DR. GYURCSEK ISTVÁN

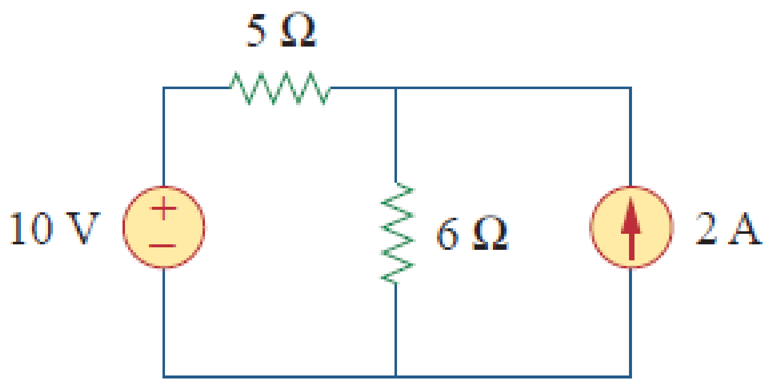
Exercises using Basic Laws of Electrical Circuits

Sources and additional materials (recommended)

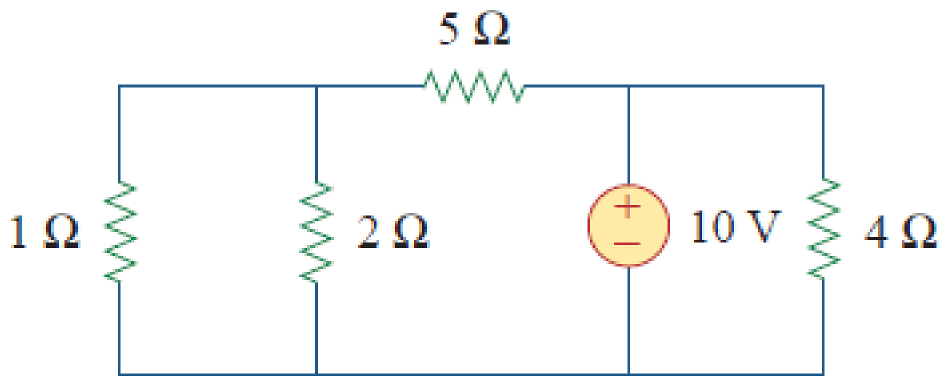
- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

Basic Laws

LAW.01
Determine the number of branches and nodes in the circuit.
Identify which elements are in series and which are in parallel.



LAW.02
How many branches and nodes does the circuit?
Identify the elements that are in series and in parallel.



Basic Laws

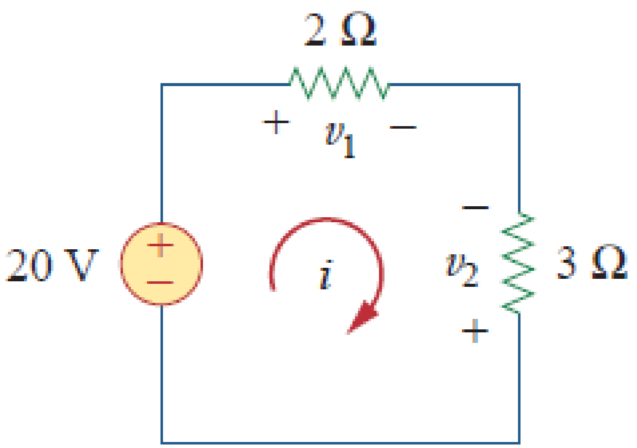
LAW.03 – Find voltages v_1 and v_2.

Solution:

$$v_1 = 2 \cdot i\,, \qquad v_2 = -3 \cdot i$$

$$-20 + v_1 - v_2 = 0$$

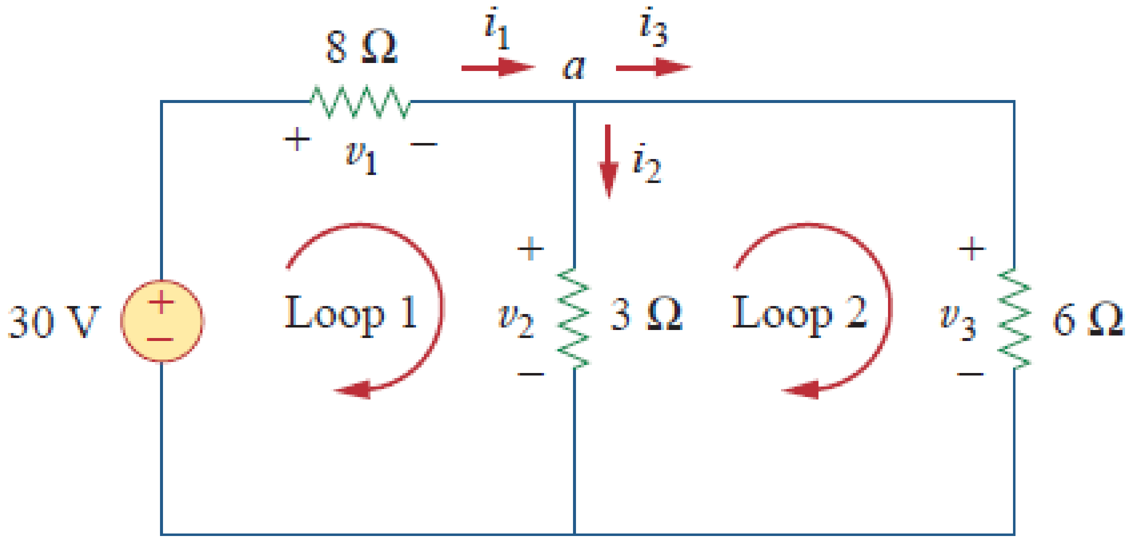
$$-20 + 2 \cdot i - (-3 \cdot i) = 0$$



$$i = \frac{20}{5} = 4\ \mathrm{A} \;\rightarrow\; v_1 = 2 \cdot 4 = 8\ \mathrm{V}\,, \qquad v_2 = -3 \cdot 4 = -12\ \mathrm{V}$$

Basic Laws

LAW.04 – Find the currents and voltages in the circuit.



Solution:

$$v_1 = 8 \cdot i_1 \qquad (\boldsymbol{node\ a}):\ i_1 - i_2 - i_3 = 0$$

$$v_2 = 3 \cdot i_2 \qquad (\boldsymbol{loop1}):\ -30 + v_1 + v_2 = 0$$

$$v_3 = 6 \cdot i_3 \qquad -30 + 8 \cdot i_1 + 3 \cdot i_2 = 0$$

$$i_1 = \frac{30 - 3 \cdot i_2}{8}$$

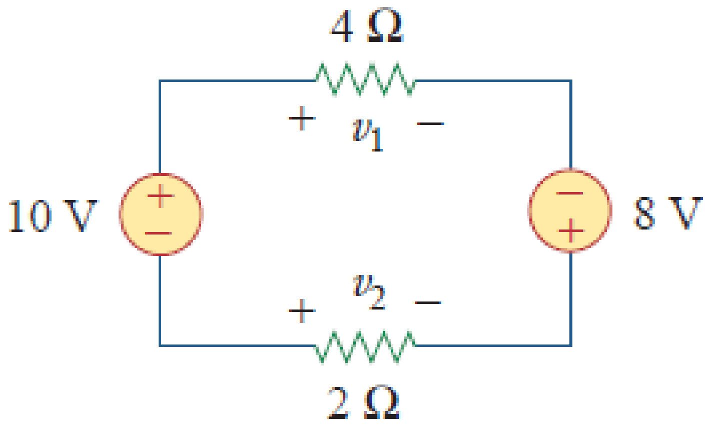
$$(\boldsymbol{loop2}):\ -v_2 + v_3 = 0 \rightarrow v_2 = v_3 \qquad (\boldsymbol{node\ a}):\ \frac{30 - 3 \cdot i_2}{8} - i_2 - 0.5 \cdot i_2 = 0$$

$$6 \cdot i_3 = 3 \cdot i_2 \rightarrow i_3 = 0.5 \cdot i_2 \qquad i_2 = 2\ A, \quad i_3 = 1\ A, \quad v_1 = 24\ V, \quad v_2 = v_3 = 6\ V$$

Basic Laws

LAW.05

Find v_1 and v_2 in the circuit.



Solution:

$$-10 + v_1 - 8 - v_2 = 0$$

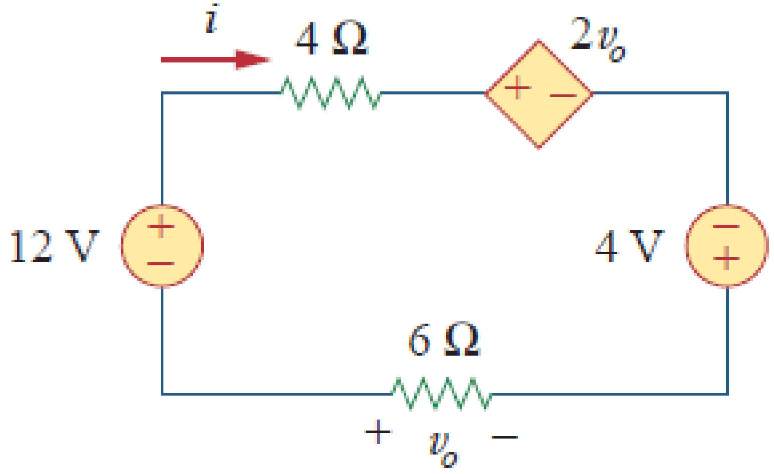
$$v_1 = 4i\,, \qquad v_2 = -2i$$

$$-18 + 6i = 0 \rightarrow i = 3\ A$$

$$v_1 = 4 \cdot 3 = 12\ V\,, \qquad v_2 = -2 \cdot 3 = -6\ V$$

LAW.06

Find v_0 and i in the circuit.



Solution:

$$-12 + 4i + 2v_0 - 4 + 6i = 0$$

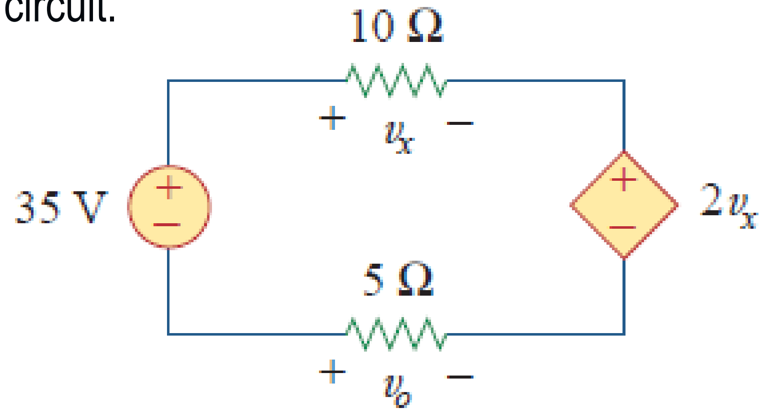
$$v_0 = -6i$$

$$-16 + 10i - 12i = 0$$

$$i = -8\ A \rightarrow v_0 = 48\ V$$

Basic Laws

LAW.07 Find v_x and v_0 in the circuit.



Solution:

$$-35 + v_x + 2v_x - v_0 = 0$$

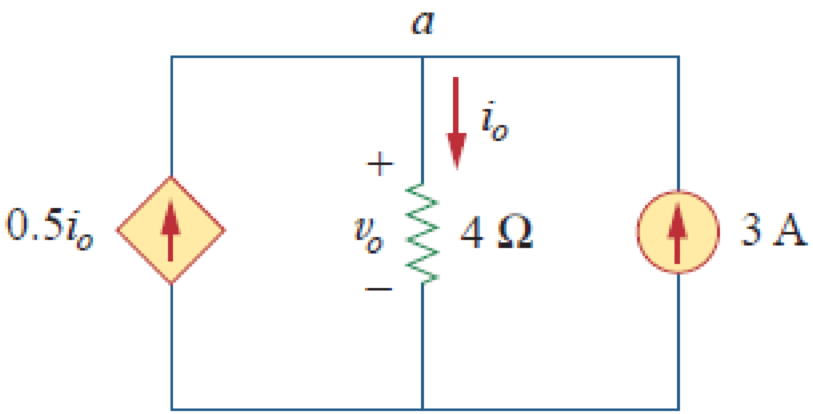
$$v_x = 10i\,, \qquad v_0 = -5i$$

$$-35 + 30i + 5i = 0 \rightarrow i = 1\ A$$

$$v_x = 10 \cdot 1 = 10\ V\,, \qquad v_0 = -5 \cdot 1 = -5\ V$$

LAW.08

Find i_0 and v_0 in the circuit.



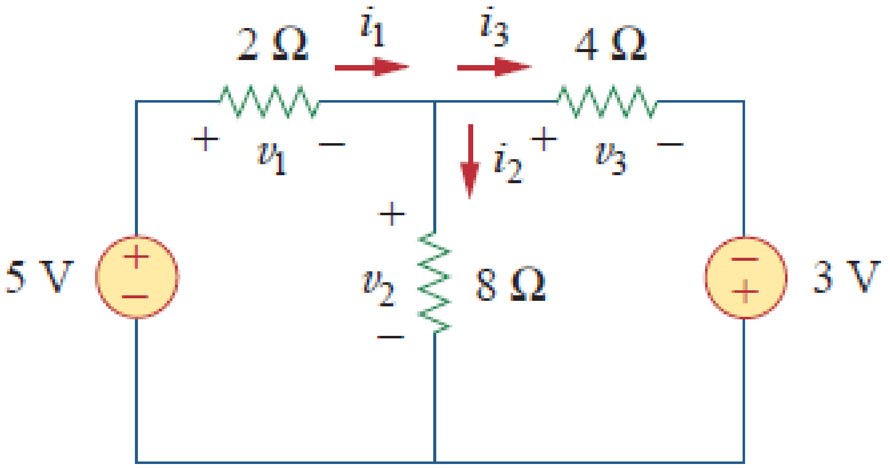
Solution:

$$3 + 0.5i_0 = i_0 \rightarrow i_0 = 6\ \mathrm{A}$$

$$v_0 = 4i_0 = 24\ \mathrm{V}$$

Basic Laws

LAW.09 – Obtain the currents and voltages in the circuit.



Solution: (1): $-5 + v_1 + v_2 = 0$

(2): $-3 - v_2 + v_3 = 0$

(3): $i_1 = i_2 + i_3$

(4)(5)(6): $v_1 = 2i_1 \,, v_2 = 8i_2 \,, v_3 = 4i_3$

(1): $2i_1 + 8i_2 = 5$

(2): $-8i_2 + 4i_3 = 3$

(3): $i_1 = i_2 + i_3$

(1)(3): $2i_2 + 2i_3 + 8i_2 = 5$

(2): $-8i_2 + 4i_3 = 3 \rightarrow i_3 = \dfrac{3 + 8i_2}{4}$

$$10i_2 + \frac{3 + 8i_2}{2} = 5$$

$$20i_2 + 3 + 8i_2 = 10 \rightarrow i_2 = 0.25\ A$$

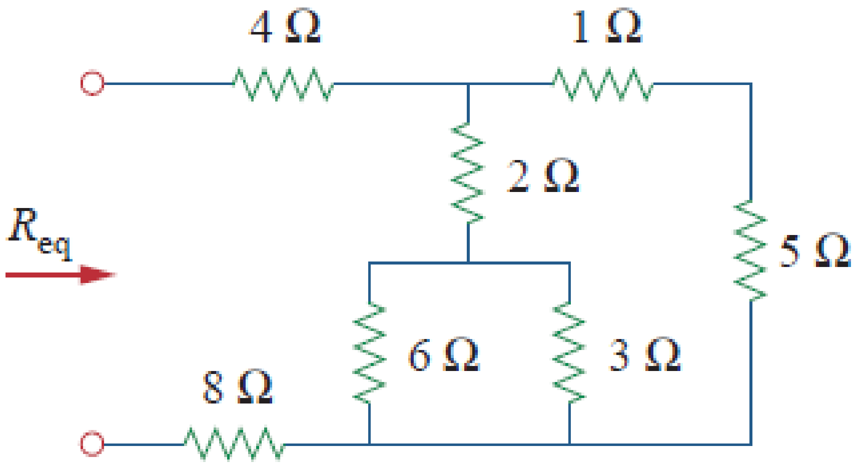
$$i_3 = 1.25\ A \,, \qquad i_1 = 1.5\ A$$

$$v_1 = 3\ V \,, \qquad v_2 = 2\ V \,, \qquad v_3 = 5\ V$$

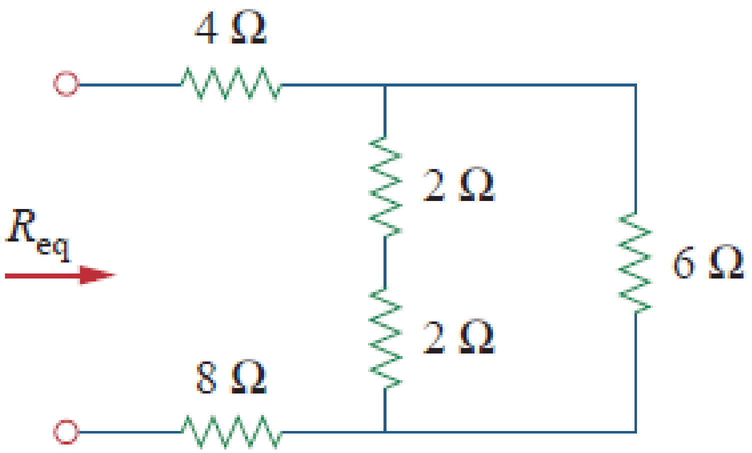
gyurcsek.istvan@mik.pte.hu

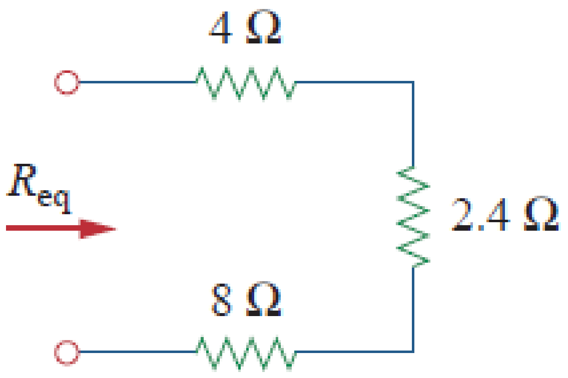
Series and Parallel Elements

LAW.10 – Find the equivalent resistance for the circuit.



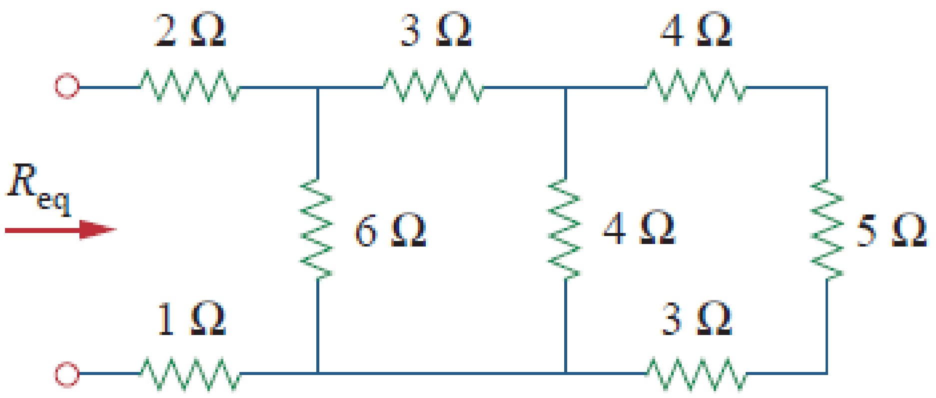
Solution:





$R_{eq} = 14.4\ \Omega$

LAW.11
Find the equivalent resistance for the circuit.



Solution: $R_{eq} = 6\ \Omega$

Series and Parallel Elements

LAW.12 – Find R_{ab} for the circuit.

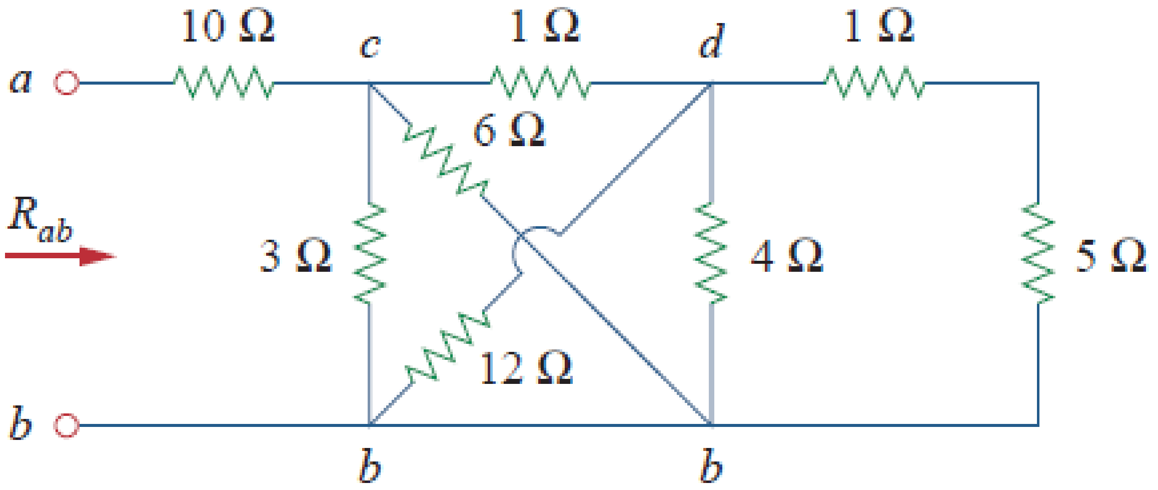
Solution*:*

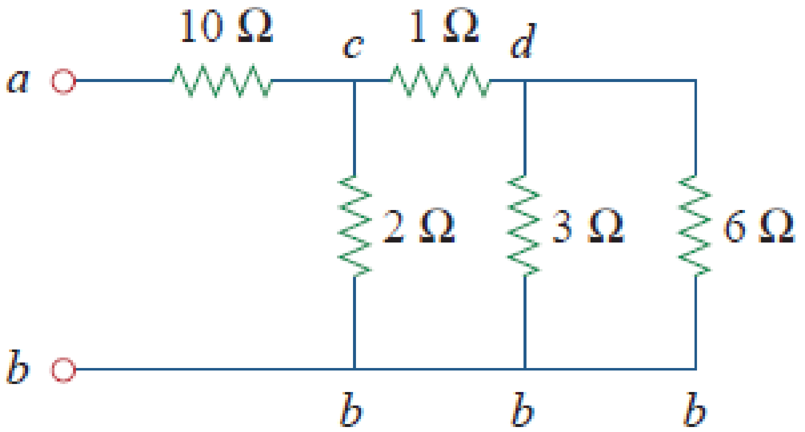
$$3 \times 6 = \frac{3 \cdot 6}{3+6} = 2\ \Omega$$

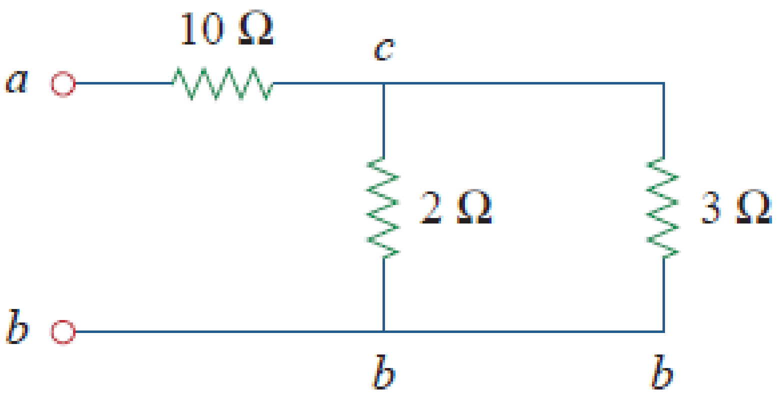
$$12 \times 4 = \frac{12 \cdot 4}{12+4} = 3\ \Omega$$

$$2 \times 3 = \frac{2 \cdot 3}{2+3} = 1.2\ \Omega$$

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

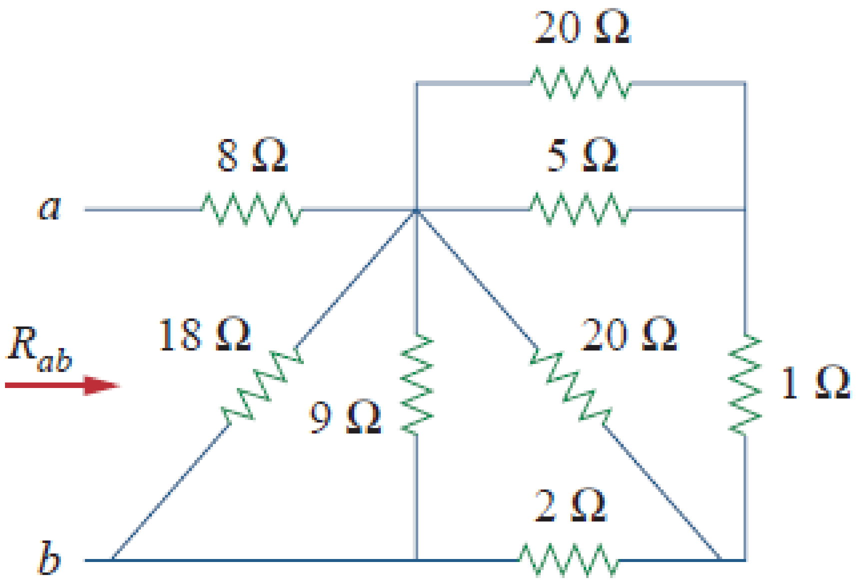






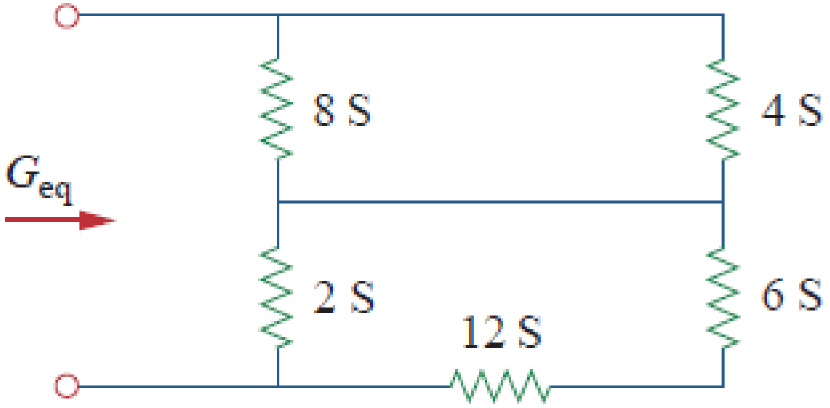
Series and Parallel Elements

LAW.13 – Find R_{ab} for the circuit.



Solution: $R_{ab} = 11\ \Omega$

LAW.14 – Calculate G_{eq} for the circuit.

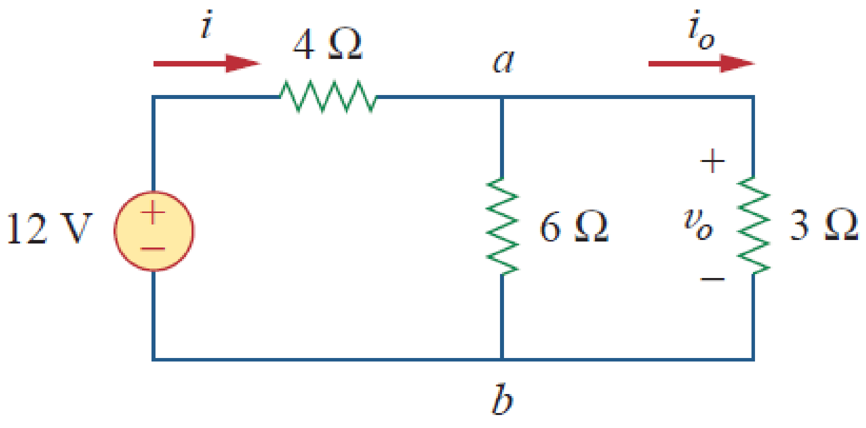


Solution: $G_{eq} = 4\ S$

Voltage and Current Divisions

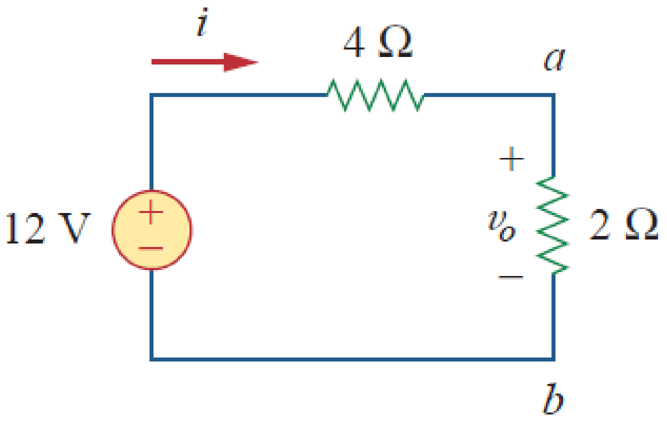
LAW.15 – Find i_0 and v_0 in the circuit. Calculate the power dissipated in the 3Ω- resistor.

Solution: $6 \times 3 = \dfrac{6 \cdot 3}{6+3} = 2\ \Omega$



$$voltage\ division \rightarrow v_0 = 12\ V \cdot \frac{2\ \Omega}{(2+4)\ \Omega} = 4\ V$$

$$i_0 = \frac{v_0}{3} = 1.33\ A$$



$$current\ division \rightarrow i_0 = i \cdot \frac{6\ \Omega}{(6+3)\ \Omega}$$

$$i = \frac{12\ V}{(4+2)\ \Omega} = 2\ A \rightarrow i_0 = 2\ A \cdot \frac{6\ \Omega}{(6+3)\ \Omega} = 1.33\ A$$

$$p_0 = v_0 \cdot i_0 = 4\ \cdot 1.33 = 5.33\ W$$

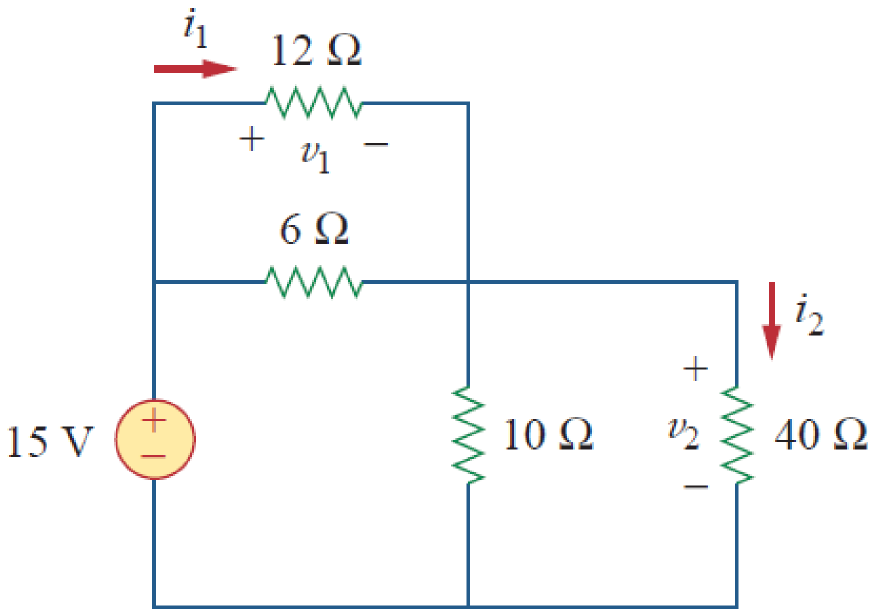
Voltage and Current Divisions

LAW.16 – Find v_1 and v_2 in the circuit. Also calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.

Solution:

$$(v_1 = 5\,V, i_1 = 416.7\,mA, p_1 = 2.08\,W)$$

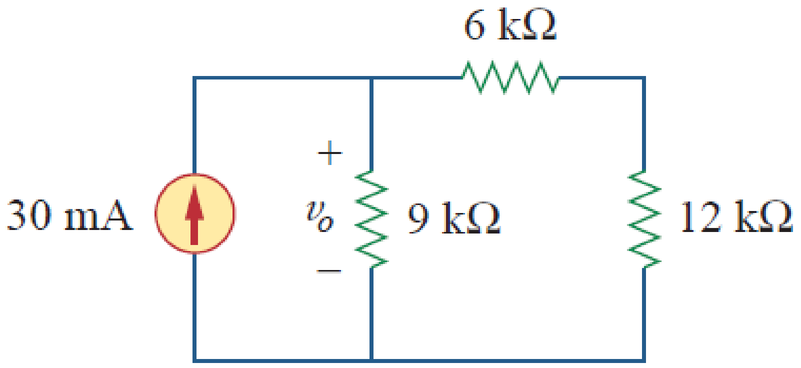
$$(v_2 = 10\,V, i_2 = 250\,mA, p_2 = 2.5\,W)$$

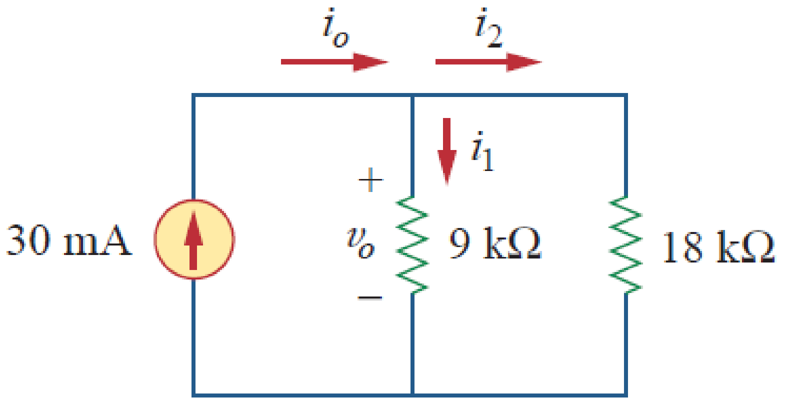


Voltage and Current Divisions

LAW.17 – For the circuit shown in Figure determine: (a) the voltage v_0, (b) the power supplied by the current source, (c) the power absorbed by each resistor.

Solution:





$$i_1 = \frac{18}{9+18} 30 = 20\ mA$$

$$i_2 = \frac{9}{9+18} 30 = 10\ mA$$

$$v_0 = 9 \cdot i_1 = 18 \cdot i_2 = 180\ V \qquad p_0 = v_0 \cdot i_0 = 180 \cdot 30 = 5.4\ W$$

$$p_{12} = i_2 (i_2 \cdot R) = i_2^2 \cdot R = (20 \cdot 10^{-3})^2 \cdot 12 \cdot 10^3 = 1.2\ W$$

$$supplied\ power \rightarrow 5.4\ W$$

$$p_6 = i_2^2 \cdot R = (20 \cdot 10^{-3})^2 \cdot 6 \cdot 10^3 = 0.6\ W$$

$$absorbed\ power \rightarrow (1.2 + 0.6 + 3.6) = 5.4\ W$$

$$p_9 = \frac{v_0^2}{R} = \frac{(180)^2}{9 \cdot 10^3} = 3.6\ W\ or\ p_9 = v_0 \cdot i_1 = 180 \cdot 20 \cdot 10^{-3} = 3.6\ W$$

gyurcsek.istvan@mik.pte.hu

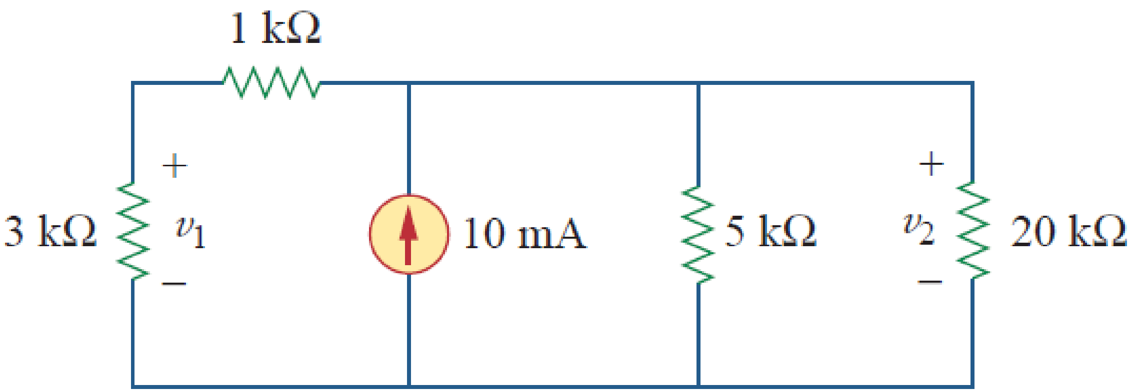
Voltage and Current Divisions

LAW.18 – For the circuit shown in Figure, find:

(a) v_1 and v_2

(b) power dissipated in the 3-kΩ and 20-k Ω resistors

(c) power supplied by the current source.



Solution:

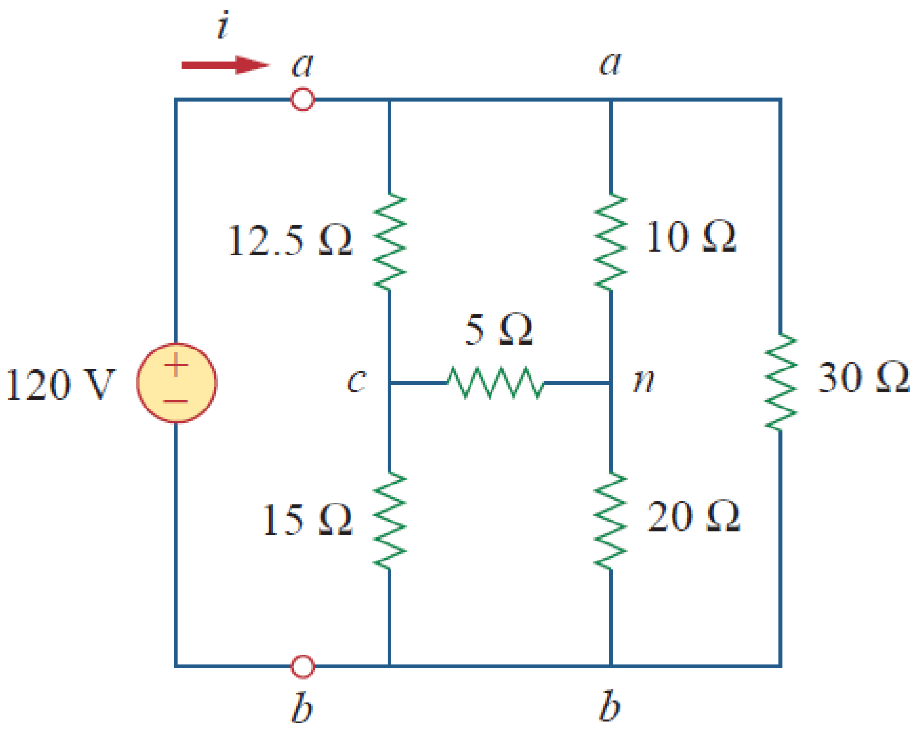
(a) 15 V, 20 V

(b) 75 mW, 20 mW

(c) 200 mW

Wye-Delta Conversion

LAW.19 – Obtain current *i* in the circuit.

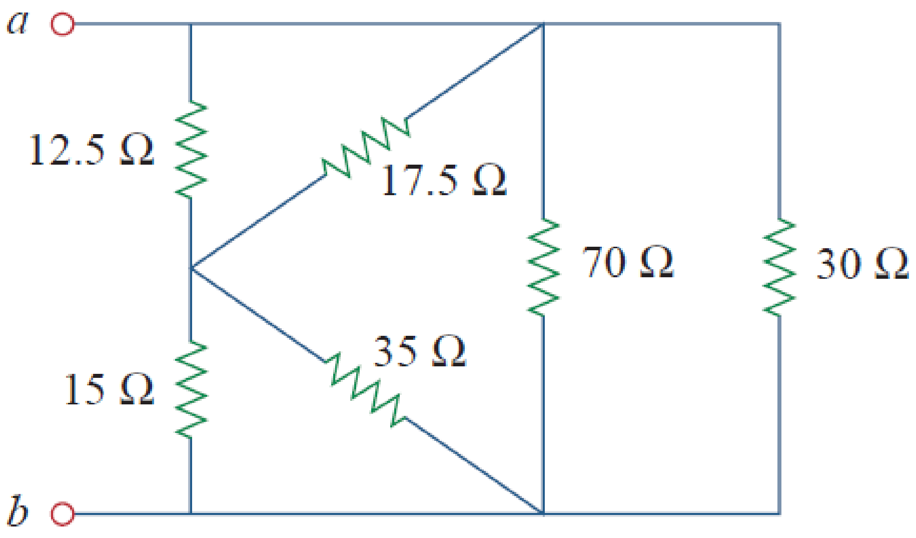


Solution: $G_\Delta = \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = 0.35\ S$

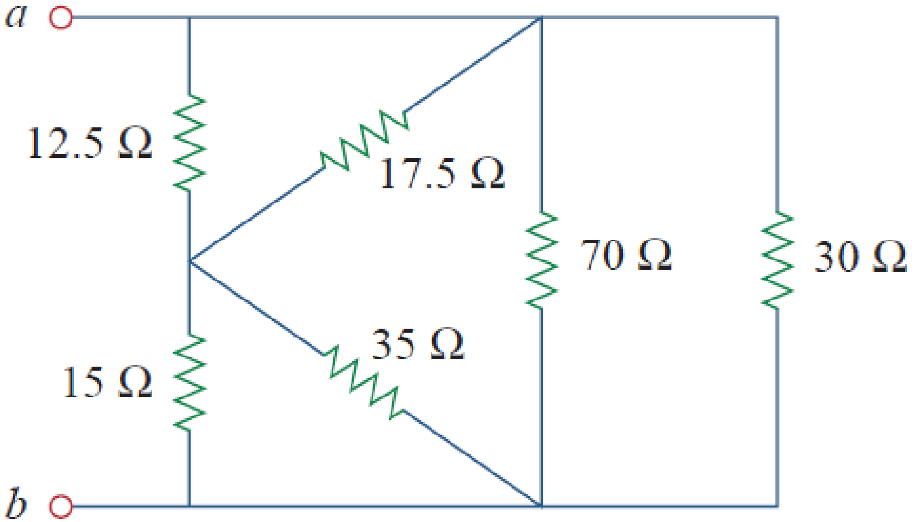
$$G_{ac} = \frac{\frac{1}{5}\cdot\frac{1}{10}}{G_\Delta} = \frac{0.02}{0.35} \rightarrow R_{ac} = \frac{0.35}{0.02} = 17.5\ \Omega$$

$$G_{bc} = \frac{\frac{1}{5}\cdot\frac{1}{20}}{G_\Delta} = \frac{0.01}{0.35} \rightarrow R_{bc} = \frac{0.35}{0.01} = 35\ \Omega$$

$$G_{ab} = \frac{\frac{1}{10}\cdot\frac{1}{20}}{G_\Delta} = \frac{0.005}{0.35} \rightarrow R_{ab} = \frac{0.35}{0.005} = 70\ \Omega$$



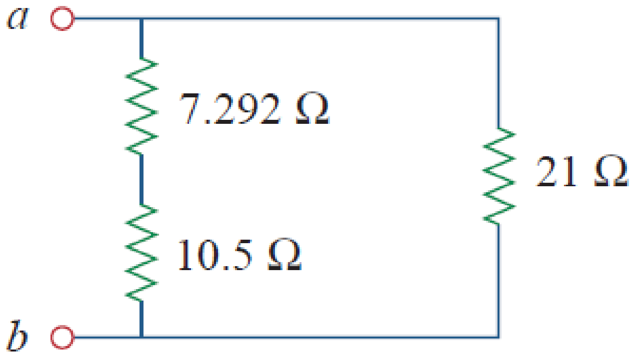
Wye-Delta Conversion



$$70 \times 30 = \frac{70 \cdot 30}{100} = 21\ \Omega$$

$$12.5 \times 17.5 = \frac{12.5 \cdot 17.5}{30} = 7.292\ \Omega$$

$$15 \times 35 = \frac{15 \cdot 35}{50} = 10.5\ \Omega$$

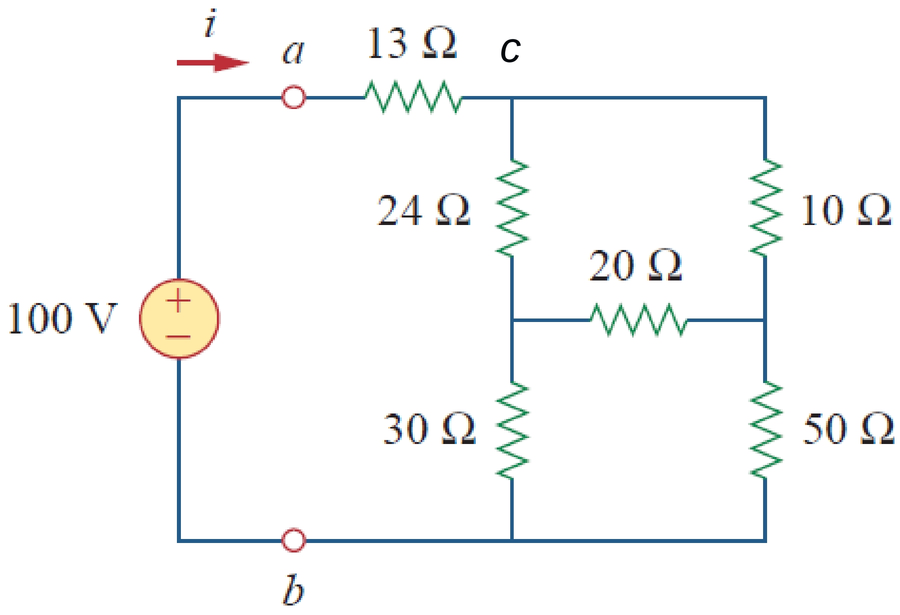


$$R_{ab} = (7.292 + 10.5) \times 21 = \cdots = 9.632\ \Omega$$

$$i = \frac{120\ V}{R_{ab}} = 12.458\ A$$

Wye-Delta Conversion

LAW.20 – Obtain current *i* in the circuit.



Solution 1*:*

$$G_\Delta = \frac{1}{20} + \frac{1}{10} + \frac{1}{50} = 0.17\ S$$

$$G_1 = \frac{\frac{1}{10}\cdot\frac{1}{20}}{G_\Delta} = \frac{0.005}{0.17} \to R_1 = 34\ \Omega \to R_{1,24} = \frac{34\cdot 24}{58} = 14\ \Omega$$

$$G_2 = \frac{\frac{1}{20}\cdot\frac{1}{50}}{G_\Delta} = \frac{0.001}{0.17} \to R_2 = 170\ \Omega \to R_{2,30} = \frac{170\cdot 30}{200} = 25.5\ \Omega$$

$$G_3 = \frac{\frac{1}{10}\cdot\frac{1}{50}}{0.17} \to R_3 = 85\ \Omega \to R_{bc} = 85 \times (14 + 25.5) = \cdots = 27\ \Omega$$

$$R_{ab} = R_{bc} + 13 = 40\ \Omega \to i = \frac{100}{R_{ab}} = 2.5\ A$$

***Solution 2:* …**

$$(RR)_\Delta = 10\cdot 20 + 10\cdot 50 + 20\cdot 50 = 1700\ \Omega^2 \to R_1 = \frac{(RR)_\Delta}{50} = 34\ \Omega\,, \qquad R_2 = \frac{(RR)_\Delta}{10} = 170\ \Omega\,, \qquad R_3 = \frac{(RR)_\Delta}{20} = 85\ \Omega$$

gyurcsek.istvan@mik.pte.hu

Questions