DR. GYURCSEK ISTVÁN

Dynamic Circuits 1

First-Order Circuits

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*
- *Ormándlaky Zsolt: Átmeneti jelenségek*

Positioning of DC Transients

Static circuit
- COND1 - no storage element
- algebraic equations
- *DC circuit*

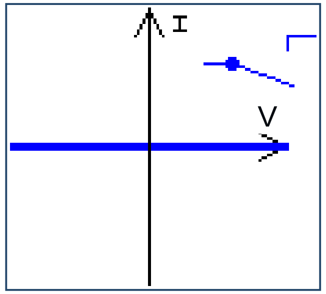
Dynamic circuit
- COND1 – capacitor *AND/OR* inductor
- COND2 – time var. excitation *AND/OR* structure
- differential equations
- *AC → part of DYN*

Transient Analysis 1

First-order circuit
- *One storage element (C or L)*
- *Characterized by first-order differential equation*

Transient in a circuit
- Change of V_x (st1)→ V_x (st2)
- Change of I_x (st1) → I_x (st2)
- X= any/all of elements

Reason for a transient can be
- Time dependent change in the source parameter
- Time dependent change in the circuit structure (i.e. switch on/off)



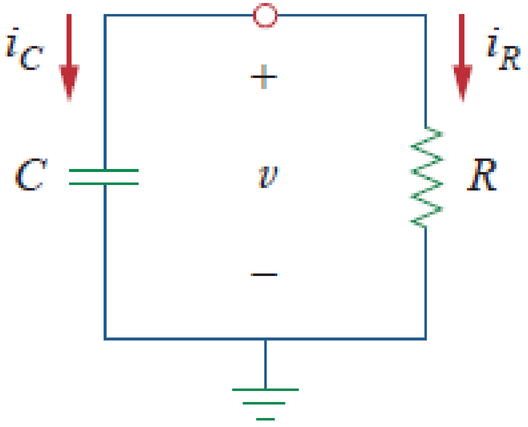
Progress in Content

- **Source-Free RC Circuits**
- Source-Free RL Circuits
- Singularity Functions
- Step Response of an RC Circuit
- Step Response of an RL Circuit

Source-Free RC Circuit

Source-free circuit

- ❑ ‚Suddenly' disconnected dc source
- ❑ Stored energy in *C* released to *R*



$$v(0) = V_0 \to w(0) = \frac{1}{2}CV_0^2$$

$$i_C + i_R = 0 \to C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\frac{dv}{dt} + \frac{v}{RC} = 0 \leftarrow 1st\ ord.eq.$$

$$\text{Solving} \to \frac{dv}{v} = -\frac{1}{RC}dt$$

$$\int\frac{dv}{v} = -\frac{1}{RC}\int dt \to \ln v = -\frac{t}{RC} + \ln A$$

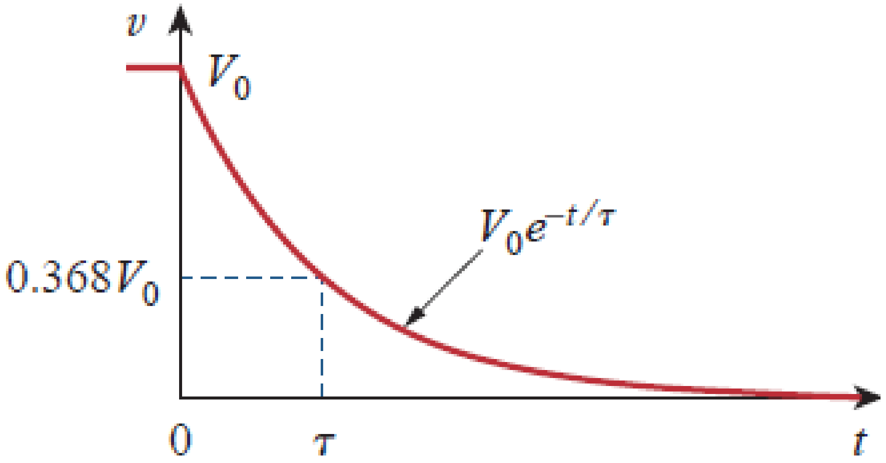
$$\ln\frac{v}{A} = -\frac{t}{RC} \to v(t) = A\,e^{-\frac{t}{RC}}$$

$$\text{Initial cond.} \to v(0) = V_0 = A$$

$$v(t) = V_0\,e^{-\frac{t}{RC}}, i(t) = \frac{v(t)}{R} = \frac{V_0}{R}\,e^{-\frac{t}{RC}}$$

Natural response

- ❑ No external excitation
- ❑ Behavior of the circuit itself



Time constant

- ❑ Time required to decay to 1/e (36.8%)
- ❑ $\tau = RC \to v(t) = V_0\,e^{-\frac{t}{\tau}}$

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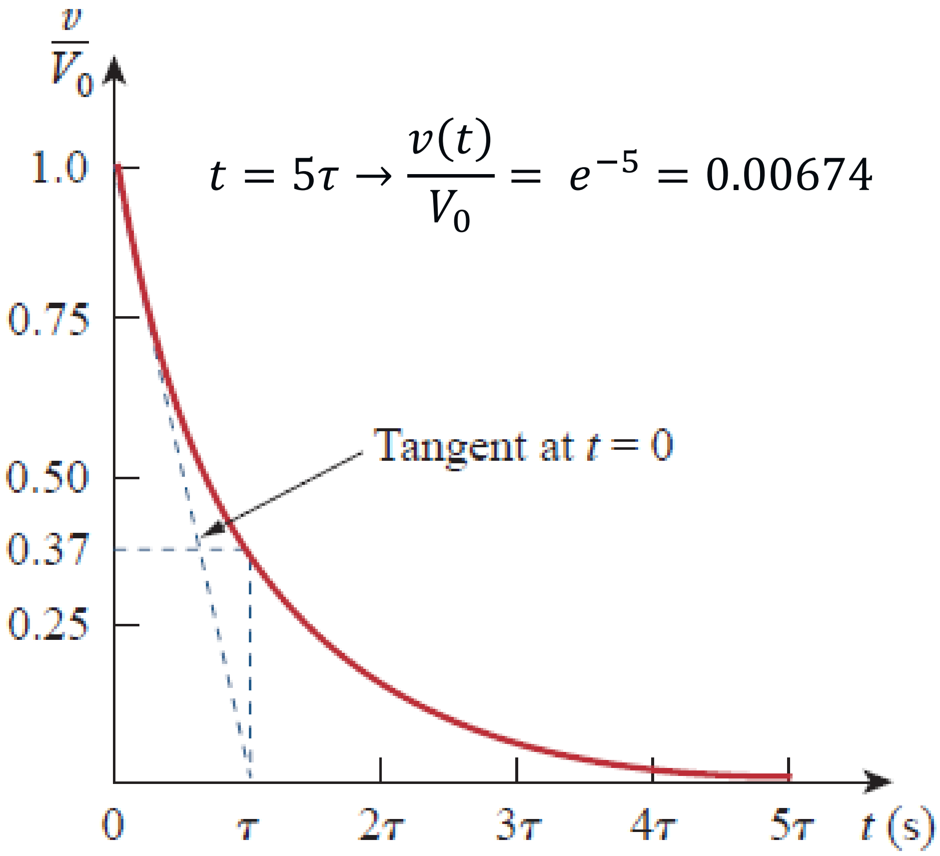
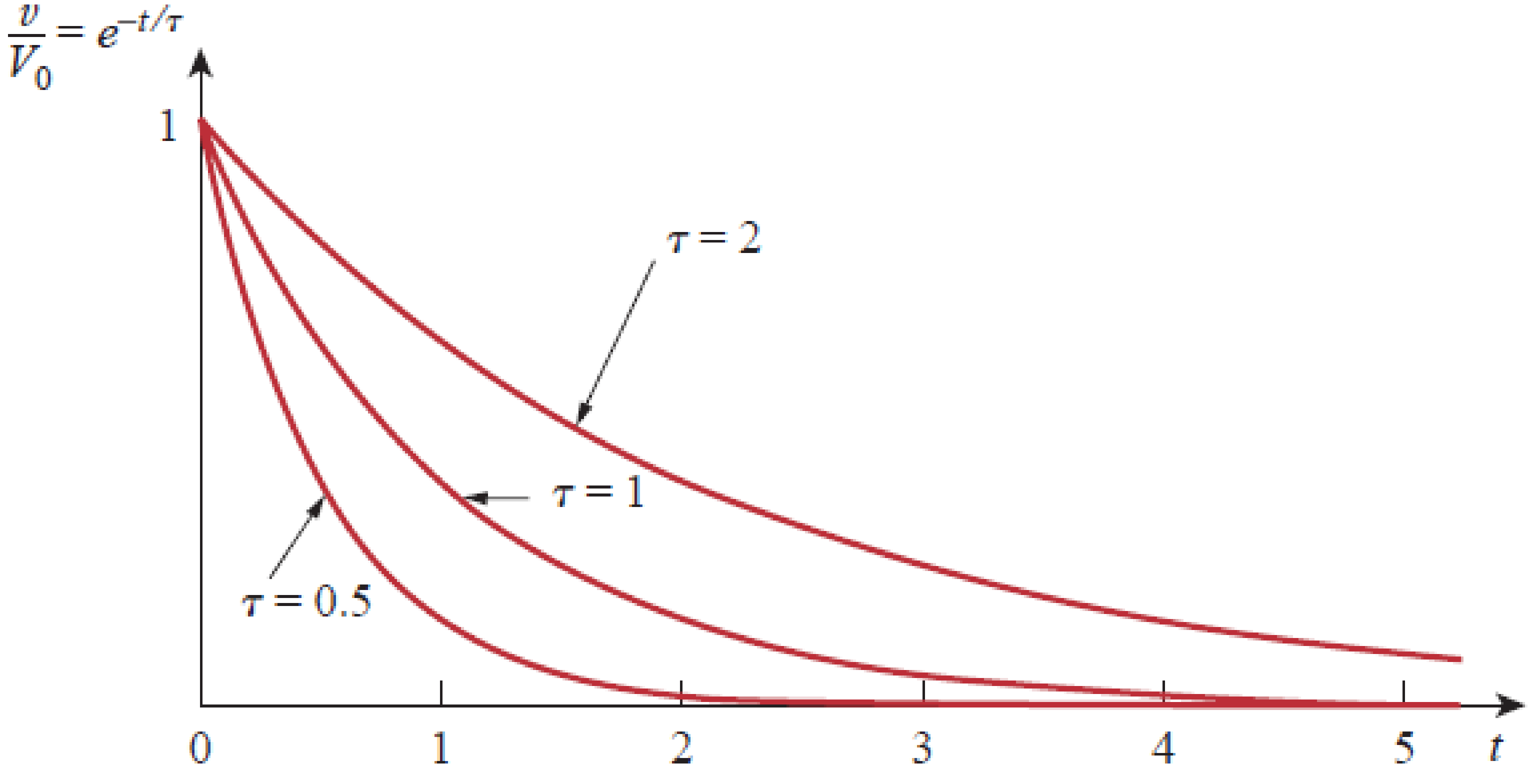
Never falls to zero but…

$$t = 5\tau \rightarrow \frac{v(t)}{V_0} = e^{-5} = 0.00674$$





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Source-Free RC Circuit

Conservation of energy $v(t) = V_0\, e^{-\frac{t}{RC}},\qquad i(t) = \frac{v(t)}{R} = \frac{V_0}{R}\, e^{-\frac{t}{RC}}$

$$p_R(t) = v(t) \cdot i(t) = \frac{V_0^2}{R}\, e^{-\frac{2t}{\tau}}$$

$$w_R(t) = \int_0^t p_R\, dt = \int_0^t \frac{V_0^2}{R}\, e^{-\frac{2t}{\tau}} dt = -\frac{\tau\, V_0^2}{2\, R}\, e^{-\frac{2t}{\tau}}\Bigg|_0^t = \frac{1}{2} C V_0^2 \left(1 - e^{-\frac{2t}{\tau}}\right)$$

$t \to \infty$ $\ldots w_R(\infty) = \frac{1}{2} C V_0^2$ Not a surprise because $\leftarrow w_C(0) = \frac{1}{2} C V_0^2$

Key to work with source-free RC circuits

- ❑ Find initial conditions
- ❑ Find RC time constant *(more than one R → Thevenin equivalent!)*

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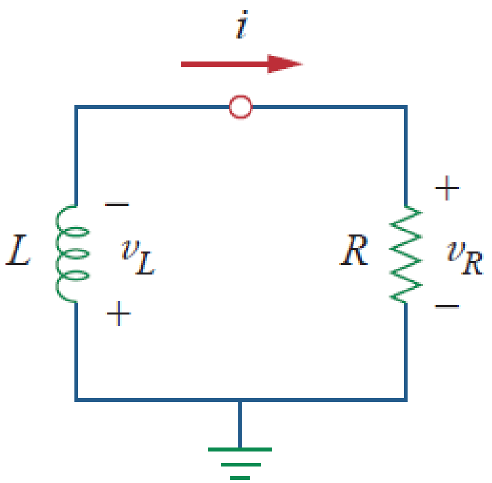
Source-Free RL Circuit



$$\frac{di}{dt} + \frac{R}{L} i = 0 \leftarrow 1st\ ord.eq.$$

Solving $\rightarrow \dfrac{di}{i} = -\dfrac{R}{L} dt$

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\frac{R}{L} \int_0^t dt \rightarrow \ln i \Big|_{I_0}^{i(t)} = -\frac{R}{L} t \Big|_0^t$$

Initial condition →

$$\ln \frac{i(t)}{I_0} = -\frac{R}{L} t \rightarrow i(t) = I_0\, e^{-\frac{R}{L} t}$$

$$i(0) = I_0 \rightarrow w(0) = \frac{1}{2} L I_0^2$$

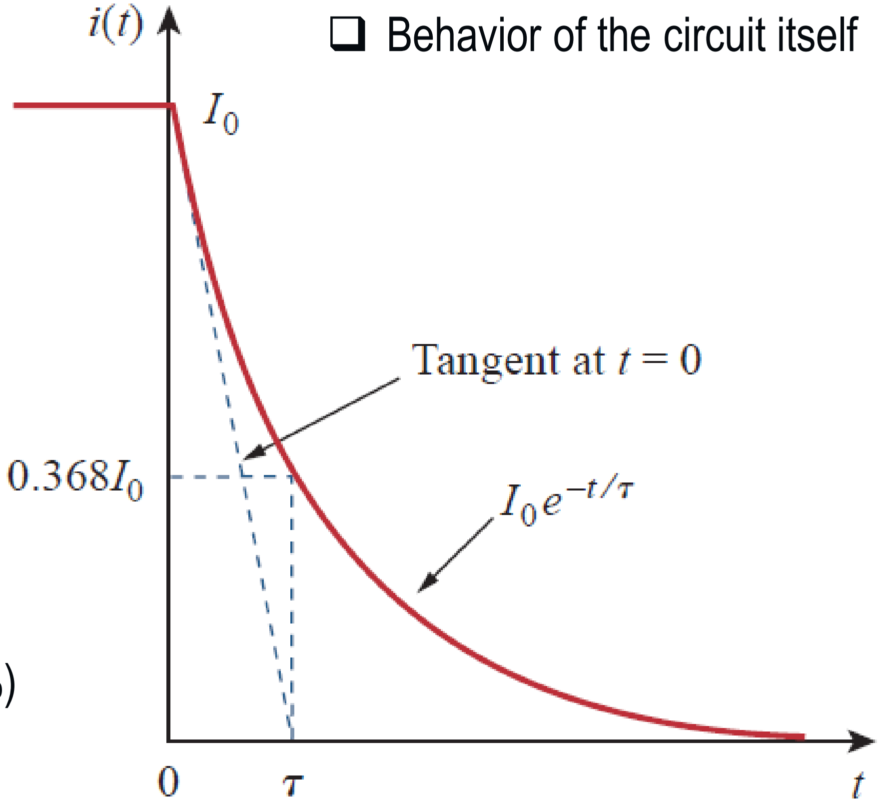
$$v_L + v_R = 0 \rightarrow L \frac{di}{dt} + Ri = 0$$

Time constant

- ❑ Time required to decay to 1/e (36.8%)
- ❑ $\tau = \dfrac{L}{R} \rightarrow i(t) = I_0\, e^{-\frac{t}{\tau}}$

Natural response

- ❑ No external excitation
- ❑ Behavior of the circuit itself



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Conservation of energy $i(t) = I_0\, e^{-\frac{t}{\tau}}$

$$v_R(t) = iR = I_0 R\, e^{-\frac{t}{\tau}} \rightarrow p_R(t) = v_R(t) \cdot i(t) = I_0^2 R\, e^{-\frac{2t}{\tau}}$$

$$w_R(t) = \int_0^t p_R\, dt = \int_0^t I_0^2 R\, e^{-\frac{2t}{\tau}}\, dt = -\frac{1}{2}\tau\, I_0^2\, R\, e^{-\frac{2t}{\tau}}\Big|_0^t = \frac{1}{2} L I_0^2 \left(1 - e^{-\frac{2t}{\tau}}\right)$$

$t \rightarrow \infty \ \ldots w_R(\infty) = \frac{1}{2} L I_0^2$ Not a surprise because $\leftarrow w_L(0) = \frac{1}{2} L I_0^2$

Key to work with source-free RL circuits

- Find initial condition $i(0) = I_0$
- Find L/R time constant *(more than one R → Thevenin equivalent!)*

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- **Singularity Functions**
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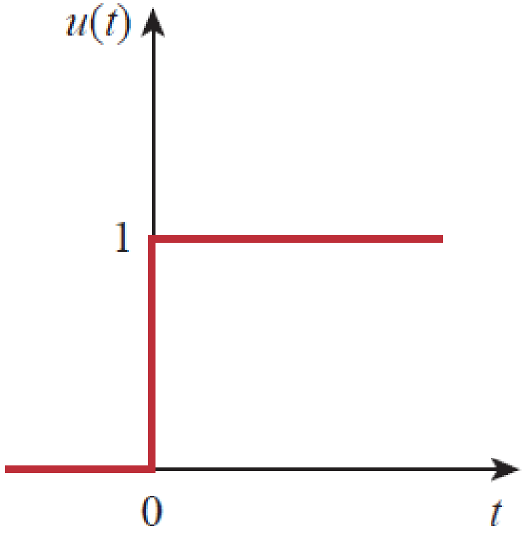
Singularity Functions (1) – Unit Step Function

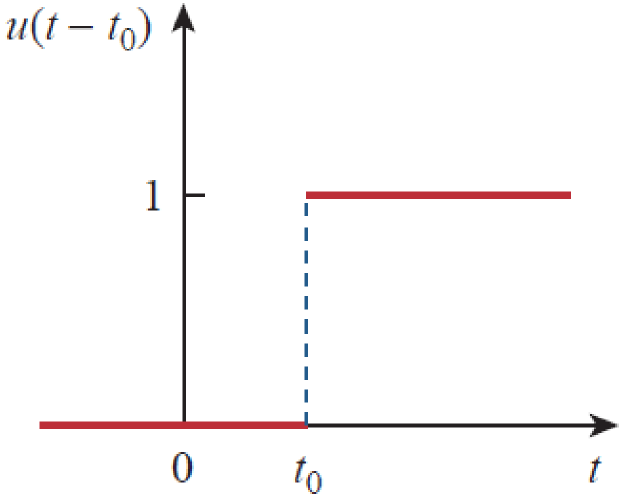
Singularity function → is discontinuous **OR** has discontinuous derivative

Unit step function



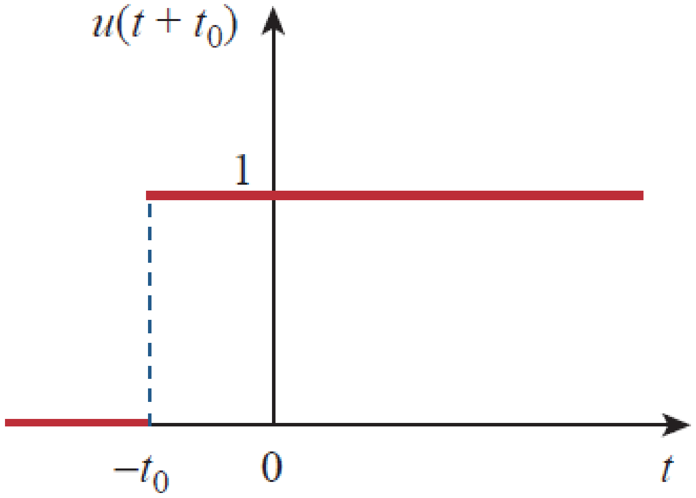
$$u(t) = \begin{cases} 0 & t < 0 \\ (undef.) & (t = 0) \\ 1 & t > 0 \end{cases}$$

Unit step delayed by t_0



$$u(t - t_0) = \begin{cases} 0 & t < t_0 \\ (undef.) & (t = t_0) \\ 1 & t > t_0 \end{cases}$$

Unit step advanced by t_0



$$u(t + t_0) = \begin{cases} 0 & t < -t_0 \\ (undef.) & (t = -t_0) \\ 1 & t \succ -t_0 \end{cases}$$

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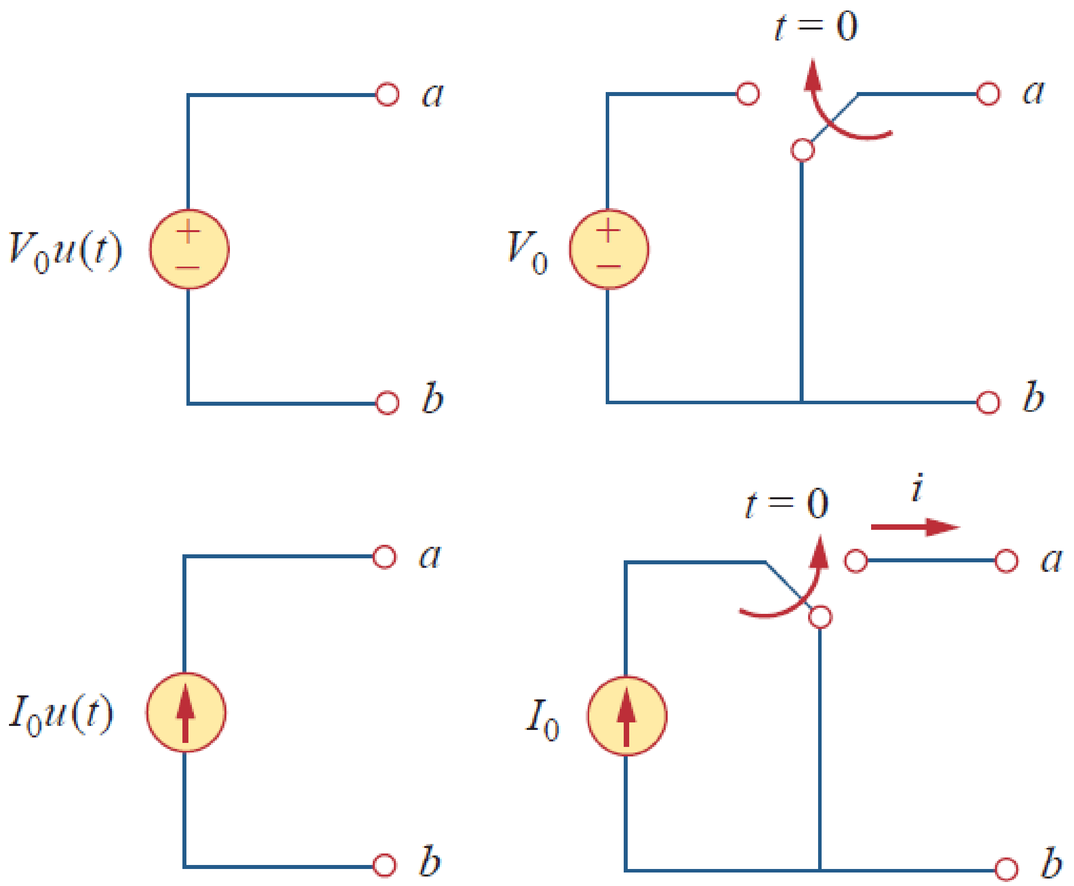
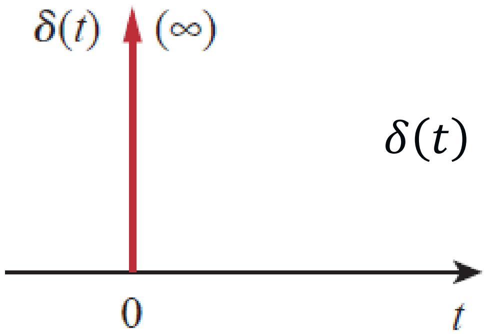
Equivalent Circuits of Sources



Singularity Functions (2) – Unit Impulse Function



$$\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0 & t < 0 \\ (undef.) & (t = 0) \\ 0 & t > 0 \end{cases}$$

Unit impulse

- ❑ Applied (resulting) shock
- ❑ Very short duration
- ❑ Unit area

$$\int_{0^-}^{0^+} \delta(t)\, dt = 1$$

Affect of unit impulse for *f(t)*

$$\int_a^b f(t)\,\delta(t-t_0)\,dt = \int_a^b f(t_0)\,\delta(t-t_0)\,dt = f(t_0)\int_a^b \delta(t-t_0)\,dt = f(t_0)$$

$\int_a^b f(t)\,\delta(t-t_0)\,dt = f(t_0)$ ← Sampling (shifting) property of impulse func.

$t_0 = 0 \rightarrow \int_a^b f(t)\,\delta(t)\,dt = f(0)$

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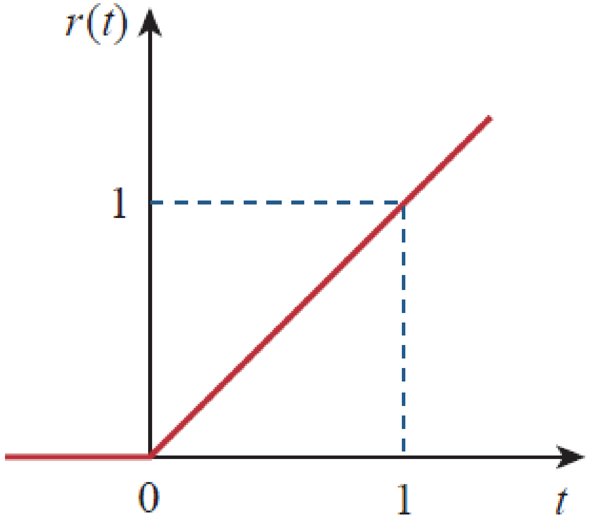
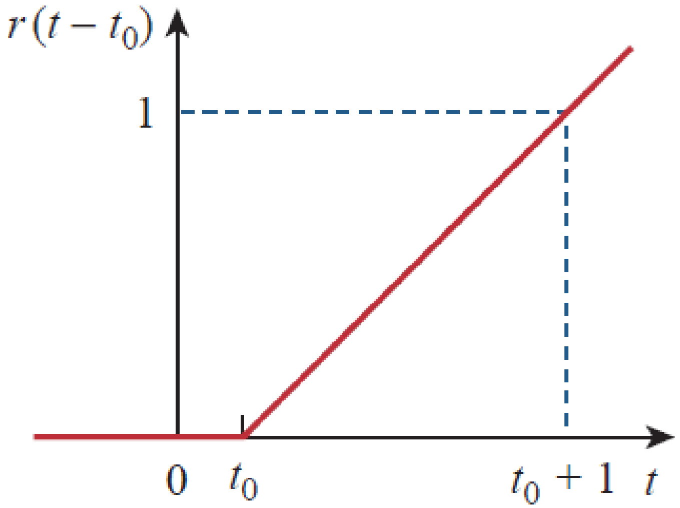
Singularity Functions (3) – Unit Ramp Function

Unit ramp function



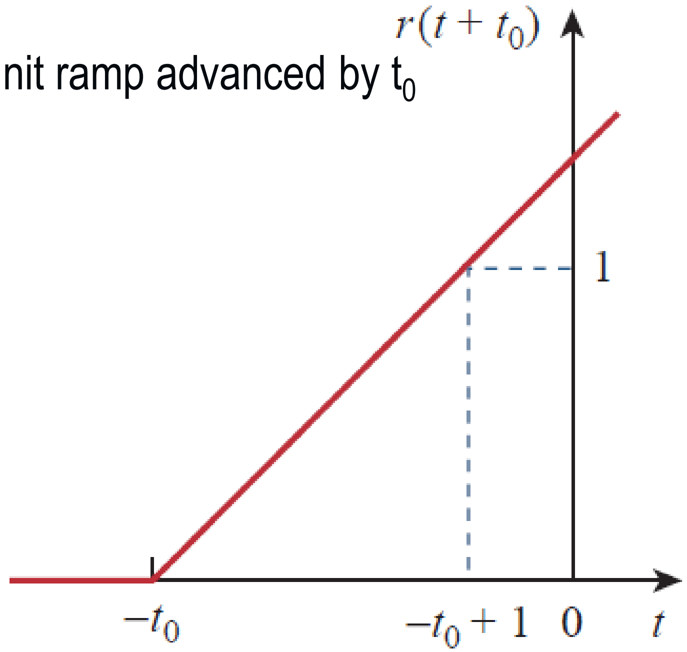
$$r(t) = \int_{-\infty}^{t} u(t)\,dt = \begin{cases} 0 & t \le 0 \\ t & t \ge 0 \end{cases}$$

Unit ramp delayed by t_0



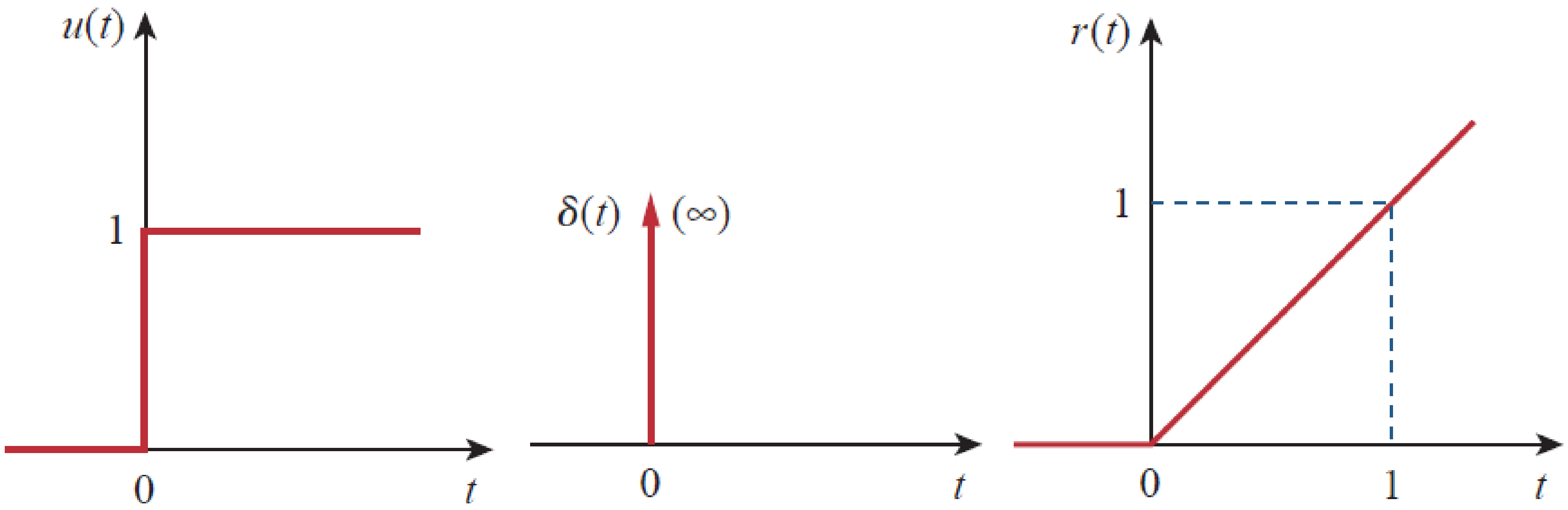
$$r(t - t_0) = \begin{cases} 0 & t \le t_0 \\ t - t_0 & t \ge t_0 \end{cases}$$

Unit ramp advanced by t_0



$$r(t + t_0) = \begin{cases} 0 & t \le -t_0 \\ t + t_0 & t \ge -t_0 \end{cases}$$

Singularity Functions – Summary



$$\delta(t) = \frac{du(t)}{dt}\,, \qquad u(t) = \frac{dr(t)}{dt} \qquad \textit{or} \qquad u(t) = \int_{-\infty}^{t} \delta(t)\,dt\,, \qquad r(t) = \int_{-\infty}^{t} u(t)\,dt$$

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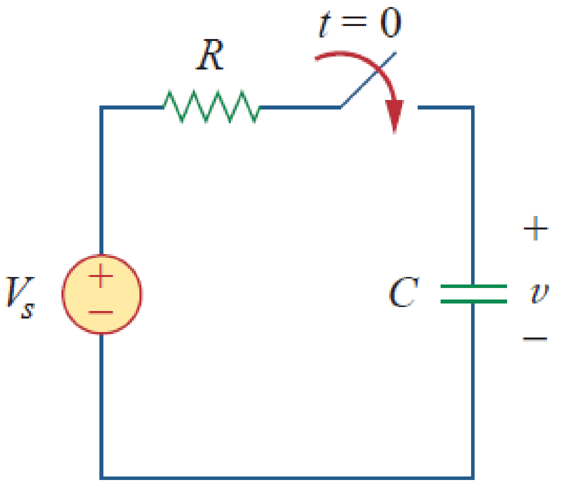
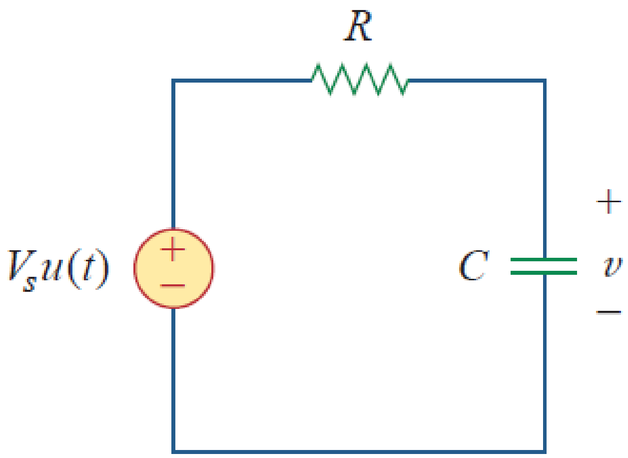
Step Response of an RC Circuit

Step response

- ‚Suddenly' applied dc source (i.e. voltage or current excitation is a step function)





$$\int_{V_0}^{v(t)} \frac{dv}{v-V_S} = \int_0^t \left(-\frac{dt}{RC}\right) \rightarrow \ln(v-V_S)\Big|_{V_0}^{v(t)} = -\frac{t}{RC}\Big|_0^t$$

$$\ln[v(t)-V_S] - \ln(V_0-V_S) = -\frac{t}{RC} + 0$$

$$\ln\frac{v(t)-V_S}{V_0-V_S} = -\frac{t}{RC} \rightarrow \frac{v(t)-V_S}{V_0-V_S} = e^{-\frac{t}{\tau}} \leftarrow \tau = RC$$

$$v(0^-) = v(0^+) = V_0$$

$$t > 0 \rightarrow \frac{dv}{dt} + \frac{v}{RC} = \frac{V_S}{RC}$$

$$v(t) = V_S + (V_0 - V_S)e^{-\frac{t}{\tau}} \leftarrow t > 0$$

$$C\frac{dv}{dt} + \frac{v - V_S\,u(t)}{R} = 0$$

$$\frac{dv}{dt} = -\frac{v-V_S}{RC} \rightarrow \frac{dv}{v-V_S} = -\frac{dt}{RC}$$

$$v(t) = \begin{cases} V_0 & t < 0 \\ V_S + (V_0 - V_S)e^{-\frac{t}{\tau}} & t > 0 \end{cases}$$

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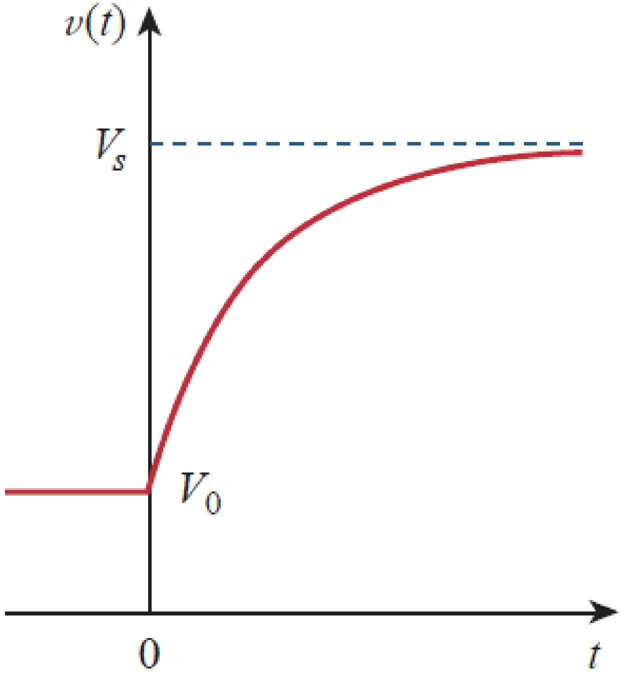
Step Response of an RC Circuit

Complete (total) response

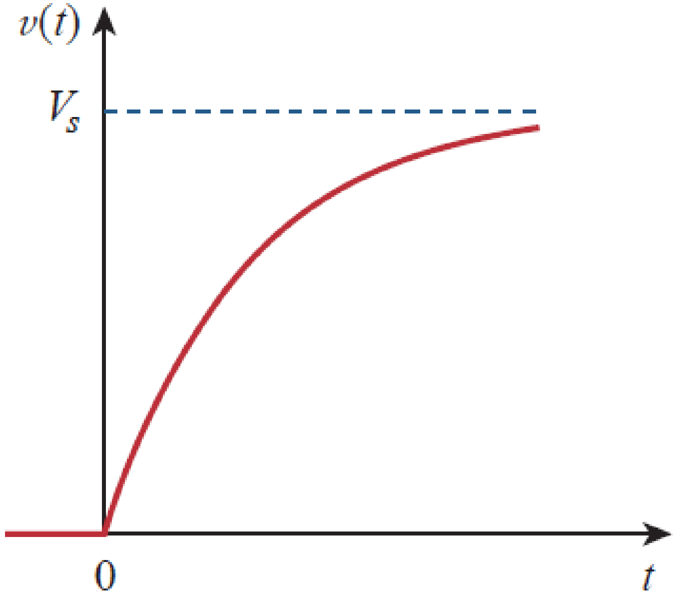
$$v(t) = \begin{cases} V_0 & t < 0 \\ V_S + (V_0 - V_S)e^{-\frac{t}{\tau}} & t > 0 \end{cases}$$



Originally uncharged capacitor

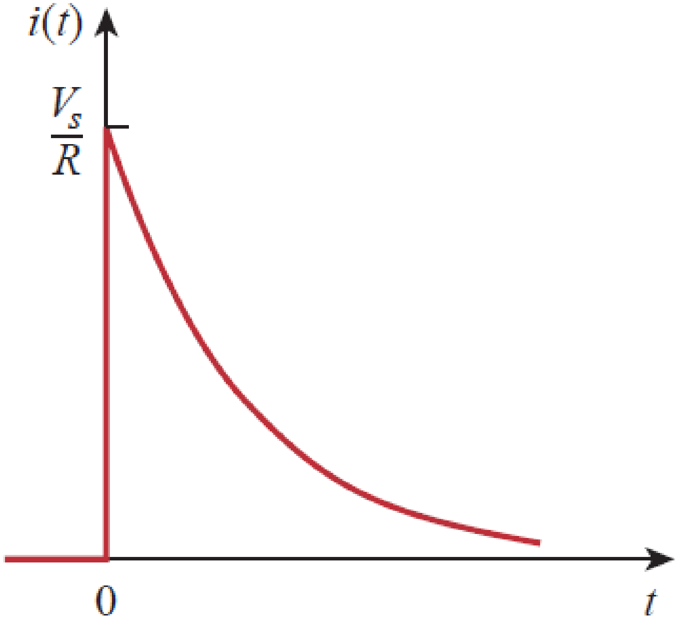
$$V_0 = 0$$

$$\rightarrow v(t) = V_S\left(1 - e^{-\frac{t}{\tau}}\right)u(t)$$



$$i(t) = C\frac{dv}{dt} = \frac{C}{\tau} V_S\, e^{-\frac{t}{\tau}} u(t)$$

$$i(t) = \frac{V_S}{R}\, e^{-\frac{t}{\tau}} u(t)$$



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Step Response of an RC Circuit

- **Decompositing way 1**

$$\rightarrow v(t) = V_S + (V_0 - V_S)e^{-\frac{t}{\tau}} = v_{ss} + v_{tr} \qquad v_{ss} = V_S\,, \qquad v_{tr} = (V_0 - V_S)e^{-\frac{t}{\tau}}$$

Complete response = steady-state response (‚*permanent part*') **+ transient response** ('*temporary part*')

- **Decompositing way 2**

$$\rightarrow v(t) = V_0\, e^{-\frac{t}{\tau}} + V_S\left(1 - e^{-\frac{t}{\tau}}\right) = v_n + v_f \qquad v_n = V_0\, e^{-\frac{t}{\tau}}\,, \qquad v_f = V_S\left(1 - e^{-\frac{t}{\tau}}\right)$$

Complete response = natural response (‚*stored energy*') **+ forced response** ('*independent source*')

$$v(t) = v_{ss} + v_{tr} = v(\infty) + [v(0^+) - v(\infty)]\, e^{-\frac{t}{\tau}}$$

Key to work with step response of an RC circuits

- Find initial condition $v(0) = V_0$
- Find steady-state (final) capacitor voltage $v(\infty)$
- Find RC time constant *(more than one R → Thevenin equivalent!)*

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- ❑ Source-Free RC Circuits
- ❑ Source-Free RL Circuits
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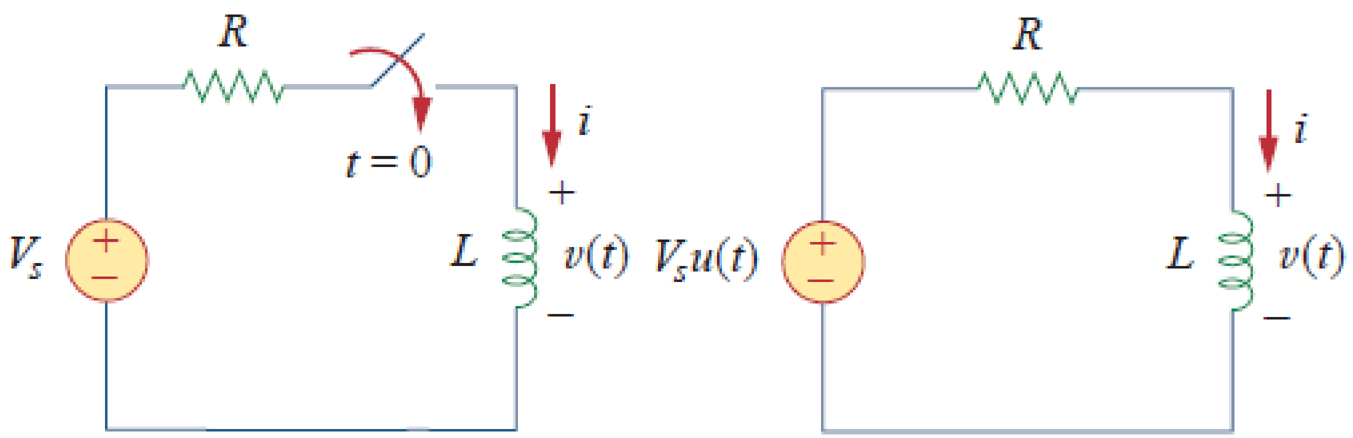
- ❑ **Step Response of an RL Circuit**

Step Response of an RL Circuit



$$i(t) = v_{ss} + v_{tr}$$

$$i_{ss} = \frac{V_S}{R}\,, \qquad i_{tr} = A\,e^{-\frac{t}{\tau}}, \qquad \tau = \frac{L}{R}$$

$$i(0^-) = i(0^+) = I_0$$

$$i(t) = \frac{V_S}{R} + A\,e^{-\frac{t}{\tau}} \rightarrow i(0) = I_0 = \frac{V_S}{R} + A\,e^0$$

$$A = I_0 - \frac{V_S}{R} \rightarrow i(t) = \frac{V_S}{R} + \left(I_0 - \frac{V_S}{R}\right) e^{-t/\tau}$$

$$i(t) = i_{ss} + i_{tr} = i(\infty) + [i(0^+) - i(\infty)]\,e^{-\frac{t}{\tau}}$$

Key to work with step response of an RL circuits

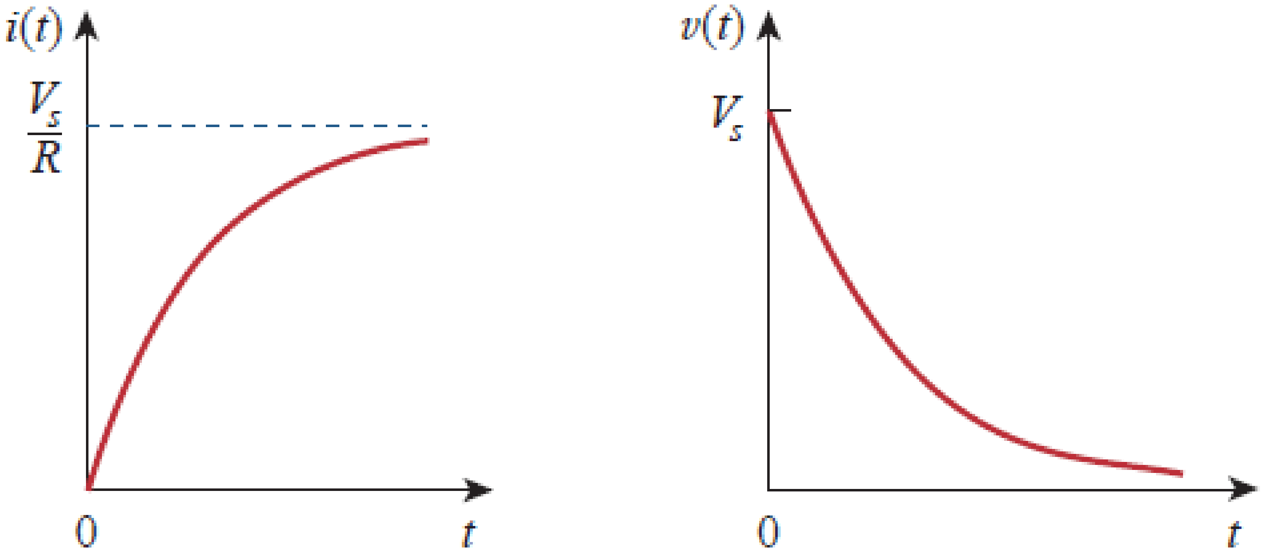
- ❑ Find initial condition $i(0) = I_0$
- ❑ Find steady-state (final) inductor current $v(\infty)$
- ❑ Find L/R time constant

  (more than one R → Thevenin equivalent!)

Originally energy-free inductor

$$I_0 = 0 \rightarrow i(t) = \begin{cases} 0 & t < 0 \\ \dfrac{V_S}{R}\left(1 - e^{-t/\tau}\right) & t > 0 \end{cases} \ \ldots or \ldots\ i(t) = \frac{V_S}{R}\left(1 - e^{-\frac{t}{\tau}}\right)u(t)$$

$$v(t) = L\frac{di}{dt} = V_S\frac{L}{\tau R}\, e^{-\frac{t}{\tau}}\, u(t) \rightarrow v(t) = \frac{V_S}{R}\, e^{-\frac{t}{\tau}}\, u(t)\,, \qquad \tau = \frac{L}{R}$$



Questions

$$i(t) = i_{ss} + i_{tr} = i(\infty) + [i(0^+) - i(\infty)]\, e^{-\frac{t}{\tau}}$$