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# Exercises using Basic Laws of Electrical Circuits

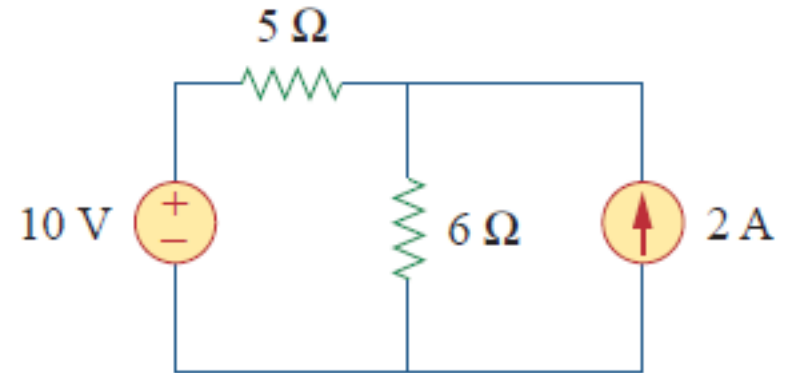
## *Sources and additional materials (recommended)*

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

## Basic Laws of Electrical Circuits

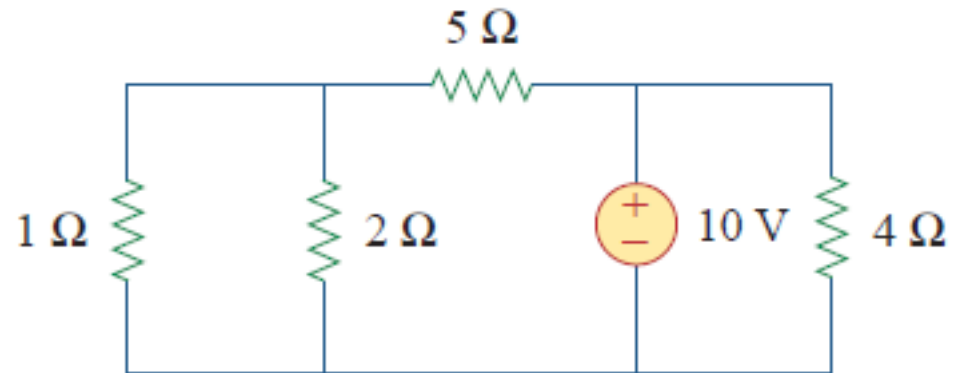
### LAW.01

Determine the number of branches and nodes in the circuit.  
Identify which elements are in series and which are in parallel.



### LAW.02

How many branches and nodes does the circuit?  
Identify the elements that are in series and in parallel.



**LAW.03** – Find voltages  $v_1$  and  $v_2$ .

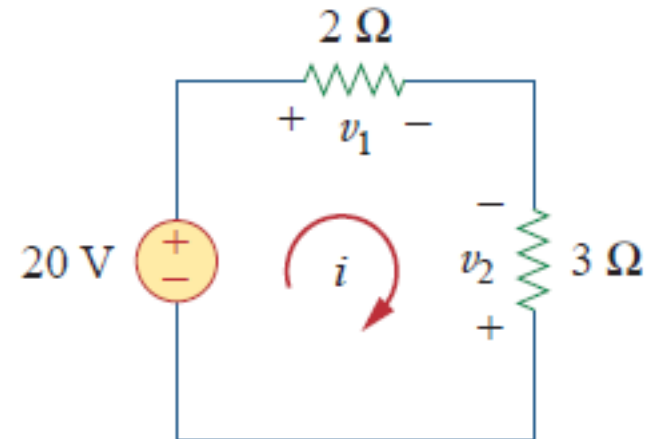
**Solution:**

$$v_1 = 2 \cdot i, \quad v_2 = -3 \cdot i$$

$$-20 + v_1 - v_2 = 0$$

$$-20 + 2 \cdot i - (-3 \cdot i) = 0$$

$$i = \frac{20}{5} = 4 \text{ A} \rightarrow v_1 = 8 \text{ V}, \quad v_2 = -12 \text{ V}$$



## Basic Laws of Electrical Circuits

**LAW.04** – Find the currents and voltages in the circuit.

**Solution:**

$$v_1 = 8 \cdot i_1 \quad (\text{node } a): i_1 - i_2 - i_3 = 0$$

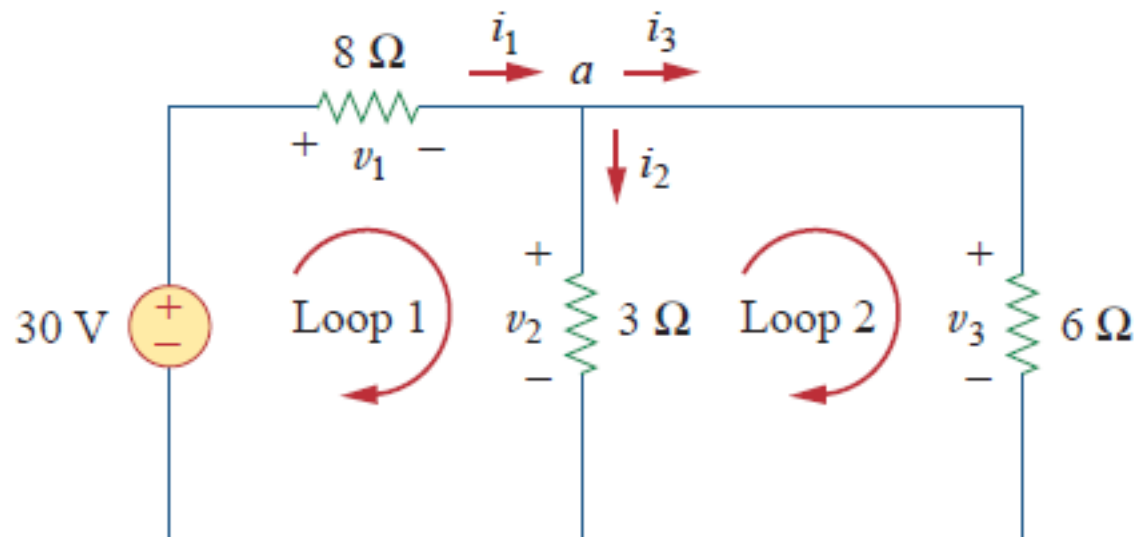
$$v_2 = 3 \cdot i_2 \quad (\text{loop1}): -30 + v_1 + v_2 = 0$$

$$v_3 = 6 \cdot i_3 \quad -30 + 8 \cdot i_1 + 3 \cdot i_2 = 0$$

$$i_1 = \frac{30 - 3 \cdot i_2}{8}$$

$$(\text{loop2}): -v_2 + v_3 = 0 \rightarrow v_2 = v_3$$

$$6 \cdot i_3 = 3 \cdot i_2 \rightarrow i_3 = 0.5 \cdot i_2$$



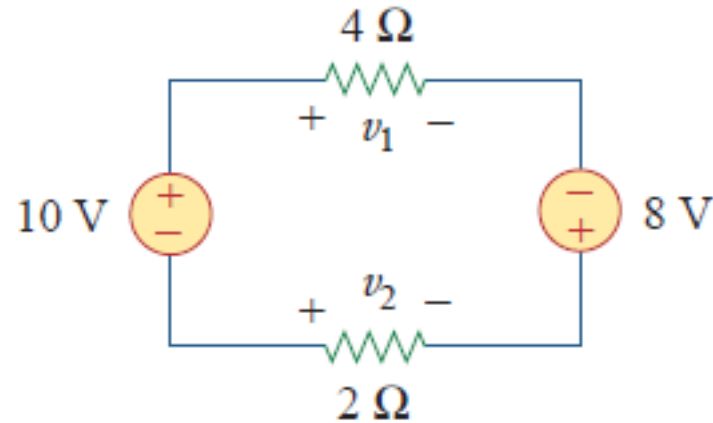
$$(\text{node } a): \frac{30 - 3 \cdot i_2}{8} - i_2 - 0.5 \cdot i_2 = 0$$

$$i_2 = 2 \text{ A}, \quad i_3 = 1 \text{ A}, \quad v_1 = 24 \text{ V}, \quad v_2 = v_3 = 6 \text{ V}$$

## Basic Laws of Electrical Circuits

### LAW.05

Find  $v_1$  and  $v_2$  in the circuit.



**Solution:**

$$-10 + v_1 - 8 - v_2 = 0$$

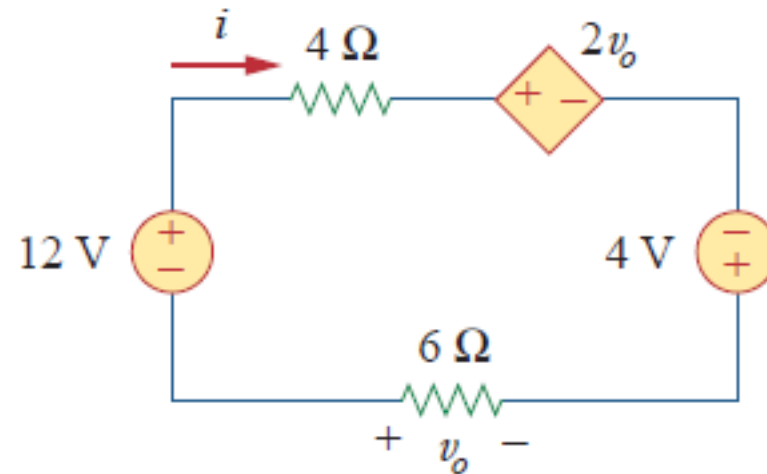
$$v_1 = 4i, \quad v_2 = -2i$$

$$-18 + 6i = 0 \rightarrow i = 3 \text{ A}$$

$$v_1 = 4 \cdot 3 = 12 \text{ V}, \quad v_2 = -2 \cdot 3 = -6 \text{ V}$$

### LAW.06

Find  $v_0$  and  $i$  in the circuit.



**Solution:**  $-12 + 4i + 2v_0 - 4 + 6i = 0$

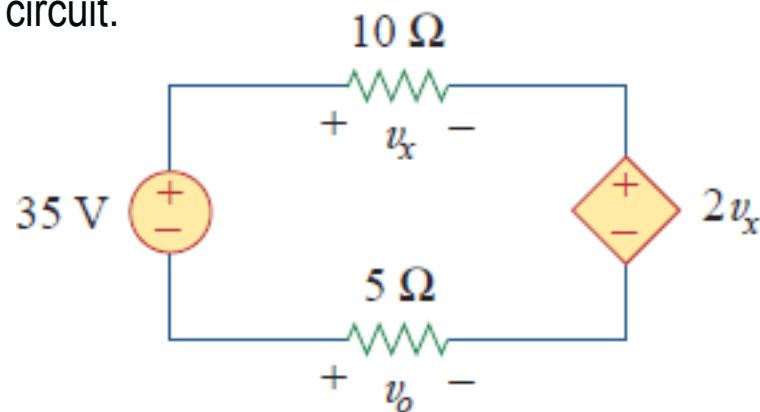
$$v_0 = -6i$$

$$-16 + 10i - 12i = 0$$

$$i = -8 \text{ A} \rightarrow v_0 = 48 \text{ V}$$

## Basic Laws of Electrical Circuits

**LAW.07** Find  $v_x$  and  $v_0$  in the circuit.



**Solution:**

$$-35 + v_x + 2v_x - v_0 = 0$$

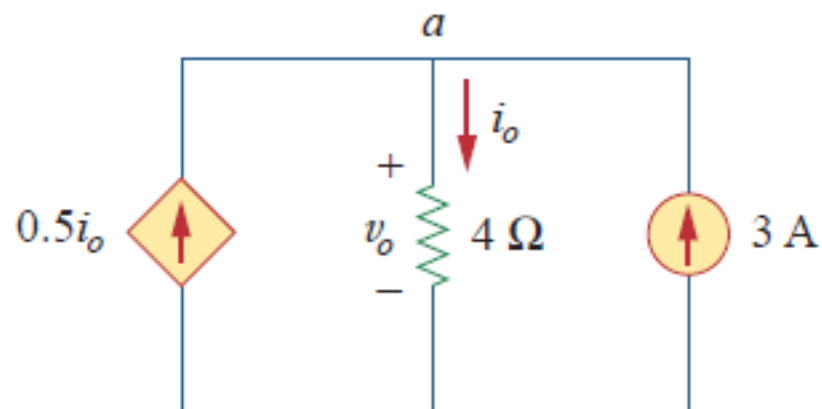
$$v_x = 10i, \quad v_0 = -5i$$

$$-35 + 30i + 5i = 0 \rightarrow i = 1 \text{ A}$$

$$v_x = 10 \cdot 1 = 10 \text{ V}, \quad v_0 = -5 \cdot 1 = -5 \text{ V}$$

**LAW.08**

Find  $i_0$  and  $v_0$  in the circuit.



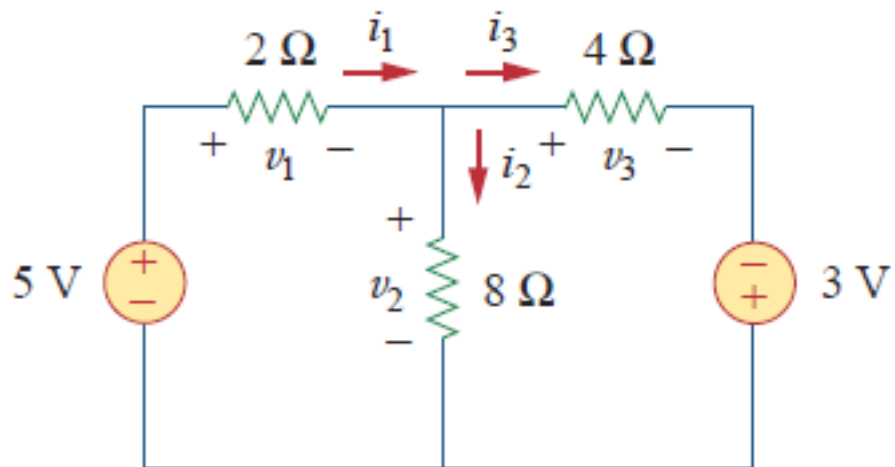
**Solution:**

$$3 + 0.5i_0 = i_0 \rightarrow i_0 = 6 \text{ A}$$

$$v_0 = 4i_0 = 24 \text{ V}$$

## Basic Laws of Electrical Circuits

**LAW.09** – Obtain the currents and voltages in the circuit.



**Solution:** (1):  $-5 + v_1 + v_2 = 0$

(2):  $-3 - v_2 + v_3 = 0$

(3):  $i_1 = i_2 + i_3$

(4)(5)(6):  $v_1 = 2i_1, v_2 = 8i_2, v_3 = 4i_3$

(1):  $2i_1 + 8i_2 = 5$

(2):  $-8i_2 + 4i_3 = 3$

(3):  $i_1 = i_2 + i_3$

(1)(3):  $2i_2 + 2i_3 + 8i_2 = 5$

(2):  $-8i_2 + 4i_3 = 3 \rightarrow i_3 = \frac{3 + 8i_2}{4}$

$$10i_2 + \frac{3 + 8i_2}{2} = 5$$

$$20i_2 + 3 + 8i_2 = 10 \rightarrow i_2 = 0.25 \text{ A}$$

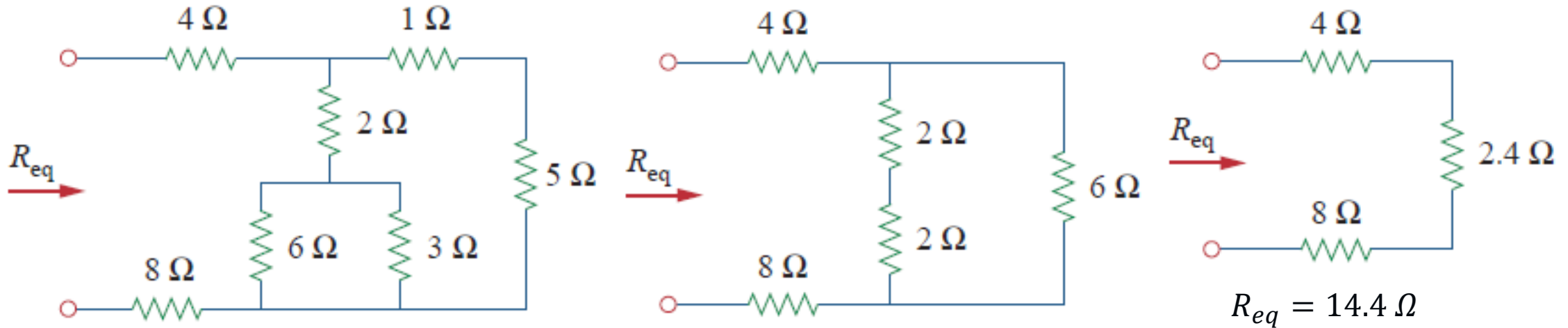
$$i_3 = 1.25 \text{ A}, \quad i_1 = 1.5 \text{ A}$$

$$v_1 = 3 \text{ V}, \quad v_2 = 2 \text{ V}, \quad v_3 = 5 \text{ V}$$

## Series and Parallel Circuit Elements

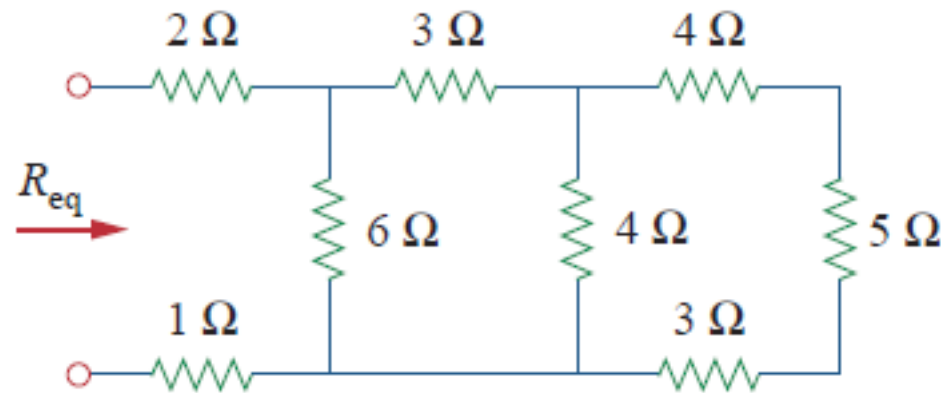
**LAW.10** – Find the equivalent resistance for the circuit.

**Solution:**



**LAW.11**

Find the equivalent resistance for the circuit.



**Solution:**  $R_{eq} = 6 \Omega$



## Series and Parallel Circuit Elements

**LAW.12** – Find  $R_{ab}$  for the circuit.

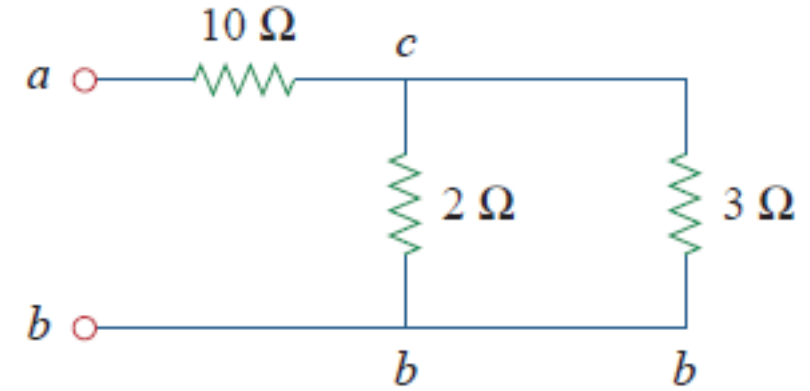
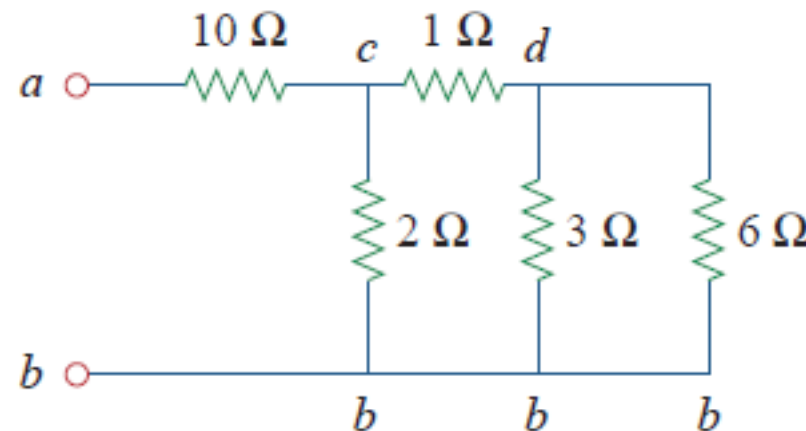
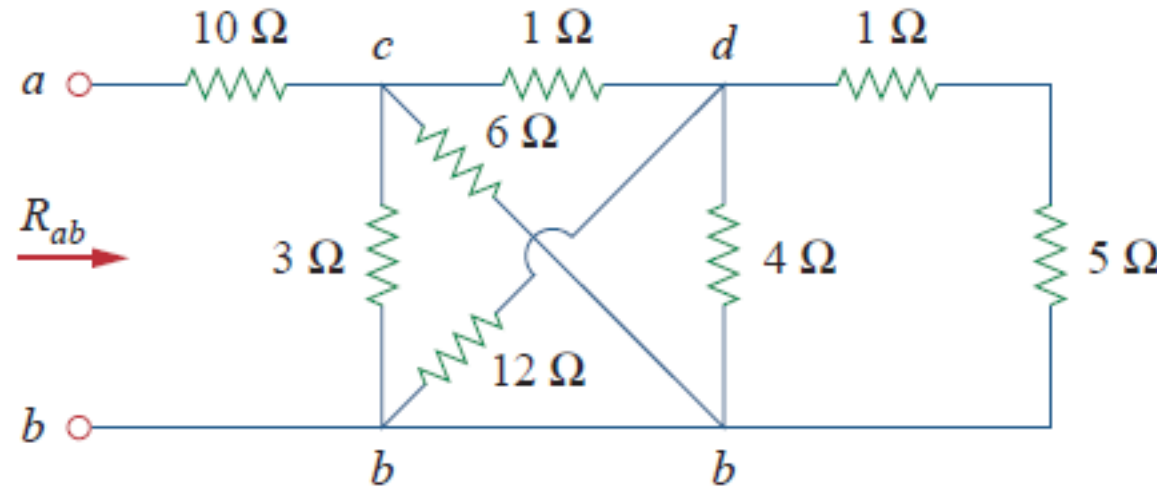
**Solution:**

$$3 \times 6 = \frac{3 \cdot 6}{3 + 6} = 2 \Omega$$

$$12 \times 4 = \frac{12 \cdot 4}{12 + 4} = 3 \Omega$$

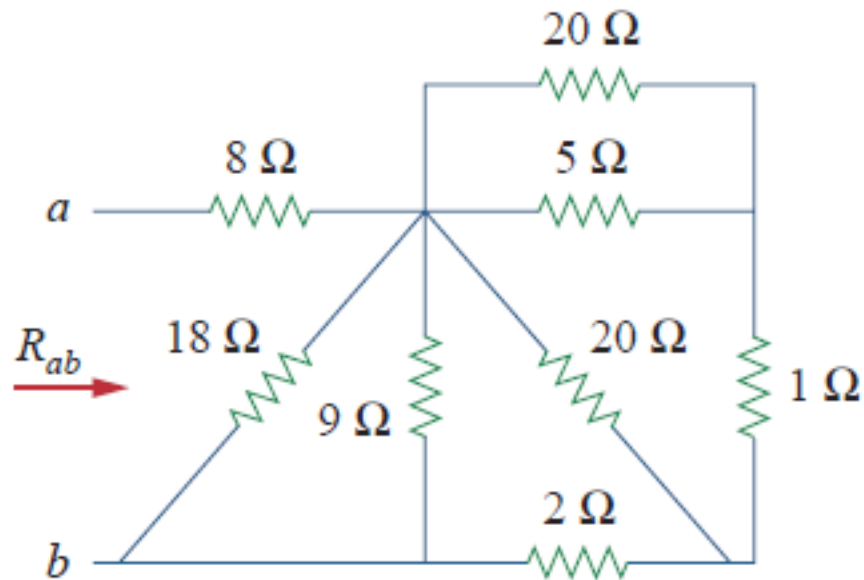
$$2 \times 3 = \frac{2 \cdot 3}{2 + 3} = 1.2 \Omega$$

$$R_{ab} = 10 + 1.2 = 11.2 \Omega$$



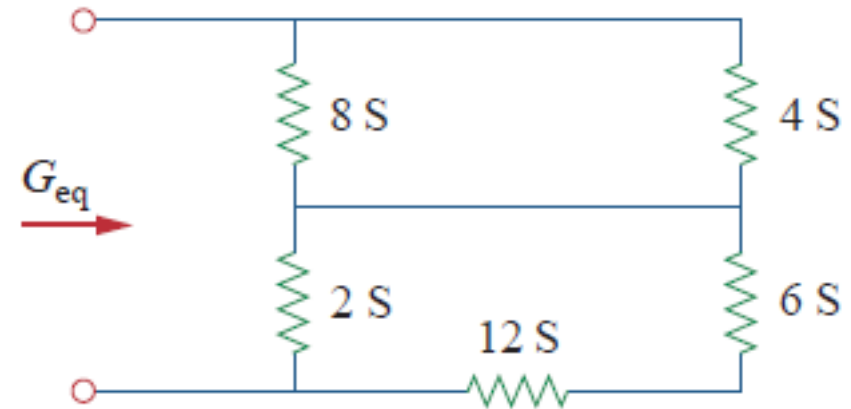
## Series and Parallel Circuit Elements

**LAW.13** – Find  $R_{ab}$  for the circuit.



**Solution:**  $R_{ab} = 11 \Omega$

**LAW.14** – Calculate  $G_{eq}$  for the circuit.



**Solution:**  $G_{eq} = 4 S$

## Voltage and Current Divisions

**LAW.15** – Find  $i_0$  and  $v_0$  in the circuit. Calculate the power dissipated in the  $3\Omega$ - resistor.

**Solution:**  $6 \times 3 = \frac{6 \cdot 3}{6 + 3} = 2 \Omega$

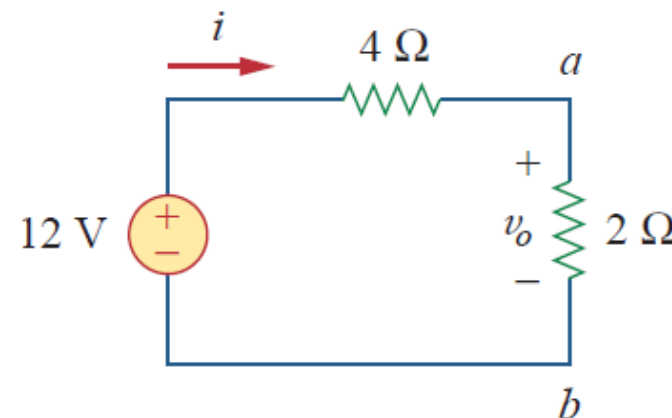
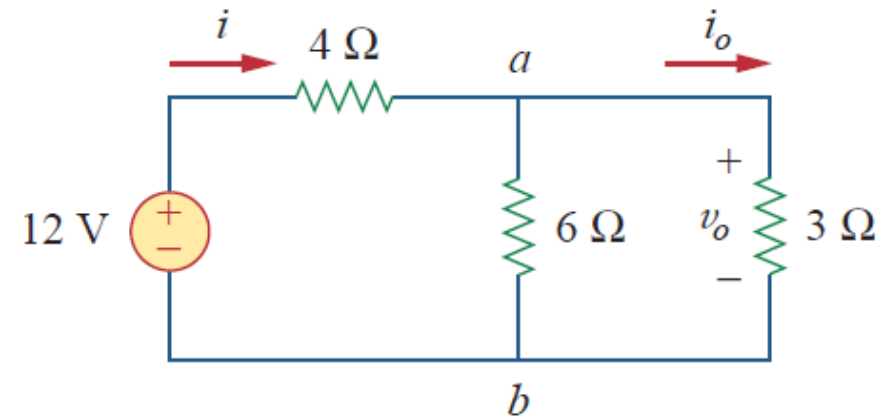
*voltage division*  $\rightarrow v_0 = 12 V \cdot \frac{2 \Omega}{(2 + 4) \Omega} = 4 V$

$i_0 = \frac{v_0}{3} = 1.33 A$

*current division*  $\rightarrow i_0 = i \cdot \frac{6 \Omega}{(6 + 3) \Omega}$

$i = \frac{12 V}{(4 + 2) \Omega} = 2 A \rightarrow i_0 = 2 A \cdot \frac{6 \Omega}{(6 + 3) \Omega} = 1.33 A$

$p_0 = v_0 \cdot i_0 = 4 \cdot 1.33 = 5.33 W$



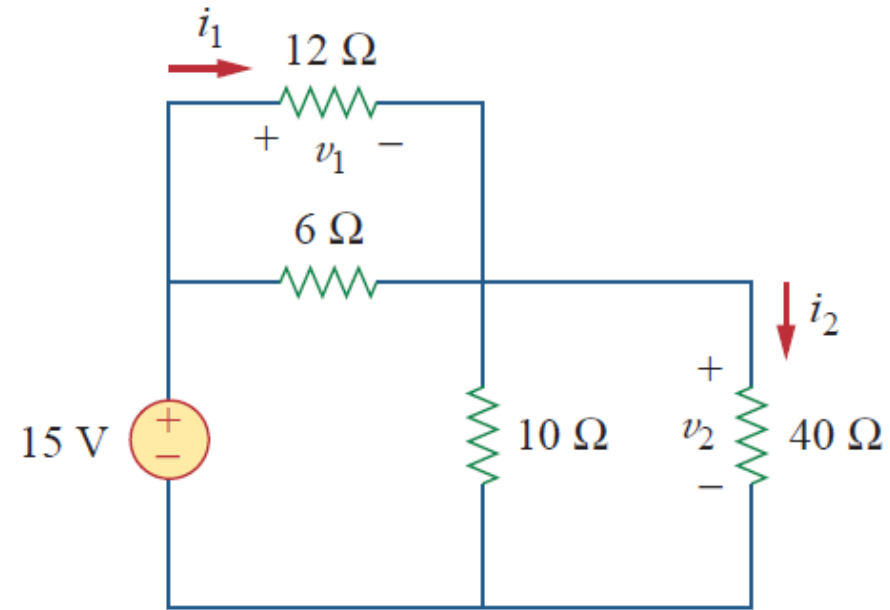
# Voltage and Current Divisions

**LAW.16** – Find  $v_1$  and  $v_2$  in the circuit.  
Also calculate  $i_1$  and  $i_2$  and the power dissipated in the 12- $\Omega$  and 40- $\Omega$  resistors.

**Solution:**

$(v_1 = 5\text{ V}, i_1 = 416.7\text{ mA}, p_1 = 2.08\text{ W})$

$(v_2 = 10\text{ V}, i_2 = 250\text{ mA}, p_2 = 2.5\text{ W})$



## Voltage and Current Divisions

**LAW.17** – For the circuit shown in Figure determine: (a) the voltage  $v_0$ , (b) the power supplied by the current source, (c) the power absorbed by each resistor.

**Solution:**

$$i_1 = \frac{18}{9 + 18} 30 = 20 \text{ mA}$$

$$i_2 = \frac{9}{9 + 18} 30 = 10 \text{ mA}$$

$$v_0 = 9 \cdot i_1 = 18 \cdot i_2 = 180 \text{ V}$$

$$p_0 = v_0 \cdot i_0 = 180 \cdot 30 = 5.4 \text{ W}$$

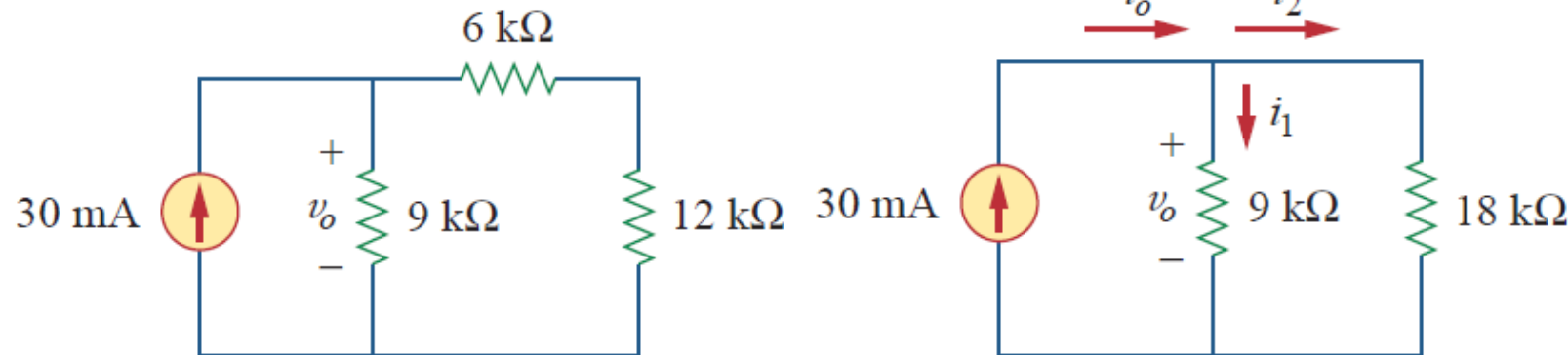
$$p_{12} = i_2(i_2 \cdot R) = i_2^2 \cdot R = (20 \cdot 10^{-3})^2 \cdot 12 \cdot 10^3 = 1.2 \text{ W}$$

*supplied power*  $\rightarrow 5.4 \text{ W}$

$$p_6 = i_2^2 \cdot R = (20 \cdot 10^{-3})^2 \cdot 6 \cdot 10^3 = 0.6 \text{ W}$$

*absorbed power*  $\rightarrow (1.2 + 0.6 + 3.6) = 5.4 \text{ W}$

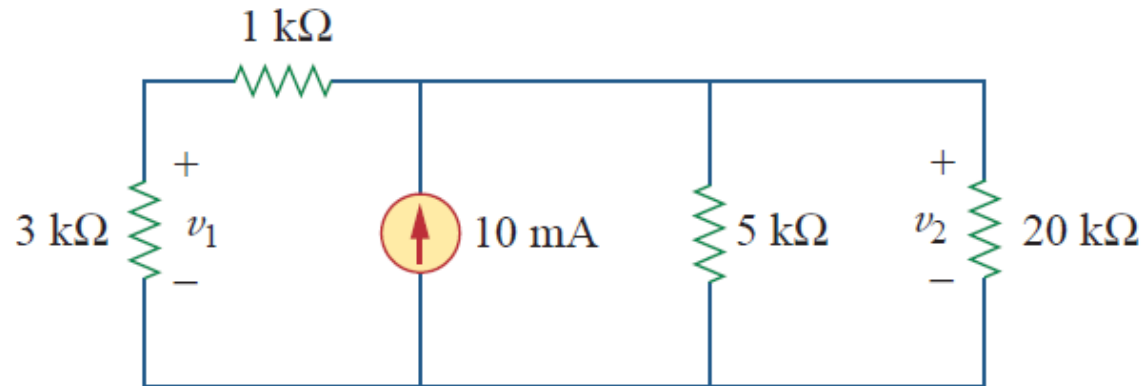
$$p_9 = \frac{v_0^2}{R} = \frac{(180)^2}{9 \cdot 10^3} = 3.6 \text{ W} \text{ or } p_9 = v_0 \cdot i_1 = 180 \cdot 20 \cdot 10^{-3} = 3.6 \text{ W}$$



## Voltage and Current Divisions

**LAW.18** – For the circuit shown in Figure, find:

- $v_1$  and  $v_2$
- power dissipated in the  $3\text{-k}\Omega$  and  $20\text{-k}\Omega$  resistors
- power supplied by the current source.

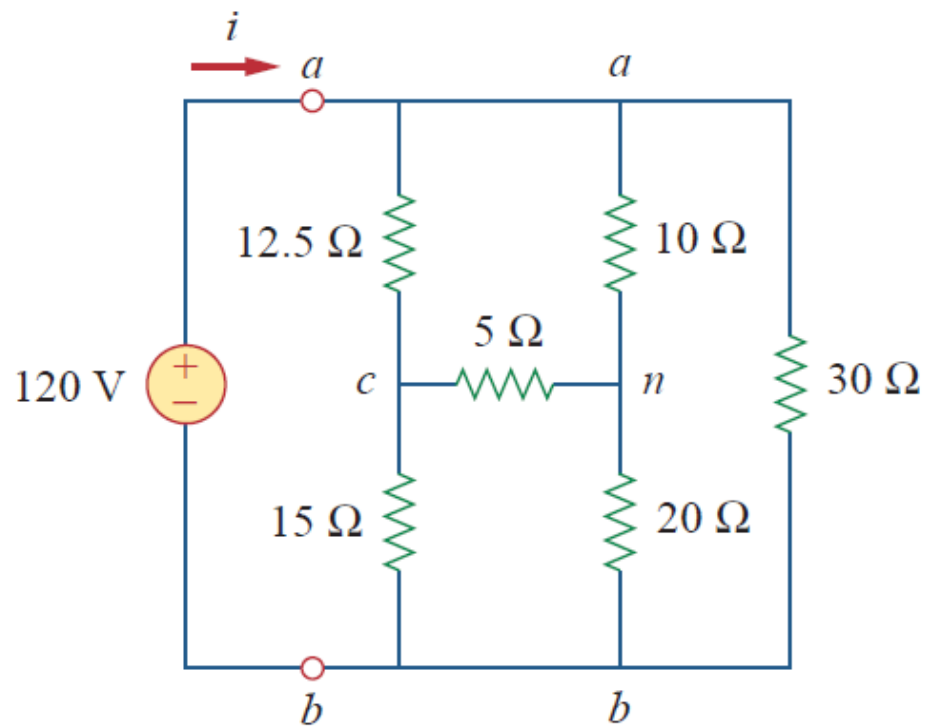


**Solution:**

- $15\text{ V}$ ,  $20\text{ V}$
- $75\text{ mW}$ ,  $20\text{ mW}$
- $200\text{ mW}$

## Wye-Delta Conversion

**LAW.19** – Obtain current  $i$  in the circuit.

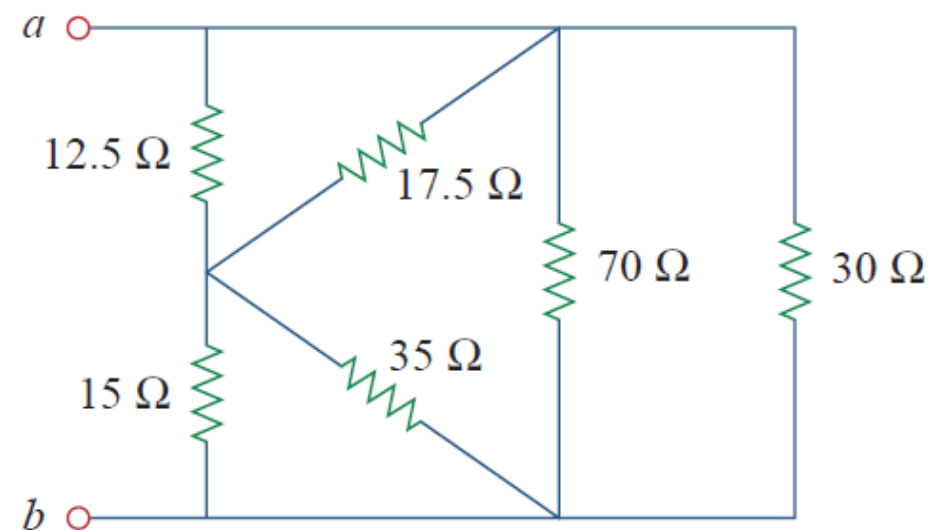


**Solution:** 
$$G_{\Delta} = \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = 0.35 \text{ S}$$

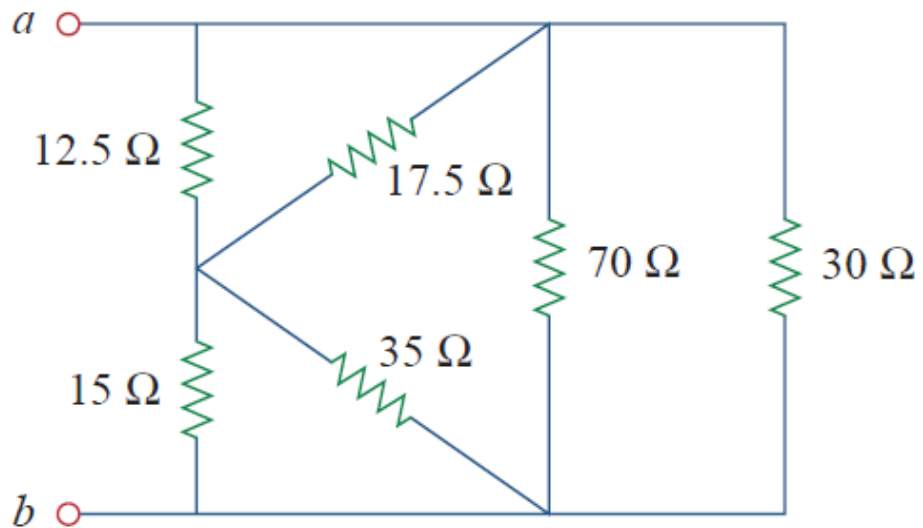
$$G_{ac} = \frac{\frac{1}{5} \cdot \frac{1}{10}}{G_{\Delta}} = \frac{0.02}{0.35} \rightarrow R_{ac} = \frac{0.35}{0.02} = 17.5 \Omega$$

$$G_{bc} = \frac{\frac{1}{5} \cdot \frac{1}{20}}{G_{\Delta}} = \frac{0.01}{0.35} \rightarrow R_{bc} = \frac{0.35}{0.01} = 35 \Omega$$

$$G_{ab} = \frac{\frac{1}{10} \cdot \frac{1}{20}}{G_{\Delta}} = \frac{0.005}{0.35} \rightarrow R_{ab} = \frac{0.35}{0.005} = 70 \Omega$$



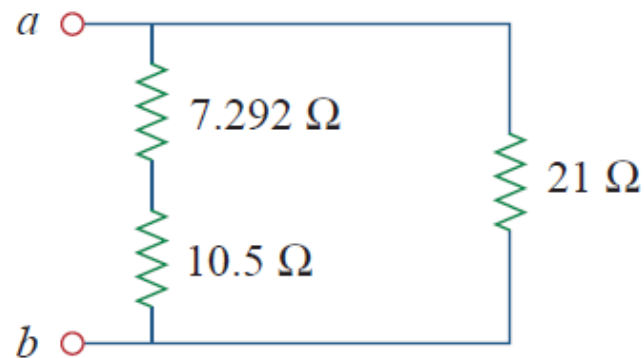
## Wye-Delta Conversion



$$70 \times 30 = \frac{70 \cdot 30}{100} = 21 \Omega$$

$$12.5 \times 17.5 = \frac{12.5 \cdot 17.5}{30} = 7.292 \Omega$$

$$15 \times 35 = \frac{15 \cdot 35}{50} = 10.5 \Omega$$



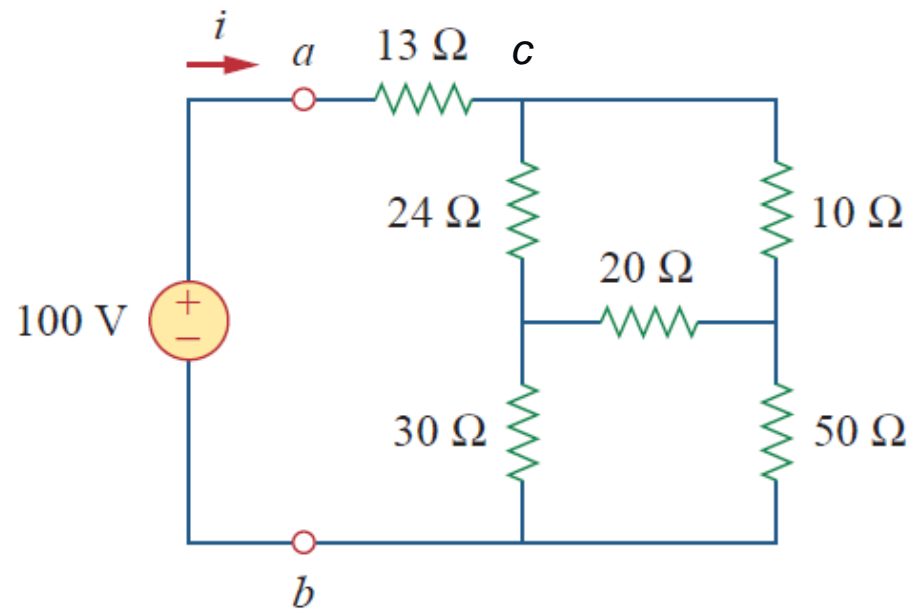
$$R_{ab} = (7.292 + 10.5) \times 21 = \dots = 9.632 \Omega$$

$$i = \frac{120 V}{R_{ab}} = 12.458 A$$



## Wye-Delta Conversion

LAW.20 – Obtain current  $i$  in the circuit.



**Solution 1:**

$$G_{\Delta} = \frac{1}{20} + \frac{1}{10} + \frac{1}{50} = 0.17 \text{ S}$$

$$G_1 = \frac{\frac{1}{10} \cdot \frac{1}{20}}{G_{\Delta}} = \frac{0.005}{0.17} \rightarrow R_1 = 34 \Omega \rightarrow R_{1,24} = \frac{34 \cdot 24}{58} = 14 \Omega$$

$$G_2 = \frac{\frac{1}{20} \cdot \frac{1}{50}}{G_{\Delta}} = \frac{0.001}{0.17} \rightarrow R_2 = 170 \Omega \rightarrow R_{2,30} = \frac{170 \cdot 30}{200} = 25.5 \Omega$$

$$G_3 = \frac{\frac{1}{10} \cdot \frac{1}{50}}{0.17} \rightarrow R_3 = 85 \Omega \rightarrow R_{bc} = 85 \times (14 + 25.5) = \dots = 27 \Omega$$

$$R_{ab} = R_{bc} + 13 = 40 \Omega \rightarrow i = \frac{100}{R_{ab}} = 2.5 \text{ A}$$

**Solution 2:**

$$(RR)_{\Delta} = 10 \cdot 20 + 10 \cdot 50 + 20 \cdot 50 = 1700 \rightarrow R_1 = \frac{(RR)_{\Delta}}{50} = 34 \Omega, \quad R_2 = \frac{(RR)_{\Delta}}{10} = 170 \Omega, \quad R_3 = \frac{(RR)_{\Delta}}{20} = 85 \Omega$$

# Questions

