



DR. GYURCSEK ISTVÁN

Theorems in Circuit Analysis - Examples

Sources and additional materials (recommended)

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Simonyi K.: Villamosságatan. AK Budapest 1983, ISBN:9630534134*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságatan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságatan 2. TK Budapest 2002, ISBN:9631026043*

THO.01 – Find I_0 when $v_S = 12\text{ V}$ and $v_S = 24\text{ V}$.

Solution

Loop1: $12i_1 - 4i_2 + v_S = 0$

Loop2: $-4i_1 + 16i_2 - 3v_x - v_S = 0$

But: $v_x = 2i_1$

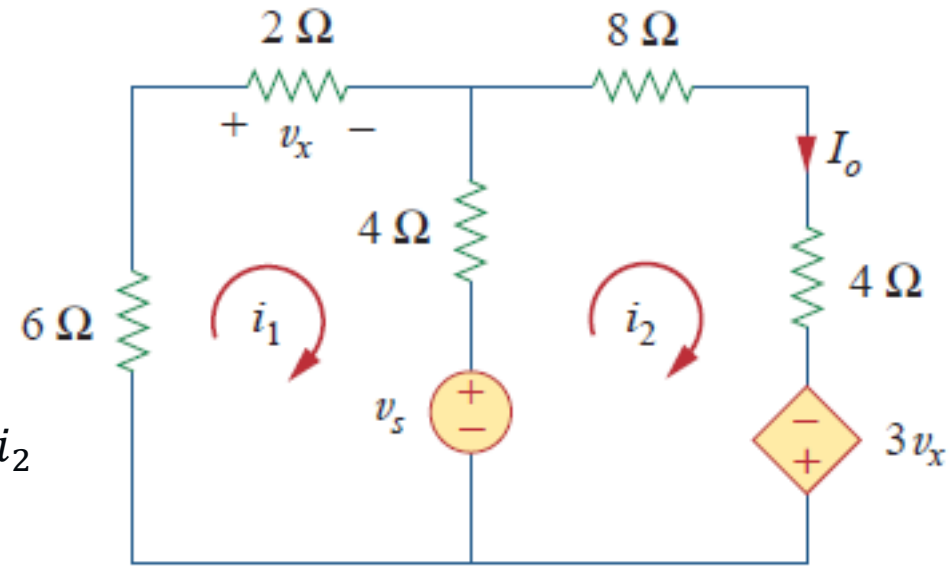
(A): $-10i_1 + 16i_2 - v_S = 0$

(Loop1 + A): $2i_1 + 12i_2 = 0 \rightarrow i_1 = -6i_2$

(\rightarrow Loop1): $-76i_2 + v_S = 0 \rightarrow i_2 = \frac{v_S}{76}$

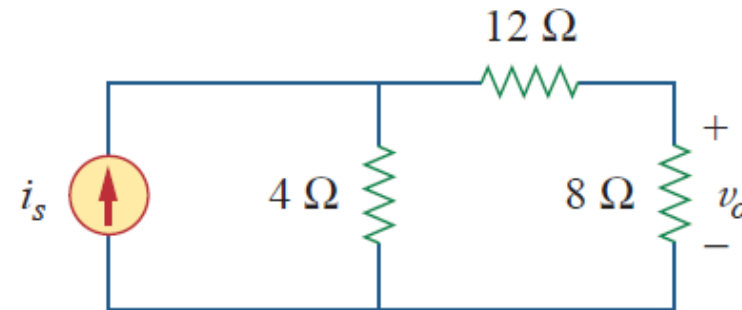
$v_S = 12\text{ V} \rightarrow I_0 = \frac{12}{76} = 157.89\text{ mA}$

$v_S = 24\text{ V} \rightarrow I_0 = \frac{24}{76} = 315.78\text{ mA}$



Two time higher $v_S \rightarrow$ two times higher I_0

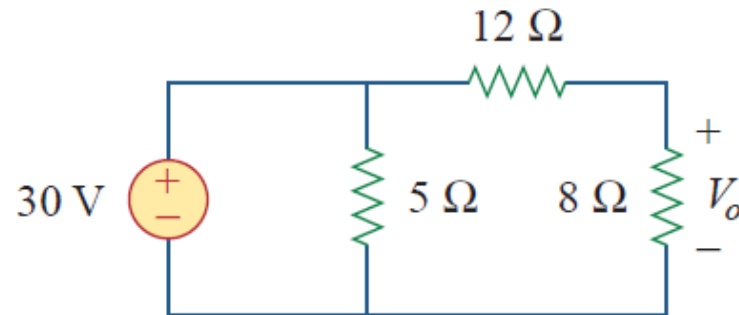
THO.02 – Find v_0 when i_s is 15 A and i_s is 30 A.



Solution
$$i_{01} = i_{s1} \frac{4}{4 + 20} = 2.5 \text{ A} \rightarrow v_{01} = 8i_{01} = 20 \text{ V}$$

$$i_{s2} = 2 \cdot i_{s1} \rightarrow v_{02} = 2 \cdot v_{01} = 40 \text{ V}$$

THO.03 – Assume $v_0 = 1$ V and use linearity to calculate actual value of v_0 .

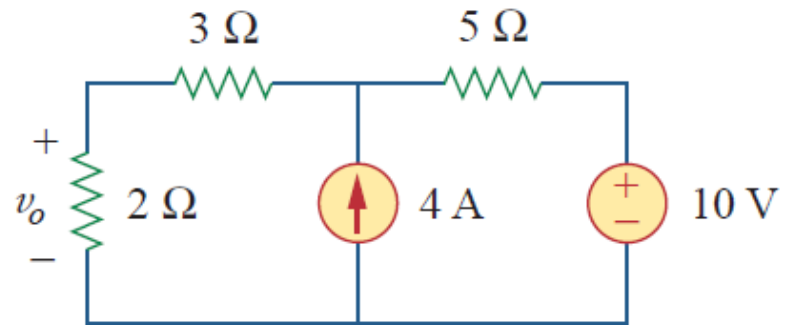


Solution $v_0 = 12$ V

Superposition Principle



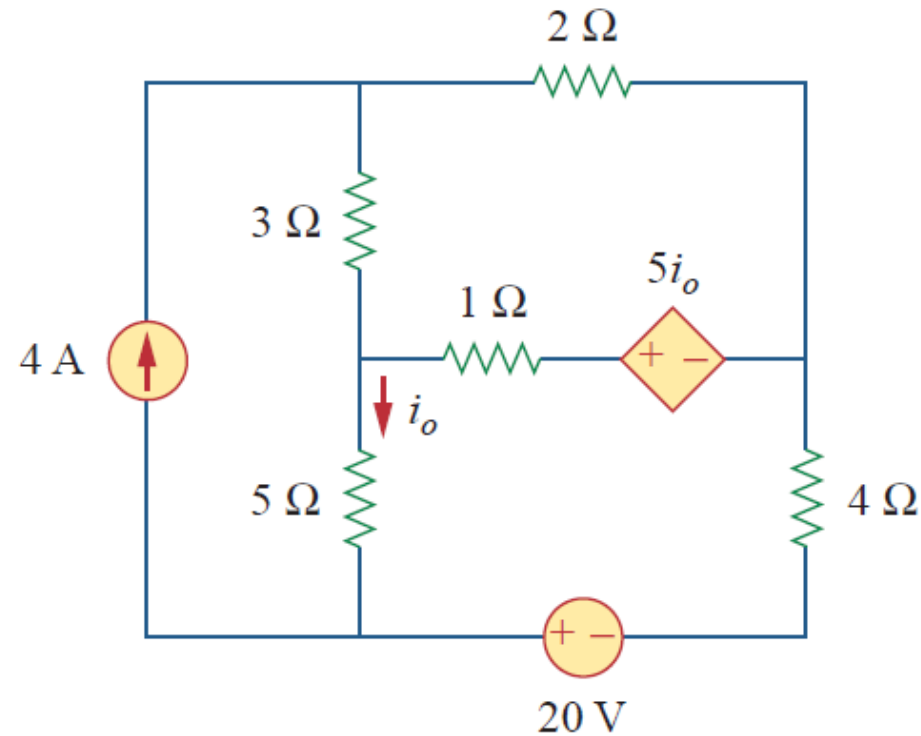
THO.04 – Using the superposition principle, find v_o in the circuit.



Solution $v_o = 6 V$

Superposition Principle

THO.05 – Using the superposition principle, find i_o in the circuit.

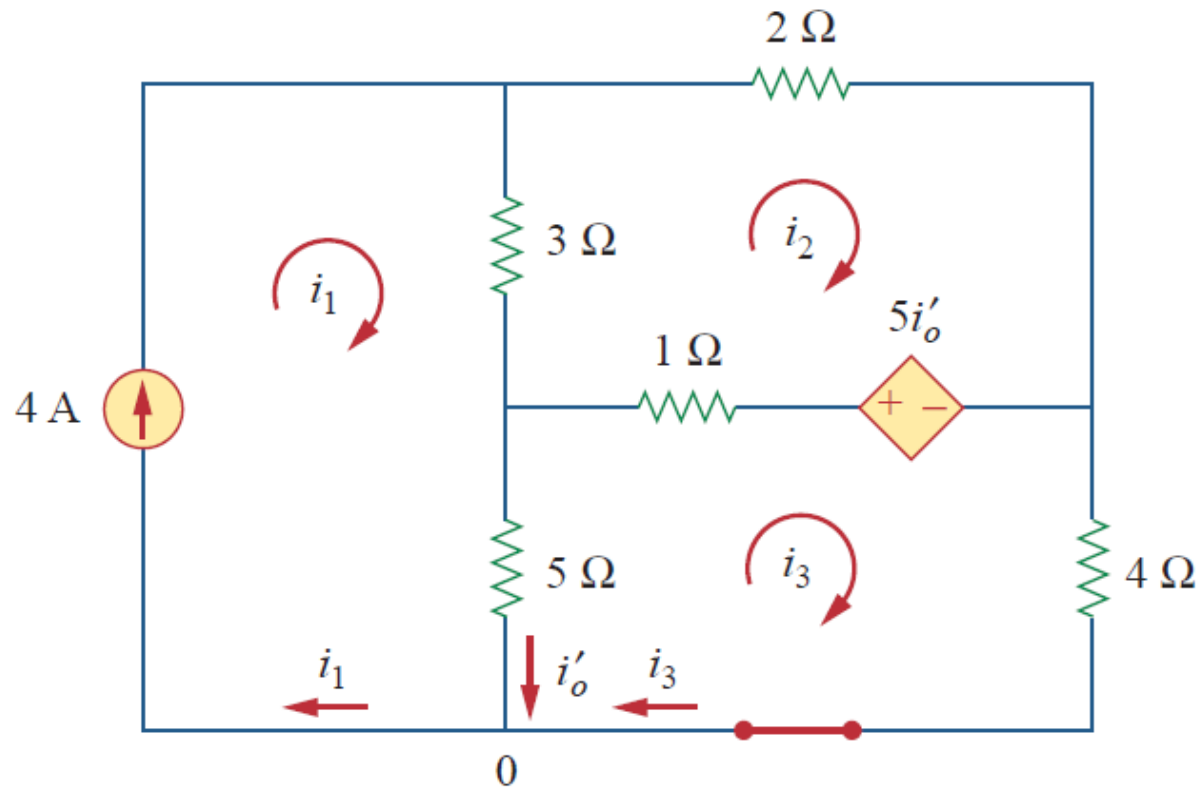


Solution

$$i_o = i'_o + i''_o$$

Superposition Principle

$$i'_0 = ?$$



$$(\text{loop 1}): i_1 = 4$$

$$(\text{loop 2}): -3i_1 + 6i_2 - 1i_3 - 5i'_0 = 0$$

$$(\text{loop 3}): -5i_1 - 1i_2 + 10i_3 + 5i'_0 = 0$$

$$(\text{node 0}): i_3 = i_1 - i'_0 = 4 - i'_0$$

$$(\text{loop 1})(\text{node 0}) \rightarrow (\text{loop 2}):$$

$$3i_2 - 2i'_0 = 8$$

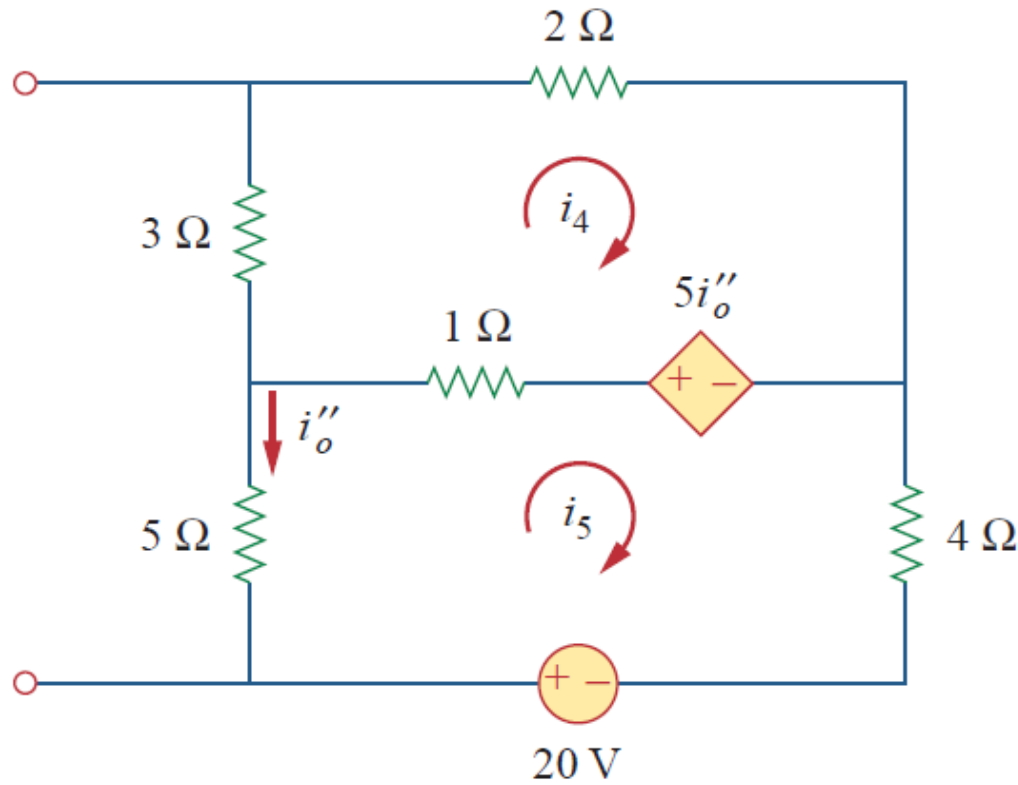
$$(\text{loop 1})(\text{node 0}) \rightarrow (\text{loop 3}):$$

$$i_2 + 5i'_0 = 20$$

$$i'_0 = \frac{52}{17} \text{ A}$$

Superposition Principle

$i''_0 = ?$



(loop 4): $6i_4 - i_5 - 5i''_0 = 0$

(loop 5): $-i_4 + 10i_5 - 20 + 5i''_0 = 0$

(but): $i_5 = -i''_0$

(but) → (loop 4): $6i_4 - 4i''_0 = 0$

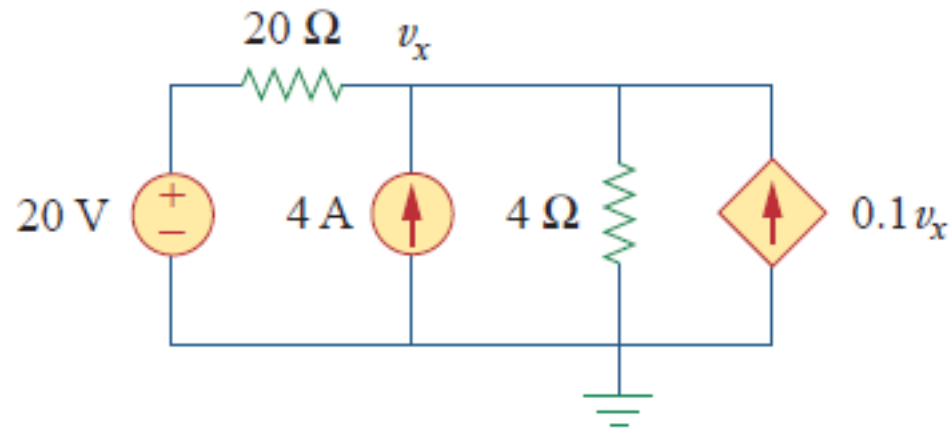
(but) → (loop 5): $i_4 + 5i''_0 = -20$

$$i''_0 = -\frac{60}{17} A$$

$$i_0 = i'_0 + i''_0 = \frac{52}{17} - \frac{60}{17} = -\frac{8}{17} = -0.47 A$$

Superposition Principle

THO.06 – Using the superposition principle, find v_x in the circuit.



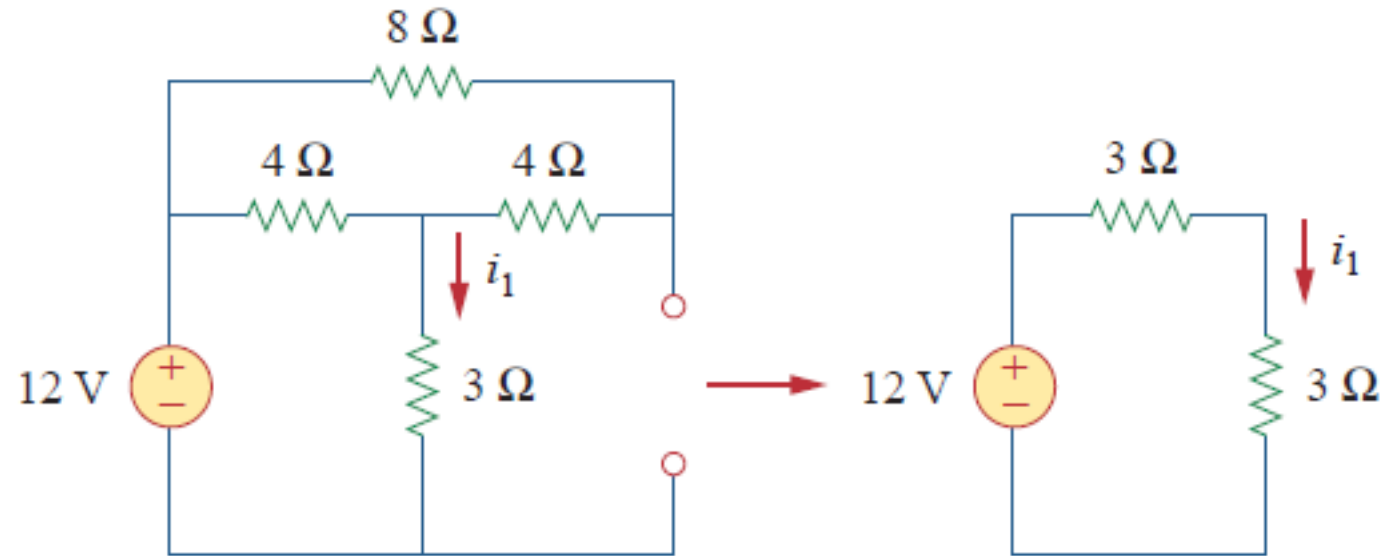
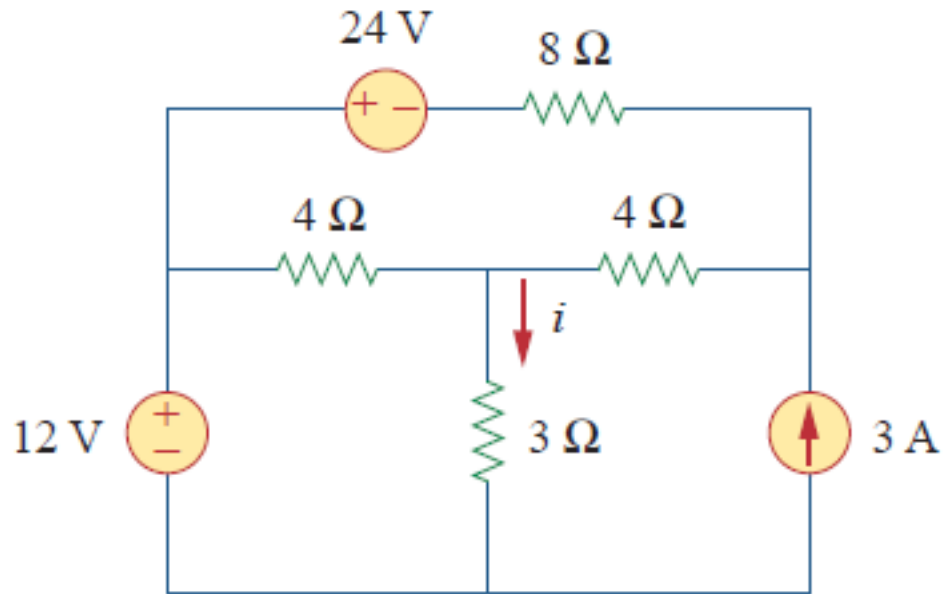
Solution $v_x = 25 V$

Superposition Principle

THO.07 – Using the superposition principle, find i in the circuit.

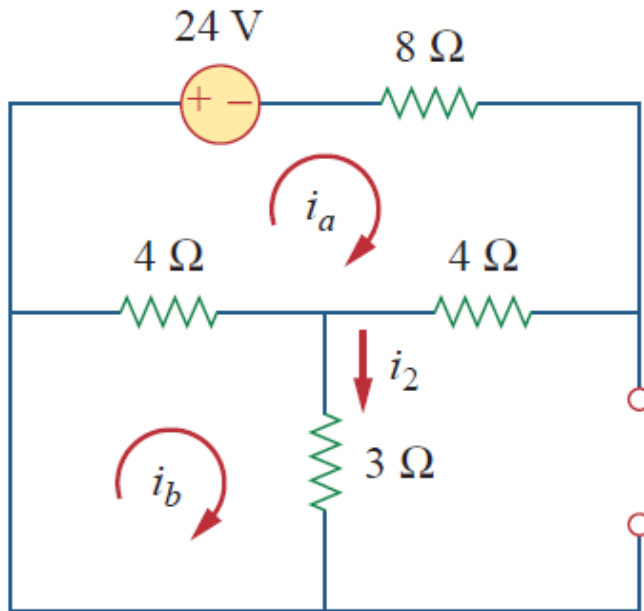
Solution

$$i = i_1 + i_2 + i_3, \quad i_1 = ?$$



$$i_1 = \frac{12}{6} = 2 \text{ A}$$

Superposition Principle



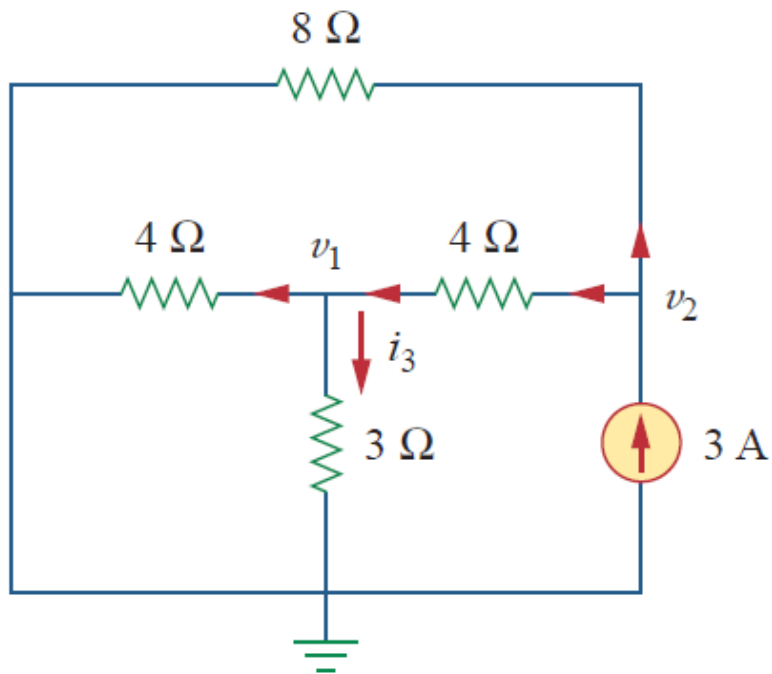
$i_2 = ?$

$$12i_a - 4i_b + 24 = 0 \rightarrow (1): 4i_a - i_b = -6$$

$$7i_b - 4i_a = 0 \rightarrow (2): i_a = \frac{7}{4}i_b$$

$$(2) \rightarrow (1): i_2 = i_b = -1$$

Superposition Principle



$$i_3 = ?$$

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \rightarrow (1): 24 = 3v_2 - 2v_1$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \rightarrow (2): v_2 = \frac{10}{3}v_1$$

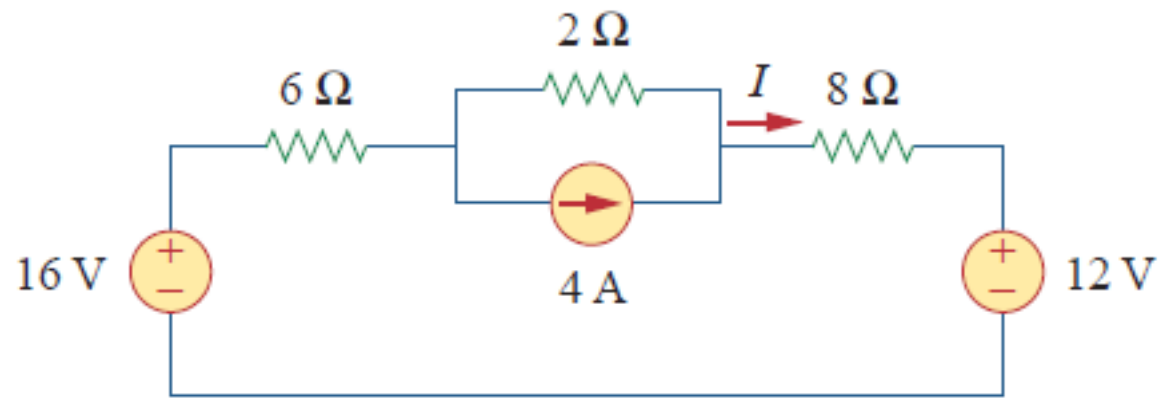
$$(2) \rightarrow (1): v_1 = 3 \rightarrow i_3 = \frac{v_3}{3} = 1 A$$

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2 A$$

Superposition Principle

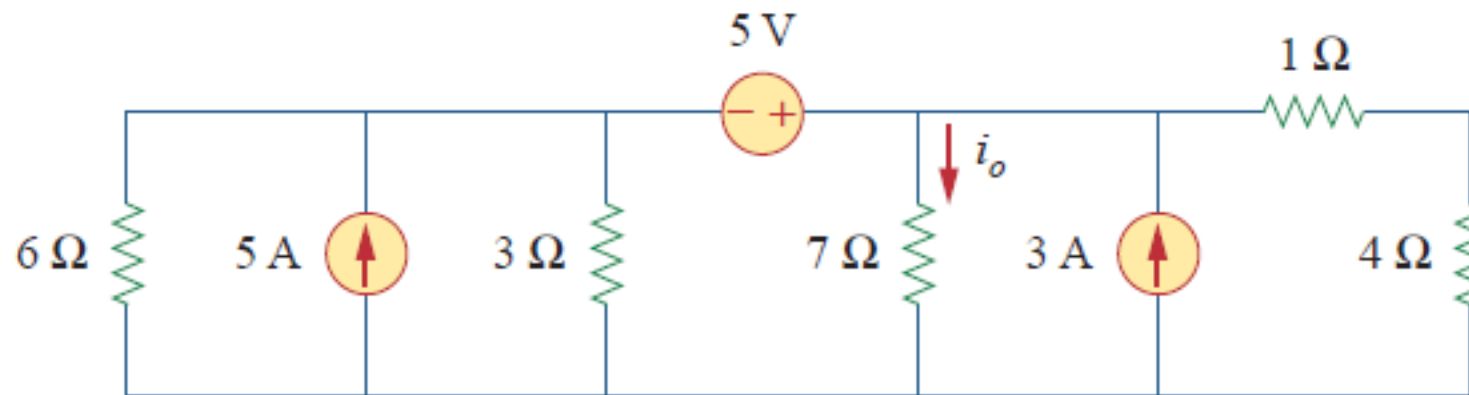


THO.08 – Using the superposition principle find I in the circuit.



Solution $I = 0.75 \text{ A}$

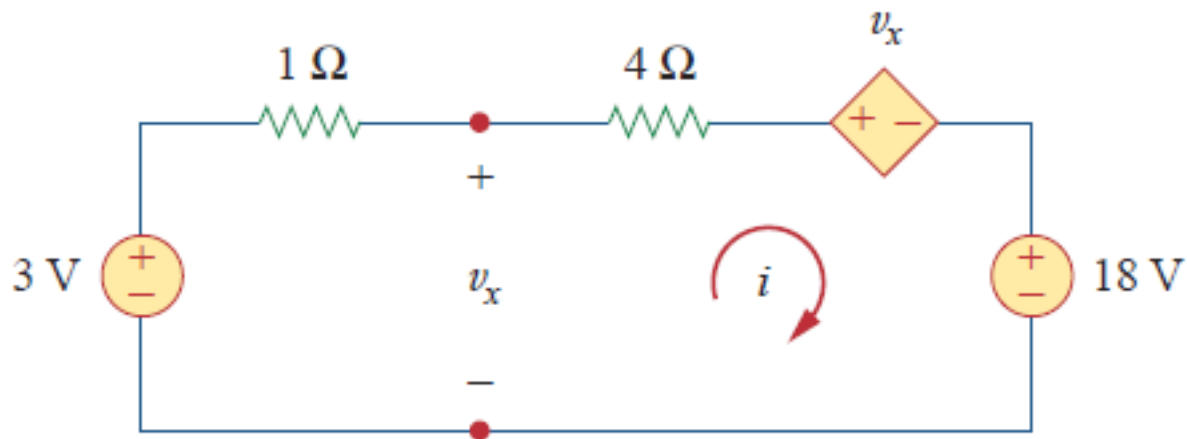
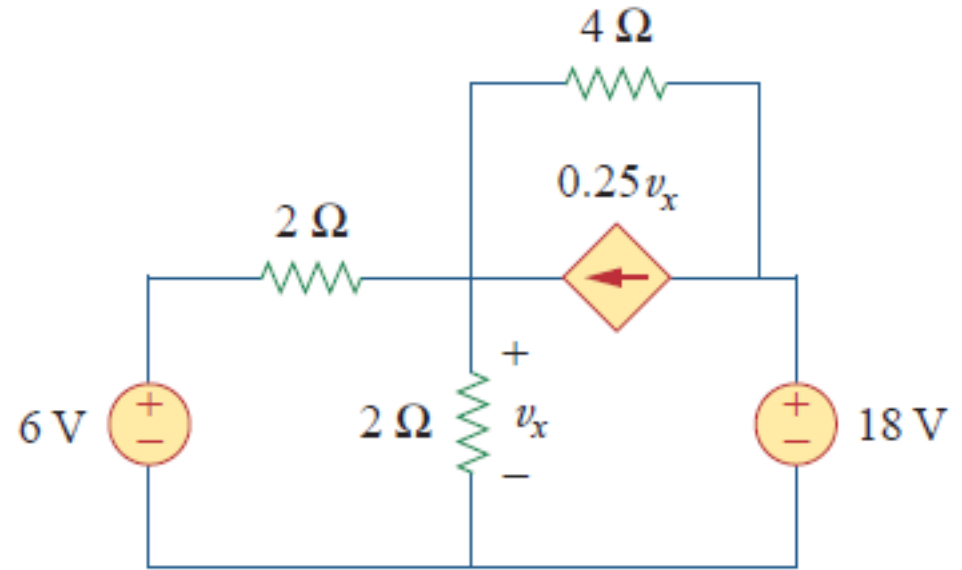
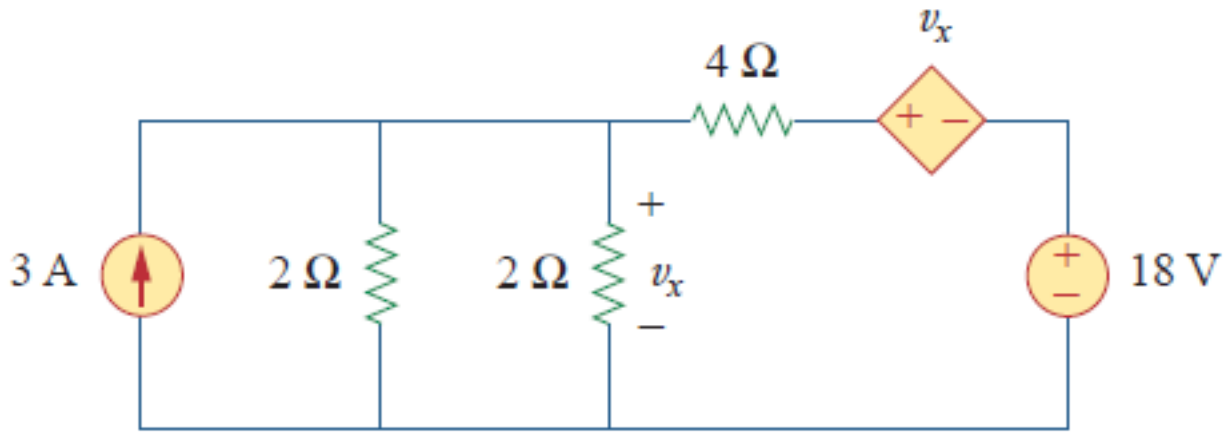
THO.09 – Using the source transformation find i_o in the circuit.



Solution $I = 1.78 \text{ A}$

Source Transformation

THO.10 – Using the source transformation find v_x in the circuit.



(Loop 1): $-3 + 5i + v_x + 18 = 0$

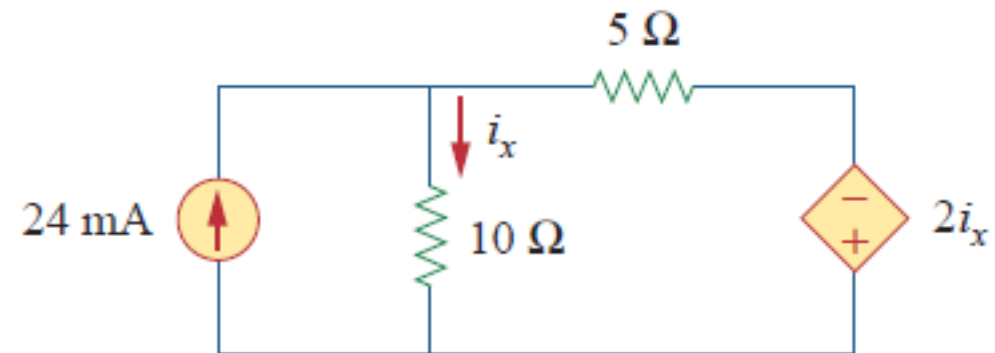
(Loop 2): $-3 + 1i + v_x = 0$

(Loop 2) → (Loop 1): $15 + 5i + 3 - i = 0 \rightarrow i = -4.5 A$

or ... (Loop 3): $-v_x + 4i + v_x + 18 = 0 \rightarrow i = -4.5 A$

$v_x = 3 - i = 7.5 V$

THO.11 – Using the source transformation find i_x in the circuit.



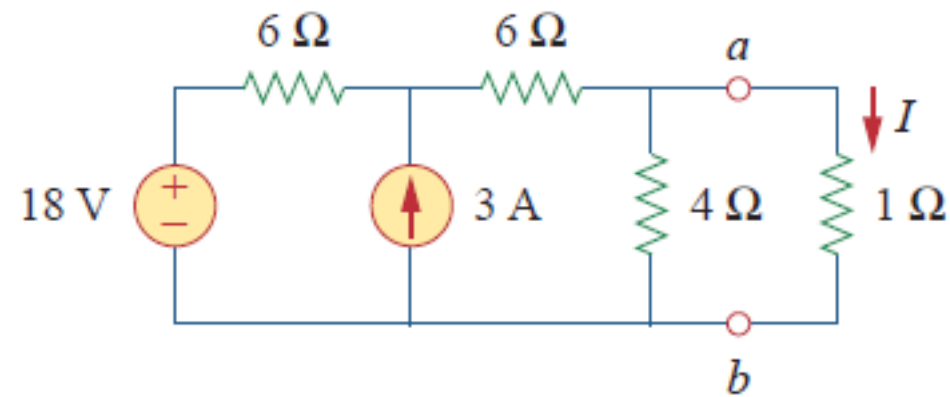
Solution

$$I = 7.056 \text{ mA}$$

Thevenin's Theorem



THO.12 – Find the Thevenin equivalent circuit to the left of the terminals and find I .

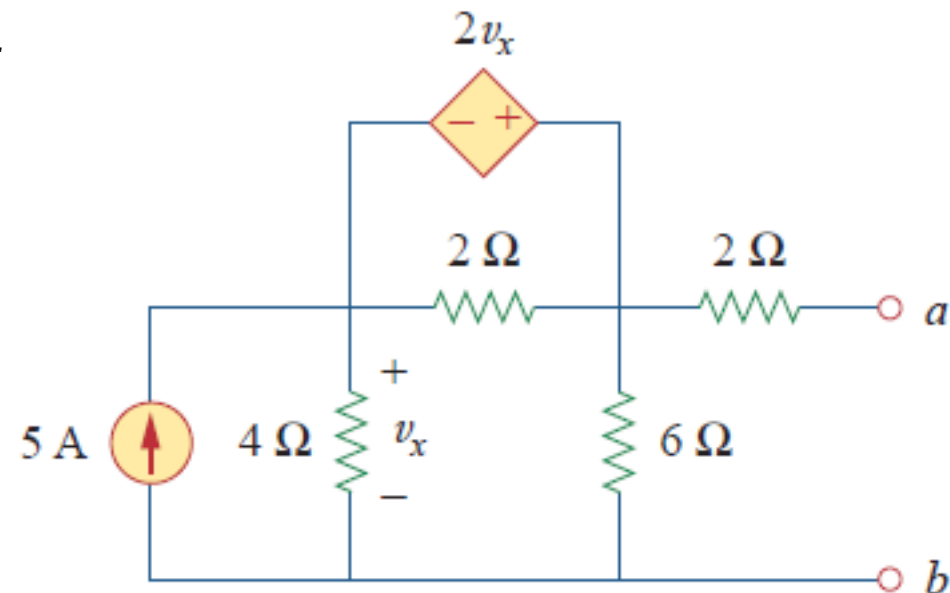
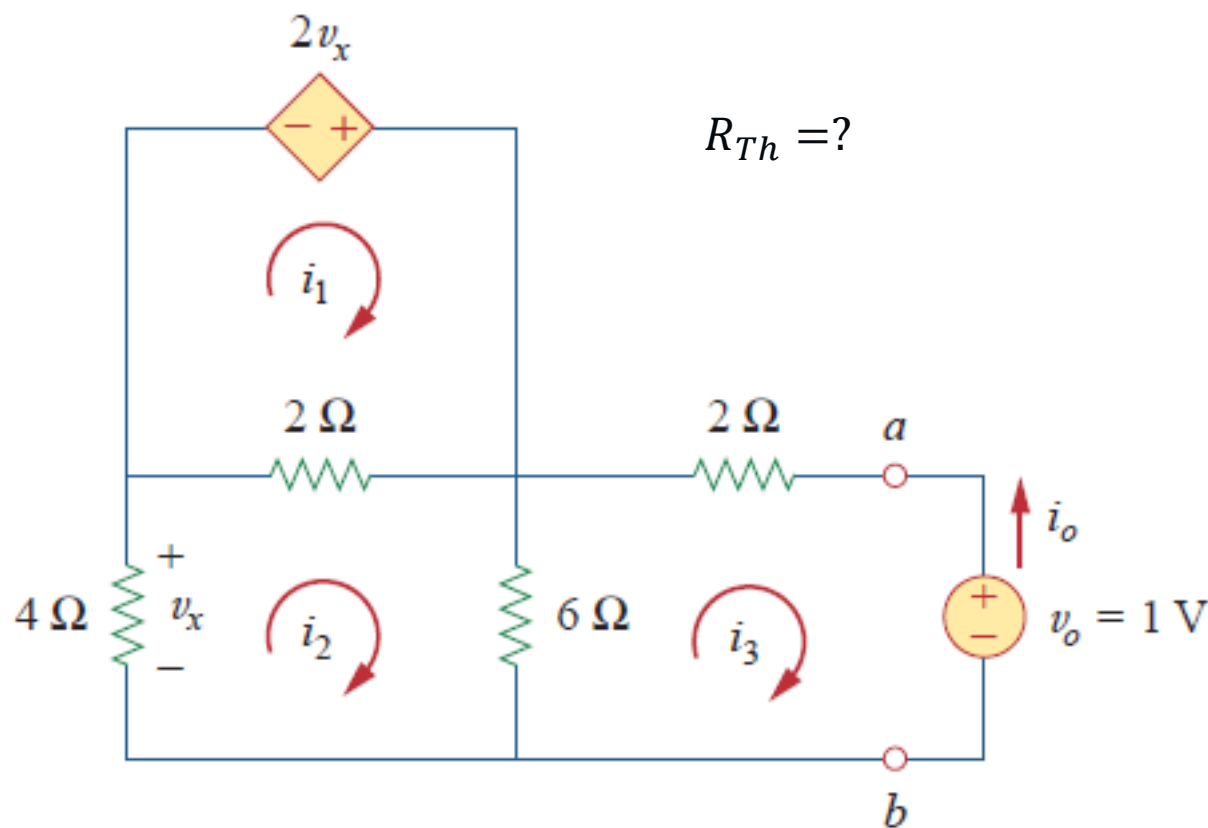


Solution $V_{Th} = 9 V$, $R_{Th} = 3 \Omega$, $I = 2.25 A$

Thevenin's Theorem

THO.13 – Find the Thevenin equivalent circuit at the terminals a - b .

Solution



(Loop 1): $-2v_x + 2(i_1 - i_2) = 0 \rightarrow v_x = i_1 - i_2$

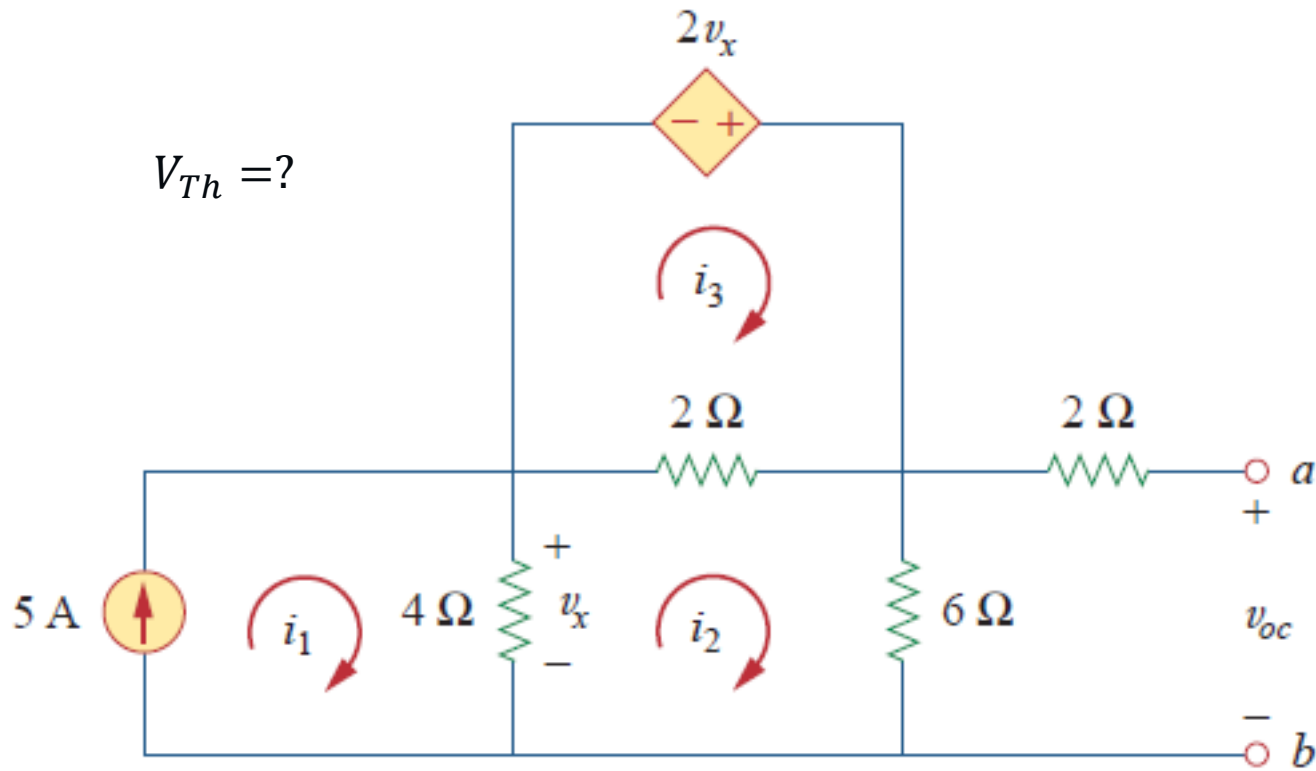
(but): $-4i_2 = v_x = i_1 - i_2 \rightarrow i_1 = -3i_2$

(Loop 2): $4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$

(Loop 3): $6(i_3 - i_2) + 2i_3 + 1 = 0$

$$i_3 = -\frac{1}{6} A \rightarrow i_o = -i_3 = \frac{1}{6} A \rightarrow R_{Th} = \frac{1 V}{i_o} = 6 \Omega$$

Thevenin's Theorem



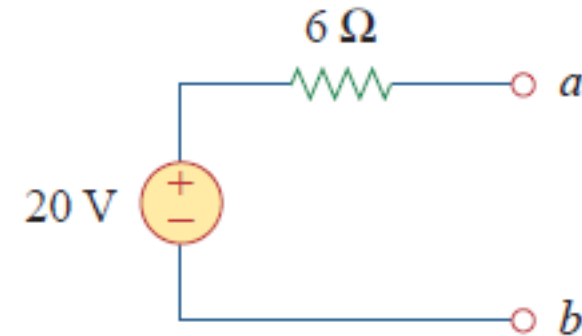
(Loop 1): $i_1 = 5 A$

(Loop 3): $-2v_x + 2(i_3 - i_2) = 0 \rightarrow v_x = i_3 - i_2$

(Loop 2): $4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$
 $\rightarrow 12i_2 - 4i_1 - 2i_3 = 0$

(but): $4(i_1 - i_2) = v_x$

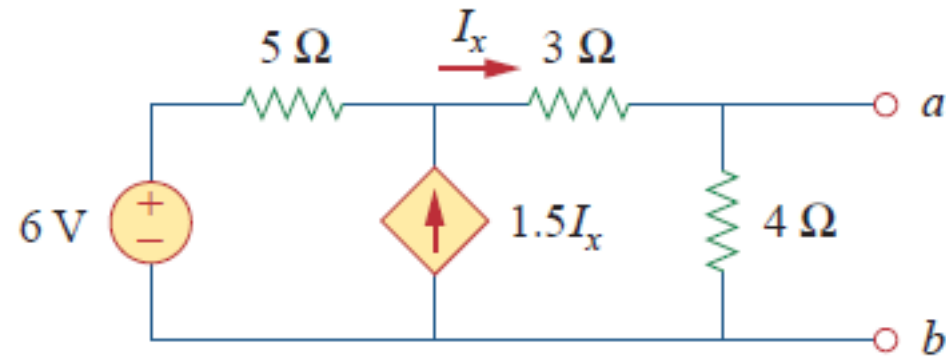
$i_2 = \frac{10}{3} A \rightarrow V_{Th} = v_{oc} = 6i_2 = 20 V$



Thevenin's Theorem



THO.14 – Find the Thevenin equivalent circuit at the terminals a - b .

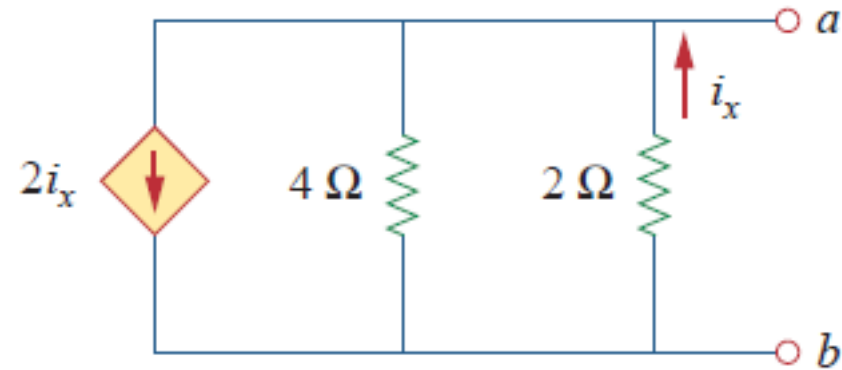
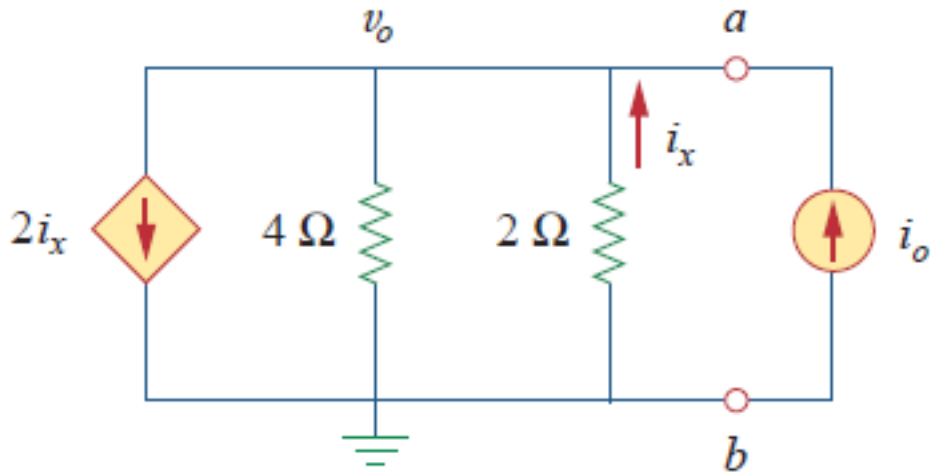


Solution $V_{Th} = 5.33 \text{ V}$, $R_{Th} = 0.44 \Omega$

Thevenin's Theorem – Negative Resistance Simulation

THO.15 – Find the Thevenin equivalent circuit at the terminals a - b .

Solution $i_0 = 1 A$



$$(2 \rightarrow 1): -\frac{2v_0}{2} + \frac{(v_0 - 0)}{4} + \frac{(v_0 - 0)}{2} - 1 = 0 \rightarrow v_0 = -4 V$$

$$(1): 2i_x + \frac{(v_0 - 0)}{4} + \frac{(v_0 - 0)}{2} - 1 = 0$$

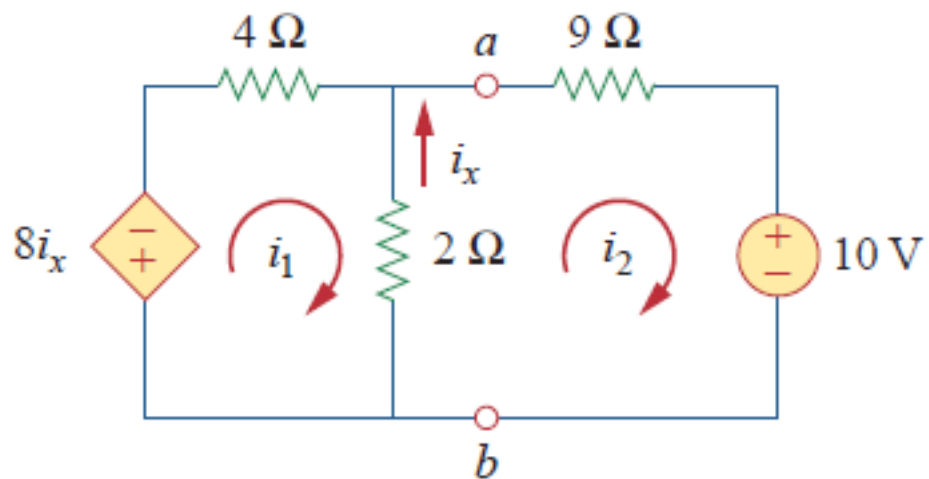
$$(2): i_x = \frac{(0 - v_0)}{2}$$

$$R_{Th} = \frac{v_0}{1} = -4 \Omega \text{ (dependent source supplies the power)}$$

Dependent source and resistors can be used to simulate negative resistance.

Thevenin's Theorem – Negative Resistance

Check... (supplying w. 10 V – 9 Ω)



$$(1): 8i_x + 4i_1 + 2(i_1 - i_2) = 0$$

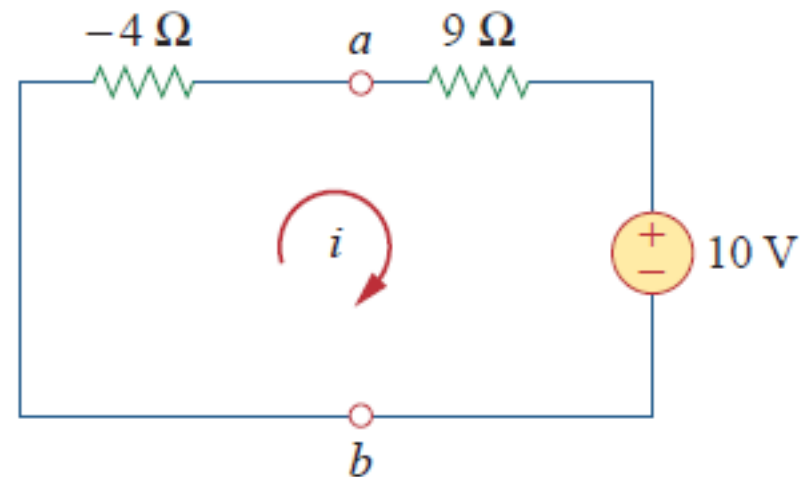
$$(2): 2(i_2 - i_1) + 9i_2 + 10 = 0$$

$$(a): i_x = i_2 - i_1$$

$$(a \rightarrow 1): -2i_1 + 6i_2 = 0 \rightarrow i_1 = 3i_2$$

$$(2): -2i_1 + 11i_2 = -10$$

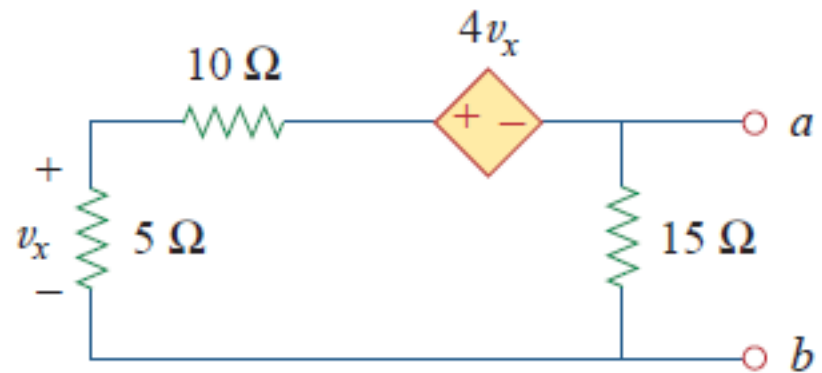
$$-6i_2 + 11i_2 = -10 \rightarrow i_2 = -\frac{10}{5} = -2 \text{ A}$$



$$i = -\frac{10 \text{ V}}{9 + (-4)} = -\frac{10}{5} = -2 \text{ A} \quad \boxed{\text{O.K.}}$$

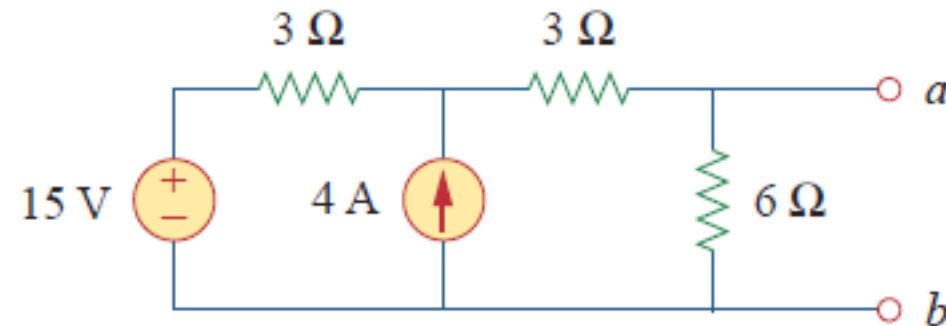
Thevenin's Theorem – Negative Resistance

THO.16 – Obtain the Thevenin equivalent of the circuit.



Solution $V_{Th} = 0 V$, $R_{Th} = -7.5 \Omega$

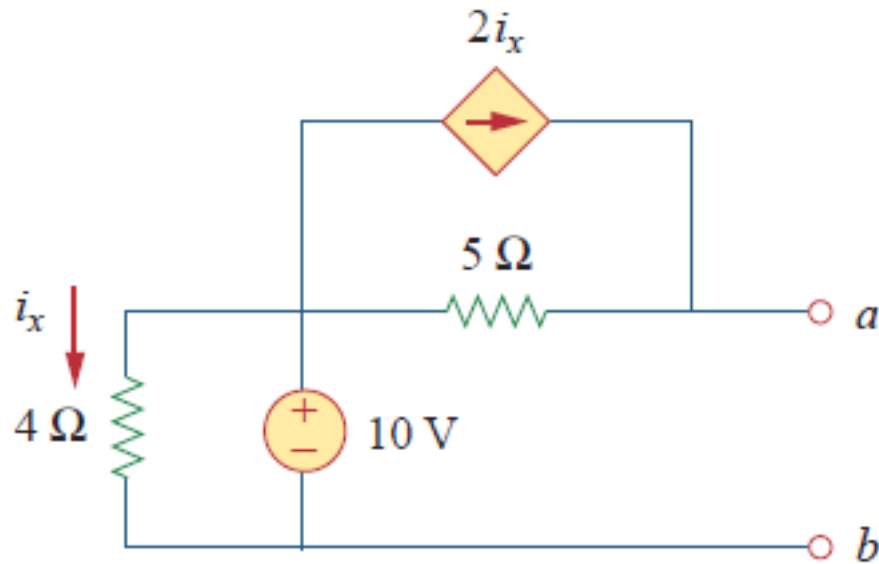
THO.17 – Find the Norton equivalent circuit.



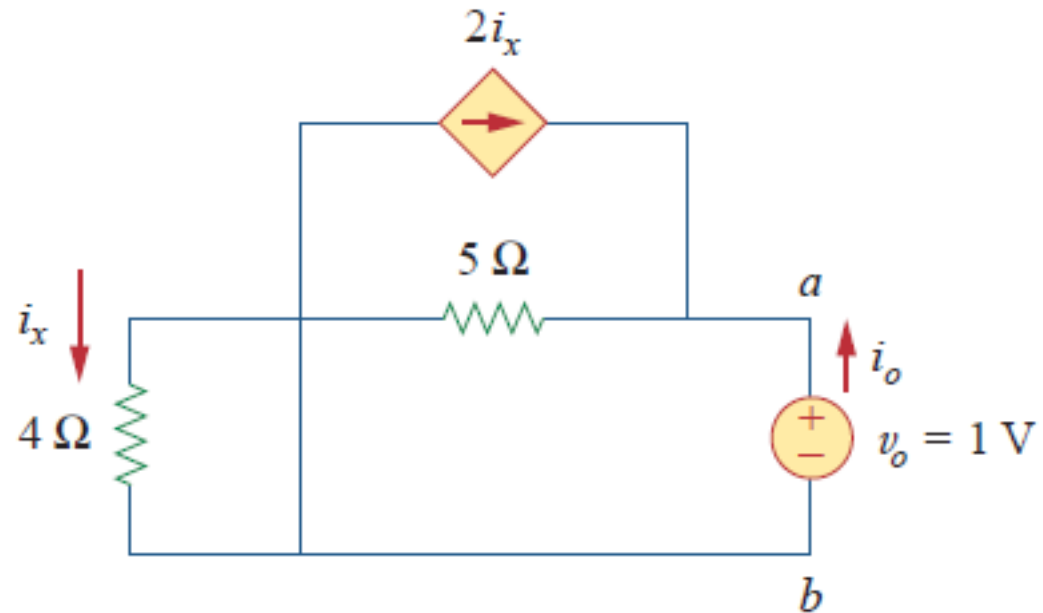
Solution $R_N = 3 \Omega$, $I_N = 4.5 A$

Norton's Theorem

THO.18 – Find the Norton equivalent circuit.

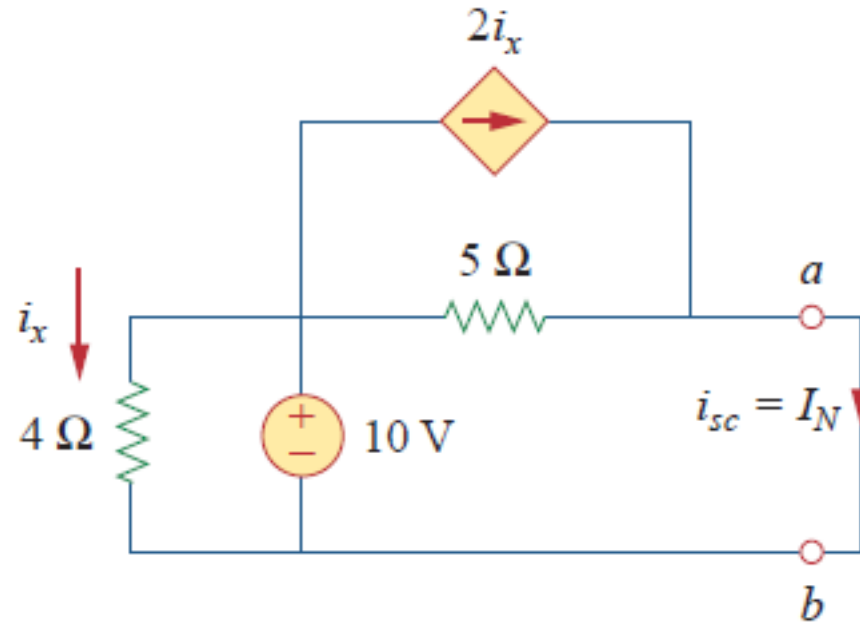


Solution



$$R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5 \Omega$$

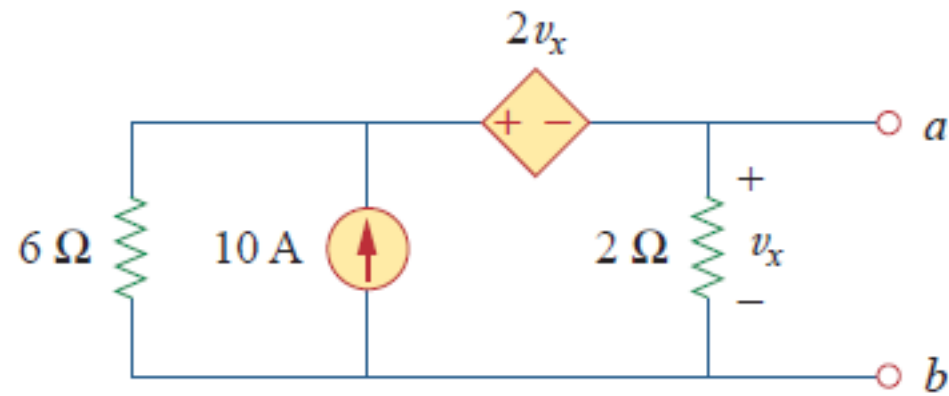
Norton's Theorem



$$i_x = \frac{10}{4} = 2.5 \text{ A}$$

$$i_{sc} = i_N = \frac{10}{5} + 2i_x = 7 \text{ A}$$

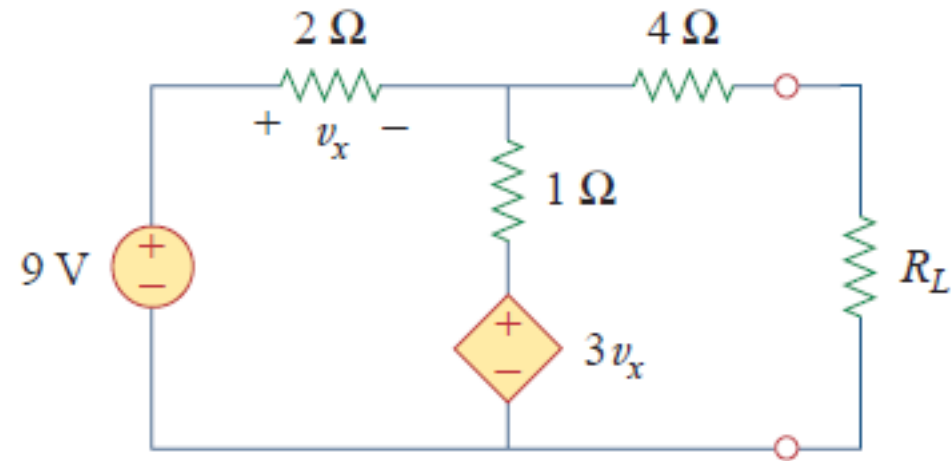
THO.19 – Find the Norton equivalent circuit.



Solution $R_N = 1 \Omega$, $I_N = 10 A$

Maximum Power Transfer

THO.20 – Find R_L for maximum power transfer. Find the maximum power.



Solution $R_L = 4.22 \Omega$, $p_{max} = 2.9 W$

Applications – Real Sources

THO.21 – The terminal voltage of a voltage source is 12 V when connected to a 2-W load. When the load is disconnected, the terminal voltage rises to 12.4 V.

- (a) Calculate the Thevenin equivalent source voltage v_S and internal resistance R_S
 (b) Determine the voltage when an 8- Ω load is connected to the source.

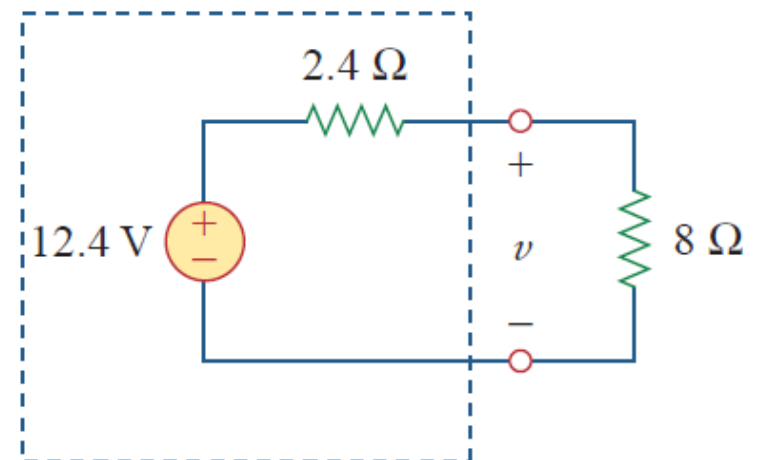
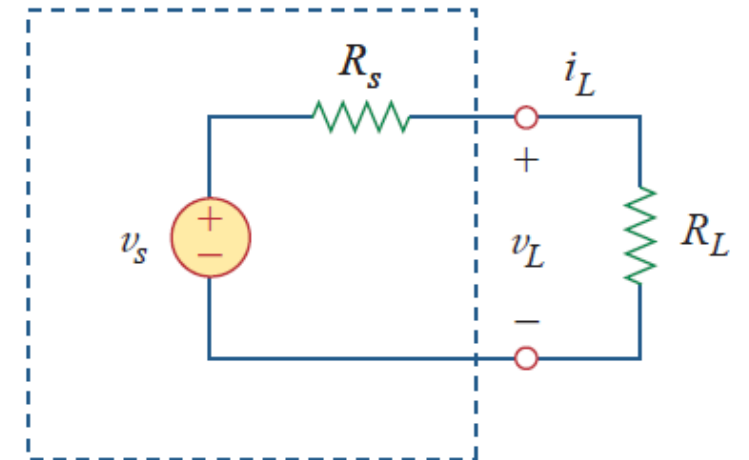
Solution $v_S = v_{OC} = 12.4 \text{ V}$

$$p_L = \frac{v_L^2}{R_L} \rightarrow R_L = \frac{v_L^2}{p_L} = \frac{144}{2} = 72 \ \Omega$$

$$i_L = \frac{v_L}{R_L} = \frac{12}{72} = \frac{1}{6} \text{ A}$$

$$12.4 - 12 = 0.4 = R_S \cdot i_L \rightarrow R_S = \frac{0.4}{i_L} = 2.4 \ \Omega$$

$$v = \frac{8}{8 + 2.4} \cdot 12.4 = 9.538 \text{ V}$$





THO.22

The measured open-circuit voltage across a certain amplifier is 9 V. The voltage drops to 8 V when a 20- Ω loudspeaker is connected to the amplifier. Calculate the voltage when a 10- Ω loudspeaker is used instead.

Solution 7.2 V

Questions

