



DR. GYURCSEK ISTVÁN

Exercises with Circuit Elements

Sources and additional materials (recommended)

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*



BAS.01

Determine the current in a circuit if a charge of 0.035C passes a given point in 70ms (milliseconds).

Solution
$$I = \frac{Q}{t} = \frac{35mC}{70ms} = 0.5A = 500mA$$

BAS.02

How much charge is represented by 4.600 electrons?

Solution

$$e = -1.602 \cdot 10^{-19} C \rightarrow Q = e \cdot n = -1.602 \cdot 10^{-19} \cdot 4600 = -7369 \cdot 10^{-19} C = -736.9 aC$$

BAS.03

The total charge entering a terminal is given by the function of $q = 5 \cdot t \cdot \sin(4\pi \cdot t)$ mC where t is the time measured in seconds. Calculate the current at $t=0.5$ s.

Solution
$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA} \quad t = 0.5 \text{ s} \rightarrow i = 31.42 \text{ mA}$$

BAS.04

Determine the total charge entering a terminal between $t=1$ s and $t=2$ s if the current passing the terminal is

$$i = (3t^2 - t) \text{ mA}$$

Solution
$$q = \int_{t=1}^2 i \cdot dt = \int_{t=1}^2 (3t^2 - t) \cdot dt = \left[t^3 - \frac{t^2}{2} \right]_1^2 = \left[(8 - 2) - \left(1 - \frac{1}{2} \right) \right] \cdot 10^{-3} = 5.5 \text{ mC}$$

Power and Energy

BAS.05 – An electric heater consumes 1.8MJ when connected to a 230 V supply for 30 minutes. Find the power rating of the heater and the current taken from the supply.

Solution $P = \frac{W}{t} = \frac{1.8 \cdot 10^6}{30 \cdot 60} = 1000 \text{ W} = 1 \text{ kW}$ $P = V \cdot I \rightarrow I = \frac{P}{V} = \frac{1000}{230} = 4.35 \text{ A}$

BAS.06 – An energy sources forces a constant current of 2A for 10 s to flow through a light bulb. If 4.6 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution $\Delta Q = I \cdot \Delta t = 2 \cdot 10 = 20 \text{ C}$ $V = \frac{\Delta W}{\Delta Q} = \frac{4.6 \cdot 10^3}{20} = 230 \text{ V}$

BASS.07

Find the power delivered to an element at $t=3 \text{ ms}$ if its current is $i = 5 \cos(60 \cdot \pi \cdot t) \text{ A}$ and its voltage is

(a) $v = 3i$
(b) $v = 3 \frac{di}{dt}$

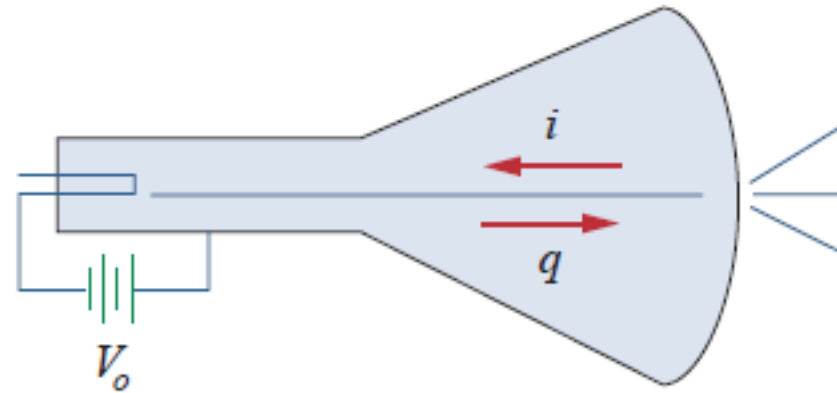
Solution:

(a) $v = 3i = 15 \cos(60\pi t) \text{ V}$, $p = vi = 75 \cos^2(60\pi t) \text{ W} \rightarrow t=3 \text{ ms} \rightarrow p = 75 \cos^2(60\pi \cdot 3 \cdot 10^{-3}) = 53.48 \text{ W}$

(b) $v = 3 \frac{di}{dt} = 3 \cdot (-60\pi) \cdot 5 \sin(60\pi t) = -900\pi \sin(60\pi t) \text{ V}$, $p = vi = -4500\pi \cdot \sin(60\pi t) \cos(60\pi t) \text{ W} \rightarrow t=3 \text{ ms} \rightarrow p = -4500\pi \cdot \sin(0.18\pi) \cos(0.18\pi) = -6.396 \text{ W}$

Power and Energy

BAS.08 – The electron beam in a TV picture tube (CRT) carries 10^{15} electrons per second. Determine the voltage V_0 needed to accelerate the electron beam to achieve 4 W.



Solution $q = n \cdot e$ $e = -1.6 \cdot 10^{-19} \text{ C}$

$$i = \frac{dq}{dt} = e \frac{dn}{dt} = -1.6 \cdot 10^{-19} \cdot 10^{15} = -1.6 \cdot 10^{-4} = -160 \mu\text{A}$$

$$p = V_0 \cdot i \rightarrow V_0 = \frac{p}{i} = \frac{4}{1.6 \cdot 10^{-4}} = 25 \text{ kV}$$

BAS.09 – Calculate the power in the electron beam in a TV picture tube if the beam carries 10^{13} electrons per second and is passing through plates maintained at a potential difference of 30 kV.

Solution $p = V_0 \cdot i = 30 \cdot 10^3 \cdot 1.6 \cdot 10^{-19} \cdot 10^{13} = 48 \text{ mW}$

BAS.10 – Calculate the power supplied or absorbed by each element.

Solution

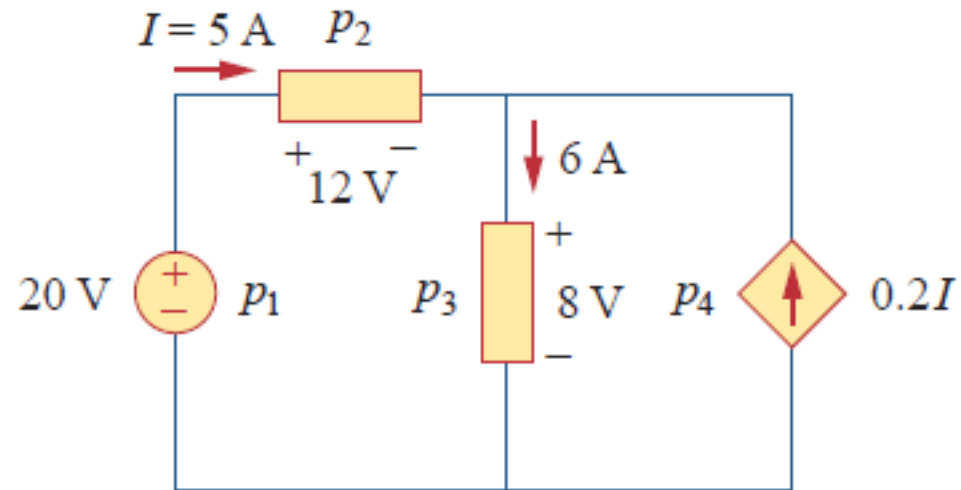
$$p_1 = 20 \cdot (-5) = -100 \text{ W}$$

$$p_2 = 12 \cdot 5 = 60 \text{ W}$$

$$p_3 = 8 \cdot 6 = 48 \text{ W}$$

$$p_4 = 8 \cdot (-0.2I) = -8 \text{ W}$$

$$p_1 + p_2 + p_3 + p_4 = -100 + 60 + 48 - 8 = 0$$



BAS.11 – Draw the characteristics of a Thevenin and a Norton generator.

BAS.12 – Compute the power absorbed or supplied by each component.

Solution

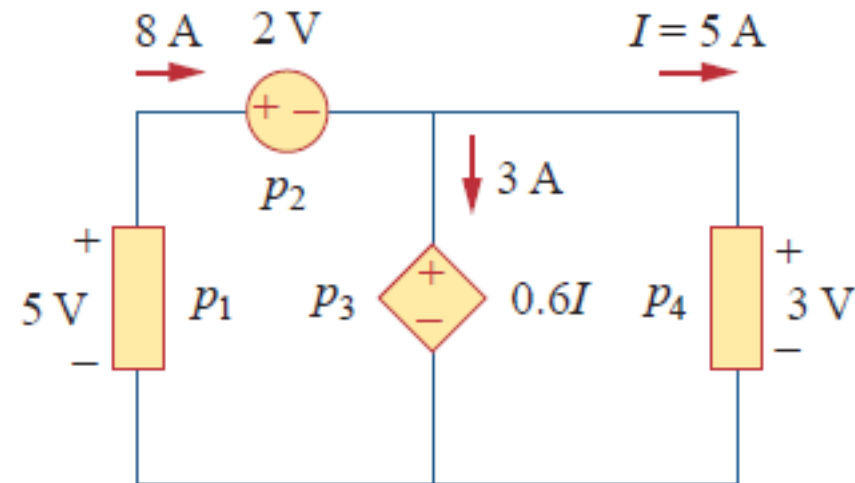
$$p_1 = 5 \cdot (-8) = -40 \text{ W}$$

$$p_2 = 2 \cdot 8 = 16 \text{ W}$$

$$p_3 = 3 \cdot 0.6 \cdot 5 = 9 \text{ W}$$

$$p_4 = 3 \cdot 5 = 15 \text{ W}$$

$$\sum_{i=1}^4 p_i = -40 + 16 + 9 + 15 = 0$$



Circuit Elements

BAS.13 – A light bulb draws 0.5 A current at an input voltage of 230 V. Find the resistance of the filament and the power dissipated.

Solution $R = \frac{v}{i} = \frac{230}{0.5} = 460 \Omega$

$$p = v \cdot i = 230 \cdot 0.5 = 115 \text{ W} \text{ or } p = i^2 \cdot R = 0.5^2 \cdot 460 = 115 \text{ W} \text{ or } p = \frac{v^2}{R} = \frac{230^2}{460} = 115 \text{ W}$$

BAS.14 – Calculate current I , conductance G , and power P .

Solution $i = \frac{v}{R} = \frac{30}{5 \cdot 10^3} = 6 \text{ mA}$

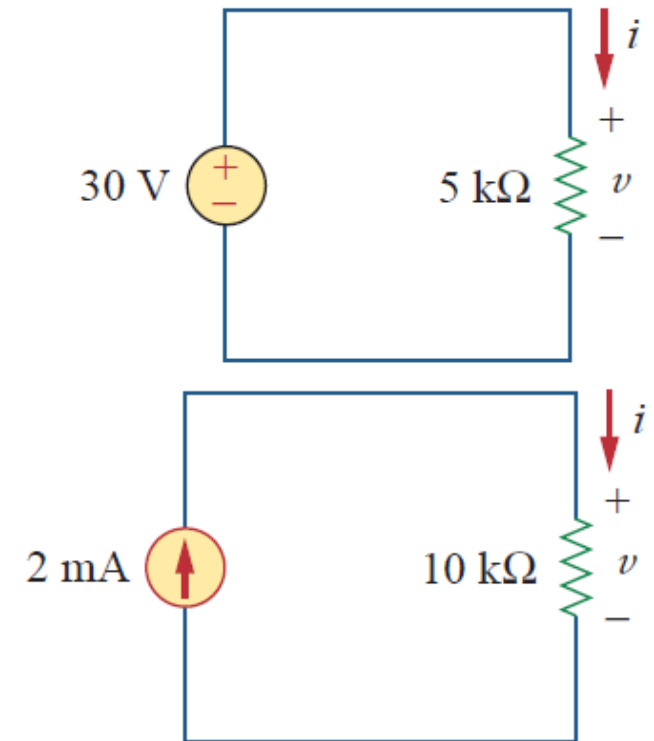
$$G = \frac{1}{R} = \frac{1}{5 \cdot 10^3} = 200 \mu\text{S}$$

$$p = v \cdot i = 30 \cdot 6 \cdot 10^{-3} = 180 \text{ mW}$$

$$p = i^2 \cdot R = (6 \cdot 10^{-3})^2 \cdot 5 \cdot 10^3 = 180 \text{ mW} \quad p = \frac{v^2}{R} = \frac{(30)^2}{5 \cdot 10^3} = 180 \text{ mW}$$

BAS.15 – Calculate the voltage and the conductance of the load. Determine the total dissipated power in the circuit.

Solution $v=20 \text{ V}$, $G=100 \mu\text{S}$, $p=40 \text{ mW}$.



Circuit Elements

BAS.16 – (a) Calculate the charge stored on a 100nF capacitor with 20 V across it. (b) Find the stored energy.

Solution (a) $q = C \cdot v = 100 \cdot 10^{-9} \cdot 20 = 2 \mu\text{C}$ (b) $w = \frac{1}{2} C \cdot v^2 = \frac{1}{2} \cdot 100 \cdot 10^{-9} \cdot 400 = 20 \mu\text{J}$

BAS.17 – The voltage across a 5 μF capacitor is: $v(t) = 10 \cos(6000t) \text{ V}$. Calculate the current through it.

Solution $i(t) = C \frac{dv}{dt} = 5 \cdot 10^{-6} \frac{d}{dt} \{10 \cos(6000t)\} = -5 \cdot 10^{-6} \cdot 6000 \cdot 10 \cdot \sin(6000t) = -0.3 \cdot \sin(6000t) \text{ A}$

BAS.18 – Determine the voltage across a 2 μF capacitor if the current through it is $i(t) = 6 \cdot e^{-3000t} \text{ mA}$. Assume that the initial capacitor voltage is zero.

Solution $v = \frac{1}{C} \int_0^t i \cdot dt + v(0), \quad v(0) = 0$

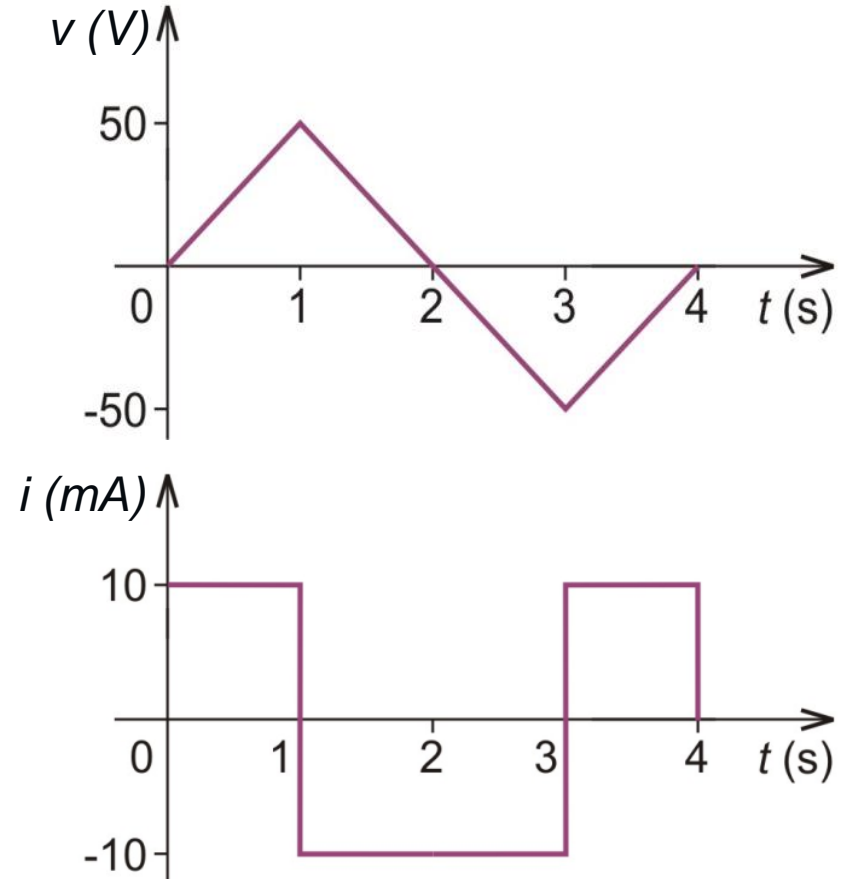
$$v = \frac{1}{2 \cdot 10^{-6}} \int_0^t 6 \cdot e^{-3000t} \cdot 10^{-3} \cdot dt = 1 \cdot (1 - e^{-3000t}) \text{ V}$$

BAS.18 – Determine the current through a $200 \mu\text{F}$ capacitor whose voltage $v(t)$ is shown in the Figure →

Solution:

$$v(t) = \begin{cases} 50t \text{ V} & \leftarrow 0 \text{ s} \leq t < 1 \text{ s} \\ (100 - 50t) \text{ V} & \leftarrow 1 \text{ s} \leq t < 3 \text{ s} \\ (-200 + 50t) \text{ V} & \leftarrow 3 \text{ s} \leq t < 4 \text{ s} \\ 0 \text{ V} & \leftarrow \textit{otherwise} \end{cases}$$

$$i(t) = \begin{cases} 10 \text{ mA} & \leftarrow 0 \text{ s} \leq t < 1 \text{ s} \\ -10 \text{ mA} & \leftarrow 1 \text{ s} \leq t < 3 \text{ s} \\ 10 \text{ mA} & \leftarrow 3 \text{ s} \leq t < 4 \text{ s} \\ -10 \text{ mA} & \leftarrow \textit{otherwise} \end{cases}$$



Circuit Elements

BAS.20 – The current through a 100mH inductors is $i(t) = 10 \cdot t \cdot e^{-5t}$ A. Find its voltage and the energy stored in it.

Solution $v = L \frac{di}{dt} = 0.1 \frac{d}{dt} (10 \cdot t \cdot e^{-5t}) = e^{-5t} + t \cdot (-5) \cdot e^{-5t} = (1 - 5t) \cdot e^{-5t} \text{ V}$

$$w = \frac{1}{2} Li^2 = \frac{1}{2} (0.1) 100 \cdot t^2 \cdot e^{-10t} = 5t^2 e^{-10t} \text{ J}$$

BAS.21 – Find the current through a 500 mH inductor if its voltage is
Also find the energy stored within $0 < t < 1$ s

$$v(t) = \begin{cases} 0 \text{ V} & t \leq 0 \text{ s} \\ 30t^2 \text{ V} & t > 0 \text{ s} \end{cases}$$

Solution $i = \frac{1}{L} \int_{t_0}^t v(t) \cdot dt + I(t_0) = 2 \int_0^t 30t^2 dt + 0 = 20 t^3 \text{ A}$

$$p = v \cdot i = 600t^5 \text{ W} \rightarrow w = \int_0^1 p \cdot dt = \int_0^1 600t^5 \cdot dt = 100 \text{ J}$$

$$\text{Alternatively} \rightarrow w = \frac{1}{2} Li^2 \Big|_{(t=1)} - \frac{1}{2} Li^2 \Big|_{(t=0)} = \frac{1}{2} \cdot 0.5 \cdot (20 \cdot 1^3)^2 - 0 = 100 \text{ J}$$

