics**
- ❑ Two-Port Interconnections
- ❑ Bartlett’s Bisection Theorem
- ❑ Two-Ports with Finite Terminations
- ❑ Applications

Relationship between Parameters - Examples

If 2 sets exist → relation can be calculated

Example process 1 ‚y' params from ‚z' params

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [z] \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow A^{-1} = \frac{adjoint\ A}{\Delta} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc}$$

$$y = z^{-1} = \frac{\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}}{z_{11}z_{22} - z_{12}z_{21}} = \begin{bmatrix} \frac{z_{22}}{\Delta} & -\frac{z_{12}}{\Delta} \\ -\frac{z_{21}}{\Delta} & \frac{z_{11}}{\Delta} \end{bmatrix}$$

Example process 2 ‚h' params from ‚z' params

$$(1):\ V_1 = z_{11} \cdot I_1 + z_{12} \cdot I_2$$

$$(2):\ V_2 = z_{21} \cdot I_1 + z_{22} \cdot I_2$$

$$(2):\ \rightarrow I_2 = -\frac{z_{21}}{z_{22}} I_1 + \frac{1}{z_{22}} V_2$$

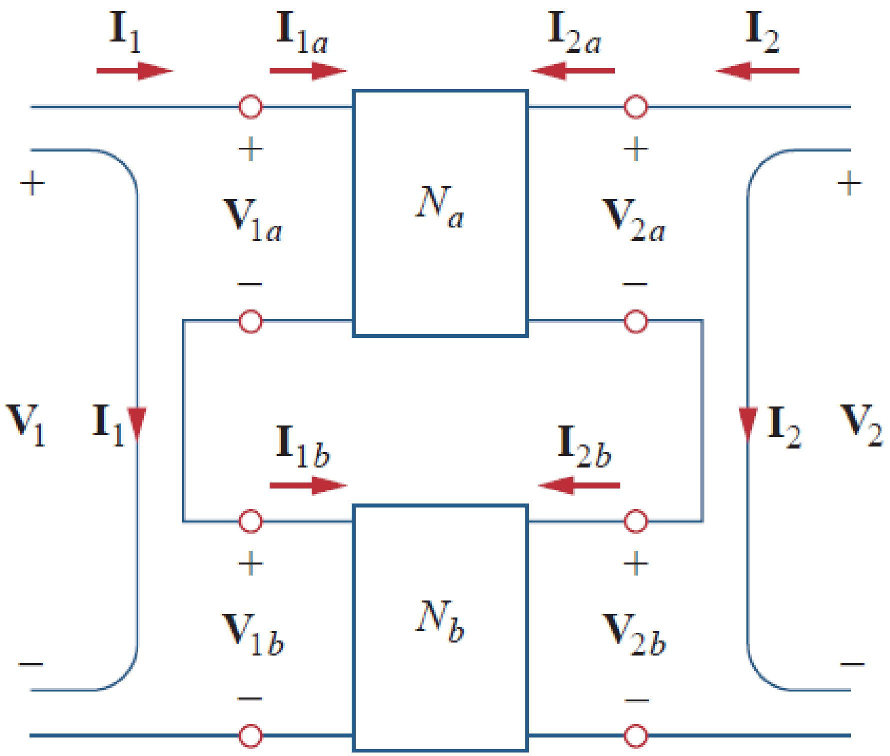
$$(I_2 \rightarrow 1):\ V_1 = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}} I_1 + \frac{z_{12}}{z_{22}} V_2$$

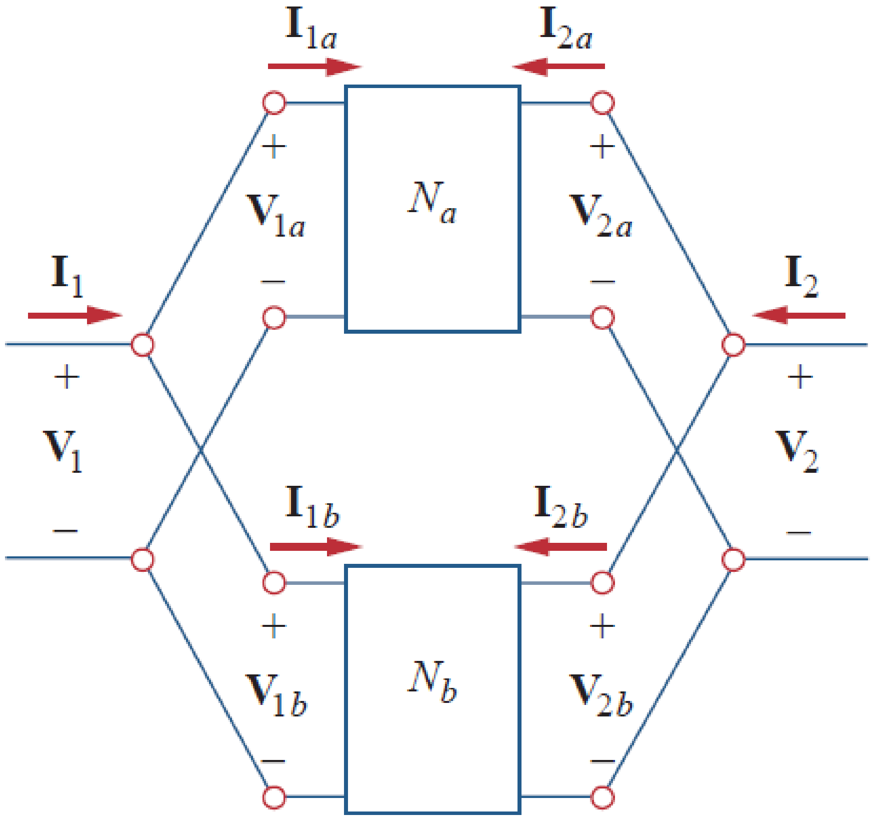
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{\Delta}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Other relationships… *https://en.wikipedia.org/wiki/Two-port_network*

Progress in Content

Series and Parallel Interconnection of Two-Port Networks

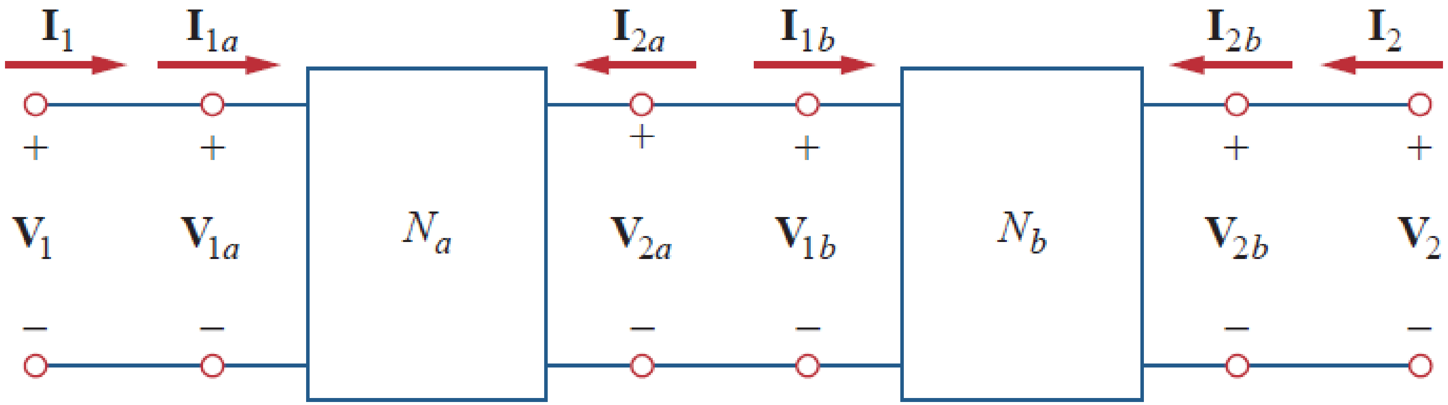




… $\rightarrow [\boldsymbol{z}] = [\boldsymbol{z}_a] + [\boldsymbol{z}_b]$

… $\rightarrow [\boldsymbol{y}] = [\boldsymbol{y}_a] + [\boldsymbol{y}_b]$

Cascaded Interconnection of Two-Port Networks



... $\rightarrow [\boldsymbol{T}] = [\boldsymbol{T}_a] \cdot [\boldsymbol{T}_b]$

... $\rightarrow [\boldsymbol{t}] = [\boldsymbol{t}_a] \cdot [\boldsymbol{t}_b]$

Questions

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- ❑ Two-Port Charactristics
- ❑ Relations between Characteristics
- ❑ Two-Port Interconnections
- ❑ **Bartlett’s Bisection Theorem**
- ❑ Two-Ports with Finite Terminations
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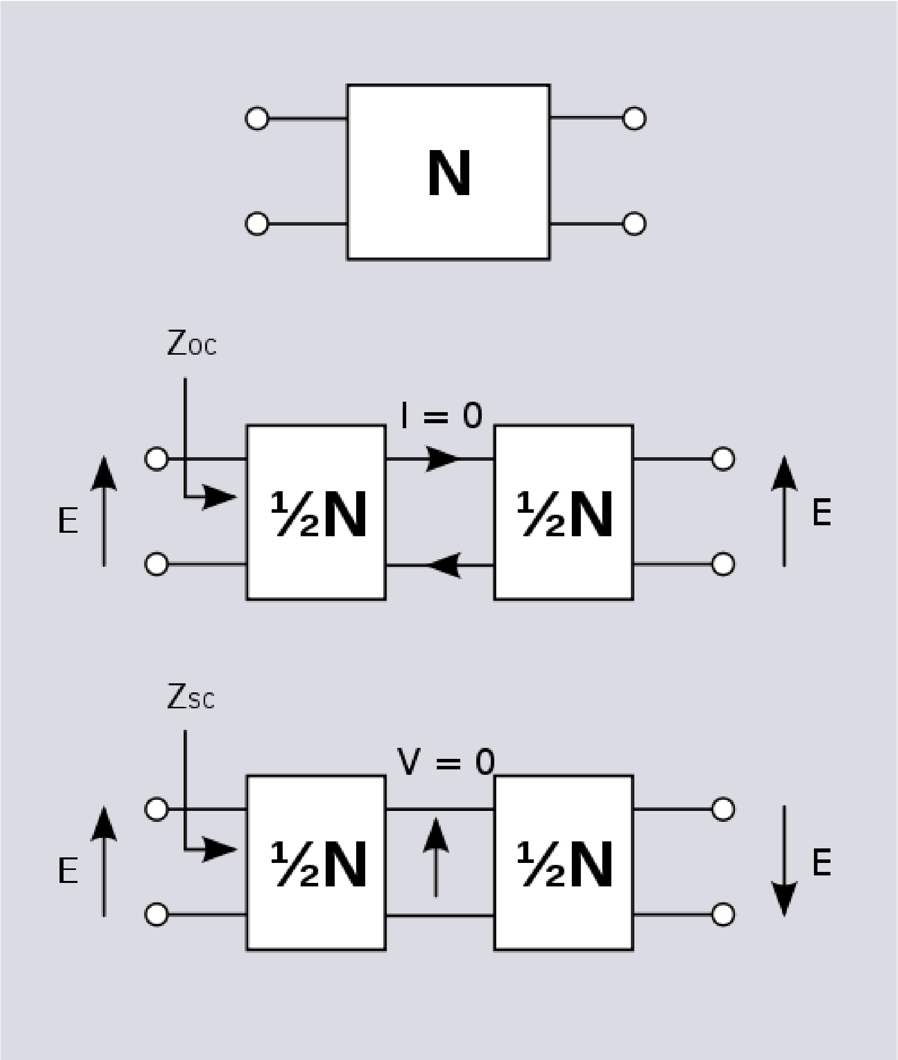
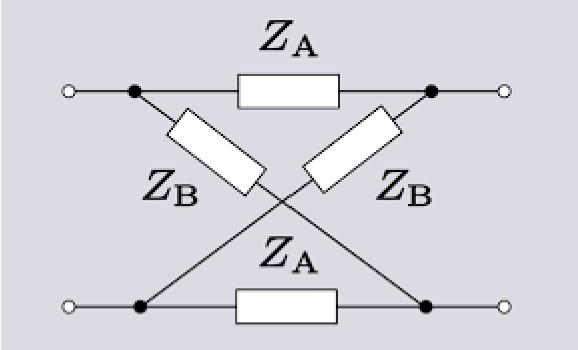
Bartlett's BisectionTheorem - Lattice Network Equivalent



Symmetric network

- ❑ 2 nw. parameters only
- ❑ ½ circuit for calculation
- ❑ Latice equivalent circuit



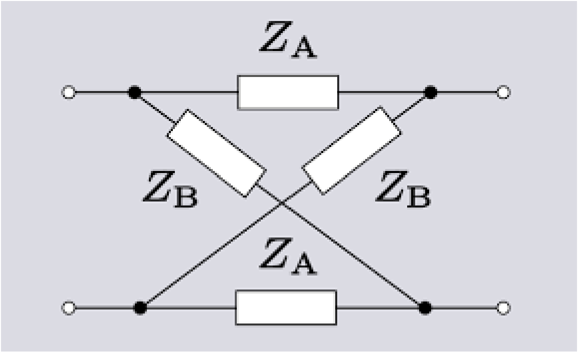
$$Z_A = Z_{sc(of\ halfsection)} = Z_{11} - Z_{12}$$

$$Z_B = Z_{oc(of\ halfsection)} = Z_{11} + Z_{12}$$

Proving Bartlett’s BisectionTheorem

$$Z_A = Z_{11} - Z_{12}$$

$$Z_B = Z_{11} + Z_{12}$$



$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2$$

$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = (Z_A + Z_B) \times (Z_A + Z_B) = \frac{Z_A + Z_B}{2}$$

$$V_2 = V_B - V_A = \frac{I_1}{2} \cdot Z_B - \frac{I_1}{2} \cdot Z_A = \frac{I_1}{2} \cdot (Z_B - Z_A)$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = (Z_A + Z_B) \times (Z_A + Z_B) = \frac{Z_A + Z_B}{2}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{I_1 \cdot (Z_B - Z_A)}{2 \cdot I_1} = \frac{Z_B - Z_A}{2} = Z_{12}$$

$$V_A = \frac{I_1}{2} \cdot Z_A \,, \qquad V_B = \frac{I_1}{2} \cdot Z_B$$

$$Z_{11} + Z_{12} = \frac{2 \cdot Z_B}{2} = Z_B \,, \qquad Z_{11} - Z_{12} = \frac{2 \cdot Z_A}{2} = Z_A$$

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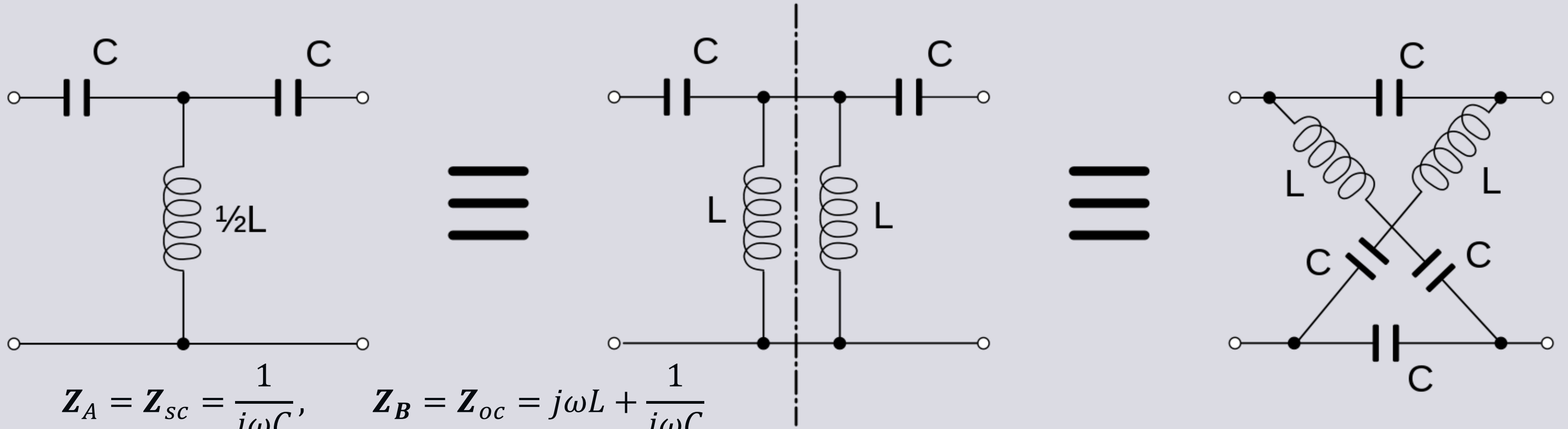
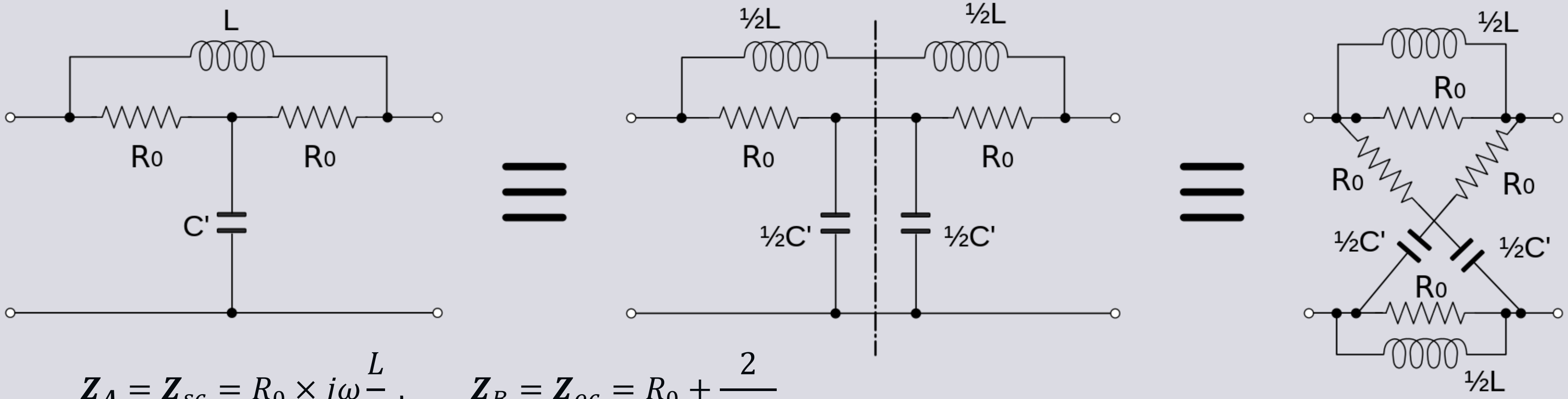
Examples: Lattice equivalents of T-section high-pass, Zobel bridged -T low-pass filters



$$\boldsymbol{Z}_A = \boldsymbol{Z}_{sc} = \frac{1}{j\omega C}, \qquad \boldsymbol{Z}_{\boldsymbol{B}} = \boldsymbol{Z}_{oc} = j\omega L + \frac{1}{j\omega C}$$



$$\boldsymbol{Z}_{\boldsymbol{A}} = \boldsymbol{Z}_{sc} = R_0 \times j\omega\frac{L}{2}\,, \qquad \boldsymbol{Z}_B = \boldsymbol{Z}_{oc} = R_0 + \frac{2}{j\omega C}$$

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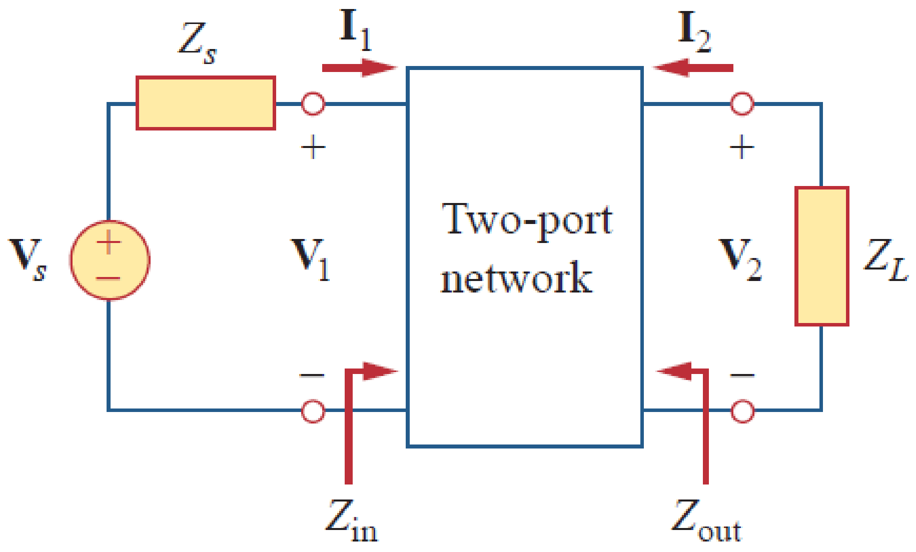
Two-Port with Finite Terminations



Any of the out of 6 params set can be used.
(→ h params … most useful)

Most important parameters…

$$A_v = \frac{V_2(s)}{V_1(s)} \leftarrow voltage\ gain$$

$$A_i = \frac{I_2(s)}{I_1(s)} \leftarrow current\ gain$$

$$z_{in} = \frac{V_1(s)}{I_1(s)} \leftarrow input\ imp.$$

$$z_{out} = \left.\frac{V_2(s)}{I_2(s)}\right|_{V_S=0} \leftarrow output\ imp.$$

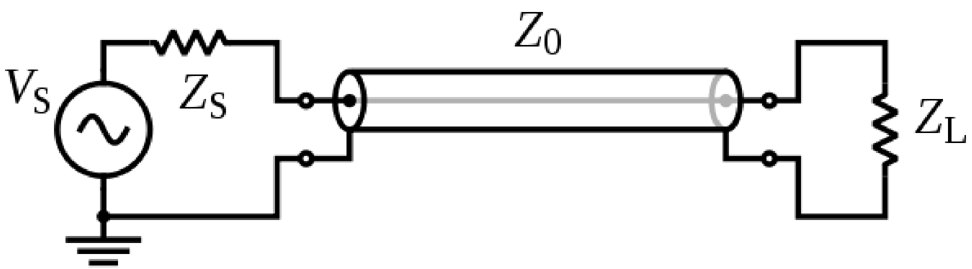
Terminated Transmission Line 1

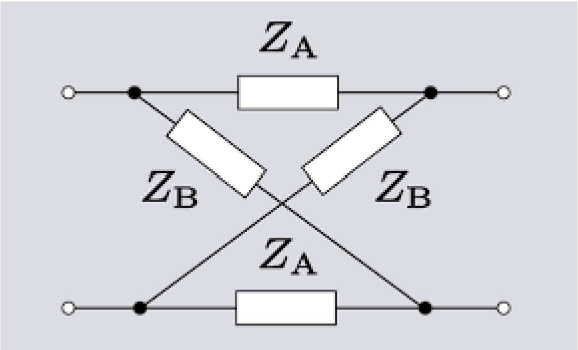
Symmetric network → 2 network parameters only → *[wave impedance]*, *[wave transmission coefficient]*

Wave impedance *(characteristic impedance)* *(https://en.wikipedia.org/wiki/Characteristic_impedance)*

$$\boldsymbol{Z}_{01}(def) = \left.\frac{\boldsymbol{V}_1}{\boldsymbol{I}_1}\right|_{refl=0} = \sqrt{\boldsymbol{Z}_{1oc}\cdot\boldsymbol{Z}_{1sc}} = \sqrt{\frac{\boldsymbol{Z}_{11}}{\boldsymbol{Y}_{11}}}$$

$$\boldsymbol{Z}_{02}(def) = \left.\frac{\boldsymbol{V}_2}{\boldsymbol{I}_2}\right|_{refl=0} = \sqrt{\boldsymbol{Z}_{2oc}\cdot\boldsymbol{Z}_{2sc}} = \sqrt{\frac{\boldsymbol{Z}_{22}}{\boldsymbol{Y}_{22}}}$$





Properties

- ❑ Ideal transmission line → $\boldsymbol{Z_0} = Z_0\ (real)$
- ❑ Transparent network → $\boldsymbol{Z_L} = \boldsymbol{Z}_0 \rightarrow \boldsymbol{Z_{in}} = \boldsymbol{Z}_0$
- ❑ Refl. free connection → $\boldsymbol{Z_L} = \boldsymbol{Z_S} = \boldsymbol{Z}_0$
- ❑ Lattice equivalent → $\boldsymbol{Z}_0 = \sqrt{\boldsymbol{Z}_A\cdot\boldsymbol{Z}_B}$ $\boldsymbol{Z}_0 = \sqrt{\boldsymbol{Z}_{OC}\cdot\boldsymbol{Z}_{SC}} \leftarrow \boldsymbol{Z}_{OC} = \frac{\boldsymbol{Z}_A+\boldsymbol{Z}_B}{2}$, $\boldsymbol{Z}_{SC} = 2(\boldsymbol{Z}_A \times \boldsymbol{Z}_B)$

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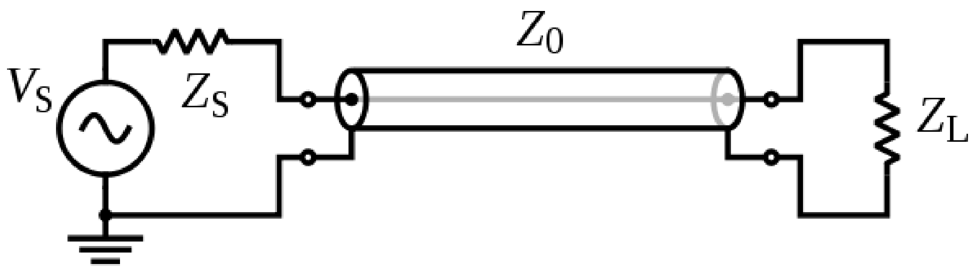
Terminated Transmission Line 2

Wave propagation coefficient *(propagation constant, transmission coefficient)*

$$\Gamma = \sqrt{\frac{P_S}{P_2}}, \qquad P_S = \frac{V_S^2}{4Z_S} \ (max.\, PWR\ transfer), \qquad P_2 = \frac{V_2^2}{Z_L}$$

$$In\ general \rightarrow \boldsymbol{\Gamma}(\omega) = \frac{\boldsymbol{V}_S}{2\boldsymbol{V}_2}\sqrt{\frac{Z_L}{Z_S}}$$

$$Z_S = Z_L = Z_0 \rightarrow \boldsymbol{\Gamma}(\omega) = \boldsymbol{\Gamma}_0(\omega) = \frac{\boldsymbol{V}_S}{2\boldsymbol{V}_2}\sqrt{\frac{Z_0}{Z_0}}$$



$$\rightarrow \boldsymbol{\Gamma}_0(\omega) = \frac{\boldsymbol{V}_S}{2\boldsymbol{V}_2} = \frac{\boldsymbol{V}_1}{\boldsymbol{V}_2} = \frac{1}{\boldsymbol{H}(\omega)}$$

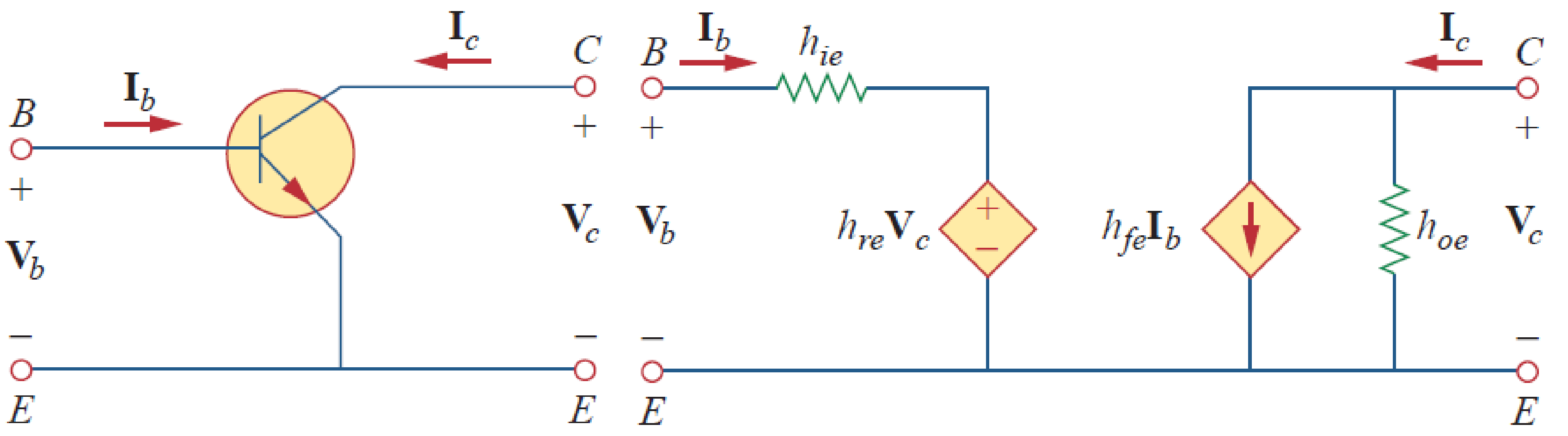
- ❑ Wave transmission *(hullámátvitel)* $\rightarrow \boldsymbol{g}_0 = \ln \boldsymbol{\Gamma}_0 = \ln\left(\Gamma_0 e^{j\boldsymbol{b}_0}\right) = a_0 + jb_0$
- ❑ Wave attenaution *(hullámcsillapítás)* $\rightarrow a_0 = \ln \boldsymbol{\Gamma}_0 \ [Np]$
- ❑ Wave phase *(hullámforgatás)* $\rightarrow b_0\ [rad]$ *https://en.wikipedia.org/wiki/Propagation_constant*

Progress in Content

- Two-Port Charactristics
- Relations between Characteristics
- Two-Port Interconnections
- Bartlett’s Bisection Theorem
- Two-Ports with Finite Terminations
- **Applications**

Application 1 – Isolating Amplifier (Common Emitter Circuit)

Goal is to find the ‚most important params' voltage and current gains, input and output impedances.



$$V_b = h_{ie} \cdot I_b + h_{re} \cdot V_c$$

$$I_c = h_{fe} \cdot I_b + h_{oe} \cdot V_c$$

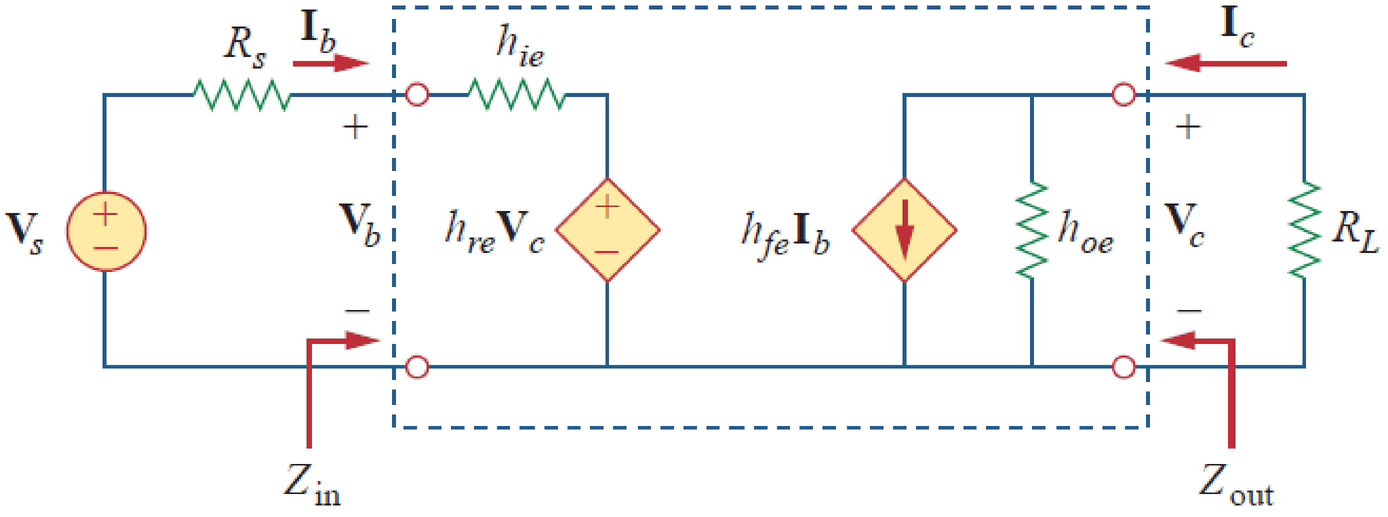
$h_{ie}(= h_{11}) \rightarrow$ *Base input impedance*

$h_{oe}(= h_{22}) \rightarrow$ *Output admittance*

$h_{re}(= h_{12}) \rightarrow$ *Reverse voltage feedback ratio*

$h_{fe}(= h_{21}) \rightarrow$ *Base − collector current gain*

Application 1 – Isolating Amplifier (Common Emitter Circuit)



$$\dots \rightarrow \boldsymbol{A}_v = \frac{\boldsymbol{V}_c}{\boldsymbol{V}_b} = \frac{-h_{fe}R_L}{h_{ie} + \left(h_{ie}h_{oe} - h_{re}h_{fe}\right)R_L}$$

$$\dots \rightarrow \boldsymbol{z}_{in} = \frac{\boldsymbol{V}_b}{\boldsymbol{I}_b} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L}$$

$$\dots \rightarrow \boldsymbol{A}_i = \frac{\boldsymbol{I}_c}{\boldsymbol{I}_b} = \frac{h_{fe}}{1 + h_{oe}R_L}$$

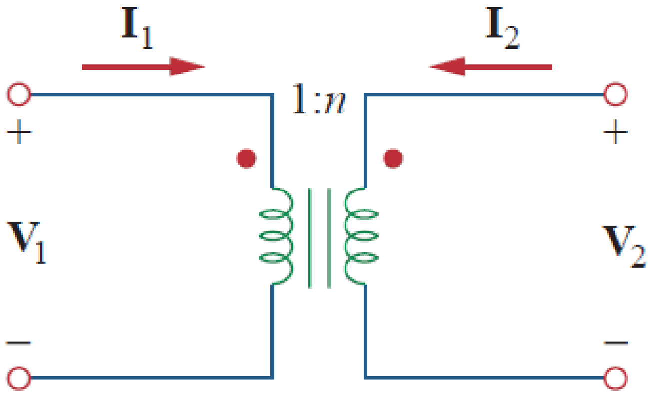
$$\dots \rightarrow \boldsymbol{z}_{out} = \left.\frac{\boldsymbol{V}_C}{\boldsymbol{I}_c}\right|_{\boldsymbol{V}_S=0} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}}$$

Application 2 – Ideal Transformer

Ideal transformer → has no z params *(4th supplementary LTI element supplementing R-L-C)*

Transmission characteristic

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} {}^{1}\!/_{n} & 0 \\ 0 & -n \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$



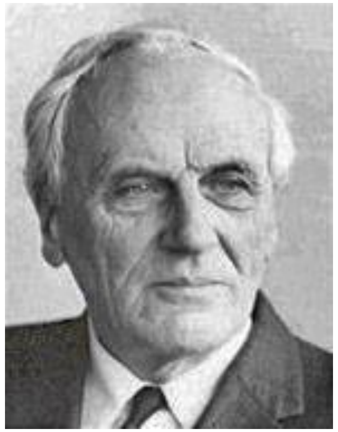
Energy-free device!

$$P = V_1 \cdot I_1 + V_2 \cdot I_2 = V_1 \cdot I_1 + n \cdot V_1 \left(-\frac{1}{n} I_1\right) = 0$$

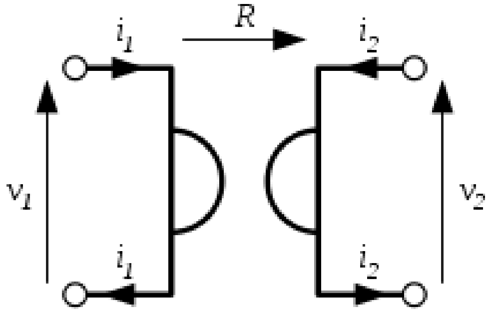
Impedance transformer

$$Z_{in} = \frac{V_1}{I_1} = \frac{{V_2}/{n}}{n \cdot I_2} = \frac{1}{n^2} \cdot \frac{V_2}{I_2} \to Z_{in} = \frac{1}{n^2} \cdot Z_L$$

Application 3 – Gyrator



$$V_1 = -R \cdot I_2 \qquad V_2 = R \cdot I_1 \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & -R \\ R & 0 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



- ❑ Energy-free device →

$$P = V_1 \cdot I_1 + V_2 \cdot I_2 = V_1 \cdot I_1 + R \cdot I_1 \left(-\frac{V_1}{R}\right) = V_1 \cdot I_1 - V_1 \cdot I_1 = 0$$

Bernard Tellegen (1900-1990)
- ❑ Dutch electrical engineer
- ❑ Philips Laboraty
- ❑ Pentode(1926)
- ❑ Gyrator (1948)
- ❑ Tellegen's theorem (1952)

- ❑ Gyrator resistance R is the only parameter.
- ❑ Different z_{12} and z_{21} → non-reciprocal device
- ❑ Loaded gyrator

$$Z_{in} = \frac{V_1}{I_1} = \frac{-R \cdot I_2}{V_2/R} = -\frac{R^2}{V_2/I_2} = -\frac{R^2}{Z_L}$$

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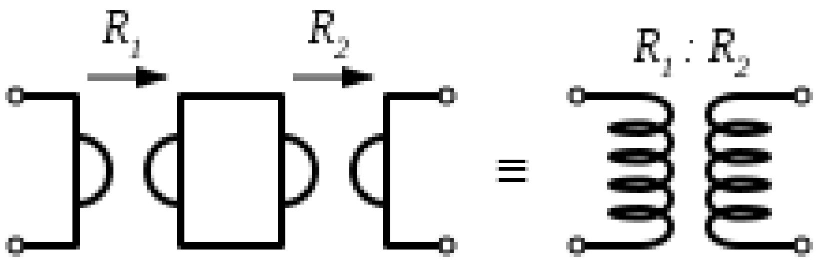
2 gyrators → ideal transformer

(3rd candidate for 4th element?)



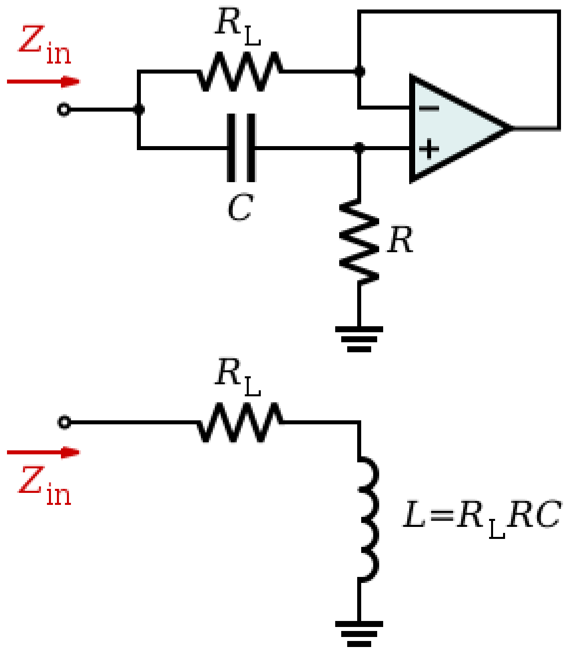
Impedance inversion

$$Z_{in} = (R_L + j\omega R_L RC) \times \left(R + \frac{1}{j\omega C}\right)$$

$$\begin{pmatrix} R \to high \\ \omega \to low \end{pmatrix} \to RC\ neglible\ in\ parallel$$

$$Z_{in} = (R_L + j\omega R_L RC)$$

Small, ideal inductor at low frequency.



Questions