



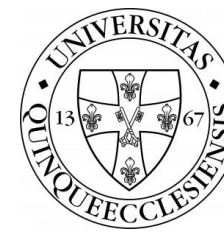
DR. GYURCSEK ISTVÁN

First-Order Dynamic Circuits

Sources and additional materials (recommended)

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- ❑ *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*
- ❑ *Ormándlaky Zsolt: Átmeneti jelenségek*

Positioning of DC Transients



Static circuit

- COND1 - no storage element
- algebraic equations
- DC circuit*

Dynamic circuit

- COND1 – capacitor *AND/OR* inductor
- COND2 – time var. excitation *AND/OR* structure
- differential equations
- AC \rightarrow part of DYN

Transient Analysis 1

First-order circuit

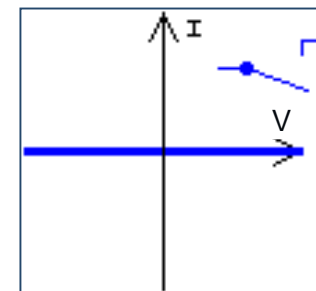
- One storage element (C or L)*
- Characterized by first-order differential equation*

Transient in a circuit

- Change of $V_x(st1) \rightarrow V_x(st2)$
- Change of $I_x(st1) \rightarrow I_x(st2)$
- X= any/all of elements

Reason for a transient can be

- Time dependent change in the source parameter
- Time dependent change in the circuit structure (i.e. switch on/off)



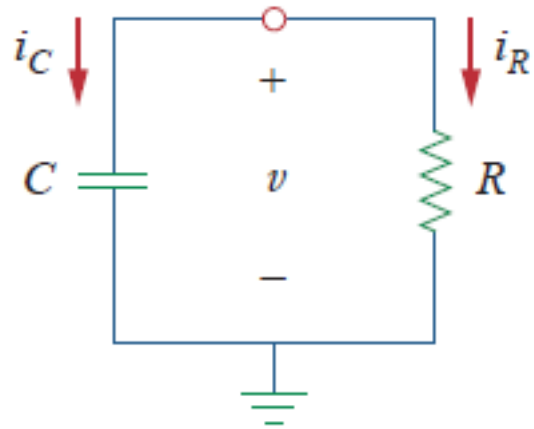


- ❑ **Source-Free RC Circuits**
- ❑ Source-Free RL Circuits
- ❑ Singularity Functions
- ❑ Step Response of an RC Circuit
- ❑ Step Response of an RL Circuit

Source-Free RC Circuit

Source-free circuit

- 'Suddenly' disconnected dc source
- Stored energy in C released to R



$$\frac{dv}{dt} + \frac{v}{RC} = 0 \leftarrow 1st\ ord.\ eq.$$

$$\text{Solving} \rightarrow \frac{dv}{v} = -\frac{1}{RC} dt$$

$$\int \frac{dv}{v} = -\frac{1}{RC} \int dt \rightarrow \ln v = -\frac{t}{RC} + \ln A$$

$$\ln \frac{v}{A} = -\frac{t}{RC} \rightarrow v(t) = A e^{-\frac{t}{RC}}$$

$$\text{Initial cond.} \rightarrow v(0) = V_0 = A$$

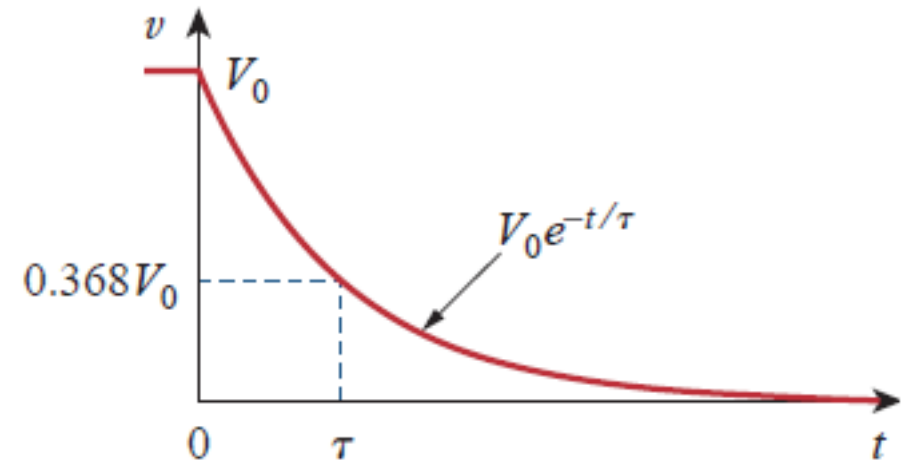
$$v(0) = V_0 \rightarrow w(0) = \frac{1}{2} CV_0^2$$

$$i_C + i_R = 0 \rightarrow C \frac{dv}{dt} + \frac{v}{R} = 0$$

$$v(t) = V_0 e^{-\frac{t}{RC}}, i(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-\frac{t}{RC}}$$

Natural response

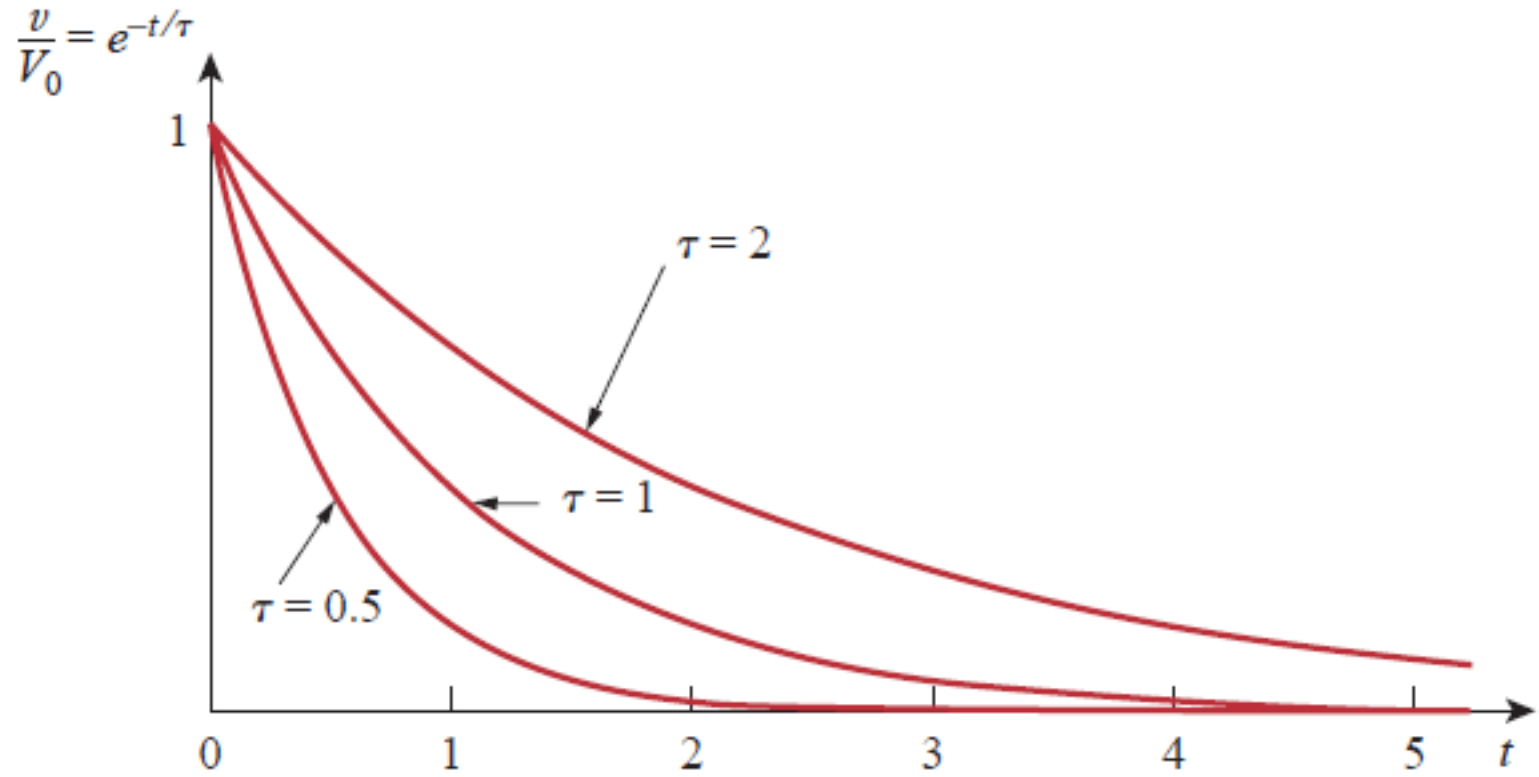
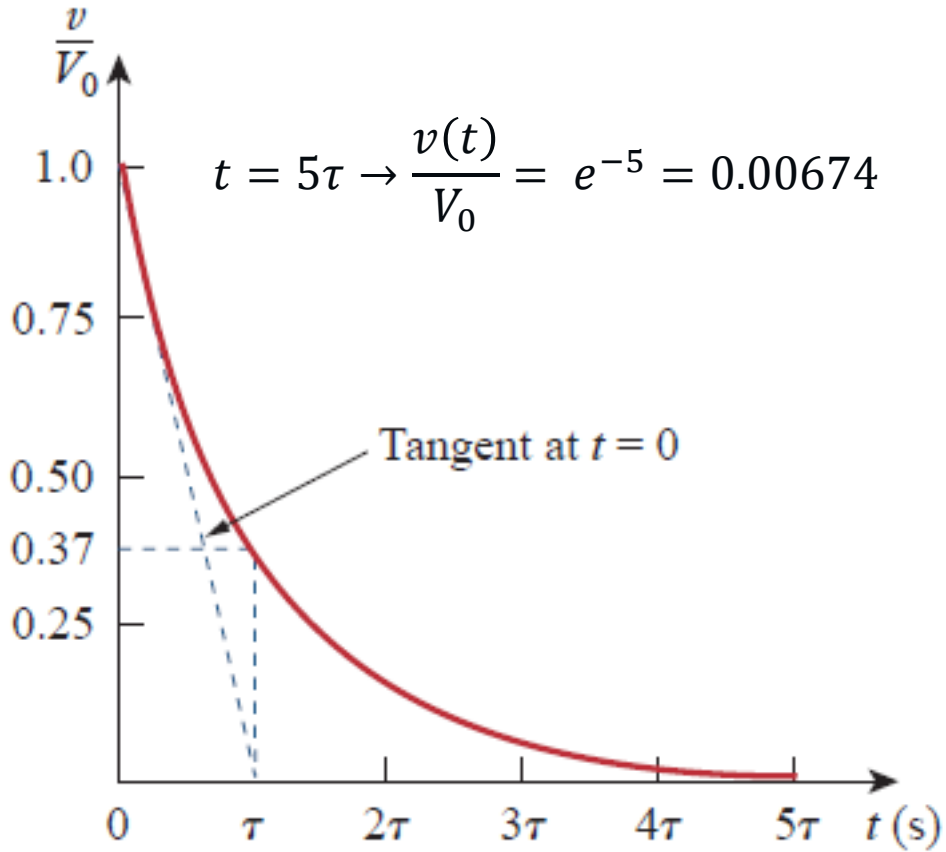
- No external excitation
- Behavior of the circuit itself



Time constant

- Time required to decay to 1/e (36.8%)
- $\tau = RC \rightarrow v(t) = V_0 e^{-\frac{t}{\tau}}$

Never falls to zero but...



Source-Free RC Circuit



Conservation of energy $v(t) = V_0 e^{-\frac{t}{RC}}, \quad i(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-\frac{t}{RC}}$

$$p_R(t) = v(t) \cdot i(t) = \frac{V_0^2}{R} e^{-\frac{2t}{\tau}}$$

$$w_R(t) = \int_0^t p_R dt = \int_0^t \frac{V_0^2}{R} e^{-\frac{2t}{\tau}} dt = -\frac{\tau V_0^2}{2R} e^{-\frac{2t}{\tau}} \Big|_0^t = \frac{1}{2} CV_0^2 (1 - e^{-\frac{2t}{\tau}})$$

$t \rightarrow \infty \dots w_R(\infty) = \frac{1}{2} CV_0^2$ Not a surprise because $\leftarrow w_C(0) = \frac{1}{2} CV_0^2$

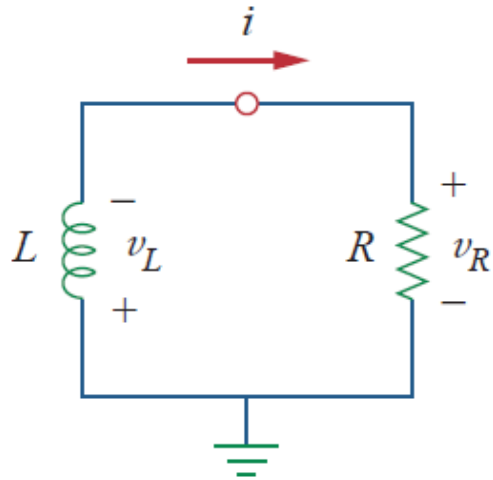
Key to work with source-free RC circuits

- Find initial conditions
- Find RC time constant (*more than one R \rightarrow Thevenin equivalent!*)



- Source-Free RC Circuits
- Source-Free RL Circuits**
- Singularity Functions
- Step Response of an RC Circuit
- Step Response of an RL Circuit

Source-Free RL Circuit



$$\frac{di}{dt} + \frac{R}{L}i = 0 \leftarrow 1st\ ord.\ eq.$$

$$\text{Solving} \rightarrow \frac{di}{i} = -\frac{R}{L}dt$$

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\frac{R}{L} \int_0^t dt \rightarrow \ln i \Big|_{I_0}^{i(t)} = -\frac{R}{L}t \Big|_0^t$$

Initial condition \rightarrow

$$i(0) = I_0 \rightarrow w(0) = \frac{1}{2}LI_0^2$$

$$v_L + v_R = 0 \rightarrow L \frac{di}{dt} + Ri = 0$$

$$\ln \frac{i(t)}{I_0} = -\frac{R}{L}t \rightarrow i(t) = I_0 e^{-\frac{R}{L}t}$$

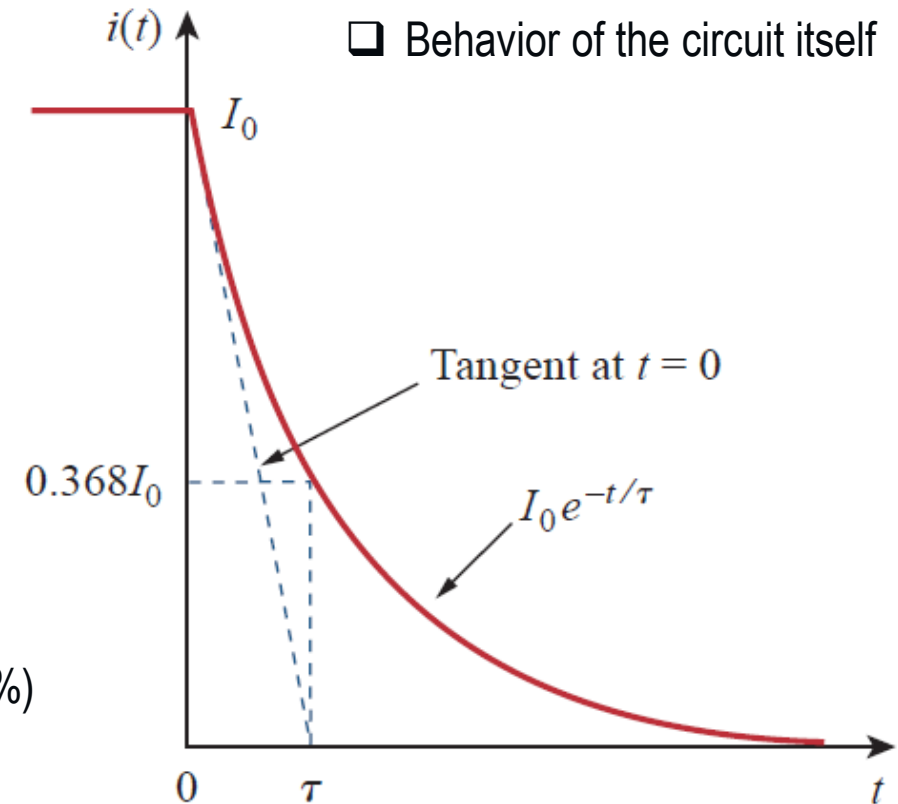
Time constant

Time required to decay to 1/e (36.8%)

$\tau = \frac{L}{R} \rightarrow i(t) = I_0 e^{-\frac{t}{\tau}}$

Natural response

- No external excitation
- Behavior of the circuit itself



Source-Free RL Circuit



Conservation of energy

$$i(t) = I_0 e^{-\frac{t}{\tau}}$$

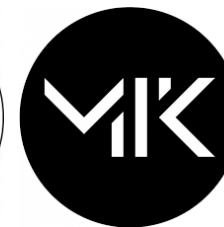
$$v_R(t) = iR = I_0 R e^{-\frac{t}{\tau}} \rightarrow p_R(t) = v_R(t) \cdot i(t) = I_0^2 R e^{-\frac{2t}{\tau}}$$

$$w_R(t) = \int_0^t p_R dt = \int_0^t I_0^2 R e^{-\frac{2t}{\tau}} dt = -\frac{1}{2} \tau I_0^2 R e^{-\frac{2t}{\tau}} \Big|_0^t = \frac{1}{2} L I_0^2 (1 - e^{-\frac{2t}{\tau}})$$

$$t \rightarrow \infty \dots w_R(\infty) = \frac{1}{2} L I_0^2 \quad \text{Not a surprise because} \quad \leftarrow w_L(0) = \frac{1}{2} L I_0^2$$

Key to work with source-free RL circuits

- ❑ Find initial condition $i(0) = I_0$
- ❑ Find L/R time constant (*more than one R* \rightarrow Thevenin equivalent!)



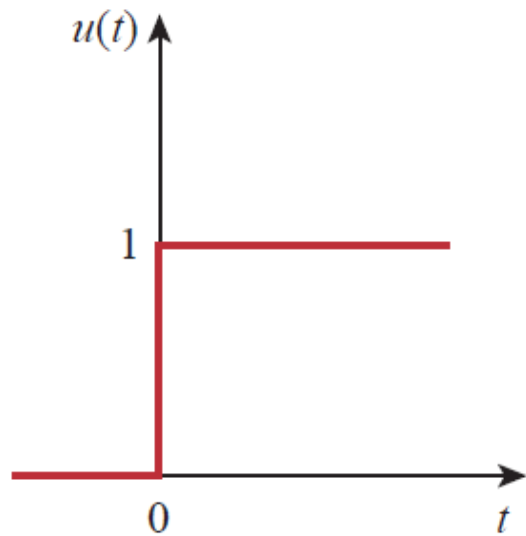
- Source-Free RC Circuits
- Source-Free RL Circuits
- Singularity Functions**
- Step Response of an RC Circuit
- Step Response of an RL Circuit

Singularity Functions 1



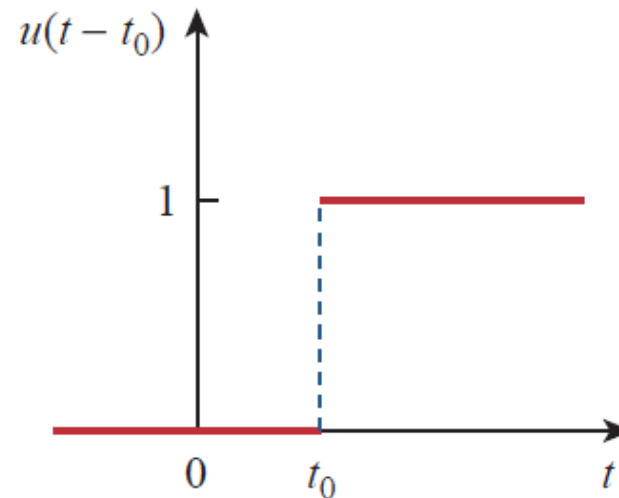
Singularity function \rightarrow is discontinuous **OR** has discontinuous derivative

Unit step function



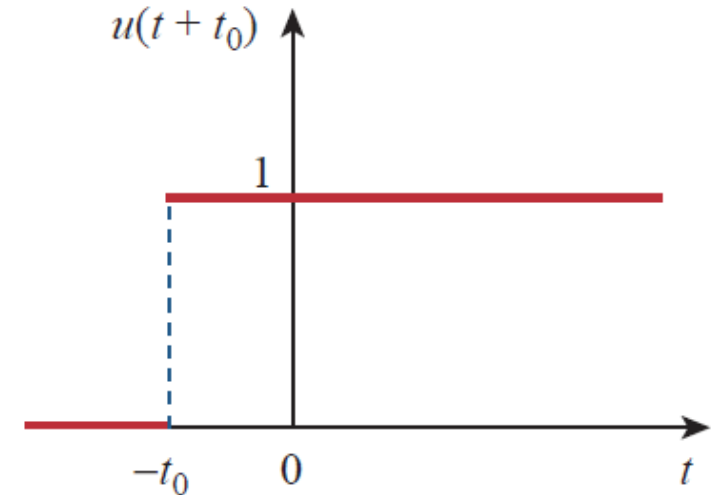
$$u(t) = \begin{cases} 0 & t < 0 \\ (\text{undef.}) & (t = 0) \\ 1 & t > 0 \end{cases}$$

Unit step delayed by t_0



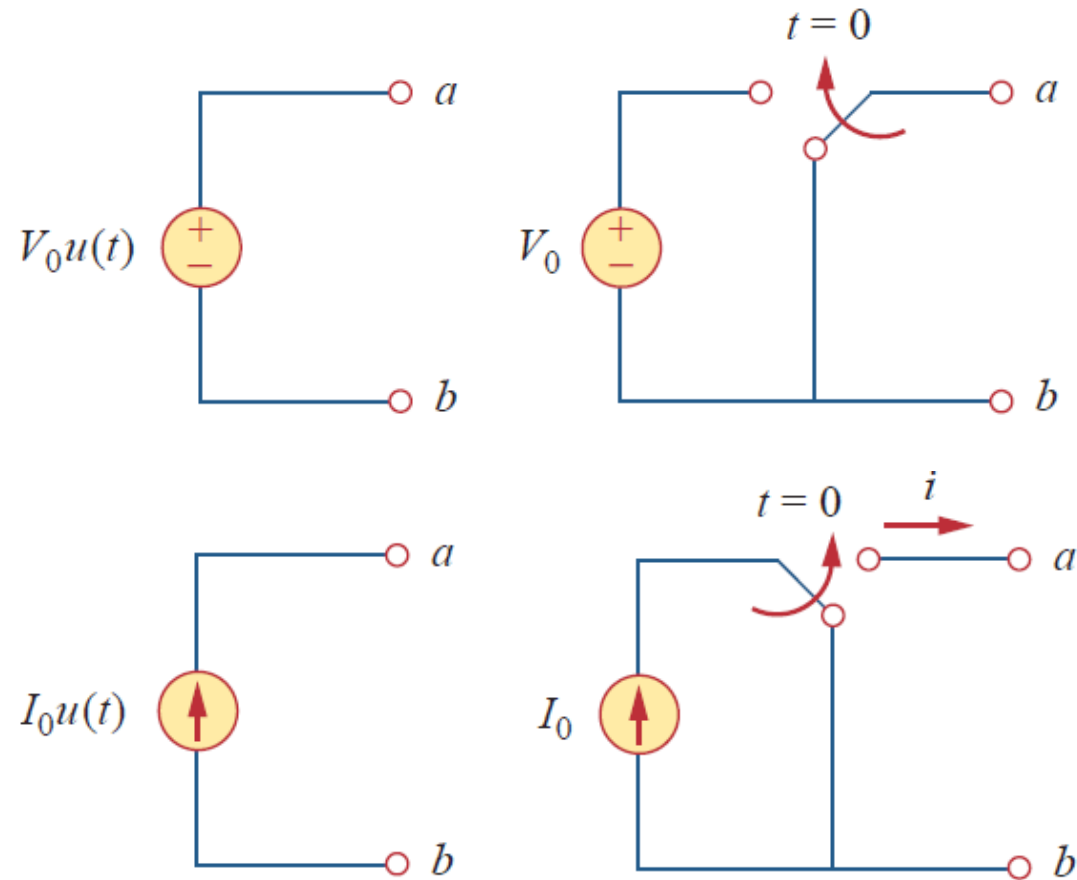
$$u(t - t_0) = \begin{cases} 0 & t < t_0 \\ (\text{undef.}) & (t = t_0) \\ 1 & t > t_0 \end{cases}$$

Unit step advanced by t_0

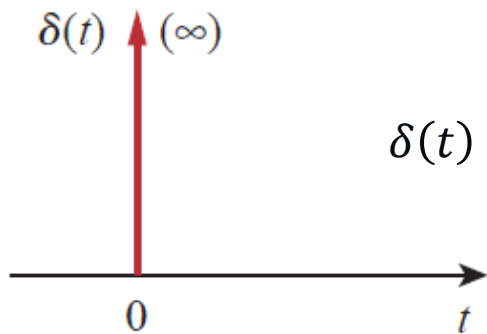


$$u(t + t_0) = \begin{cases} 0 & t < -t_0 \\ (\text{undef.}) & (t = -t_0) \\ 1 & t > -t_0 \end{cases}$$

Equivalent Circuits of Sources



Singularity Functions 2



$$\delta(t) = \frac{d}{dt} u(t) = \begin{cases} 0 & t < 0 \\ (\text{undef.}) & (t = 0) \\ 0 & t > 0 \end{cases}$$

Unit impulse

- Applied (resulting) shock
- Very short duration
- Unit area

$$\int_{0^-}^{0^+} \delta(t) dt = 1$$

Affect of unit impulse for $f(t)$

$$\int_a^b f(t) \delta(t - t_0) dt = \int_a^b f(t_0) \delta(t - t_0) dt = f(t_0) \int_a^b \delta(t - t_0) dt = f(t_0)$$

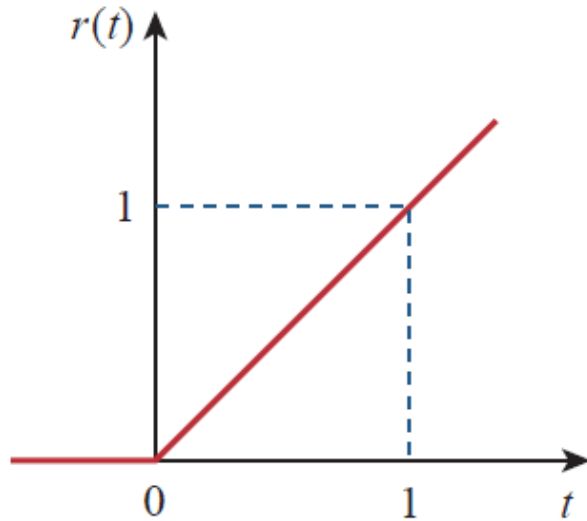
$$\int_a^b f(t) \delta(t - t_0) dt = f(t_0) \leftarrow \text{Sampling (shifting) property of impulse func.}$$

$$t_0 = 0 \rightarrow \int_a^b f(t) \delta(t) dt = f(0)$$

Singularity Functions 3

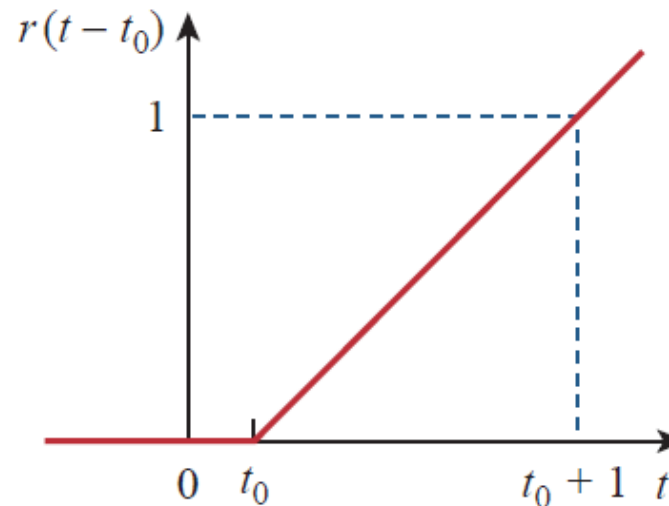


Unit ramp function



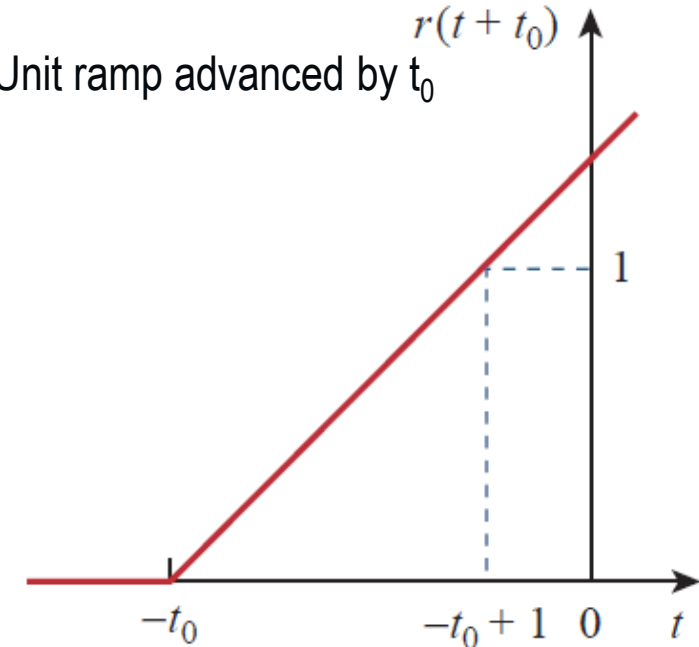
$$r(t) = \int_{-\infty}^t u(t) dt = \begin{cases} 0 & t \leq 0 \\ t & t \geq 0 \end{cases}$$

Unit ramp delayed by t_0



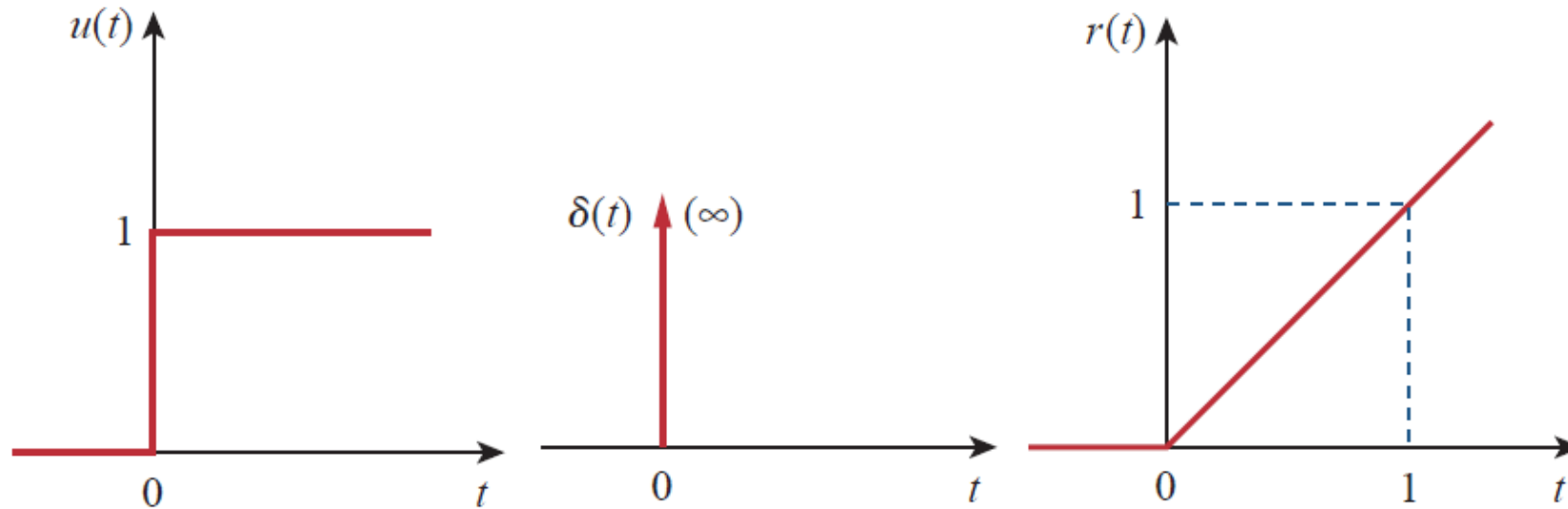
$$r(t - t_0) = \begin{cases} 0 & t \leq t_0 \\ t - t_0 & t \geq t_0 \end{cases}$$

Unit ramp advanced by t_0



$$r(t + t_0) = \begin{cases} 0 & t \leq -t_0 \\ t + t_0 & t \geq -t_0 \end{cases}$$

Summary



$$\delta(t) = \frac{du(t)}{dt}, \quad u(t) = \frac{dr(t)}{dt} \quad \text{or} \quad u(t) = \int_{-\infty}^t \delta(t) dt, \quad r(t) = \int_{-\infty}^t u(t) dt$$



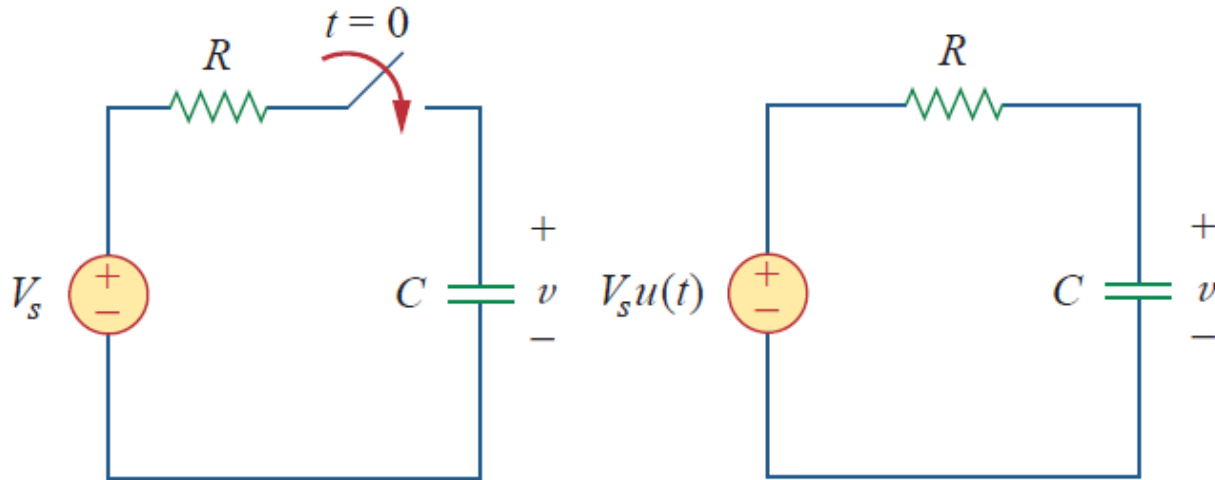
- Source-Free RC Circuits
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- Step Response of an RC Circuit**
- Step Response of an RL Circuit

Step Response of an RC Circuit



Step response

□ ,Suddenly' applied dc source (i.e. voltage or current excitation is a step function)



$$\int_{V_0}^{v(t)} \frac{dv}{v - V_S} = \int_0^t \left(-\frac{dt}{RC} \right) \rightarrow \ln(v - V_S) \Big|_{V_0}^{v(t)} = -\frac{t}{RC} \Big|_0^t$$

$$\ln[v(t) - V_S] - \ln(V_0 - V_S) = -\frac{t}{RC} + 0$$

$$\ln \frac{v(t) - V_S}{V_0 - V_S} = -\frac{t}{RC} \rightarrow \frac{v(t) - V_S}{V_0 - V_S} = e^{-\frac{t}{\tau}} \leftarrow \tau = RC$$

$$v(0^-) = v(0^+) = V_0$$

$$t > 0 \rightarrow \frac{dv}{dt} + \frac{v}{RC} = \frac{V_S}{RC}$$

$$v(t) = V_S + (V_0 - V_S)e^{-\frac{t}{\tau}} \leftarrow t > 0$$

$$C \frac{dv}{dt} + \frac{v - V_S u(t)}{R} = 0$$

$$\frac{dv}{dt} = -\frac{v - V_S}{RC} \rightarrow \frac{dv}{v - V_S} = -\frac{dt}{RC}$$

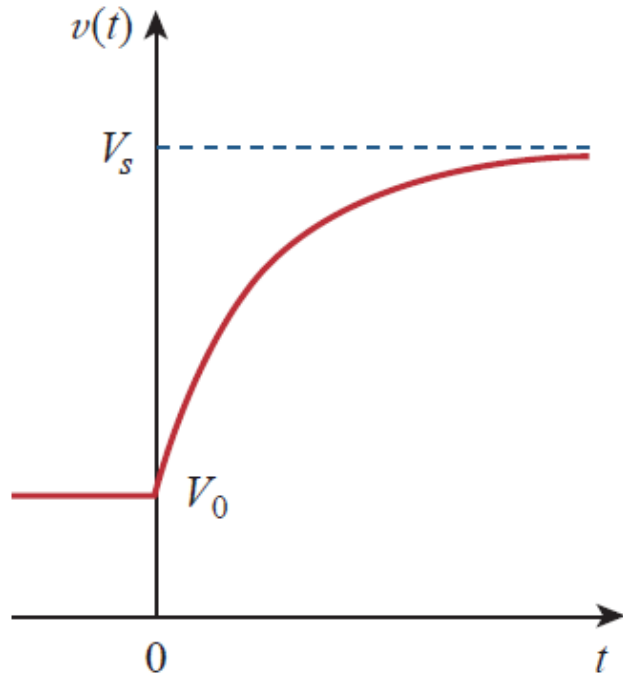
$$v(t) = \begin{cases} V_0 & t < 0 \\ V_S + (V_0 - V_S)e^{-\frac{t}{\tau}} & t > 0 \end{cases}$$

Step Response of an RC Circuit



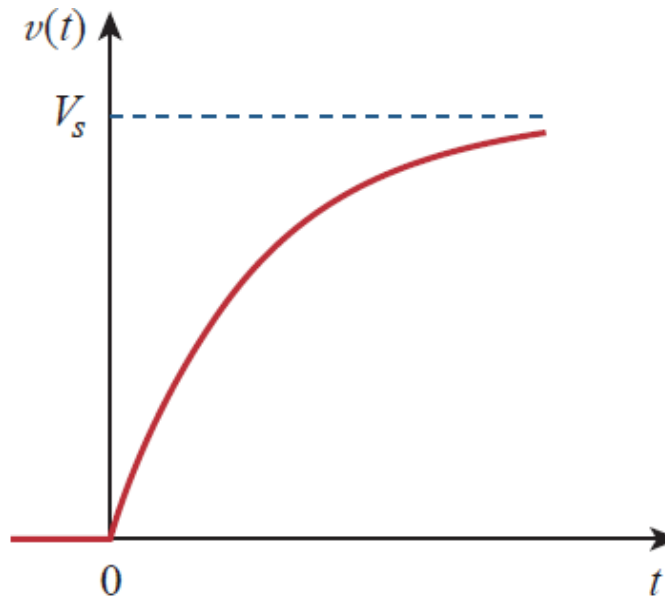
Complete (total) response

$$v(t) = \begin{cases} V_0 & t < 0 \\ V_S + (V_0 - V_S)e^{-\frac{t}{\tau}} & t > 0 \end{cases}$$



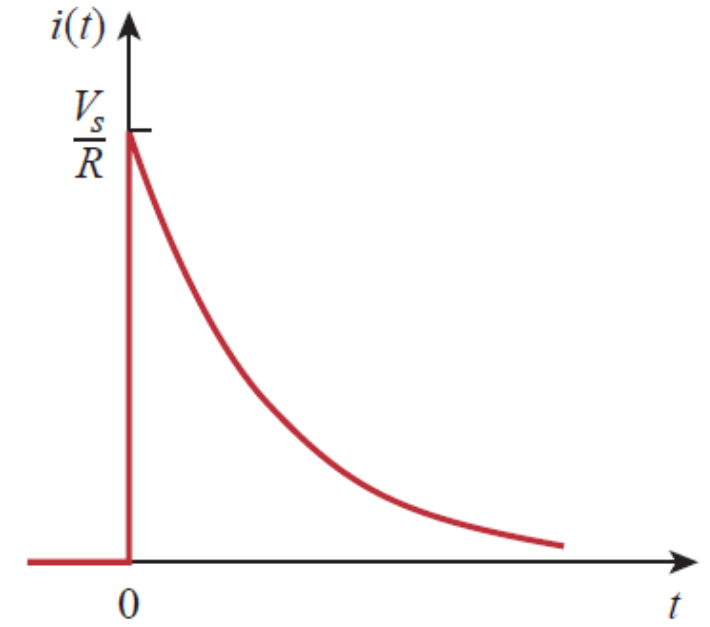
Originally uncharged capacitor

$$V_0 = 0 \rightarrow v(t) = V_S(1 - e^{-\frac{t}{\tau}})u(t)$$



$$i(t) = C \frac{dv}{dt} = \frac{C}{\tau} V_S e^{-\frac{t}{\tau}} u(t)$$

$$i(t) = \frac{V_S}{R} e^{-\frac{t}{\tau}} u(t)$$



Step Response of an RC Circuit



□ Decomposing way 1

$$\rightarrow v(t) = V_S + (V_0 - V_S)e^{-\frac{t}{\tau}} = v_{ss} + v_{tr} \quad v_{ss} = V_S, \quad v_{tr} = (V_0 - V_S)e^{-\frac{t}{\tau}}$$

Complete response = steady-state response (,permanent part') + transient response ('temporary part')

□ Decomposing way 2

$$\rightarrow v(t) = V_0 e^{-\frac{t}{\tau}} + V_S(1 - e^{-\frac{t}{\tau}}) = v_n + v_f \quad v_n = V_0 e^{-\frac{t}{\tau}}, \quad v_f = V_S(1 - e^{-\frac{t}{\tau}})$$

Complete response = natural response (,stored energy') + forced response ('independent source')

$$v(t) = v_{ss} + v_{tr} = v(\infty) + [v(0^+) - v(\infty)] e^{-\frac{t}{\tau}}$$

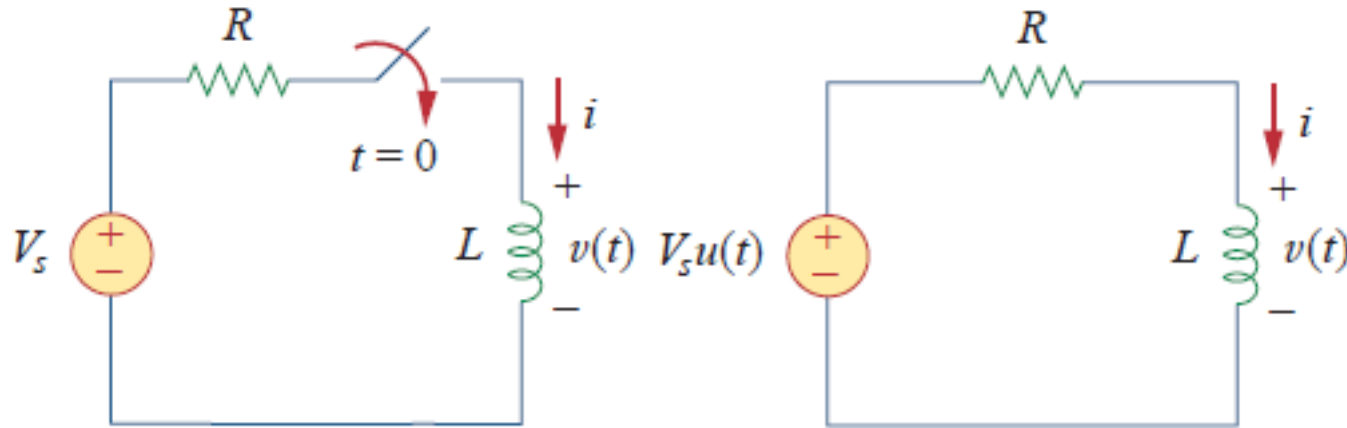
Key to work with step response of an RC circuits

- Find initial condition $v(0) = V_0$
- Find steady-state (final) capacitor voltage $v(\infty)$
- Find RC time constant (more than one R \rightarrow Thevenin equivalent!)



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Step Response of an RL Circuit



$$i(t) = v_{ss} + v_{tr}$$

$$i_{ss} = \frac{V_S}{R}, \quad i_{tr} = A e^{-\frac{t}{\tau}}, \quad \tau = \frac{L}{R}$$

$$i(0^-) = i(0^+) = I_0$$

$$i(t) = \frac{V_S}{R} + A e^{-\frac{t}{\tau}} \rightarrow i(0) = I_0 = \frac{V_S}{R} + A e^0$$

$$A = I_0 - \frac{V_S}{R} \rightarrow i(t) = \frac{V_S}{R} + \left(I_0 - \frac{V_S}{R} \right) e^{-t/\tau}$$

$$i(t) = i_{ss} + i_{tr} = i(\infty) + [i(0^+) - i(\infty)] e^{-\frac{t}{\tau}}$$

Key to work with step response of an RL circuits

- Find initial condition $i(0) = I_0$
- Find steady-state (final) inductor current $i(\infty)$
- Find L/R time constant
(more than one R \rightarrow Thevenin equivalent!)

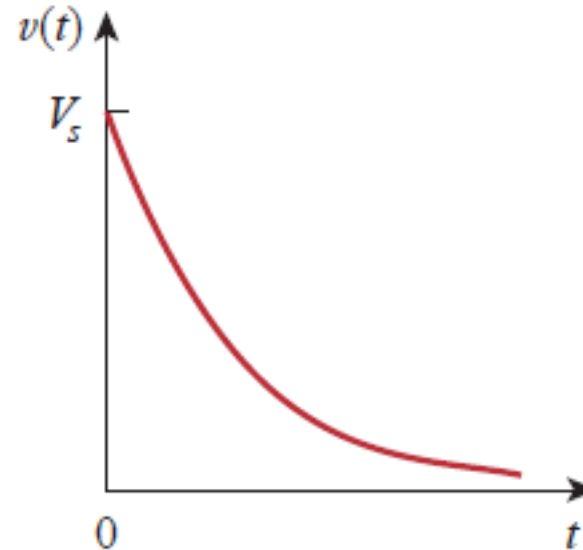
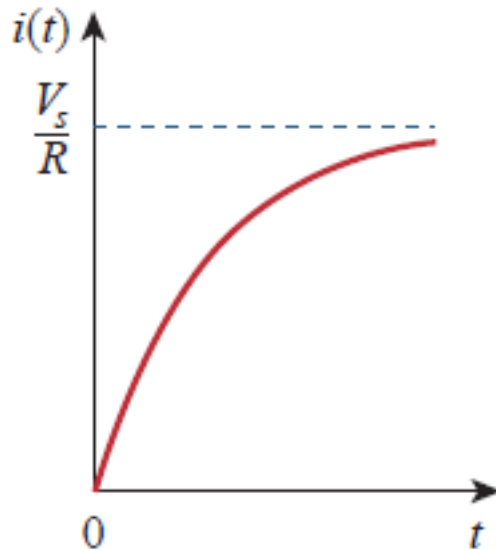
Step Response of an RL Circuit



Originally energy-free inductor

$$I_0 = 0 \rightarrow i(t) = \begin{cases} 0 & t < 0 \\ \frac{V_S}{R} (1 - e^{-t/\tau}) & t > 0 \end{cases} \dots \text{or} \dots i(t) = \frac{V_S}{R} (1 - e^{-t/\tau}) u(t)$$

$$v(t) = L \frac{di}{dt} = V_S \frac{L}{\tau R} e^{-t/\tau} u(t) \rightarrow v(t) = \frac{V_S}{R} e^{-t/\tau} u(t), \quad \tau = \frac{L}{R}$$



Questions

