



DR. GYURCSEK ISTVÁN

Temperature Transients (Device Warming)

Sources and additional materials (recommended)

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in electric circuits – an overview*
- ❑ *Ormándlaky Zsolt: Átmeneti jelenségek*



- Warming at Constant Power Condition**
- Warming at Variant Power Condition
- Example 1: Cooling, Overheating, Warm-up
- Example 2: Thermal Runaway

Device Warming – Constant Power (1)



$$P \cdot dt = c \cdot m \cdot d\vartheta + \alpha \cdot A \cdot \vartheta \cdot dt$$

$$P = c \cdot m \cdot \frac{d\vartheta}{dt} + \alpha \cdot A \cdot \vartheta$$

First order differential equation

$$x(t) = X_{st} + [x(0) - X_{st}] \cdot e^{-\frac{t}{T}}$$

$$\vartheta(t) = \vartheta_m + [\vartheta(0) - \vartheta_m] \cdot e^{-\frac{t}{T}}$$

Steady state $\rightarrow P = \alpha \cdot A \cdot \vartheta_m$

$$\vartheta_m = \frac{P}{\alpha A}; \quad (\text{device temp.} \rightarrow \vartheta_a + \vartheta_m)$$

Characteristic eq. from homogeneous diff. eq.

$$0 = \frac{c \cdot m}{-T} + \alpha \cdot A \rightarrow T = \frac{c \cdot m}{\alpha \cdot A}$$

Where...

- P – input power
- t – time
- c – specific heat
- m – mass
- ϑ – temperature difference
- α – convection factor
- A – surface
- ϑ_a – ambient temperature
- ϑ_m – steady state temp diff.

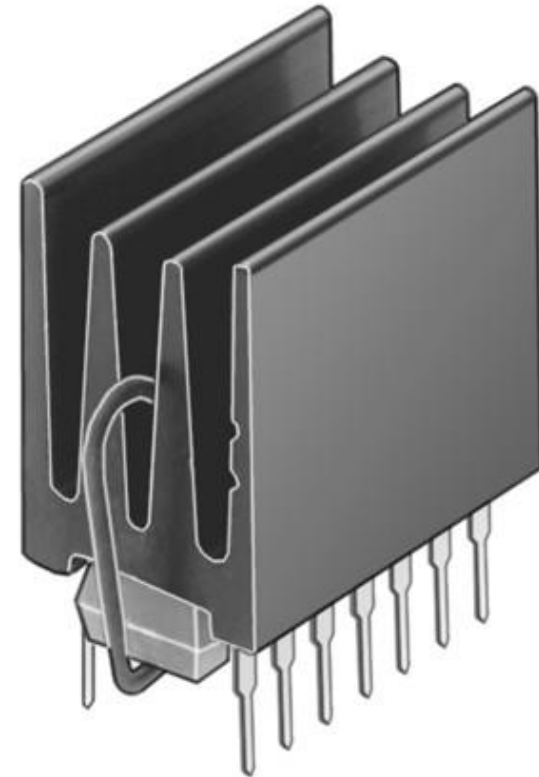
Device Warming – Constant Power (2)

$$\vartheta_{(t)} = \vartheta_m + [\vartheta_{(0)} - \vartheta_m] \cdot e^{-\frac{t}{T}}$$



$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

- ❑ 1st part → decay of initial temp. difference
- ❑ 2nd part → set (*build up*) of steady state





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Device Warming – Variant Power

$$P_{(\vartheta)} = I^2 \cdot R = I^2 \cdot R_0(1 + k \cdot \vartheta) = P_0(1 + k \cdot \vartheta)$$

$$\Rightarrow P = c \cdot m \cdot \frac{d\vartheta}{dt} + \alpha \cdot A \cdot \vartheta$$

$$P_0(1 + k \cdot \vartheta) = c \cdot m \cdot \frac{d\vartheta}{dt} + \alpha \cdot A \cdot \vartheta$$

$$P_0 + k \cdot P_0 \cdot \vartheta = c \cdot m \cdot \frac{d\vartheta}{dt} + \alpha \cdot A \cdot \vartheta$$

$$P_0 = c \cdot m \cdot \frac{d\vartheta}{dt} + (\alpha \cdot A - k \cdot P_0) \cdot \vartheta$$

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

$$0 = \frac{c \cdot m}{-T} + (\alpha \cdot A - k \cdot P_0)$$

$$T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_m = \frac{P_0}{\alpha \cdot A - k \cdot P_0}$$



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Examples – Cooling

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

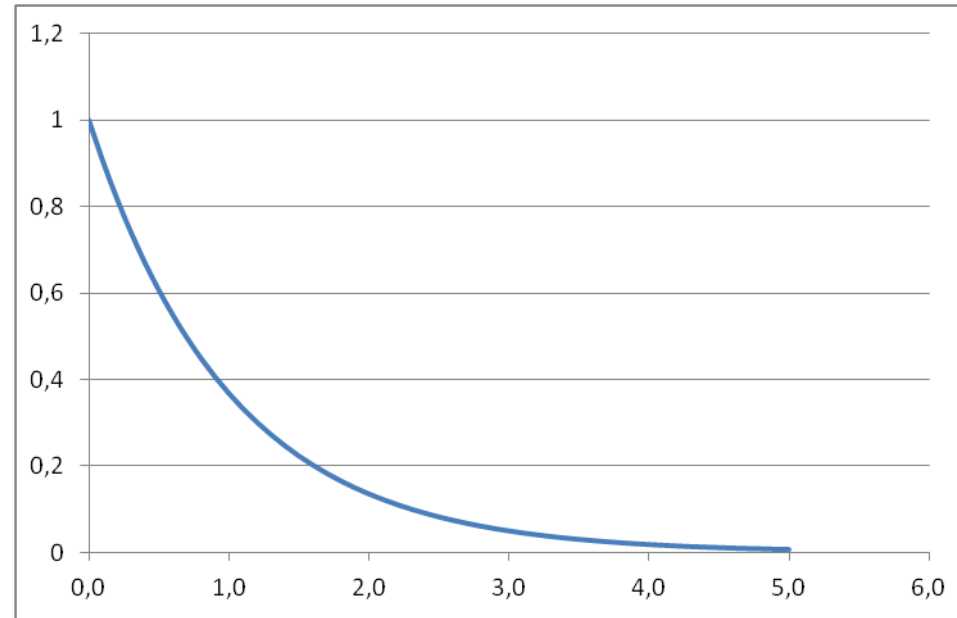
$$T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_m = \frac{P_0}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_0 > 0, P_0 = 0 \rightarrow \vartheta_m = 0$$

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}}$$

$$T = \frac{c \cdot m}{\alpha \cdot A}$$



Examples – Overheating (from steady state)

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

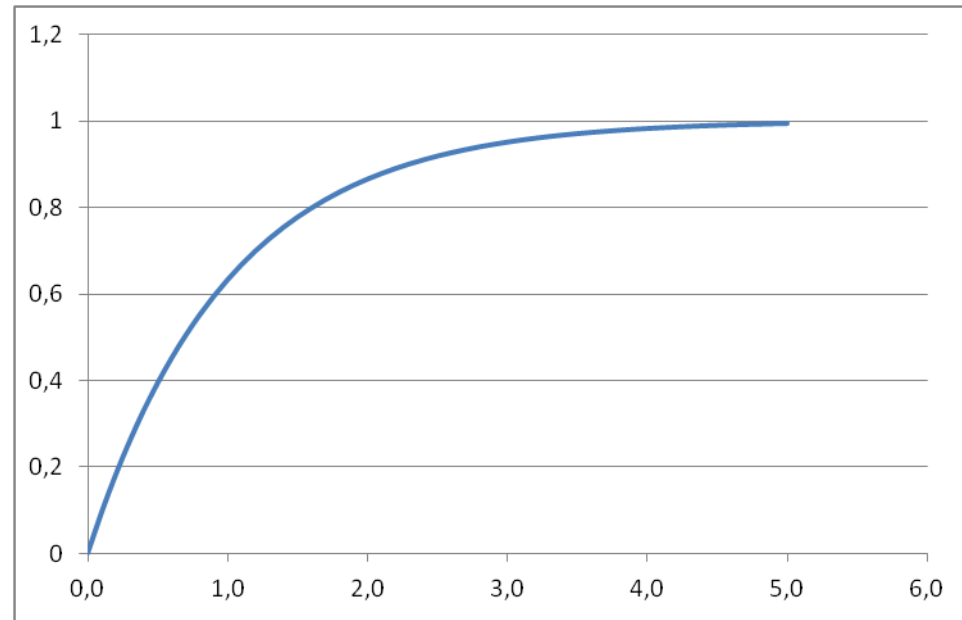
$$T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_m = \frac{P_0}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_0 = 0, P_0 \neq 0 \rightarrow \vartheta_m \neq 0$$

$$\vartheta_{(t)} = \vartheta_m \left(1 - e^{-\frac{t}{T}}\right)$$

$$T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0}$$



Examples – Overheating (from non-steady state)

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

$$T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0}$$

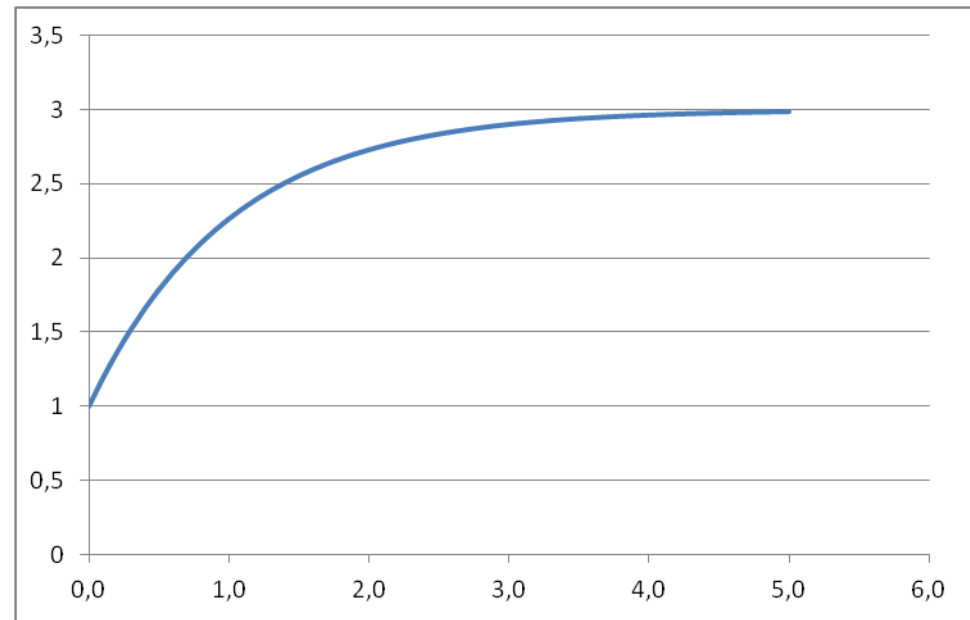
$$\vartheta_m = \frac{P_0}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_0 \neq 0, P_0 \neq 0 \rightarrow \vartheta_m \neq 0$$

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$



$$\vartheta_{(t)} = \vartheta_{(0)} + (\vartheta_m - \vartheta_{(0)}) \cdot \left(1 - e^{-\frac{t}{T}}\right)$$



Examples – Warm-up

$$\vartheta(t) = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

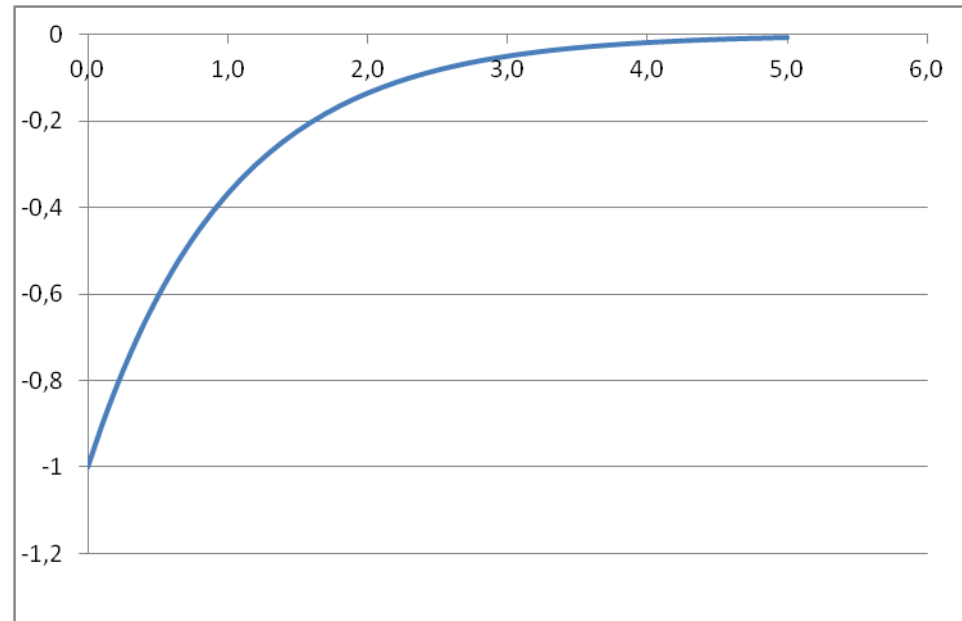
$$T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_m = \frac{P_0}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_0 < 0, P_0 = 0 \rightarrow \vartheta_m = 0$$

$$\vartheta(t) = \vartheta_{(0)} \cdot e^{-\frac{t}{T}}$$

$$T = \frac{c \cdot m}{\alpha \cdot A}$$





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Example – Thermal Runaway

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$

$$T = \frac{c \cdot m}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_m = \frac{P_0}{\alpha \cdot A - k \cdot P_0}$$

$$\vartheta_0 \neq 0, P_0 \neq 0, (\alpha \cdot A - k \cdot P_0) < 0$$

$$\vartheta_{(t)} = \vartheta_{(0)} \cdot e^{-\frac{t}{T}} + \vartheta_m \cdot \left(1 - e^{-\frac{t}{T}}\right)$$



$$\vartheta_{(t)} = \vartheta_m + (\vartheta_{(0)} - \vartheta_m) \cdot e^{-\frac{t}{T}}$$

$$T < 0, \vartheta_m < 0$$

**THERMAL
RUNAWAY!**

