



DR. GYURCSEK ISTVÁN

# Exercises using Methods in Circuit Analysis

## *Sources and additional materials (recommended)*

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

# Recall: Math



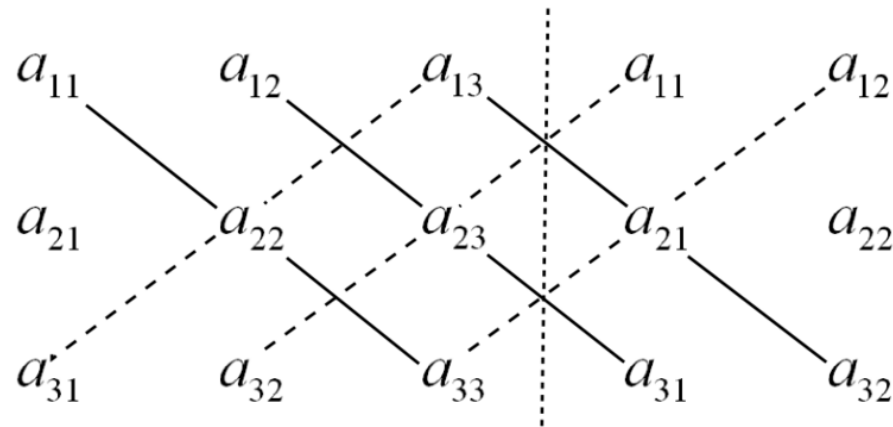
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

**Cramer's rule** ( $n \times n$ )  $\rightarrow x_1 = \frac{\Delta_1}{\Delta}$ ,  $x_2 = \frac{\Delta_2}{\Delta}$ ,  $x_3 = \frac{\Delta_3}{\Delta}$

$$\Delta = \det \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \Delta_1 = \det \begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{bmatrix} \quad \Delta_2 = \det \begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{bmatrix} \quad \Delta_3 = \det \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix}$$

**Sarrus' rule** ( $3 \times 3$ )  $\rightarrow \Delta = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$

[extension right]  
OR  
[extension down]

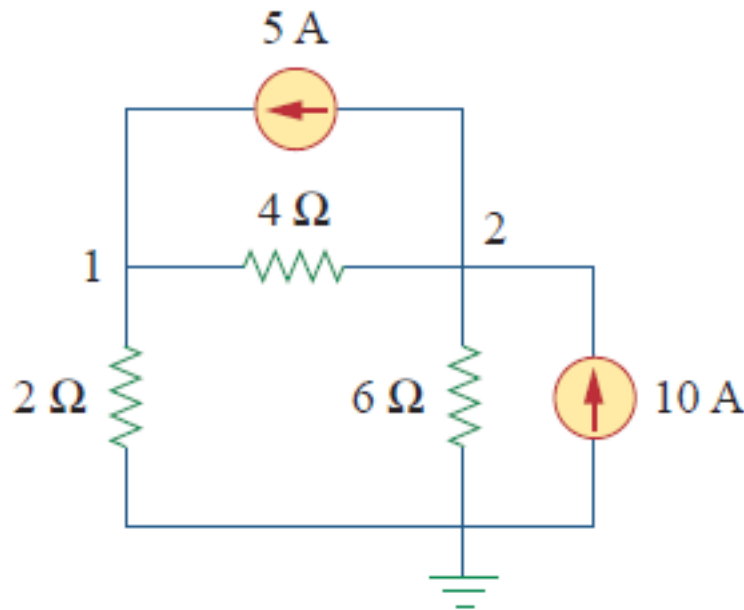


# Nodal Analysis

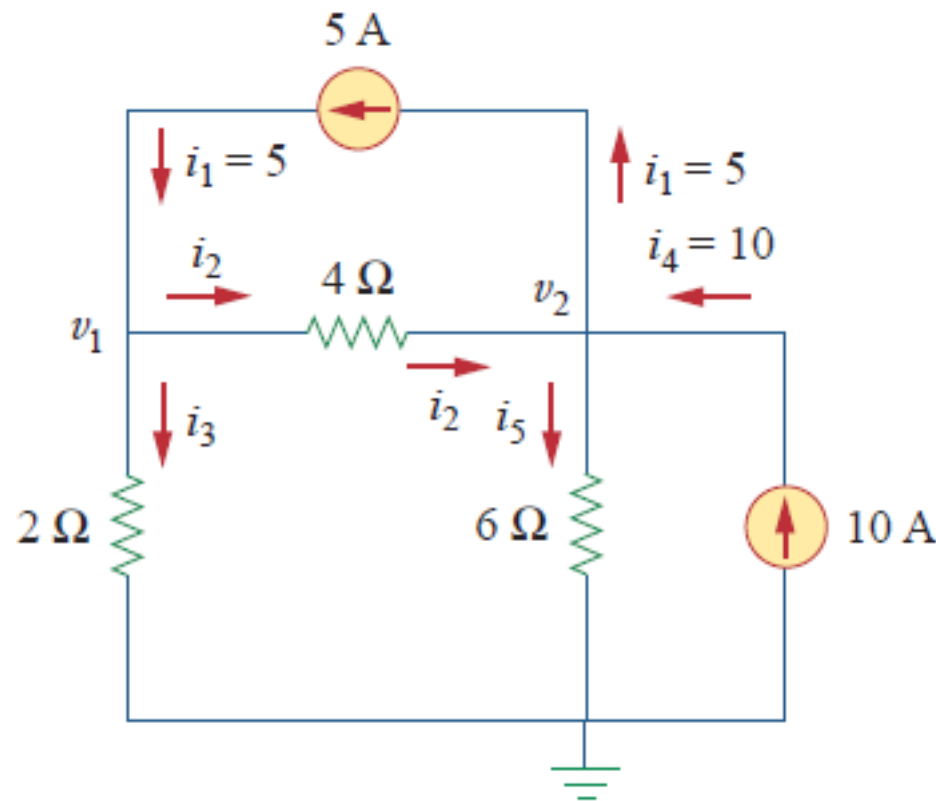


## MTD.01

Calculate the node voltages and the branch currents in the circuit.



## Solution



$$i_1 = i_2 + i_3$$

$$\rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

$$20 = v_1 - v_2 + 2v_1$$

$$3v_1 - v_2 = 20$$

$$i_2 + i_4 = i_1 + i_5$$

$$\rightarrow \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

$$3v_1 - 3v_2 + 120 = 60 + 2v_2$$

$$-3v_1 + 5v_2 = 60$$

# Nodal Analysis



$$3v_1 - v_2 = 20$$

$$-3v_1 + 5v_2 = 60$$

**Method 1**

$$4v_2 = 80 \rightarrow v_2 = 20 \text{ V}$$

$$3v_1 - 20 = 20 \rightarrow v_1 = 13.33 \text{ V}$$

**Method 2** (Cramer's rule)

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

$$\Delta = \det \begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} = 15 - 3 = 12$$

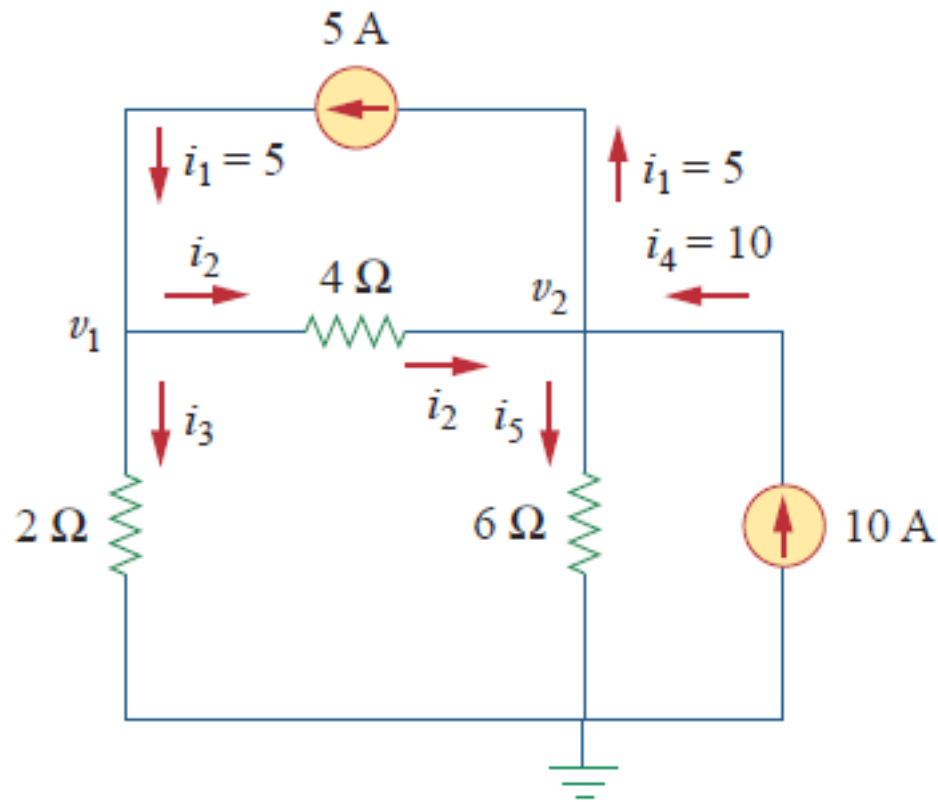
$$\Delta_1 = \det \begin{bmatrix} 20 & -1 \\ 60 & 5 \end{bmatrix} = 100 + 60 = 160$$

$$\Delta_2 = \det \begin{bmatrix} 3 & 20 \\ -3 & 60 \end{bmatrix} = 180 + 60 = 240$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{160}{12} = 13.33 \text{ V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{240}{12} = 20 \text{ V}$$

# Nodal Analysis



$$v_1 = 13.33 \text{ V}, \quad v_2 = 20 \text{ V}$$

$$i_1 = 5 \text{ A}$$

$$i_2 = \frac{v_1 - v_2}{4} = -1.67 \text{ A}$$

$$i_3 = \frac{v_1}{2} = 6.67 \text{ A}$$

$$i_4 = 10 \text{ A}$$

$$i_5 = \frac{v_2}{6} = 3.33 \text{ A}$$

**Note**  $i_2 < 0 \rightarrow$  opposite direction to the assumed

# Nodal Analysis



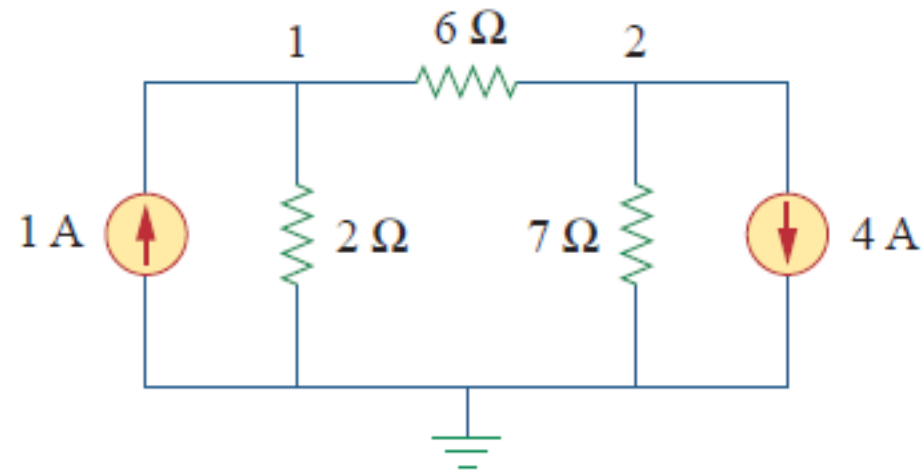
## MTD.02

Find the nodal voltages and currents through the resistors in the circuit.

### Solution

$$v_1 = -2 V, \quad v_2 = -14 V$$

$$i_2 = 1 A \text{ ('up')}, \quad i_6 = 2 A \text{ ('right')}, \quad i_7 = 2 A \text{ ('up')}$$

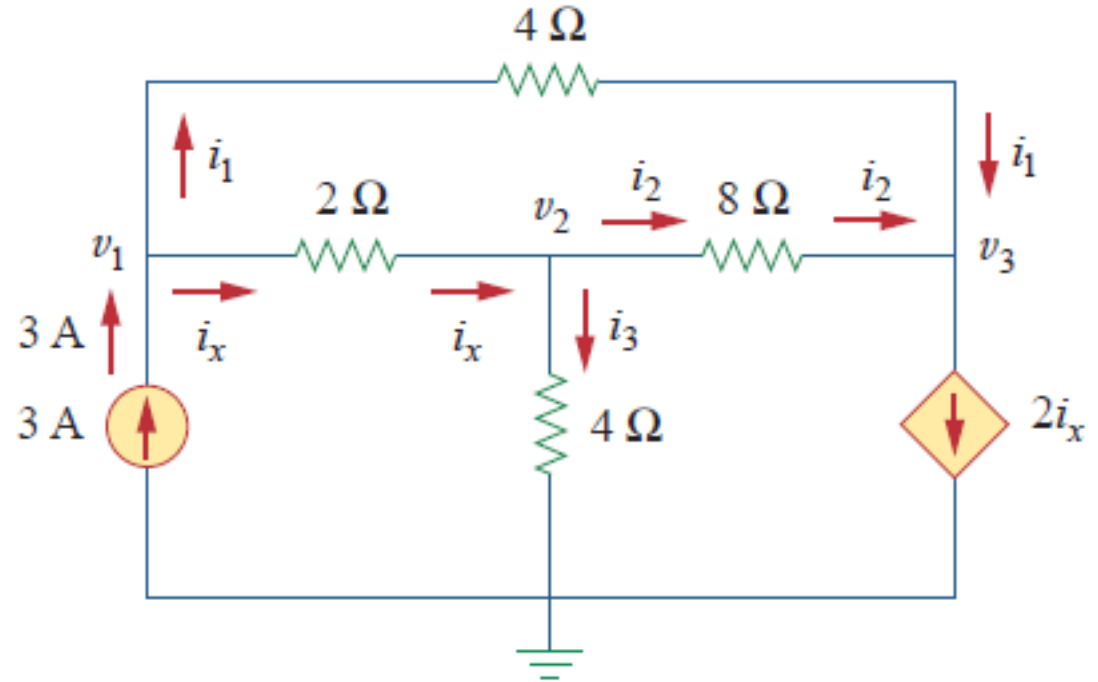
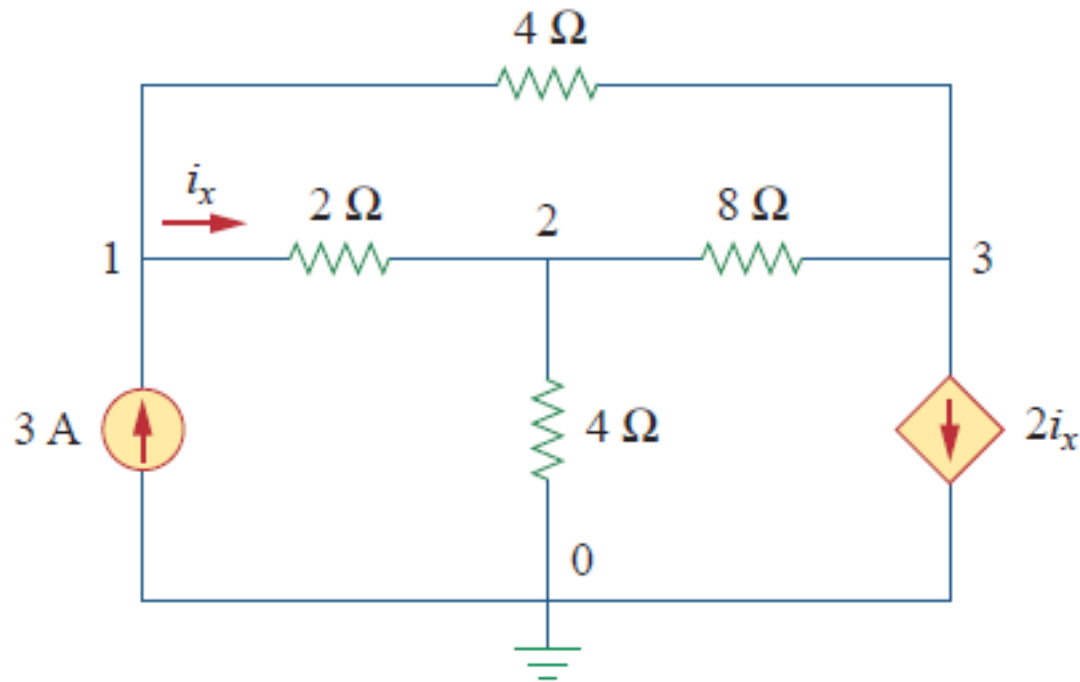


# Nodal Analysis



MTD.03 – Determine the voltages at the nodes.

Solution



$$3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2} \rightarrow 3v_1 - 2v_2 - v_3 = 12$$

$$\frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4} \rightarrow -4v_1 + 7v_2 - v_3 = 0$$

$$\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2} \rightarrow 2v_1 - 3v_2 + v_3 = 0$$

# Nodal Analysis



$$3v_1 - 2v_2 - v_3 = 12$$

$$-4v_1 + 7v_2 - v_3 = 0$$

$$2v_1 - 3v_2 + v_3 = 0$$

$$\Delta = \det \begin{bmatrix} +3 & -2 & -1 \\ -4 & +7 & -1 \\ +2 & -3 & +1 \end{bmatrix}$$

Cramer's rule

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = 21 - 12 + 4 + 14 - 9 - 8 = 10$$

$$\Delta_1 = 84 - 0 + 0 - 36 + 0 - 0 = 48$$

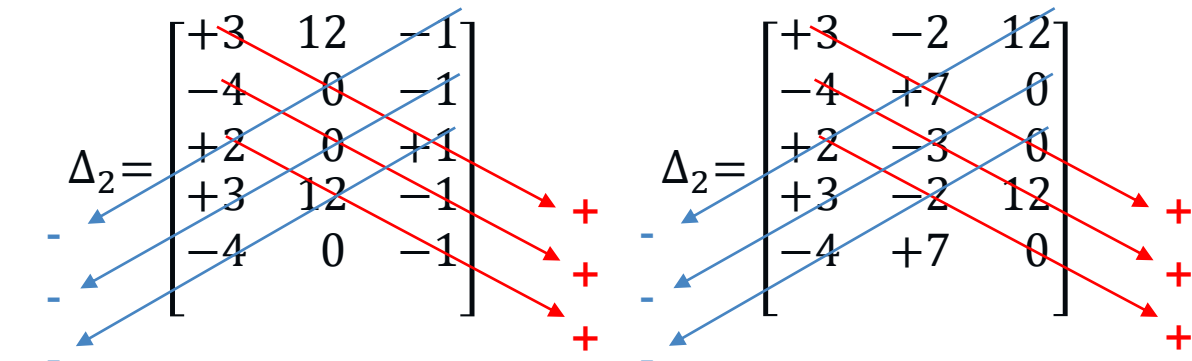
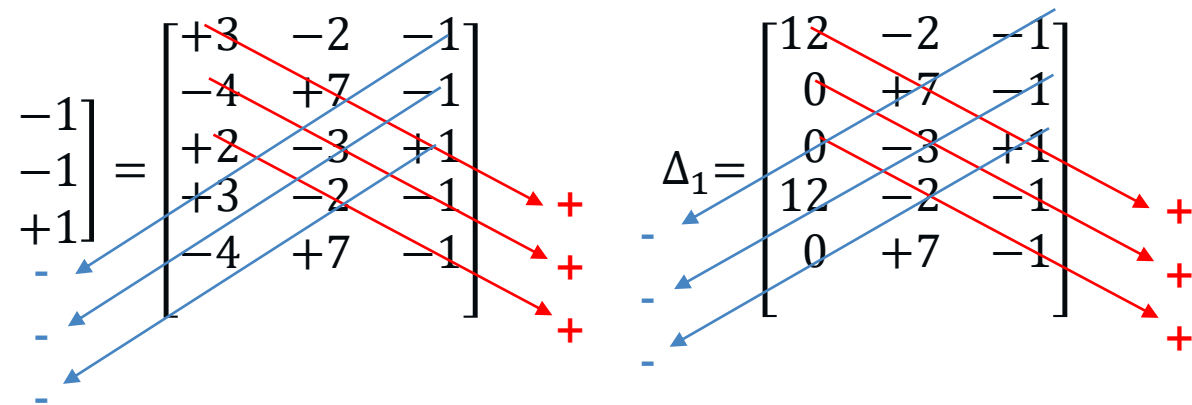
$$\Delta_2 = 0 - 0 + 0 - 0 - 24 + 48 = 24$$

$$\Delta_3 = 0 - 168 + 144 - 0 - 0 - 0 = -24$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{48}{10} = 4.8 \text{ V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{24}{10} = 2.4 \text{ V}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{-24}{10} = -2.4 \text{ V}$$



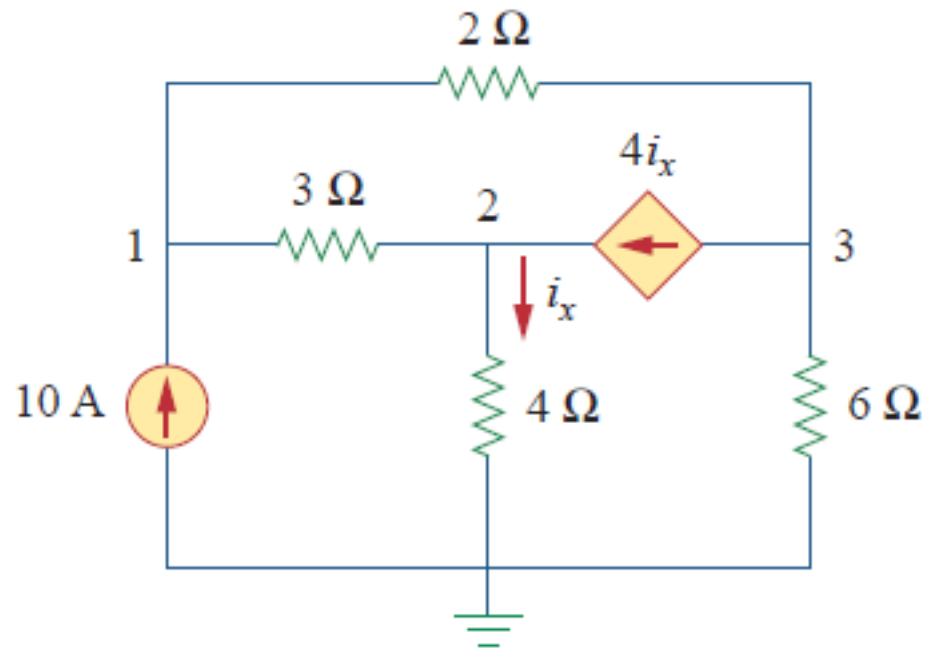


# Nodal Analysis



MTD.04 – Find node voltages and branch currents.

**Solution**

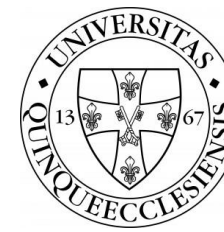


$$v_1 = 80 V, \quad v_2 = -64 V, \quad v_3 = 156 V$$

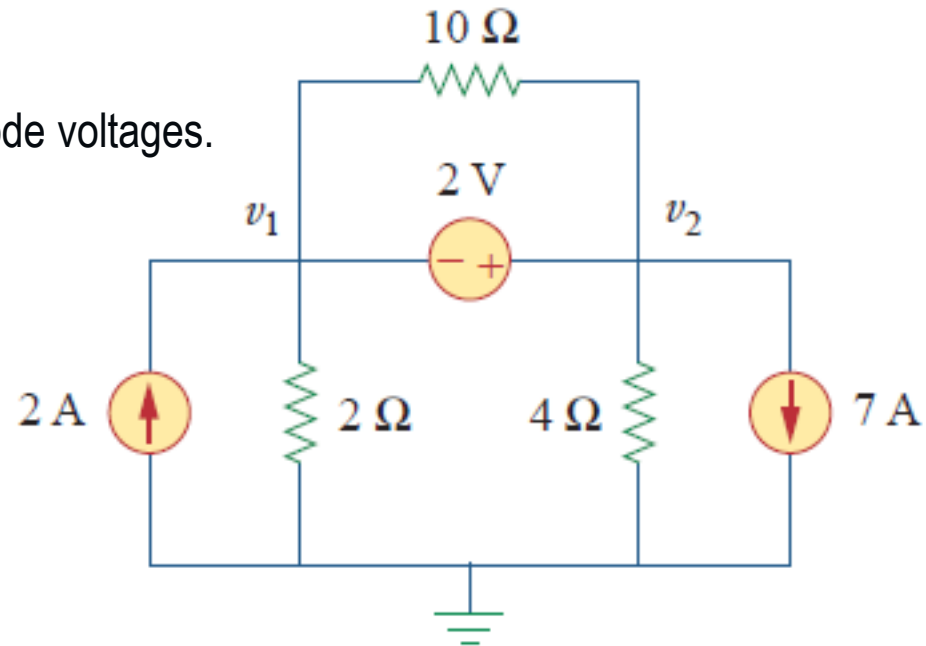
$$i_x = i_4 = -16 A \text{ (up)}, \quad i_6 = 26 A \text{ (down)}$$

$$i_3 = 48 A \text{ (right)}, \quad i_2 = 38 A \text{ (left)}$$

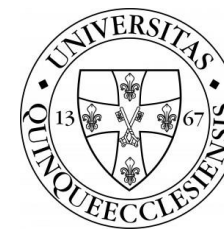
# Nodal Analysis with Super Node



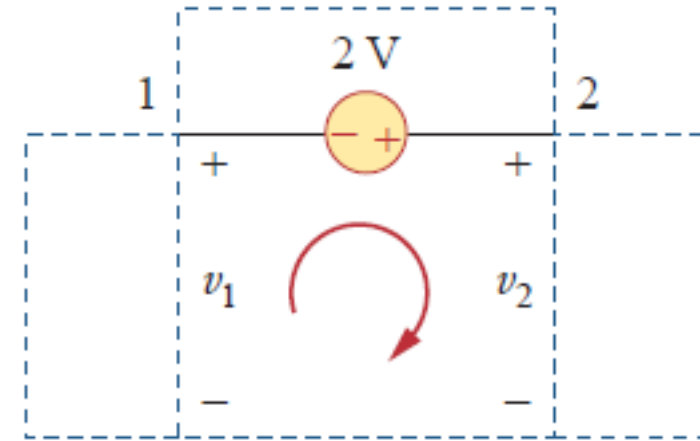
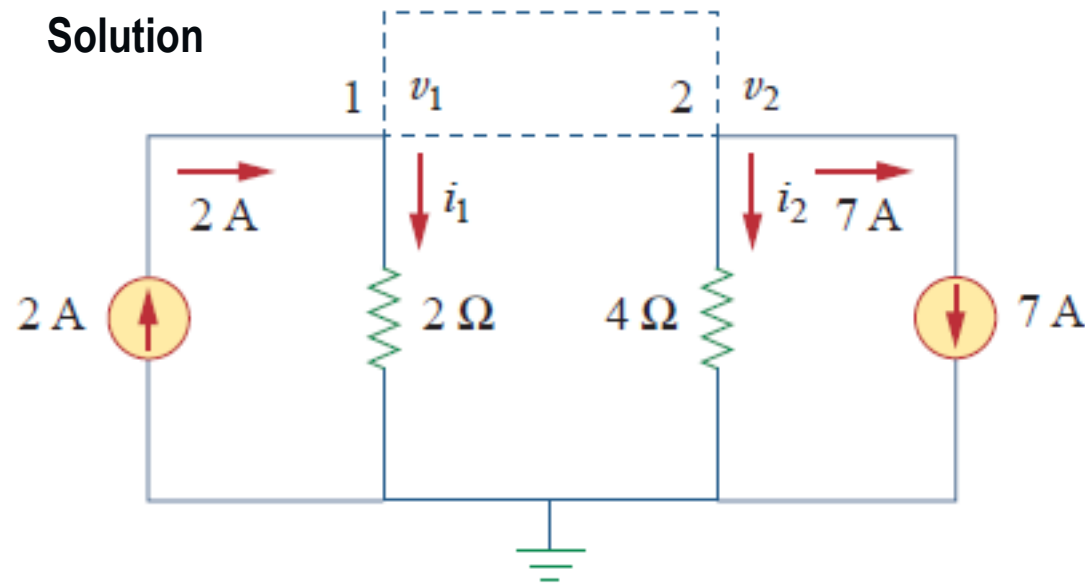
MTD.05  
Find the node voltages.



# Nodal Analysis with Super Node



Solution



$$2 = i_1 + i_2 + 7$$

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \rightarrow 8 = 2v_1 + v_2 + 28$$

$$(1): v_2 = -20 - 2v_1$$

$$-v_1 - 2 + v_2 = 0 \rightarrow (2): v_2 = v_1 + 2$$

$$(1)(2): -20 - 2v_1 = v_1 + 2$$

$$3v_1 = -22 \rightarrow v_1 = -7.33 \text{ V}, \quad v_2 = v_1 + 2 = -5.33 \text{ V}$$

# Nodal Analysis with Super Node

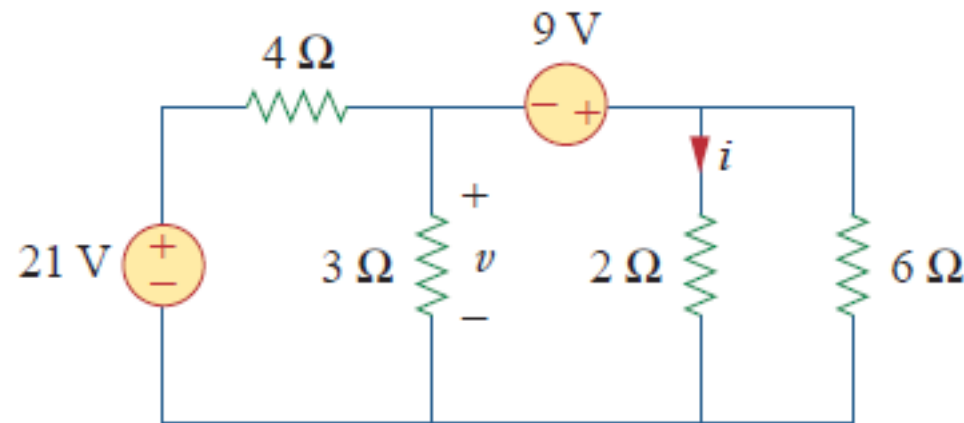


## MTD.06

Obtain  $v$  and  $i$  in the circuit.

## Solution

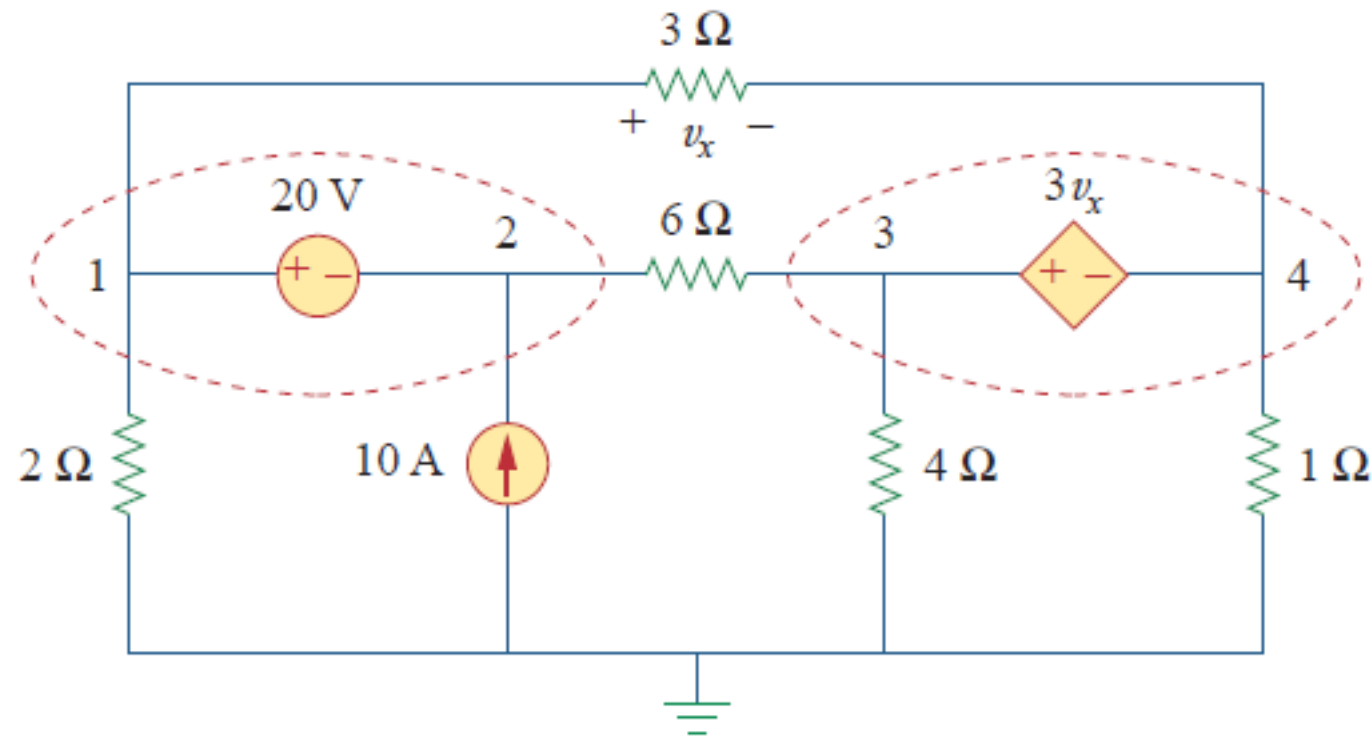
$$v = -0.6 V, \quad i = 4.2 A$$



# Nodal Analysis with Super Node



MTD.07 – Find the node voltages in the circuit.



# Nodal Analysis with Super Node



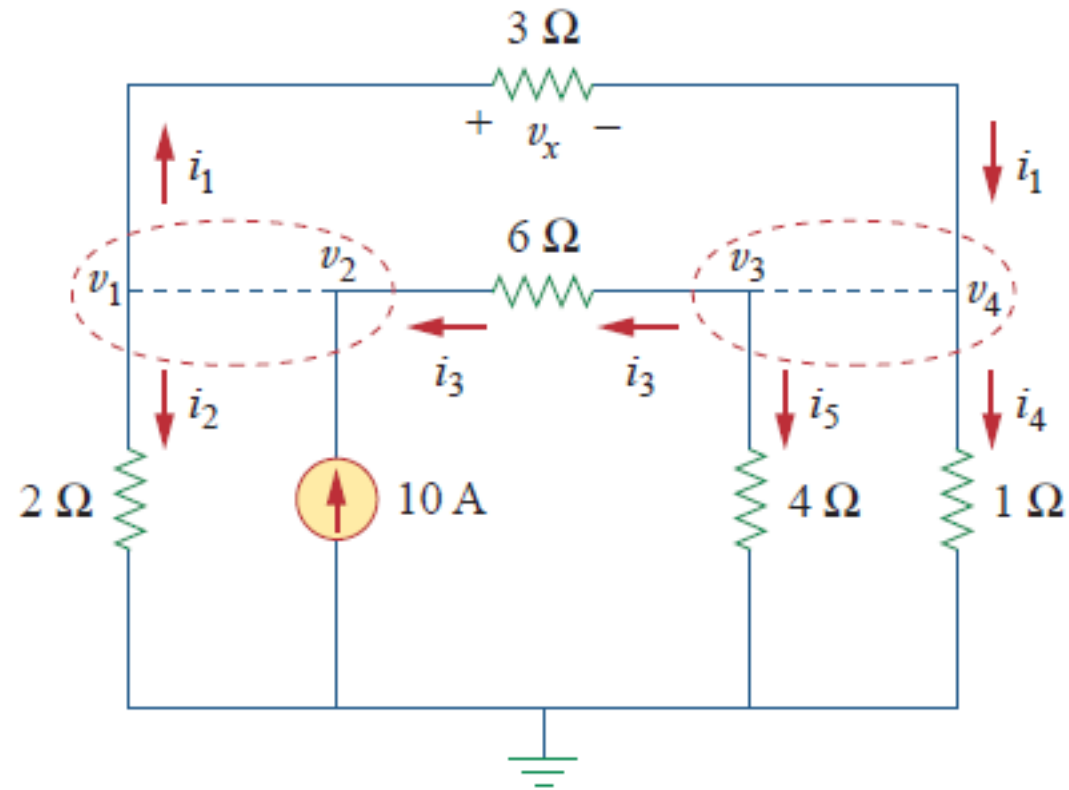
## Solution

$$i_3 + 10 = i_1 + i_2 \rightarrow \frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$$

$$(A): 5v_1 + v_2 - v_3 - 2v_4 = 60$$

$$i_1 = i_3 + i_4 + i_5 \rightarrow \frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$$

$$(B): 4v_1 + 2v_2 - 5v_3 - 16v_4 = 0$$



# Nodal Analysis with Super Node



(Loop1):  $-v_1 + 20 + v_2 = 0 \rightarrow$  (C):  $v_1 - v_2 = 20$

(Loop2): (1)  $-v_3 + 3v_x + v_4 = 0$

(2)  $v_x = v_1 - v_4$

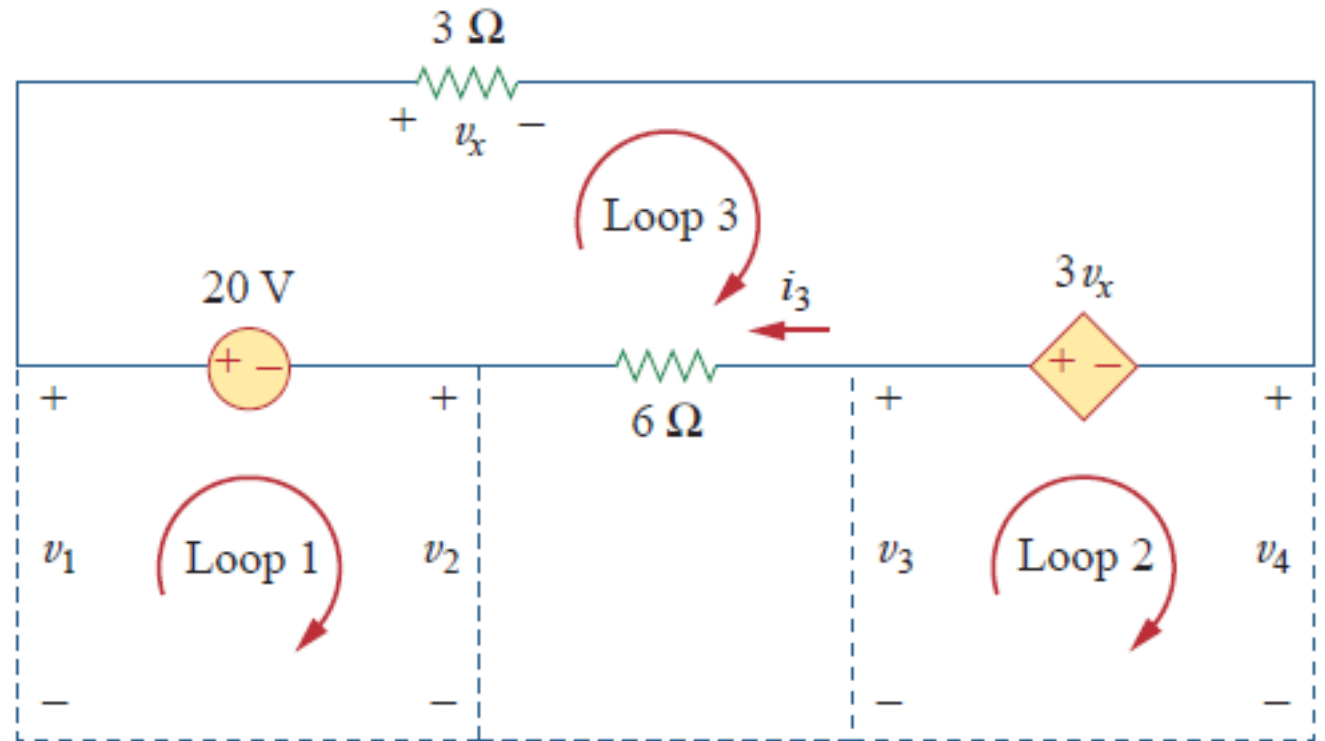
(1)(2)  $\rightarrow$  (D):  $3v_1 - v_3 - 2v_4 = 0$

(Loop3):  $v_x - 3v_x + 6i_3 - 20 = 0$

(3)  $6i_3 = v_3 - v_2$

(2)  $v_x = v_1 - v_4$

(3)(2)  $\rightarrow$  (E):  $-2v_1 - v_2 + v_3 + 2v_4 = 20$



# Nodal Analysis with Super Node



variables:  $v_1, v_2, v_3, v_4$ , equations: (A), (B), (C), (D), (E)  $\leftarrow$  1 redundant

(D) and  $v_2 = v_1 - 20 \rightarrow$  (A), (B)

$$\Delta = \det \begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} = -18$$
$$v_1 = \frac{\Delta_1}{\Delta} = \frac{-480}{-18} = 26.67 \text{ V}$$

(D):  $3v_1 - v_3 - 2v_4 = 0$

$$6v_1 - v_3 + v_3 - 2v_4 = 80$$

$$\Delta_1 = \det \begin{bmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{bmatrix} = -480$$
$$v_3 = \frac{\Delta_3}{\Delta} = \frac{-3120}{-18} = 173.33 \text{ V}$$

$$6v_1 - 5v_3 + v_3 - 16v_4 = 40$$

$$\Delta_3 = \det \begin{bmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{bmatrix} = -3120$$
$$v_4 = \frac{\Delta_4}{\Delta} = \frac{840}{-18} = -46.67 \text{ V}$$

$$\begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 40 \end{bmatrix}$$

$$\Delta_4 = \det \begin{bmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{bmatrix} = 840$$
$$v_2 = v_1 - 20 = 6.67 \text{ V}$$

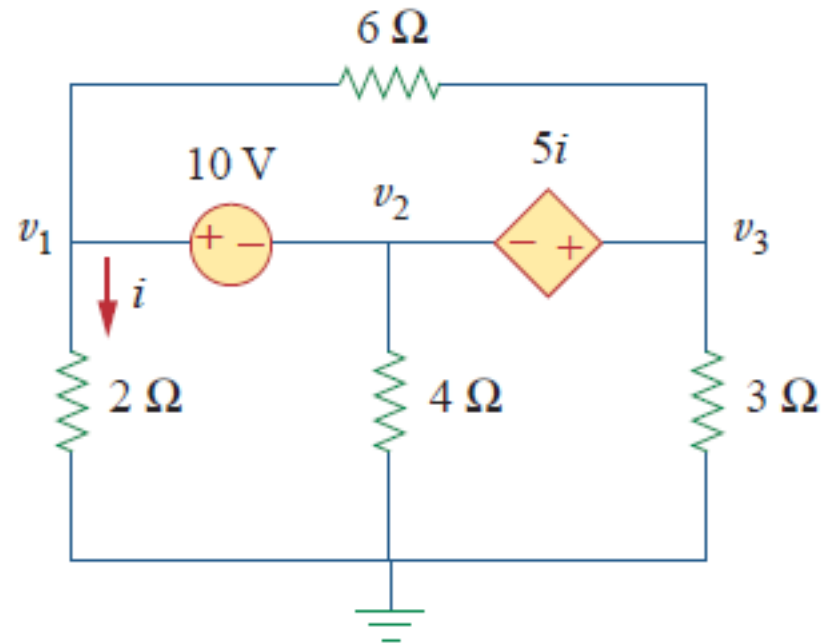
(E) can be used for cross check the result.



# Nodal Analysis with Super Node



MTD.08 – Find the nodal voltages.



**Solution**

$$v_1 = 3.04 \text{ V}$$

$$v_2 = -6.96 \text{ V}$$

$$v_3 = 0.65 \text{ V}$$

# Mesh Analysis



**MTD.09** – Find the branch currents ( $I_1, I_2, I_3$ ) using mesh analysis.

**Solution**

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0 \rightarrow 3i_1 - 2i_2 = 1$$

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0 \rightarrow -i_1 + 2i_2 = 1$$

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Delta = \det \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} = 4$$

$$\Delta_1 = \det \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix} = 4$$

$$\Delta_2 = \det \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix} = 4$$

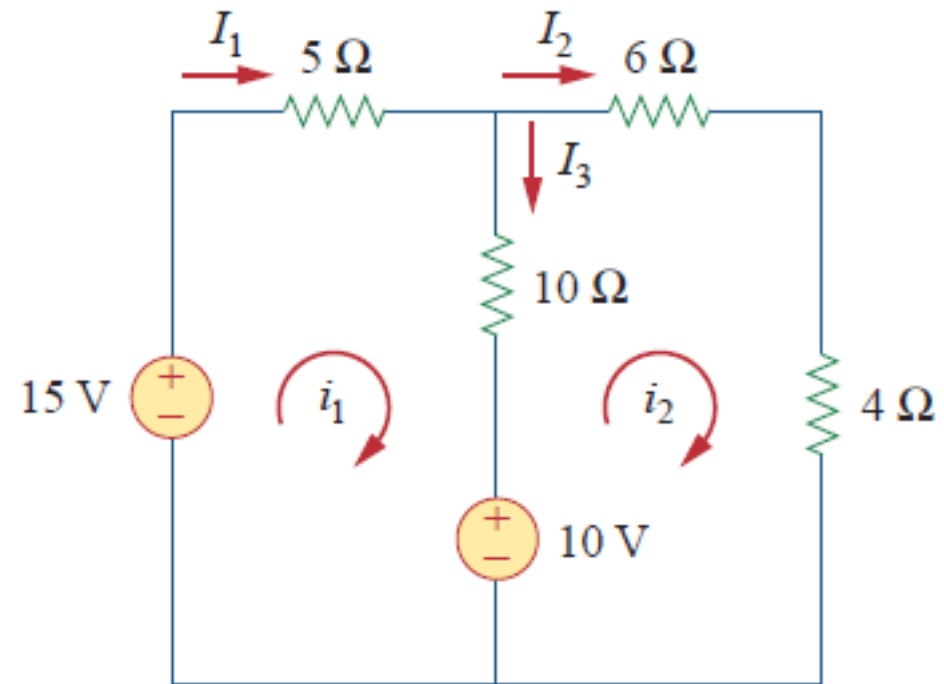
$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}$$

$$i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

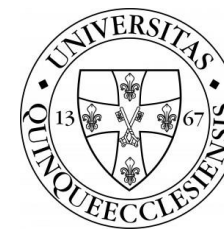
$$I_1 = i_1 = 1 \text{ A}$$

$$I_2 = i_2 = 1 \text{ A}$$

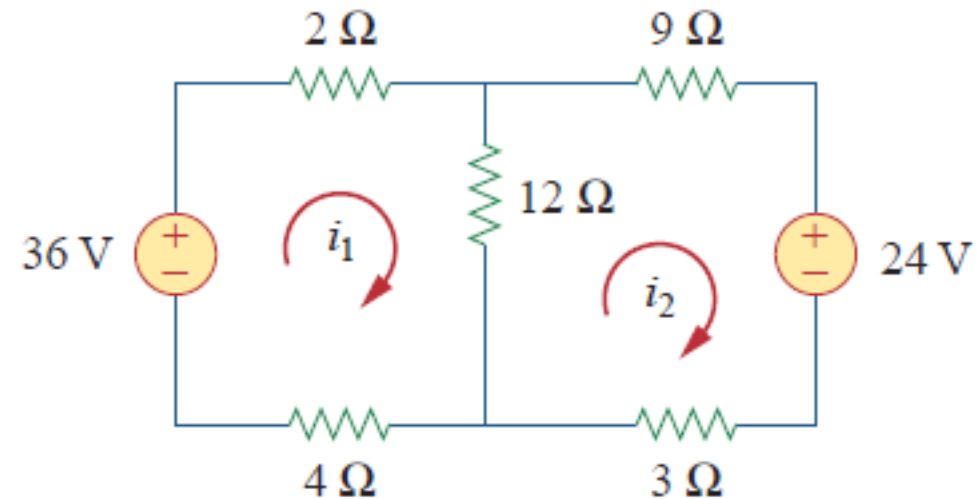
$$I_3 = i_1 - i_2 = 0$$



# Mesh Analysis



MTD.10 – Calculate the mesh currents ( $i_1, i_2$ ) of the circuit.



**Solution**  $i_1 = 2 A, \quad i_2 = 0 A$

# Mesh Analysis



MTD.11 – Calculate  $I_0$  in the circuit.

**Solution**

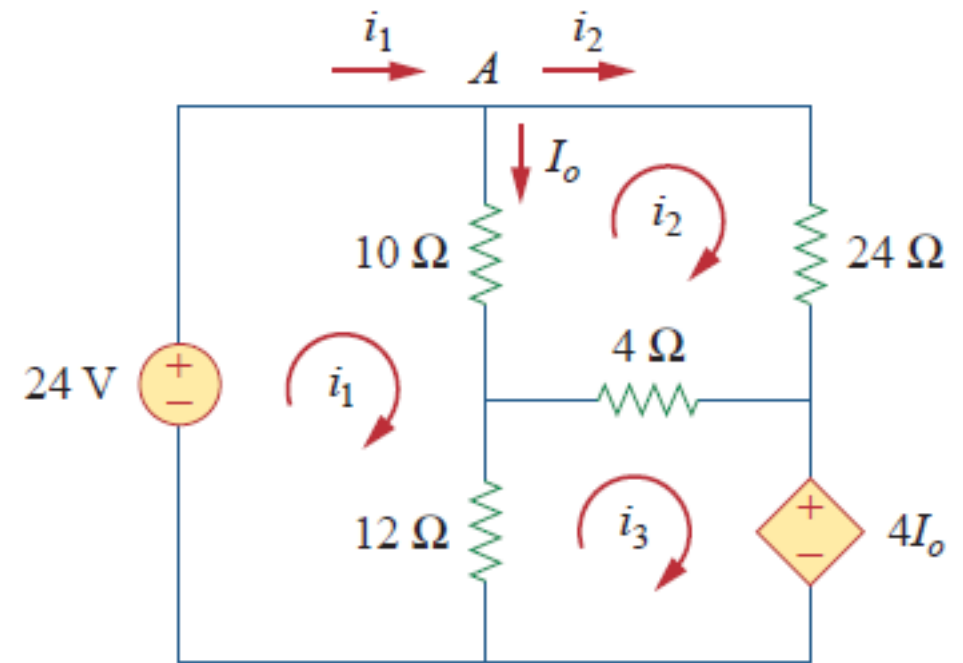
$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0 \rightarrow 11i_1 - 5i_2 - 6i_3 = 12$$

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0 \rightarrow -5i_1 + 19i_2 - 2i_3 = 0$$

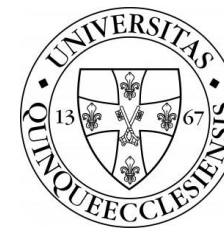
$$4I_0 + 12(i_3 - i_1) + 4(i_3 - i_2) = 0, \quad (\text{node } A): I_0 = i_1 - i_2$$

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0 \rightarrow -i_1 - i_2 + 2i_3 = 0$$

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$



# Mesh Analysis



$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \det \begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} = 192$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}$$

$$\Delta_1 = \det \begin{bmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \end{bmatrix} = 432$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A}$$

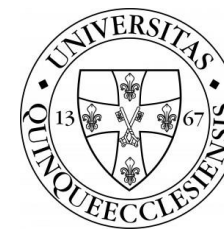
$$\Delta_2 = \det \begin{bmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \end{bmatrix} = 144$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

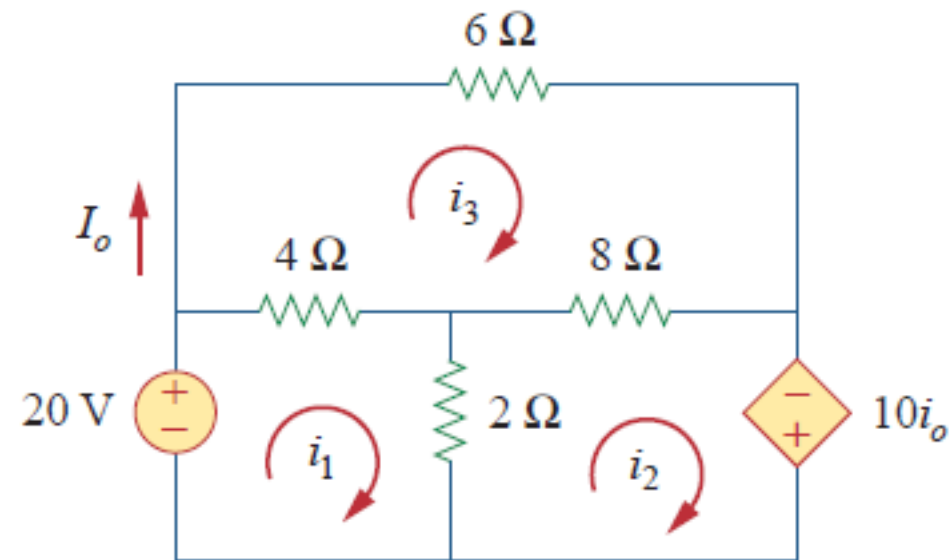
$$\Delta_3 = \det \begin{bmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \end{bmatrix} = 288$$

$$I_0 = i_1 - i_2 = 1.5 \text{ A}$$

# Mesh Analysis



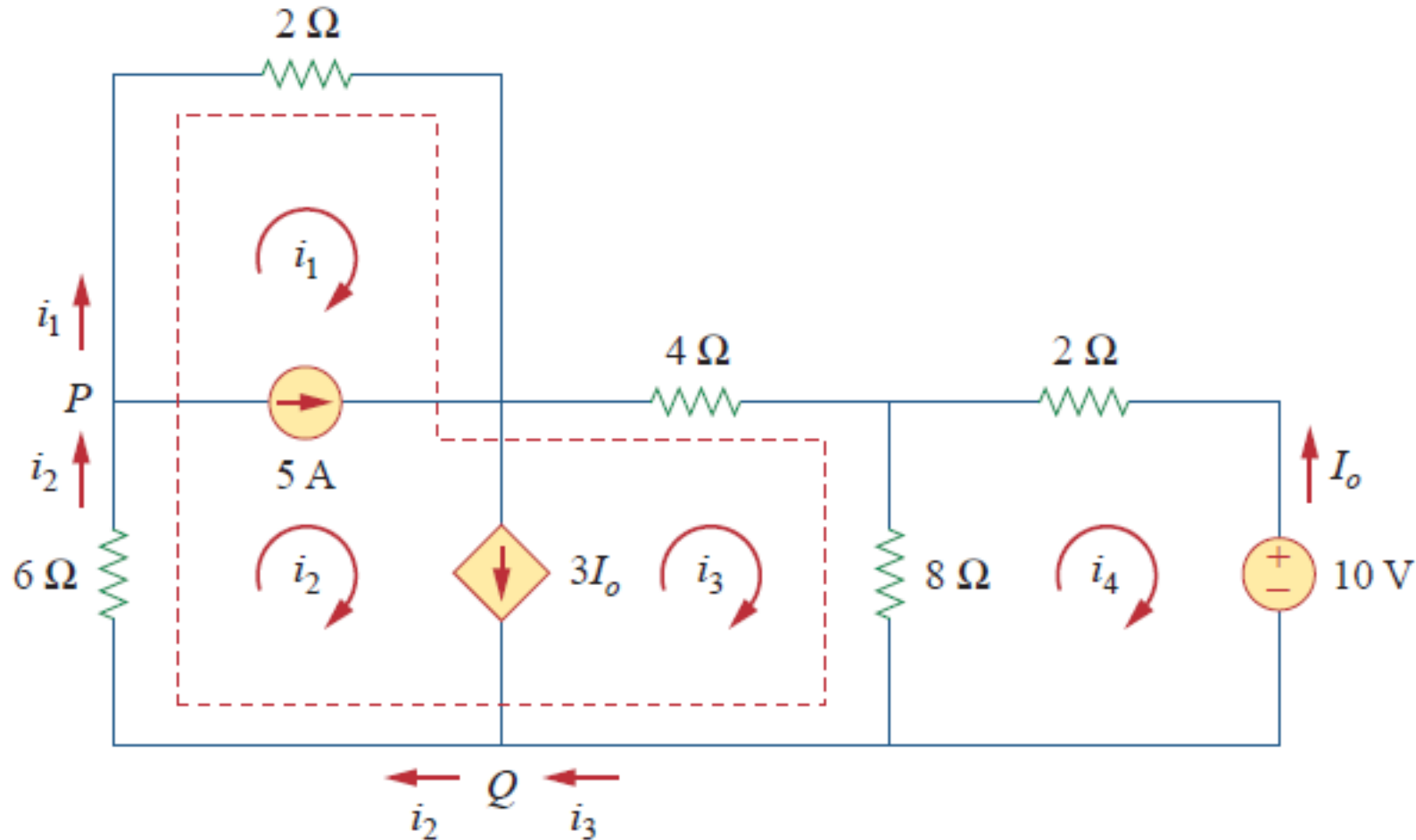
MTD.12 – Using mesh analysis find  $I_o$  in the circuit.



**Solution**  $I_o = -5A$

# Mesh Analysis with Super Mesh

MTD.13 – Find  $i_1, i_2, i_3, i_4$  using mesh analysis.



**Solution**

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$\rightarrow (1): i_1 + 3i_2 + 6i_3 - 4i_4 = 0$$

$$(node P): i_2 = i_1 + 5$$

$$(node Q): i_2 = i_3 + 3I_o$$

$$I_o = -i_4 \rightarrow i_2 = i_3 - 3i_4$$

$$(mesh 4): 2i_4 + 8(i_4 - i_3) + 10 = 0$$

$$(2): 5i_4 - 4i_3 = -5$$

(1)(2):

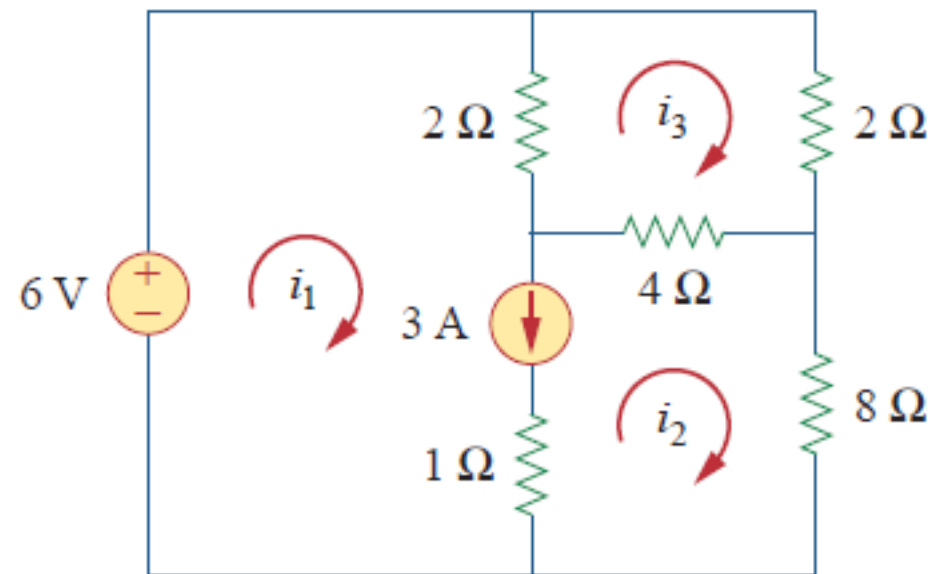
$$i_1 = -7.5 A, \quad i_2 = -2.5 A$$

$$i_3 = 3.93 A, \quad i_4 = 2.143 A$$

# Mesh Analysis



MTD.14 – Find  $i_1$ ,  $i_2$ ,  $i_3$  using mesh analysis.



**Solution**

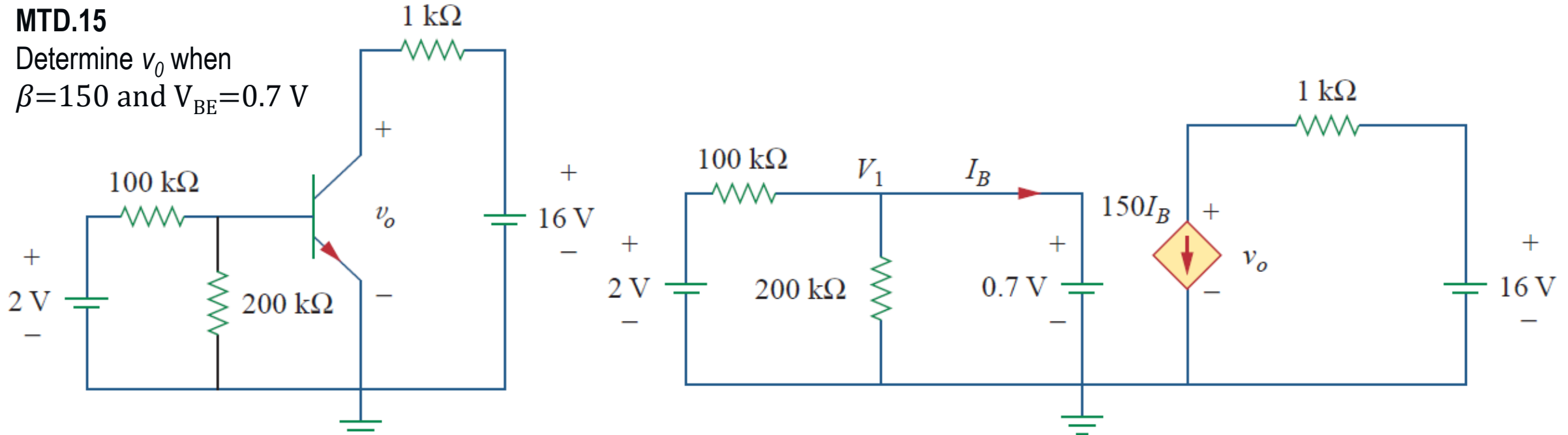
$$i_1 = 3.474 \text{ A}, \quad i_2 = 0.4737 \text{ A}, \quad i_3 = 1.1052 \text{ A}$$



# Mesh Analysis



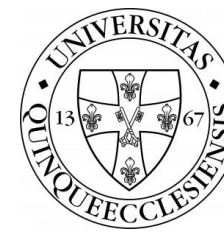
**MTD.15**  
Determine  $v_o$  when  $\beta=150$  and  $V_{BE}=0.7\text{ V}$



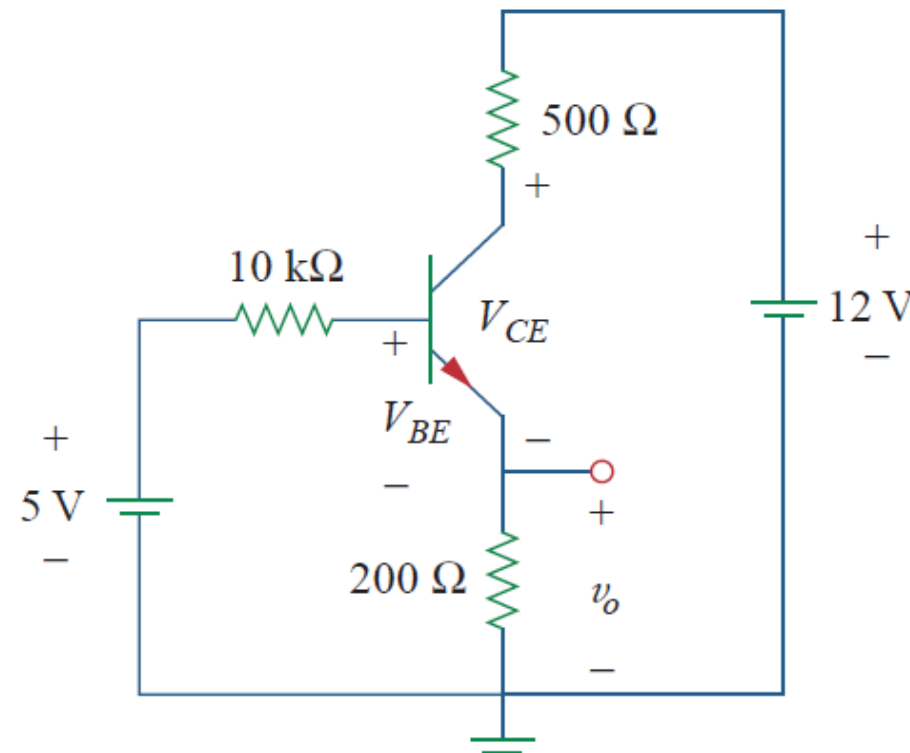
**Solution**     (*node 1*):  $V_1 = 0.7\text{ V}$ ,      $\frac{(0.7 - 2)}{100k} + \frac{0.7}{200k} + i_B = 0 \rightarrow i_B = 9.5\ \mu\text{A}$

(*node 2*):  $150I_B + \frac{(v_o - 16)}{1k} = 0 \rightarrow v_o = 16 - 150 \cdot 10^3 \cdot 9.5 \cdot 10^{-6} = 14.575\text{ V}$

# Mesh Analysis



MTD.16 (chk) – Determine  $v_o$  and  $V_{CE}$  when  $\beta=100$  and  $V_{BE}=0.7\text{ V}$



**Solution**  $v_o = 2.876\text{ V}$ ,  $V_{CE} = 1.984\text{ V}$

# Questions

