DR. GYURCSEK ISTVÁN

Temperature Transients (Device Warming)

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in electric circuits – an overview*
- *Ormándlaky Zsolt: Átmeneti jelenségek*

- **Warming at Constant Power Condition**
- Warming at Variant Power Condition
- Example 1: Cooling, Overheating, Warm-up
- Examle 2: Thermal Runaway

Device Warming, Constant Power 1

Input Energy = **Device Warming** + **Ambient Warming**

$$P \cdot dt = c \cdot m \cdot d\vartheta + \alpha \cdot A \cdot \vartheta \cdot dt$$

$$P = c \cdot m \cdot \frac{d\vartheta}{dt} + \alpha \cdot A \cdot \vartheta$$

First order differential equation

$$\boxed{x(t) = X_{st} + [x(0) \ - X_{st}] \cdot e^{-\frac{t}{T}}}$$

$$\vartheta(t) = \vartheta_m + [\vartheta(0) \ - \vartheta_m] \cdot e^{-\frac{t}{T}}$$

$$Steady\ state\ \rightarrow P = \alpha \cdot A \cdot \vartheta_m$$

$$\vartheta_m = \frac{P}{\alpha\ A}; \quad (device\ temp. \rightarrow \vartheta_a + \vartheta_m)$$

Characteristic eq. from homogeneous diff. eq.

$$0 = \frac{c \cdot m}{-T} + \alpha \cdot A \rightarrow T = \frac{c \cdot m}{\alpha \cdot A}$$

Where…

- P – *input power*
- t – *time*
- c – *specific heat*
- m – *mass*
- $\boldsymbol{\vartheta}$ – *temperature difference*
- $\boldsymbol{\alpha}$ – *convection factor*
- A – *surface*
- $\boldsymbol{\vartheta}_a$ – *ambient temperature*
- $\boldsymbol{\vartheta}_m$ – *steady state temp diff.*

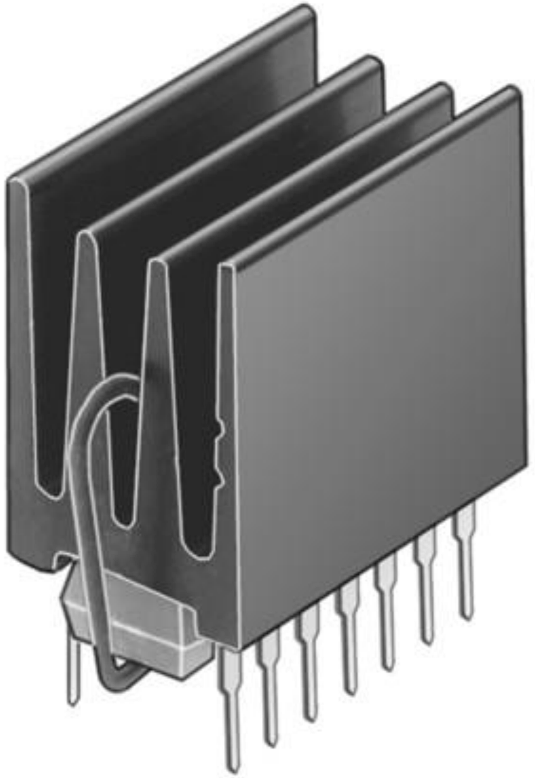
Device Warming, Constant Power 2

$$\vartheta_{(t)} = \vartheta_m + \left[\vartheta_{(0)} - \vartheta_m\right] \cdot e^{-\frac{t}{T}}$$

$$\Downarrow$$

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

- 1st part → decay of initial temp. difference
- 2nd part → set *(‚build up')* of steady state



- ❑ Warming at Constant Power Condition
- ❑ **Warming at Variant Power Condition**
- ❑ Example 1: Cooling, Overheating, Warm-up
- ❑ Examle 2: Thermal Runaway

Device Warming, Variant Power

$$P_{(\vartheta)} = I^2 \cdot R = I^2 \cdot R_0(1 + k \cdot \vartheta) = P_0(1 + k \cdot \vartheta)$$

$$P = c \cdot m \cdot \frac{d\vartheta}{dt} + \alpha \cdot A \cdot \vartheta$$

$$P_0(1 + k \cdot \vartheta) = c \cdot m \cdot \frac{d\vartheta}{dt} + \alpha \cdot A \cdot \vartheta$$

$$P_0 + k \cdot P_0 \cdot \vartheta = c \cdot m \cdot \frac{d\vartheta}{dt} + \alpha \cdot A \cdot \vartheta$$

$$P_0 = c \cdot m \cdot \frac{d\vartheta}{dt} + (\alpha \cdot A - k \cdot P_0) \cdot \vartheta$$

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

$$0 = \frac{c \cdot m}{-T} + (\alpha \cdot A - k \cdot P_0)$$

$$T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_m = \frac{P_0}{\alpha \cdot A - k \cdot P_0}$$

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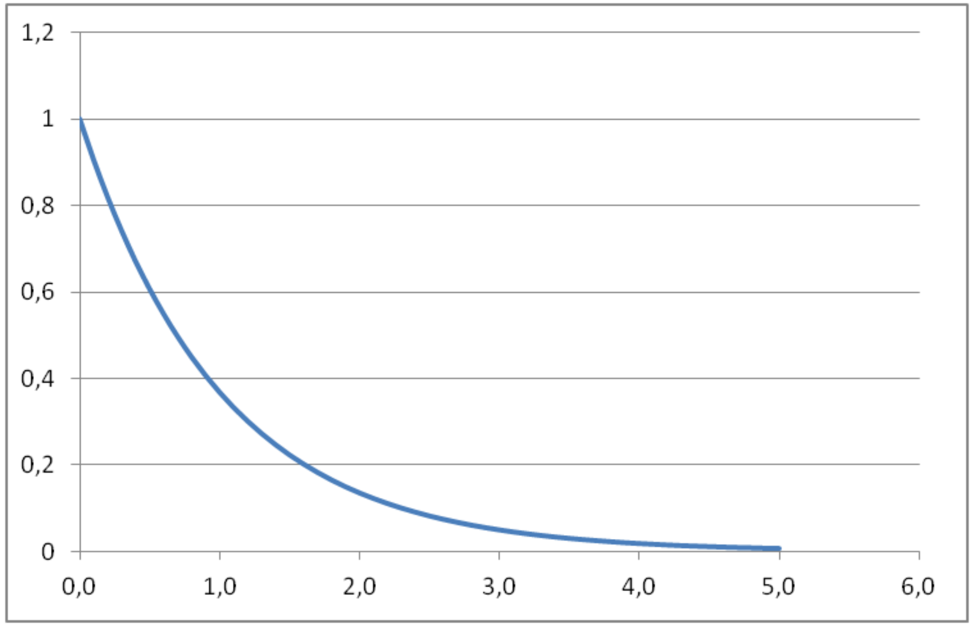
Cooling

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right) \qquad T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0} \qquad \vartheta_m = \frac{P_0}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_0 > 0, P_0 = 0 \rightarrow \vartheta_m = 0$$

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}}$$

$$T = \frac{c \cdot m}{\alpha \cdot A}$$



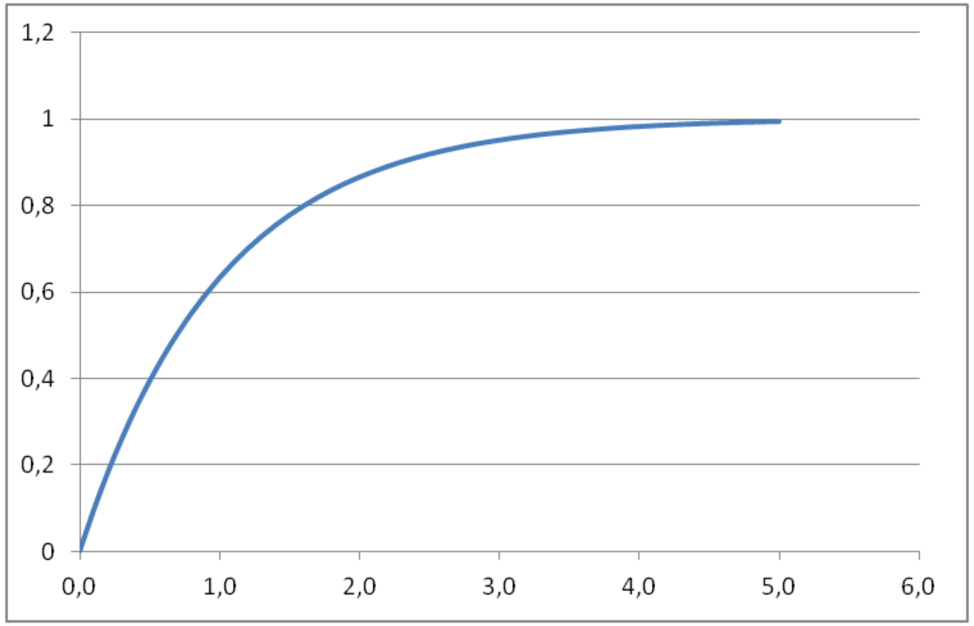
Overheating from steady state

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right) \qquad T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0} \qquad \vartheta_m = \frac{P_0}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_0 = 0, P_0 \neq 0 \rightarrow \vartheta_m \neq 0$$

$$\vartheta_{(t)} = \vartheta_m \left(1 - e^{-\frac{t}{T}}\right)$$

$$T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0}$$



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Overheating from non-steady state

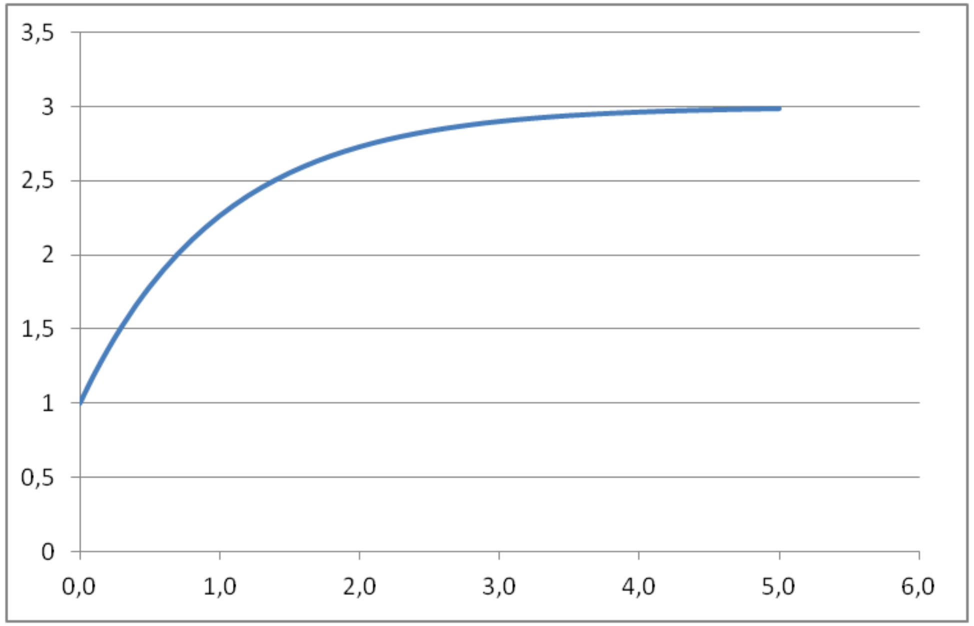
$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right) \qquad T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0} \qquad \vartheta_m = \frac{P_0}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_0 \neq 0, P_0 \neq 0 \rightarrow \vartheta_m \neq 0$$

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

$$\Downarrow$$

$$\vartheta_{(t)} = \vartheta_{(0)} + \left(\vartheta_m - \vartheta_{(0)}\right) \cdot \left(1 - e^{-\frac{t}{T}}\right)$$



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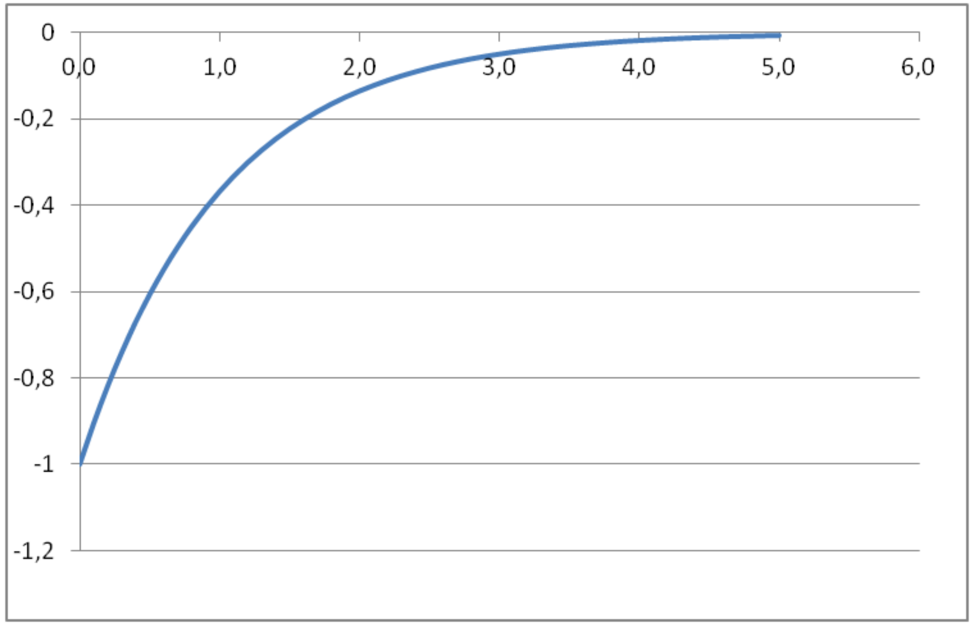
Warm-up

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right) \qquad T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0} \qquad \vartheta_m = \frac{P_0}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_0 < 0, P_0 = 0 \rightarrow \vartheta_m = 0$$

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}}$$

$$T = \frac{c \cdot m}{\alpha \cdot A}$$



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Thermal Runaway

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right) \qquad T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0} \qquad \vartheta_m = \frac{P_0}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_0 \neq 0, P_0 \neq 0, (\alpha \cdot A - k \cdot P_0) < 0$$

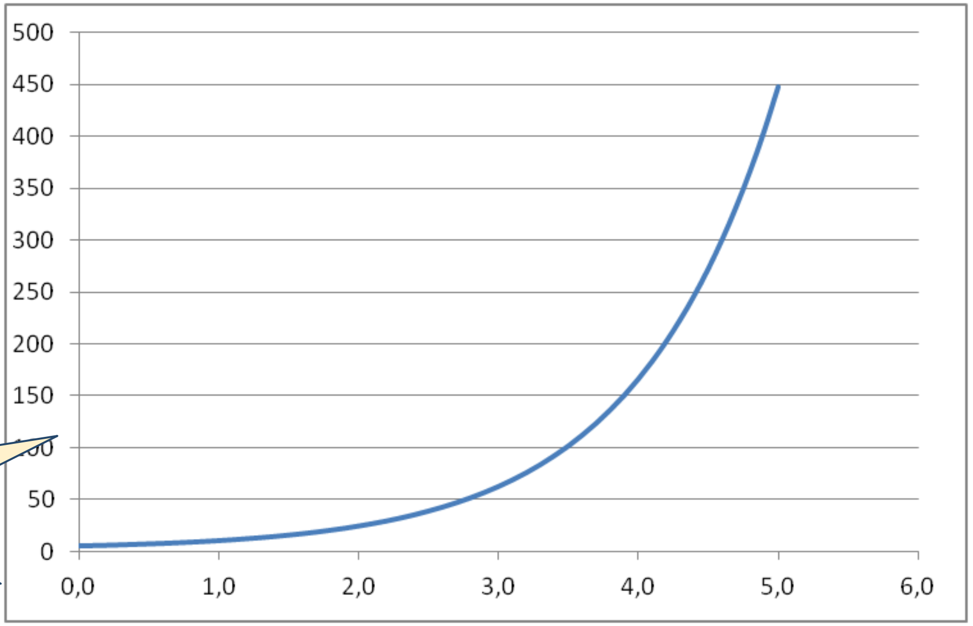
$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

$$\Downarrow$$

$$\vartheta_{(t)} = \vartheta_m + \left(\vartheta_{(0)} - \vartheta_m\right) \cdot e^{-\frac{t}{T}}$$

$$T < 0, \vartheta_m < 0 \rightarrow$$

THERMAL RUNAWAY!



Questions