DR. GYURCSEK ISTVÁN

Exercises with Circuit Elements

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

Charge and Current

BAS.01
Determine the current in a circuit if a charge of 0.035C passes a given point in 70ms (milliseconds).

Solution $I = \frac{Q}{t} = \frac{35mC}{70ms} = 0.5A = 500mA$

BAS.02
How much charge is represented by 4.600 electrons?

Solution

$$e = -1.602 \cdot 10^{-19}\,C \rightarrow Q = e \cdot n = -1.602 \cdot 10^{-19} \cdot 4600 = -7369 \cdot 10^{-19}\,C = -736.9\,aC$$

Charge and Current

BAS.03
The total charge entering a terminal is given by the function of $\mathrm{q} = 5 \cdot \mathrm{t} \cdot \sin(4\pi \cdot \mathrm{t})\ \mathrm{mC}$ where t is the time measured in seconds. Calculate the current at t=0.5 s.

Solution $$i = \frac{dq}{dt} = \frac{d}{dt}(5t\,sin\,4\pi t\,) = (5\,sin\,4\pi t + 20\pi t\,cos\,4\pi t)\ mA \qquad t = 0.5\ s\ \rightarrow i = 31.42\ mA$$

BAS.04
Determine the total charge entering a terminal between t=1 s and t=2 s if the current passing the terminal is

$i = (3t^2 - t)\ mA$

Solution $$q = \int_{t=1}^{2} i \cdot dt = \int_{t=1}^{2} (3t^2 - t) \cdot dt = \left[t^3 - \frac{t^2}{2}\right]_1^2 = \left[(8-2) - \left(1 - \frac{1}{2}\right)\right] \cdot 10^{-3} = 5.5mC$$

Power and Energy

BAS.05 – An electric heater consumes 1.8MJ when connected to a 230 V supply for 30 minutes. Find the power rating of the heater and the current taken from the supply.

Solution $P = \frac{W}{t} = \frac{1.8 \cdot 10^6}{30 \cdot 60} = 1000\, W = 1\, kW$ $\quad P = V \cdot I \rightarrow I = \frac{P}{V} = \frac{1000}{230} = 4.35\, A$

BAS.06 – An energy sources forces a constant current of 2A for 10 s to flow through a light bulb. If 4.6 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution $\Delta Q = I \cdot \Delta t = 2 \cdot 10 = 20\, C$ $\quad V = \frac{\Delta W}{\Delta Q} = \frac{4.6 \cdot 10^3}{20} = 230V$

BASS.07

Find the power delivered to an element at t=3 ms if its current is $i = 5\, cos(60 \cdot \pi \cdot t)\, A$ and its voltage is

$(a)\, v = 3i$

(b) $v = 3\frac{di}{dt}$

Solution:

(a) $v = 3i = 15cos(60\pi t)\, V, p = vi = 75cos^2(60\pi t)\, W$ → t=3 ms → $p = 75cos^2(60\pi \cdot 3 \cdot 10^{-3}) = 53.48\, W$

(b) $v = 3\frac{di}{dt} = 3 \cdot (-60\pi) \cdot 5sin(60\pi t) = -900\pi sin(60\pi t)\ \ V, p = vi = -4500\pi \cdot sin(60\pi t)cos(60\pi t)\ \ W$ →

→ t=3 ms → $p = -4500\pi \cdot sin(0.18\pi)cos(0.18\pi) = -6.396\, W$

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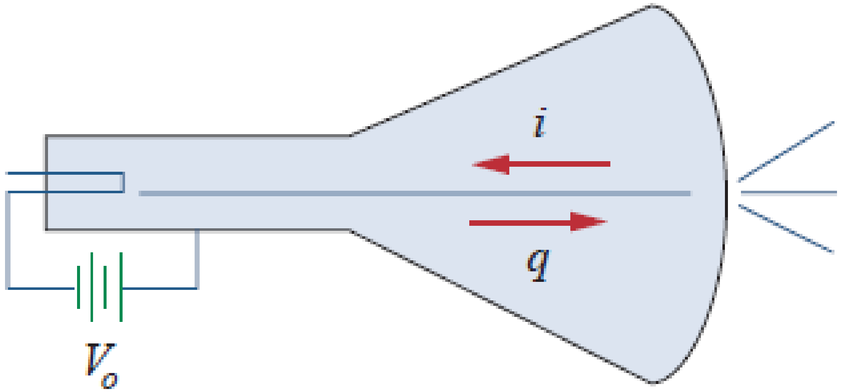
Power and Energy

BAS.08 – The electron beam in a TV picture tube (CRT) carries 10^{15} electrons per second. Determine the voltage V_0 needed to accelerate the electron beam to achieve 4 W.



Solution $q = n \cdot e$ $\quad e = -1.6 \cdot 10^{-19}\ C$

$$i = \frac{dq}{dt} = e\frac{dn}{dt} = -1.6 \cdot 10^{-19} \cdot 10^{15} = -1.6 \cdot 10^{-4} = -160\ \mu A$$

$$p = V_0 \cdot i \rightarrow V_0 = \frac{p}{i} = \frac{4}{1.6 \cdot 10^{-4}} = 25\ kV$$

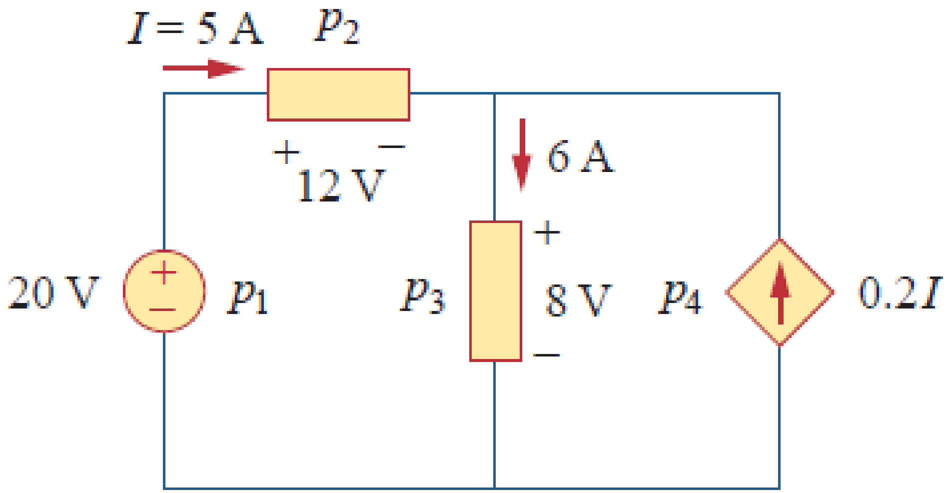
BAS.09 – Calculate the power in the electron beam in a TV picture tube if the beam carries 10^{13} electrons per second and is passing through plates maintained at a potential difference of 30 kV.

Solution $p = V_0 \cdot i = 30 \cdot 10^3 \cdot 1.6 \cdot 10^{-19} \cdot 10^{13} = 48\ mW$

Power and Energy

BAS.10 – Calculate the power supplied or absorbed by each element.

Solution



$$p_1 = 20 \cdot (-5) = -100\ W$$

$$p_2 = 12 \cdot 5 = 60\ W$$

$$p_3 = 8 \cdot 6 = 48\ W$$

$$p_4 = 8 \cdot (-0.2I) = -8\ W$$

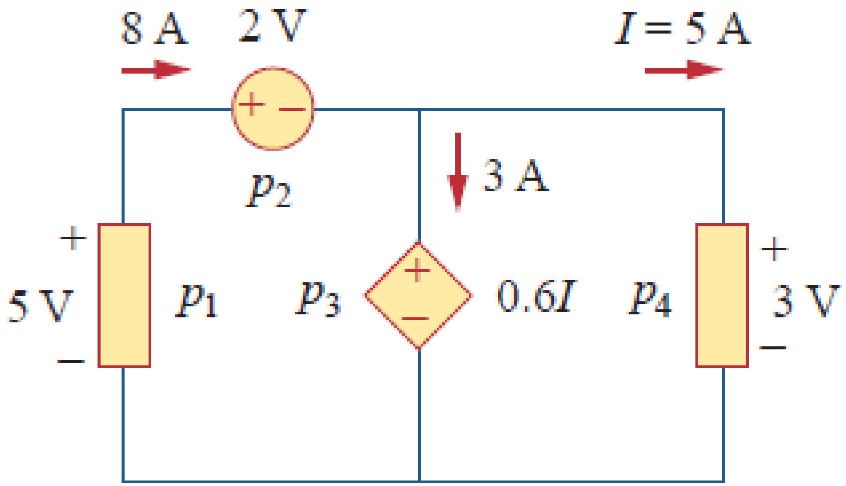
$$p_1 + p_2 + p_3 + p_4 = -100 + 60 + 48 - 8 = 0$$

Power and Energy

BAS.11 – Draw the characteristics of a Thevenin and a Norton generator.

BAS.12 – Compute the power absorbed or supplied by each component.

Solution



$$p_1 = 5 \cdot (-8) = -40\ W$$

$$p_2 = 2 \cdot 8 = 16\ W$$

$$p_3 = 3 \cdot 0.6 \cdot 5 = 9\ W$$

$$p_4 = 3 \cdot 5 = 15\ W$$

$$\sum_{i=1}^{4} p_i = -40 + 16 + 9 + 15 = 0$$

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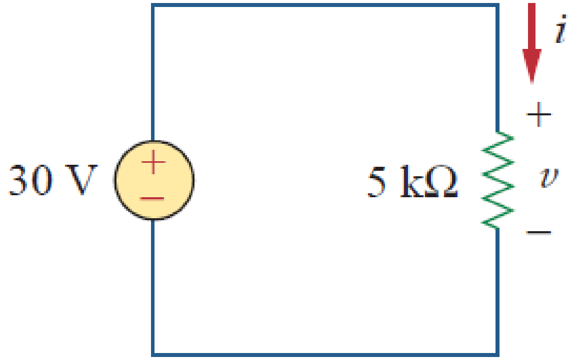
Circuit Elements

BAS.13 – A light bulb draws 0.5 A current at an input voltage of 230 V. Find the resistance of the filament and the power dissipated.

Solution $$R = \frac{v}{i} = \frac{230}{0.5} = 460\Omega$$

$$p = v \cdot i = 230 \cdot 0.5 = 115\ W\ or\ p = i^2 \cdot R = 0.5^2 \cdot 460 = 115\ W\ or\ p = \frac{v^2}{R} = \frac{230^2}{460} = 115\ W$$

BAS.14 – Calculate current I, conductance G, and power P.

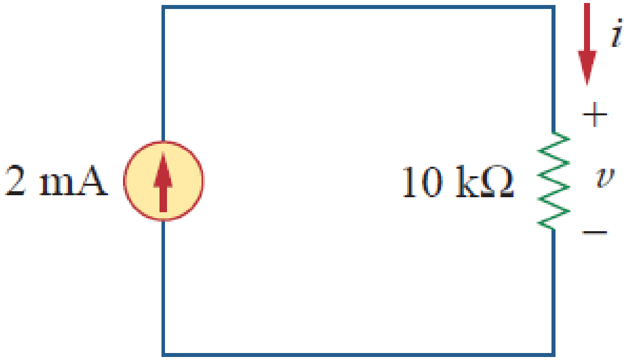


Solution $$i = \frac{v}{R} = \frac{30}{5 \cdot 10^3} = 6\ mA \qquad G = \frac{1}{R} = \frac{1}{5 \cdot 10^3} = 200\ \mu S$$

$$p = v \cdot i = 30 \cdot 6 \cdot 10^{-3} = 180\ mW$$

$$p = i^2 \cdot R = (6 \cdot 10^{-3})^2 \cdot 5 \cdot 10^3 = 180\ mW \qquad p = \frac{v^2}{R} = \frac{(30)^2}{5 \cdot 10^3} = 180\ mW$$

BAS.15 – Calculate the voltage and the conductance of the load.
Determine the total dissipated power in the circuit.



Solution v=20 V, G=100 µS, p=40 mW.

Circuit Elements

BAS.16 – (a) Calculate the charge stored on a 100nF capacitor with 20 V across it. (b) Find the stored energy.

Solution $(a)\ q = C \cdot v = 100 \cdot 10^{-9} \cdot 20 = 2\ \mu C$ $\quad (b)\ w = \frac{1}{2} C \cdot v^2 = \frac{1}{2} \cdot 100 \cdot 10^{-9} \cdot 400 = 20 \mu J$

BAS.17 – The voltage across a $5\mu\mathrm{F}$ capacitor is: $v(t) = 10\ cos(6000t)\ V$. Calculate the current through it.

Solution $i(t) = C \frac{dv}{dt} = 5 \cdot 10^{-6} \frac{d}{dt} \{10\ cos(6000t)\} = -5 \cdot 10^{-6} \cdot 6000 \cdot 10 \cdot sin(6000t) = -0.3 \cdot sin(6000t)\ A$

BAS.18 – Determine the voltage across a $2\ \mu\mathrm{F}$ capacitor if the current through it is $i(t) = 6 \cdot e^{-3000t}\ mA$
Assume that the initial capacitor voltage is zero.

Solution

$$v = \frac{1}{C} \int_0^t i \cdot dt + v(0)\,, \qquad v(0) = 0$$

$$v = \frac{1}{2 \cdot 10^{-6}} \int_0^t 6 \cdot e^{-3000t} \cdot 10^{-3} \cdot dt = 1 \cdot (1 - e^{-3000t})\ V$$

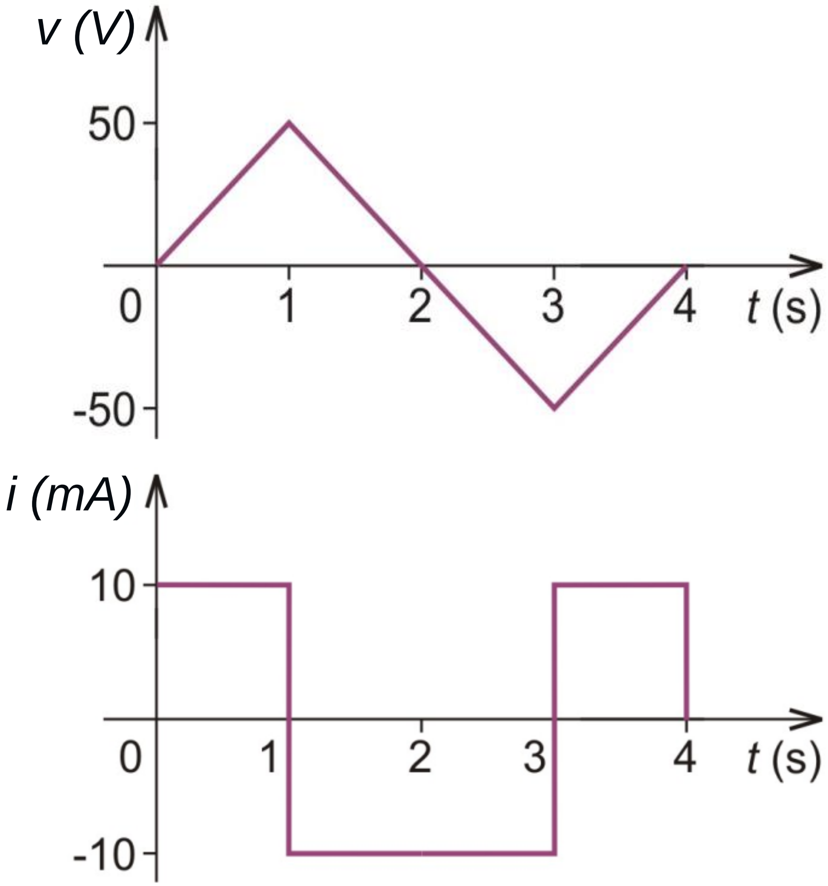
Circuit Elements

BAS.18 – Determine the current through a $200\ \mu F$ capacitor whose voltage $v(t)$ is shown in the Figure →

Solution:

$$v(t) = \begin{cases} 50t\ V & \leftarrow 0\ s \leq t < 1\ s \\ (100 - 50t)\ V & \leftarrow 1\ s \leq t < 3\ s \\ (-200 + 50t)\ V & \leftarrow 3\ s \leq t < 4\ s \\ 0\ V & \leftarrow otherwise \end{cases}$$

$$i(t) = \begin{cases} 10\ mA & \leftarrow 0\ s \leq t < 1\ s \\ -10\ mA & \leftarrow 1\ s \leq t < 3\ s \\ 10\ mA & \leftarrow 3\ s \leq t < 4\ s \\ -10\ mA & \leftarrow otherwise \end{cases}$$



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Circuit Elements

BAS.20 – The current through a 100mH inductors is $\mathrm{i(t)} = 10 \cdot \mathrm{t} \cdot \mathrm{e}^{-5\mathrm{t}}\ \mathrm{A}$. Find its voltage and the energy stored in it.

Solution $$v = L\frac{di}{dt} = 0.1\frac{d}{dt}(10 \cdot t \cdot e^{-5t}) = e^{-5t} + t \cdot (-5) \cdot e^{-5t} = (1-5t) \cdot e^{-5t}\ V$$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100 \cdot t^2 \cdot e^{-10t} = 5t^2 e^{-10t}\ J$$

BAS.21 – Find the current through a 500 mH inductor if its voltage is
Also find the energy stored within $0 < \mathrm{t} < 1s$

$$v(t) = \begin{cases} 0\ V & t \le 0\ s \\ 30t^2\ V & t > 0\ s \end{cases}$$

Solution $$i = \frac{1}{L}\int_{t_0}^{t} v(t) \cdot dt + I(t_0) = 2\int_0^t 30t^2 dt + 0 = 20\ t^3\ A$$

$$p = v \cdot i = 600t^5\ W \rightarrow w = \int_0^1 p \cdot dt = \int_0^1 600t^5 \cdot dt = 100\ J$$

$$Alternatively \rightarrow w = \frac{1}{2}Li^2\bigg|_{(t=1)} - \frac{1}{2}Li^2\bigg|_{(t=0)} = \frac{1}{2} \cdot 0.5 \cdot (20 \cdot 1^3)^2 - 0 = 100\ J$$

Questions