DR. GYURCSEK ISTVÁN

Exercises using Methods in Circuit Analysis

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

[before start]: **Maths Recall**

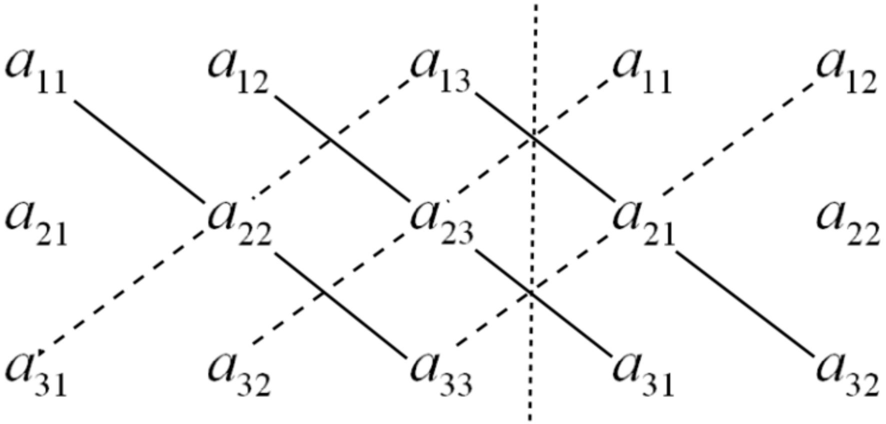
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\boldsymbol{Cramer's\ rule}\ (n \times n) \rightarrow x_1 = \frac{\Delta_1}{\Delta}, \quad x_2 = \frac{\Delta_2}{\Delta}, \quad x_3 = \frac{\Delta_3}{\Delta}$$

$$\Delta = det \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \Delta_1 = det \begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{bmatrix} \quad \Delta_2 = det \begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{bmatrix} \quad \Delta_3 = det \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix}$$

$$\boldsymbol{Sarrus' rule}\ (3 \times 3) \rightarrow \Delta = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

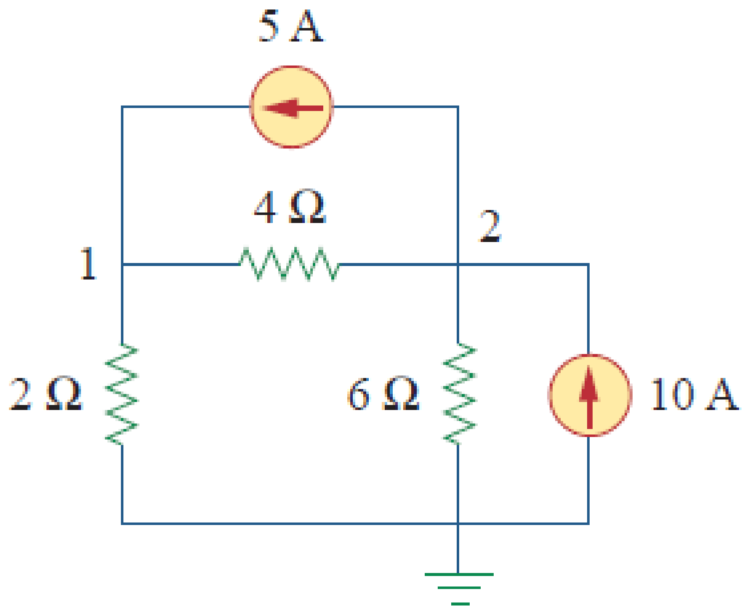
[extension right]
OR
[extension down]



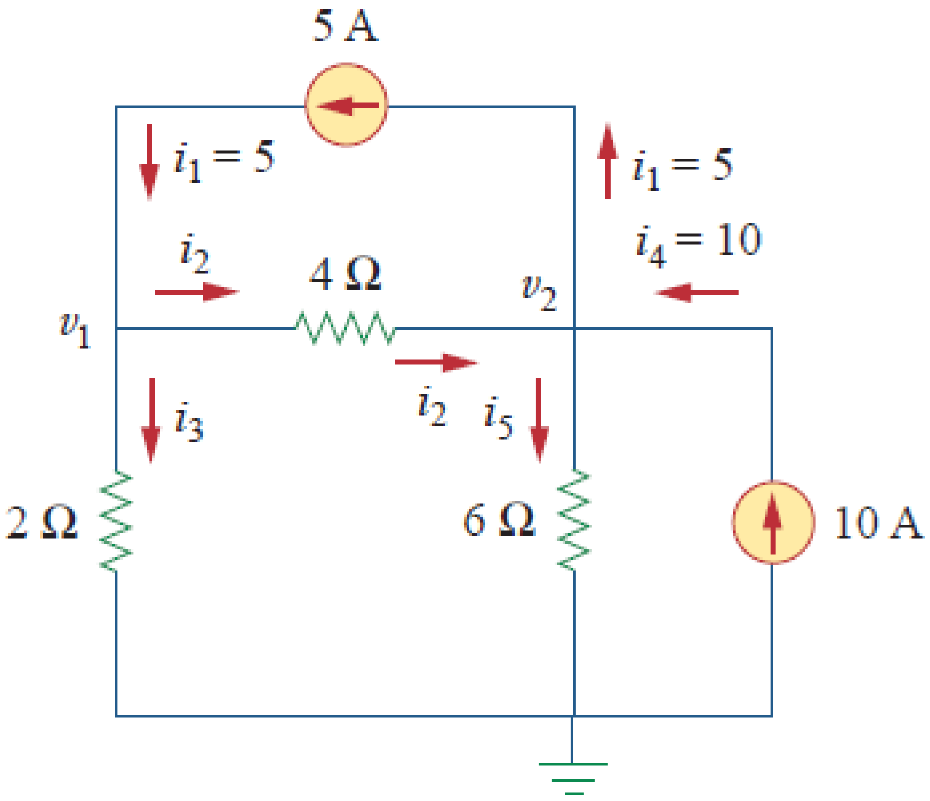
Nodal Analysis

MTD.01

Calculate the node voltages and the branch currents in the circuit,



Solution



$$i_1 = i_2 + i_3$$

$$\rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

$$20 = v_1 - v_2 + 2v_1$$

$$3v_1 - v_2 = 20$$

$$i_2 + i_4 = i_1 + i_5$$

$$\rightarrow \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

$$3v_1 - 3v_2 + 120 = 60 + 2v_2$$

$$-3v_1 + 5v_2 = 60$$

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Nodal Analysis

$$3v_1 - v_2 = 20$$

$$-3v_1 + 5v_2 = 60$$

Method 1

$$4v_2 = 80 \rightarrow v_2 = 20\ V$$

$$3v_1 - 20 = 20 \rightarrow v_1 = 13.33\ V$$

Method 2 (Cramer's rule)

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

$$\Delta_1 = det \begin{bmatrix} 20 & -1 \\ 60 & 5 \end{bmatrix} = 100 + 60 = 160$$

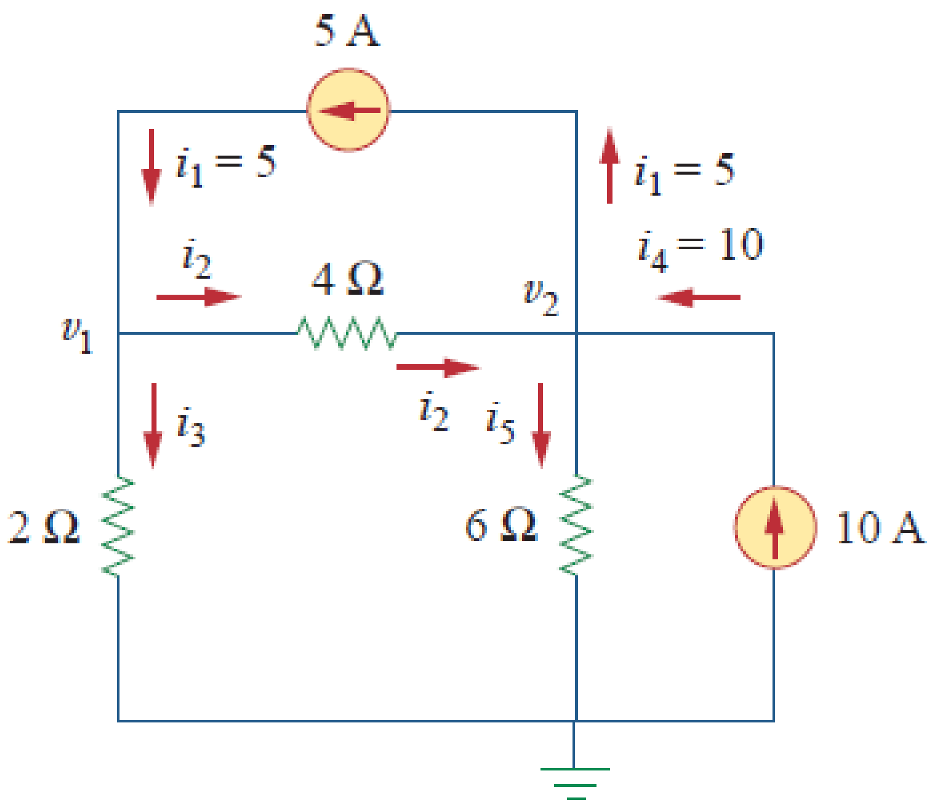
$$v_1 = \frac{\Delta_1}{\Delta} = \frac{160}{12} = 13.33\ V$$

$$\Delta = det \begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} = 15 - 3 = 12$$

$$\Delta_2 = det \begin{bmatrix} 3 & 20 \\ -3 & 60 \end{bmatrix} = 180 + 60 = 240$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{240}{12} = 20\ V$$

Nodal Analysis



$v_1 = 13.33\ V\ ,\qquad v_2 = 20\ V$

$i_1 = 5\ A$

$i_2 = \dfrac{v_1 - v_2}{4} = -1.67\ A$

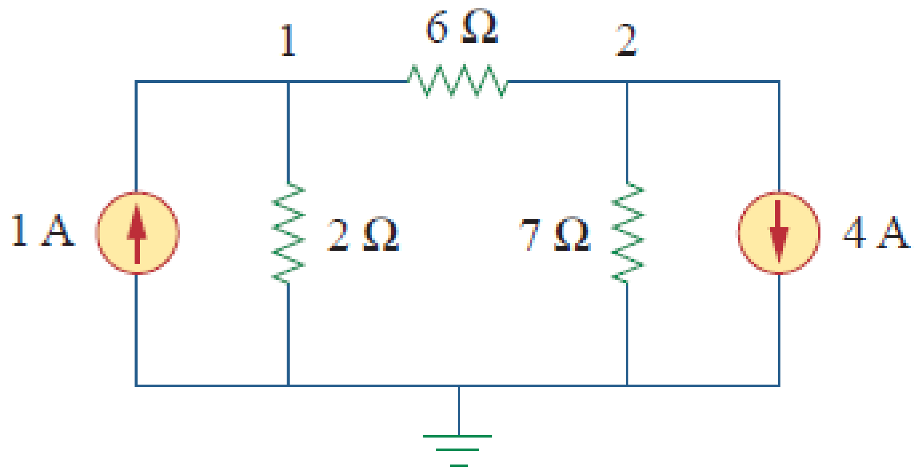
$i_3 = \dfrac{v_1}{2} = 6.67\ A$

$i_4 = 10\ A$

$i_5 = \dfrac{v_2}{6} = 3.33\ A$

Note $\quad i_2 < 0 \rightarrow opposite\ direction\ to\ the\ assumed$

MTD.02 (chk)
Find the nodal voltages and currents through the resistors in the circuit.

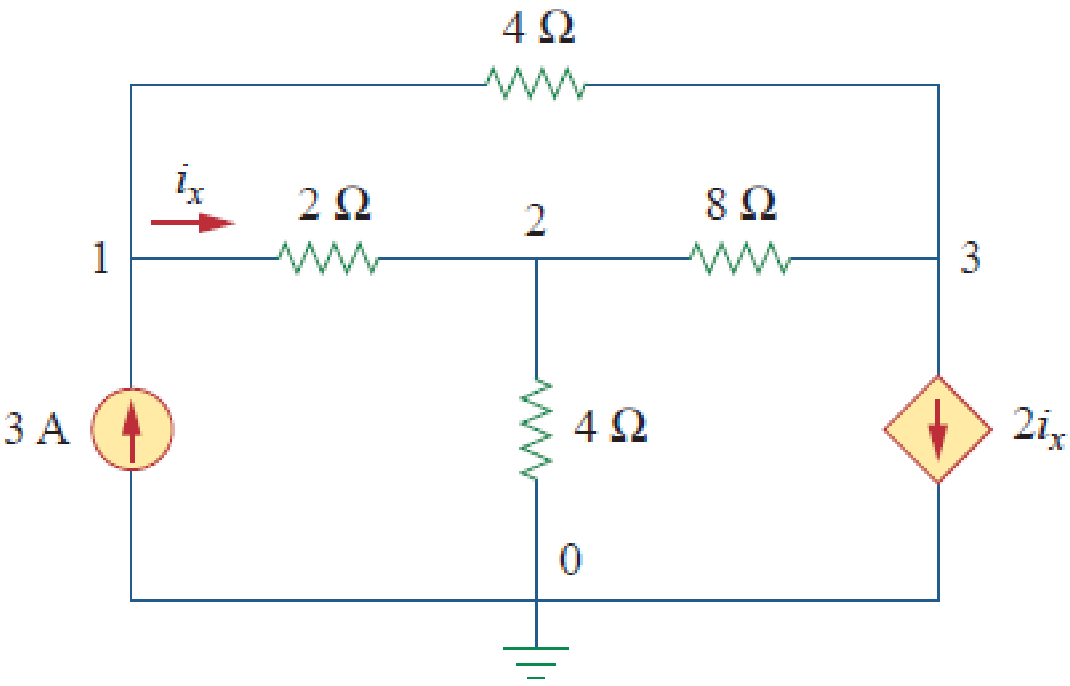


Solution

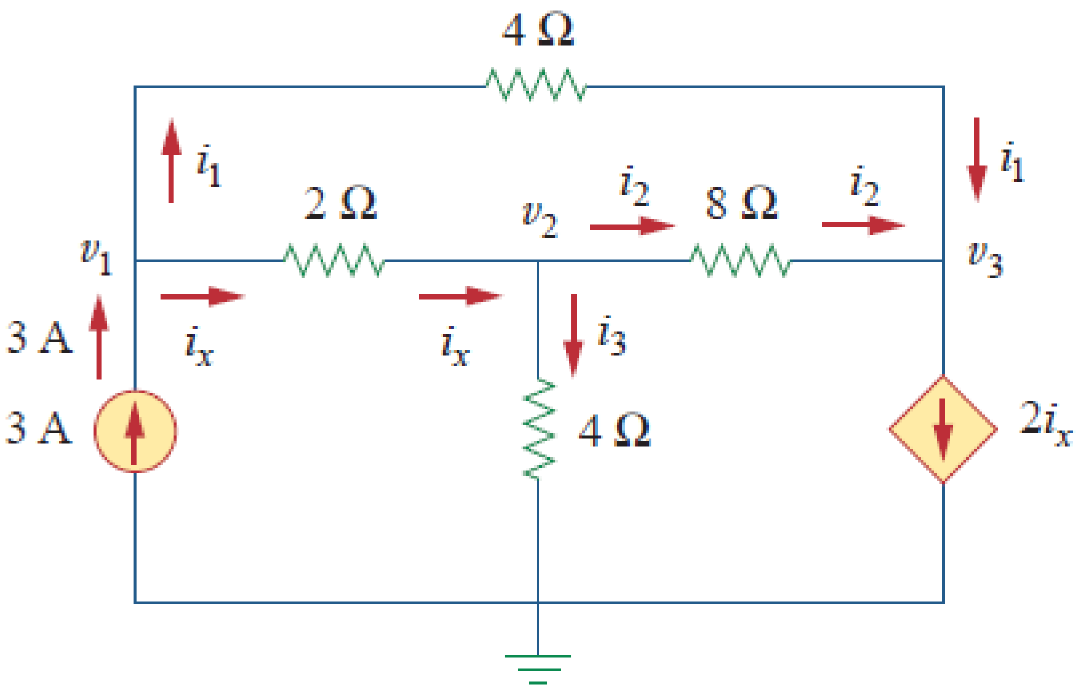
$v_1 = -2\ V\ , \qquad v_2 = -14\ V$

$i_2 = 1\ A\ ('up'), \qquad i_6 = 2\ A\ ('right'), \qquad i_7 = 2\ A\ ('up')$

MTD.03 – Determine the voltages at the nodes.



Solution



$$3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2} \rightarrow 3v_1 - 2v_2 - v_3 = 12$$

$$\frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4} \rightarrow -4v_1 + 7v_2 - v_3 = 0$$

$$\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2} \rightarrow 2v_1 - 3v_2 + v_3 = 0$$

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Nodal Analysis

$$3v_1 - 2v_2 - v_3 = 12$$
$$-4v_1 + 7v_2 - v_3 = 0$$
$$2v_1 - 3v_2 + v_3 = 0$$

Cramer's rule

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = det \begin{bmatrix} +3 & -2 & -1 \\ -4 & +7 & -1 \\ +2 & -3 & +1 \end{bmatrix} = \begin{bmatrix} +3 & -2 & -1 \\ -4 & +7 & -1 \\ +2 & -3 & +1 \\ +3 & -2 & -1 \\ -4 & +7 & -1 \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} 12 & -2 & -1 \\ 0 & +7 & -1 \\ 0 & -3 & +1 \\ 12 & -2 & -1 \\ 0 & +7 & -1 \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} +3 & 12 & -1 \\ -4 & 0 & -1 \\ +2 & 0 & +1 \\ +3 & 12 & -1 \\ -4 & 0 & -1 \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} +3 & -2 & 12 \\ -4 & +7 & 0 \\ +2 & -3 & 0 \\ +3 & -2 & 12 \\ -4 & +7 & 0 \end{bmatrix}$$

$$\Delta = 21 - 12 + 4 + 14 - 9 - 8 = 10$$

$$\Delta_1 = 84 - 0 + 0 - 36 + 0 - 0 = 48$$

$$\Delta_2 = 0 - 0 + 0 - 0 - 24 + 48 = 24$$

$$\Delta_3 = 0 - 168 + 144 - 0 - 0 - 0 = -24$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{48}{10} = 4.8\ V \qquad v_2 = \frac{\Delta_2}{\Delta} = \frac{24}{10} = 2.4\ V \qquad v_3 = \frac{\Delta_3}{\Delta} = \frac{-24}{10} = -2.4\ V$$

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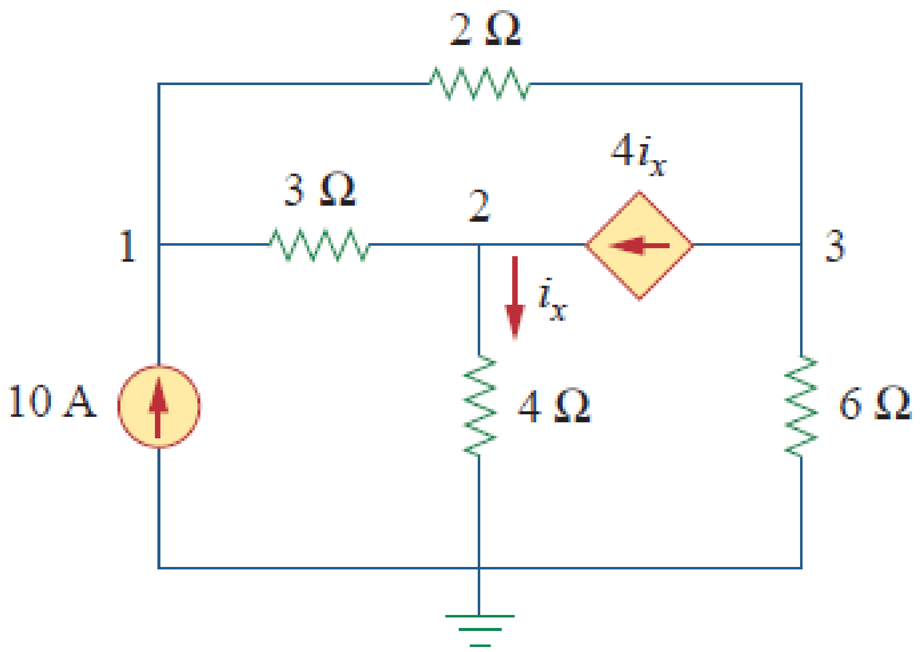
MTD.04 (chk) – Find node voltages and branch curents. **Solution**

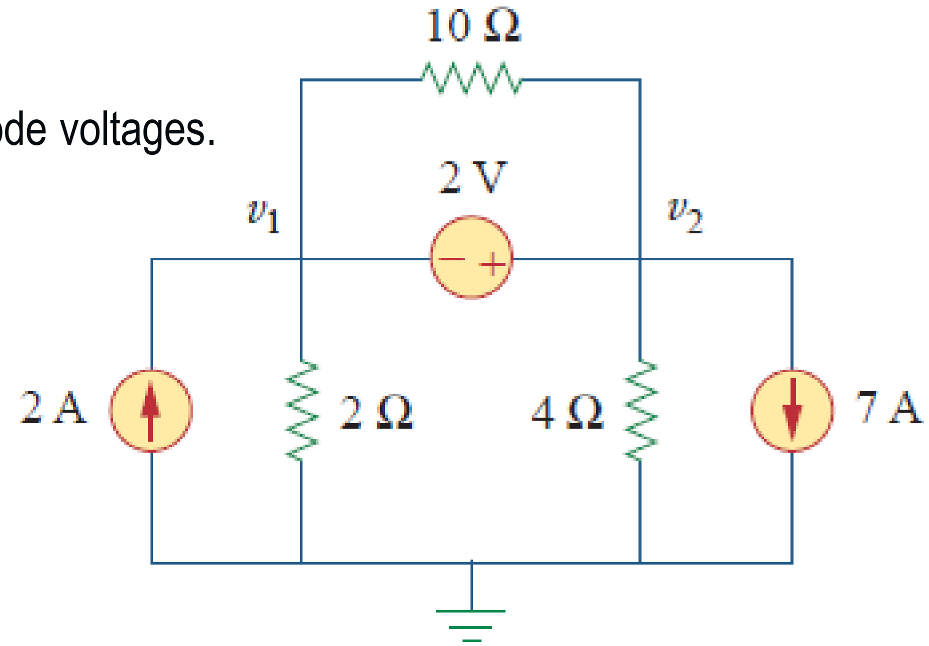


$v_1 = 80\,V\,, \qquad v_2 = -64\,V\,, \qquad v_3 = 156\,V$

$i_x = i_4 = -16\,A\,(up), \qquad i_6 = 26\,A\,(down)$

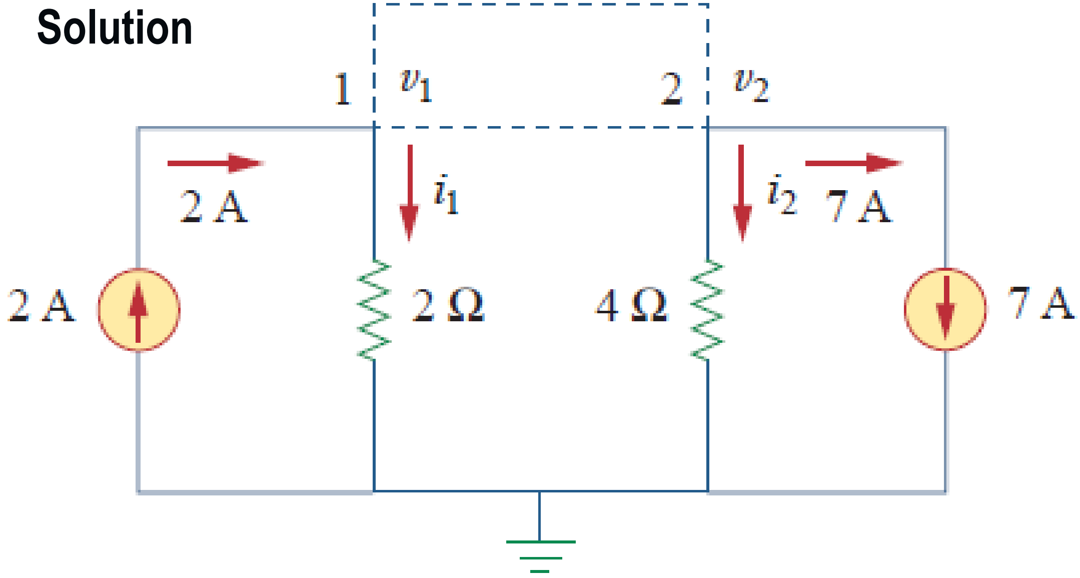
$i_3 = 48A(right)\,, \qquad i_2 = 38A\,(left)$

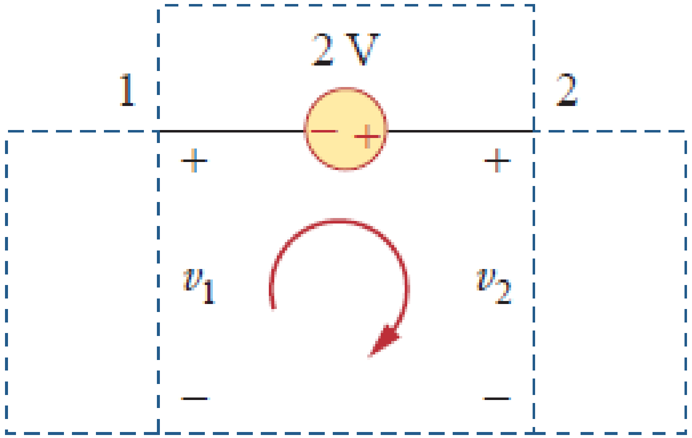
MTD.05
Find the node voltages.



Nodal Analysis with Super Node

Solution





$2 = i_1 + i_2 + 7$

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \rightarrow 8 = 2v_1 + v_2 + 28$$

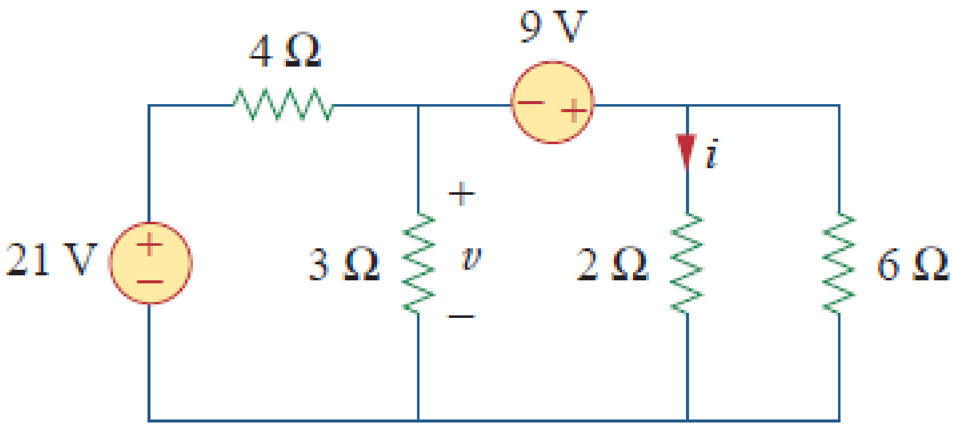
$\mathbf{(1)}$: $v_2 = -20 - 2v_1$

$-v_1 - 2 + v_2 = 0 \rightarrow \mathbf{(2)}$: $v_2 = v_1 + 2$

$\mathbf{(1)(2)}$: $-20 - 2v_1 = v_1 + 2$

$3v_1 = -22 \rightarrow v_1 = -7.33\ V$, $\quad v_2 = v_1 + 2 = -5.33\ V$

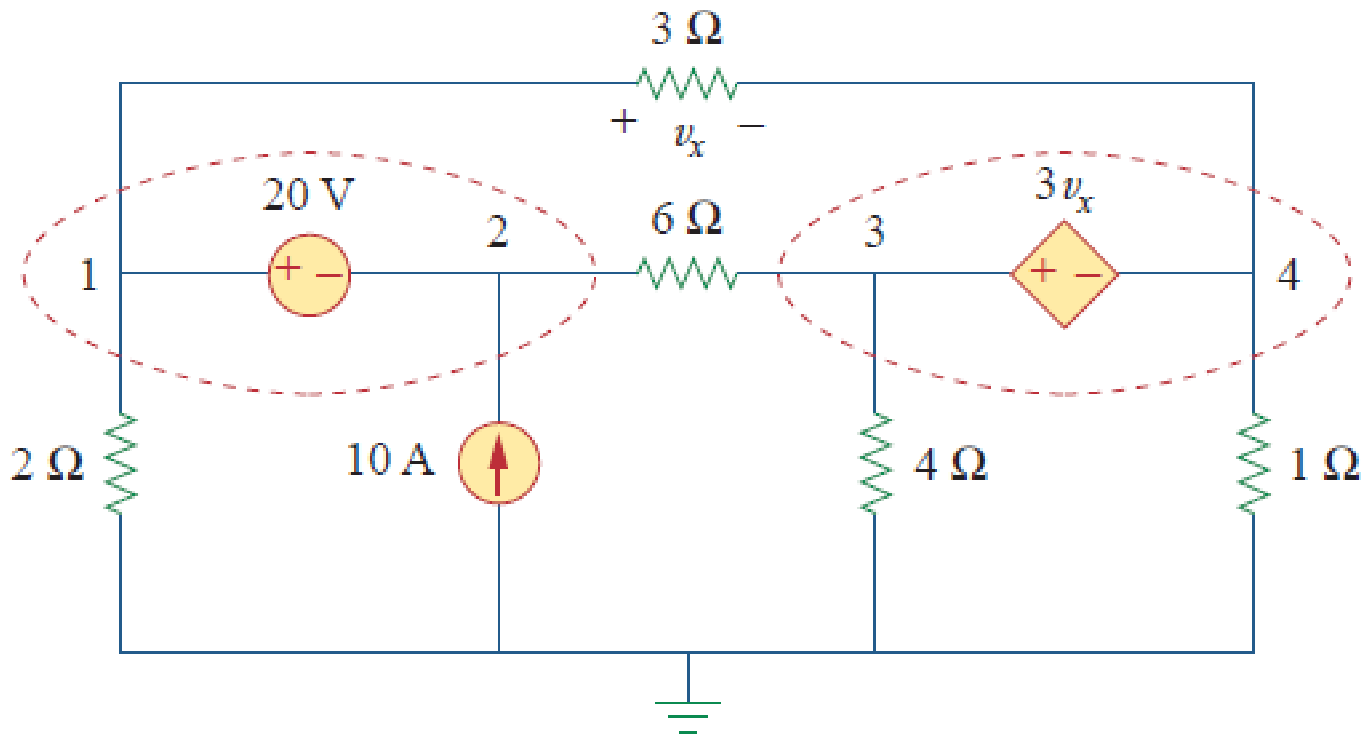
MTD.06 (chk)
Obtain *v* and *i* in the circuit.



Solution

$v = -0.6\,V\ , \qquad i = 4.2\ A$

MTD.07 – Find the node voltages in the circuit.



Nodal Analysis with Super Node

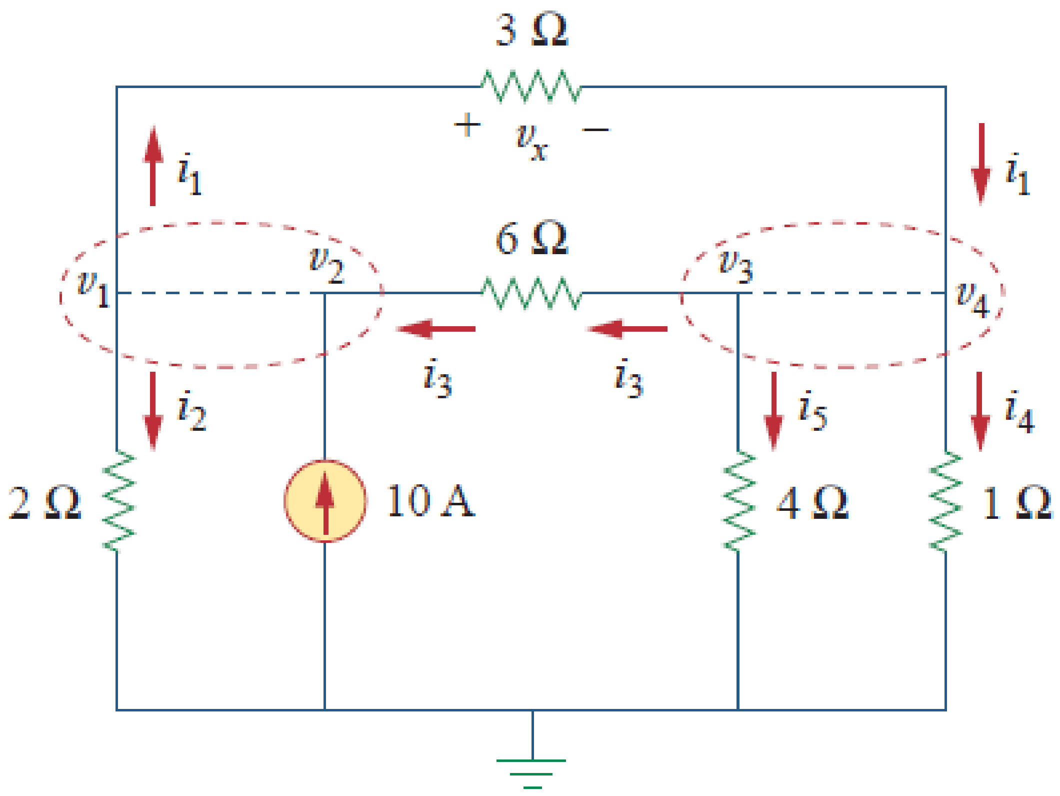
Solution

$$i_3 + 10 = i_1 + i_2 \rightarrow \frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$$

$$(A):\ 5v_1 + v_2 - v_3 - 2v_4 = 60$$

$$i_1 = i_3 + i_4 + i_5 \rightarrow \frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_4}{1} + \frac{v_3}{4}$$

$$(B):\ 4v_1 + 2v_2 - 5v_3 - 16v_4 = 0$$



Nodal Analysis with Super Node

$(\boldsymbol{Loop1})$: $-v_1 + 20 + v_2 = 0 \rightarrow (C)$: $v_1 - v_2 = 20$

$(\boldsymbol{Loop2})$: (1) $-v_3 + 3v_x + v_4 = 0$

(2) $v_x = v_1 - v_4$

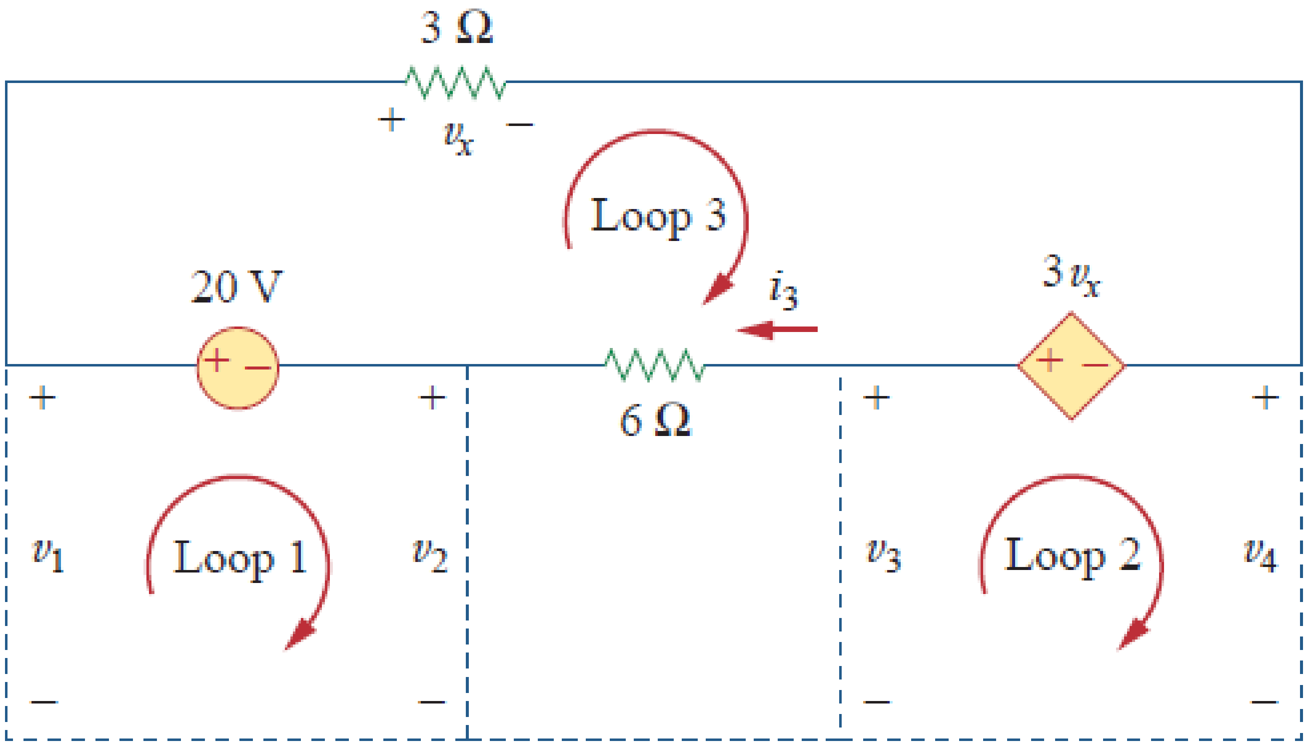
(1)(2) $\rightarrow (D)$: $3v_1 - v_3 - 2v_4 = 0$

$(\boldsymbol{Loop3})$: $v_x - 3v_x + 6i_3 - 20 = 0$

(3) $6\, i_3 = v_3 - v_2$

(2) $v_x = v_1 - v_4$

(3)(2) $\rightarrow (E)$: $-2v_1 - v_2 + v_3 + 2v_4 = 20$



Nodal Analysis with Super Node

$variables: v_1, v_2, v_3, v_4,$ $\quad equations: (A), (B), (C), (D), (E) \leftarrow 1\ redundant$

$(D)\ and\ v_2 = v_1 - 20 \rightarrow (A), (B)$

(D): $3v_1 - v_3 - 2v_4 = 0$

$6v_1 - v_3 + v_3 - 2v_4 = 80$

$6v_1 - 5v_3 + v_3 - 16v_4 = 40$

$$\begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 40 \end{bmatrix}$$

$$\Delta = det \begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} = -18$$

$$\Delta_1 = det \begin{bmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \end{bmatrix} = -480$$

$$\Delta_3 = det \begin{bmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \end{bmatrix} = -3120$$

$$\Delta_4 = det \begin{bmatrix} 3 & -1 & 0 \\ 6 & -1 & 80 \\ 6 & -5 & 40 \end{bmatrix} = 840$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{-480}{-18} = 26.67\ V$$

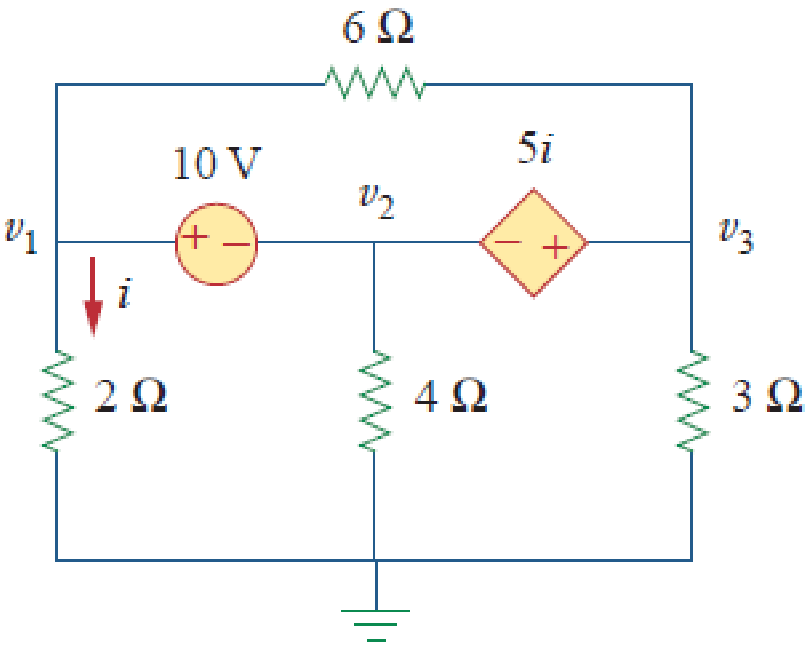
$$v_3 = \frac{\Delta_3}{\Delta} = \frac{-3120}{-18} = 173.33\ V$$

$$v_4 = \frac{\Delta_4}{\Delta} = \frac{840}{-18} = -46.67\ V$$

$$v_2 = v_1 - 20 = 6.67\ V$$

$(E)\ can\ be\ used\ for\ cross\ check\ the\ result.$

MTD.08 (chk) – Find the nodal voltages.



Solution

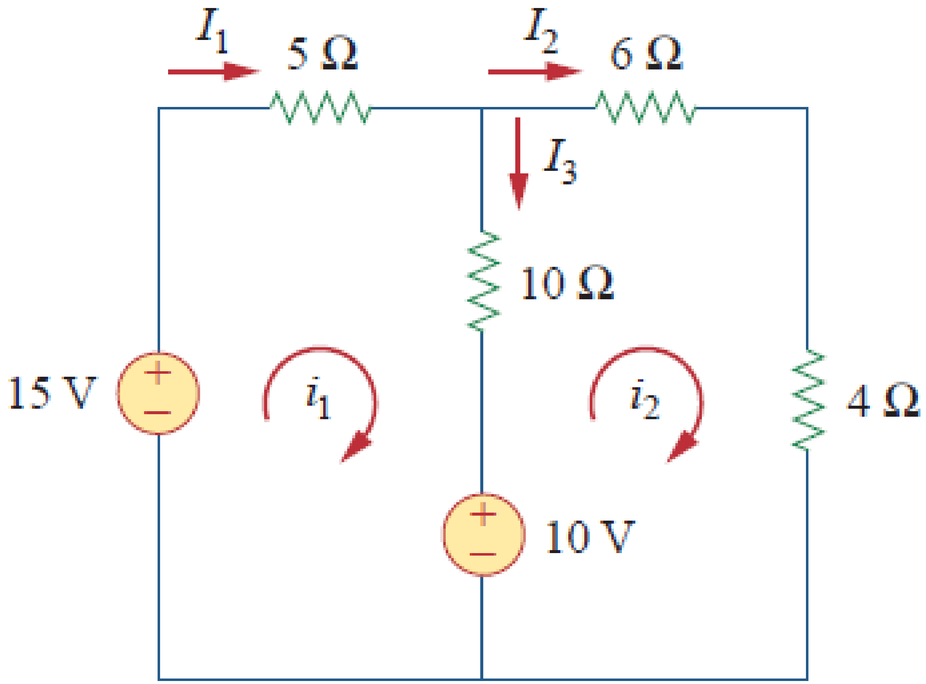
$v_1 = 3.04\ V$
$v_2 = -6.96\ V$
$v_3 = 0.65\ V$

MTD.09 – Find the branch currents (I_1, I_2, I_3) using mesh analysis.

Solution

$$-15+5i_1+10(i_1-i_2)+10=0 \rightarrow 3i_1-2i_2=1$$

$$6i_2+4i_2+10(i_2-i_1)-10=0 \rightarrow -i_1+2i_2=1$$

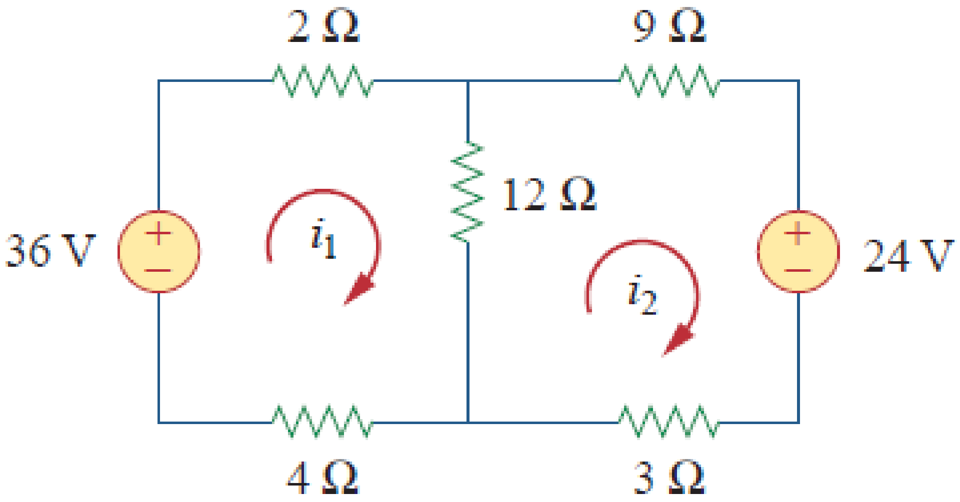


$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \Delta = det\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} = 4$$

$$\Delta_1 = det\begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix} = 4 \qquad \Delta_2 = det\begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix} = 4$$

$$i_1 = \frac{\Delta_1}{\Delta} = 1\,A \qquad i_2 = \frac{\Delta_2}{\Delta} = 1\,A \qquad I_1 = i_1 = 1\,A \qquad I_2 = i_2 = 1\,A \qquad I_3 = i_1 - i_2 = 0$$

MTD.10 – Calculate the mesh currents (i_1, i_2) of the circuit.



Solution $i_1 = 2\ A$, $i_2 = 0\ A$

MTD.11 – Calculate I_0 in the circuit. **Solution**

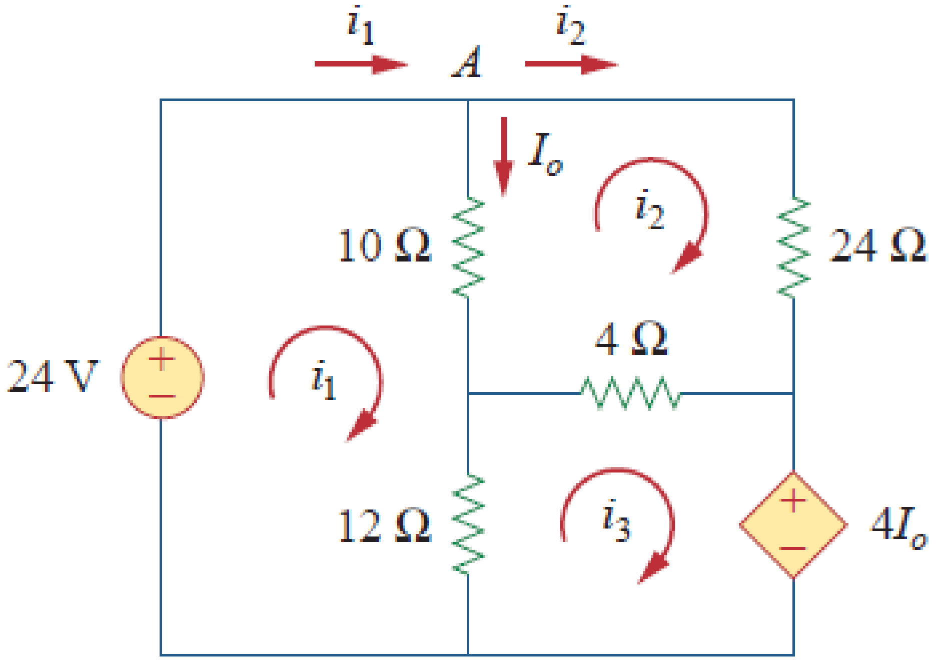
$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0 \rightarrow 11i_1 - 5i_2 - 6i_3 = 12$$

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0 \rightarrow -5i_1 + 19i_2 - 2i_3 = 0$$

$$4I_0 + 12(i_3 - i_1) + 4(i_3 - i_2) = 0\,, \qquad (\boldsymbol{node\ A}):\ I_0 = i_1 - i_2$$

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0 \rightarrow -i_1 - i_2 + 2i_3 = 0$$

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$



Mesh Analysis

$$\begin{bmatrix}11 & -5 & -6\\ -5 & 19 & -2\\ -1 & -1 & 2\end{bmatrix}\cdot\begin{bmatrix}i_1\\ i_2\\ i_3\end{bmatrix}=\begin{bmatrix}12\\ 0\\ 0\end{bmatrix}$$

$$\Delta = det\begin{bmatrix}11 & -5 & -6\\ -5 & 19 & -2\\ -1 & -1 & 2\end{bmatrix} = 192$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25\ A$$

$$\Delta_1 = det\begin{bmatrix}12 & -5 & -6\\ 0 & 19 & -2\\ 0 & -1 & 2\end{bmatrix} = 432$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75\ A$$

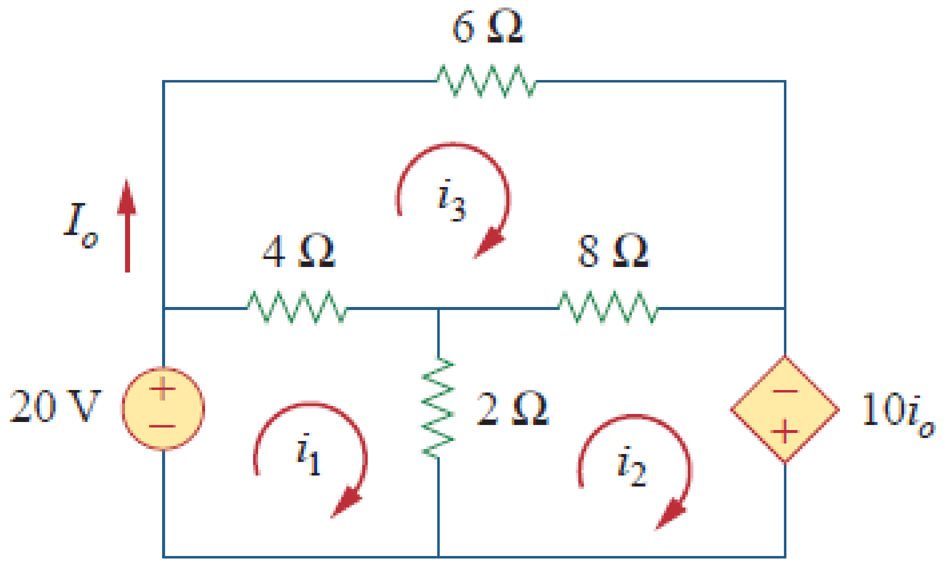
$$\Delta_2 = det\begin{bmatrix}11 & 12 & -6\\ -5 & 0 & -2\\ -1 & 0 & 2\end{bmatrix} = 144$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5\ A$$

$$\Delta_3 = det\begin{bmatrix}11 & -5 & 12\\ -5 & 19 & 0\\ -1 & -1 & 0\end{bmatrix} = 288$$

$$I_0 = i_1 - i_2 = 1.5\ A$$

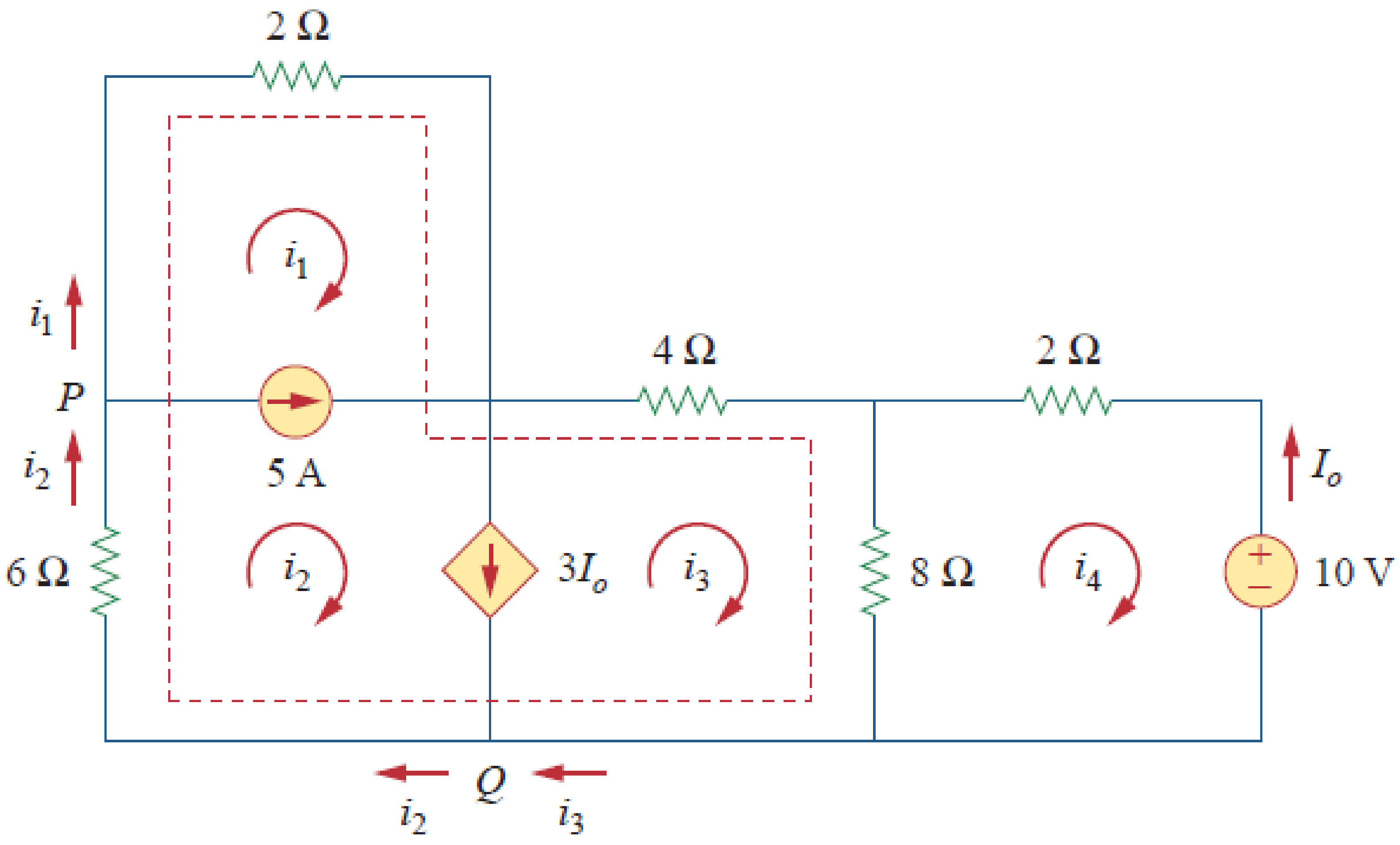
MTD.12 (chk) – Using mesh analysis find I_0 in the circuit.



Solution $I_0 = -5\ A$

Mesh Analysis with Super Mesh

MTD.13 – Find i_1, i_2, i_3, i_4 using mesh analysis.



Solution

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$\rightarrow (\mathbf{1}):\ i_1 + 3i_2 + 6i_3 - 4i_4 = 0$$

$$(\boldsymbol{node\ P}):\ i_2 = i_1 + 5$$

$$(\boldsymbol{node\ Q}):\ i_2 = i_3 + 3I_0$$

$$I_0 = -i_4 \rightarrow i_2 = i_3 - 3i_4$$

$$(\boldsymbol{mesh\ 4}): 2i_4 + 8(i_4 - i_3) + 10 = 0$$

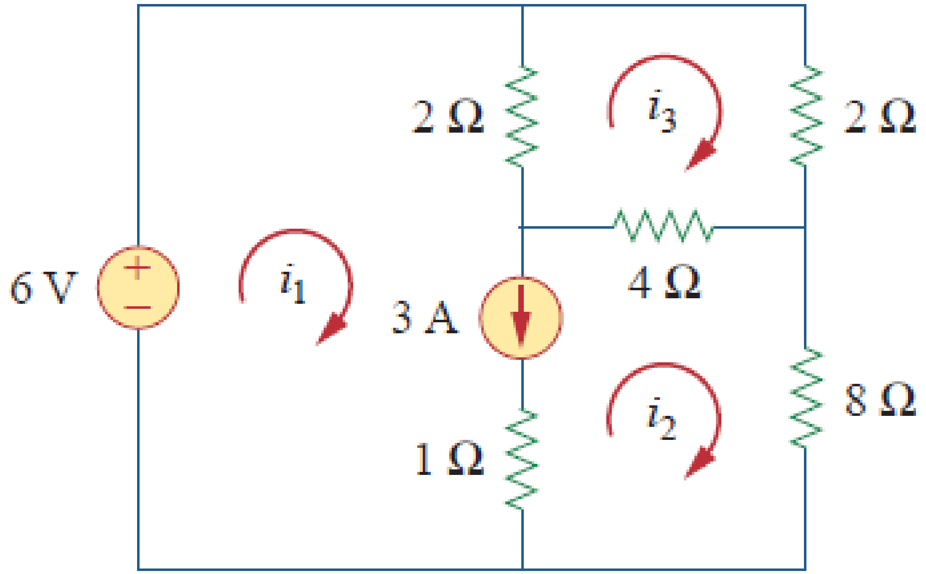
$$(\mathbf{2}):\ 5i_4 - 4i_3 = -5$$

(1)(2):

$i_1 = -7.5\ A$, $\quad i_2 = -2.5\ A$

$i_3 = 3.93\ A$, $\quad i_4 = 2.143\ A$

MTD.14 (chk) – Find i_1, i_2, i_3 using mesh analysis.



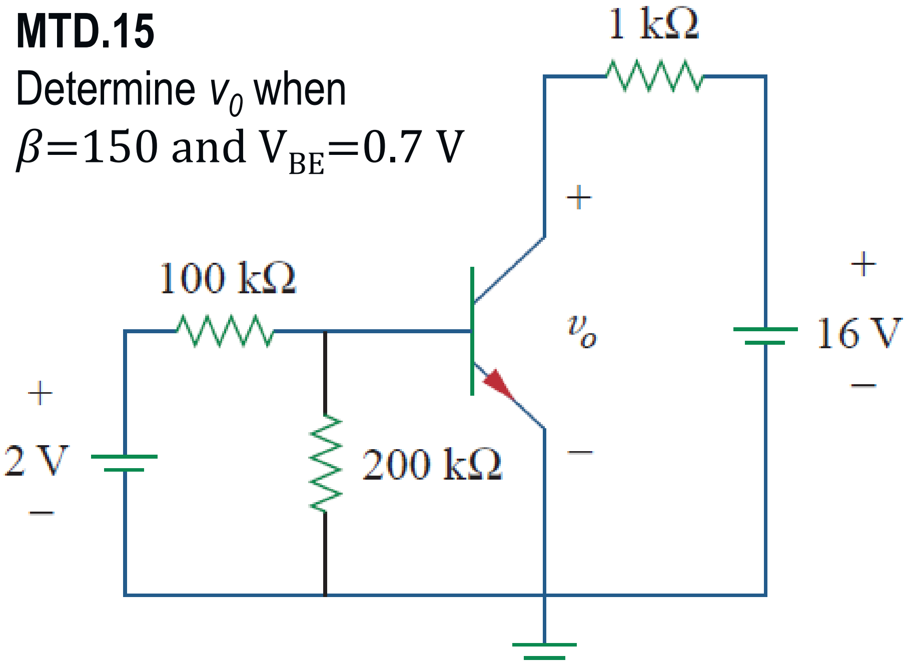
Solution

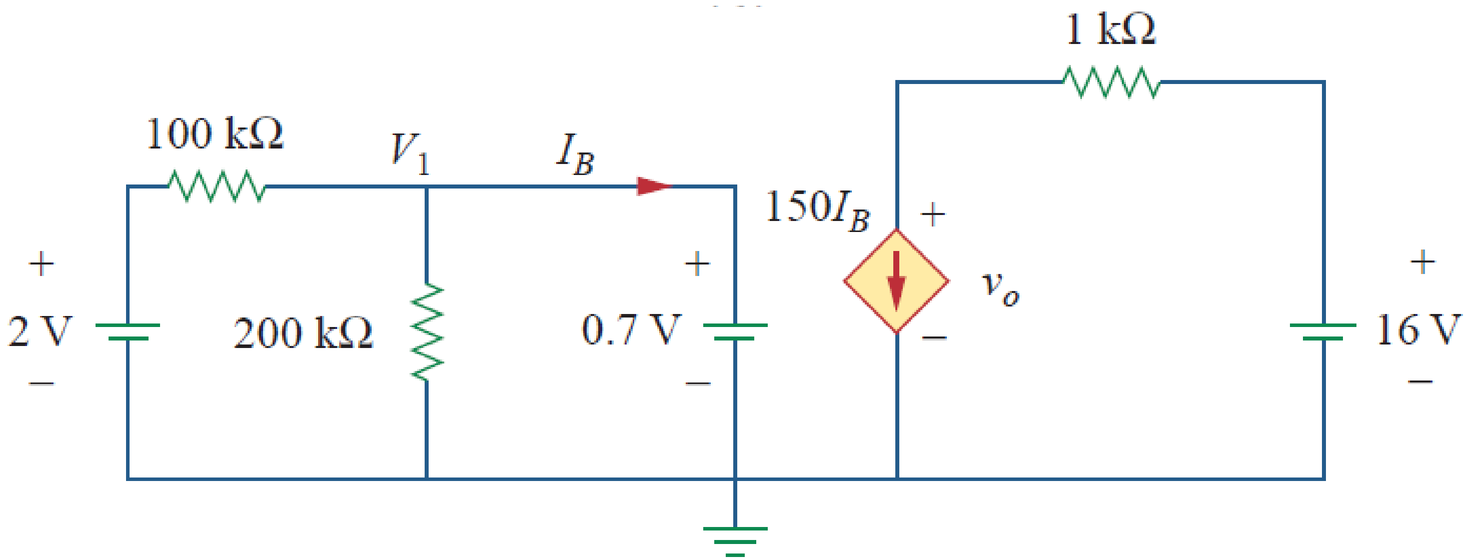
$i_1 = 3.474\ A\,,\qquad i_2 = 0.4737\ A\,,\qquad i_3 = 1.1052\ A$

MTD.15

Determine v_0 when

β=150 and V_{BE}=0.7 V



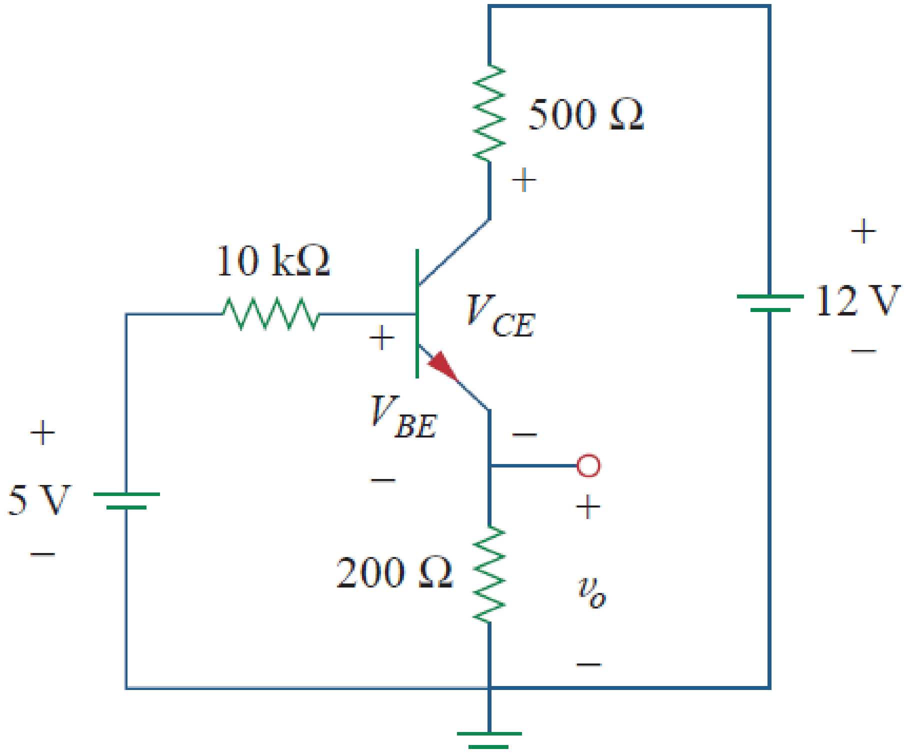


Solution

$$(\boldsymbol{node\ 1}):\ V_1 = 0.7\ V\ ,\qquad \frac{(0.7-2)}{100k} + \frac{0.7}{200k} + i_B = 0 \rightarrow i_B = 9.5\ \mu A$$

$$(\boldsymbol{node\ 2}):\ 150 I_B + \frac{(v_0 - 16)}{1k} = 0 \rightarrow v_0 = 16 - 150 \cdot 10^3 \cdot 9.5 \cdot 10^{-6} = 14.575\ V$$

MTD.16 (chk) – Determine v_0 and V_{CE} when $\beta=100$ and $V_{BE}=0.7$ V



Solution v_0 = 2.876 V, V_{CE} = 1.984 V

Questions

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