DR. GYURCSEK ISTVÁN

Exercises using Theorems in Circuit Analysis

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

Linearity

THO.01 – Find I_0 when v_S = 12 V and v_S = 24 V.

Solution

$\boldsymbol{Loop1}\colon\ 12i_1 - 4i_2 + v_S = 0$

$\boldsymbol{Loop2}\colon\ -4i_1 + 16i_2 - 3v_x - v_S = 0$

$\boldsymbol{But}\colon v_x = 2i_1$

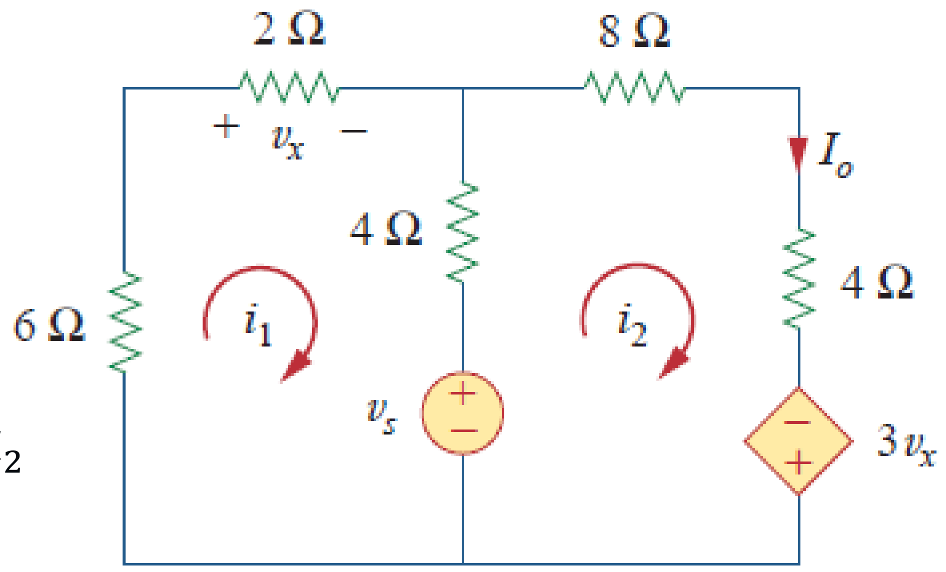
$(\boldsymbol{Loop2 \rightarrow A})\colon\ -10i_1 + 16i_2 - v_S = 0$

$(\boldsymbol{Loop1 + A})\colon\ 2i_1 + 12i_2 = 0 \rightarrow i_1 = -6i_2$

$(\rightarrow \boldsymbol{Loop1})\colon -76i_2 + v_S = 0 \rightarrow i_2 = \dfrac{v_S}{76}$

$v_S = 12\ V \rightarrow I_0 = \dfrac{12}{76} = 157.89\ mA$

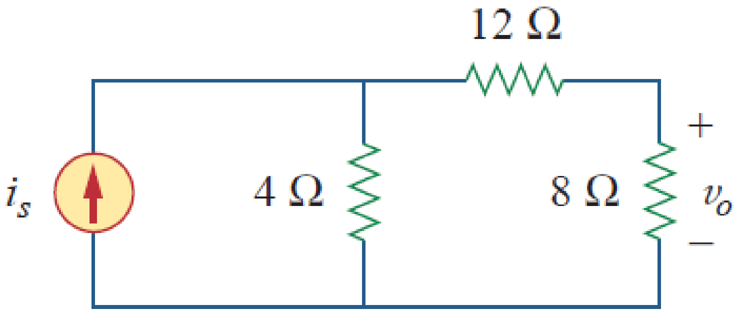
$v_S = 24\ V \rightarrow I_0 = \dfrac{24}{76} = 315.78\ mA$



Two time higher $v_S \rightarrow$ two times higher I_0

Linearity

THO.02 – Find v_0 when i_S is 15 A and i_S is 30 A.



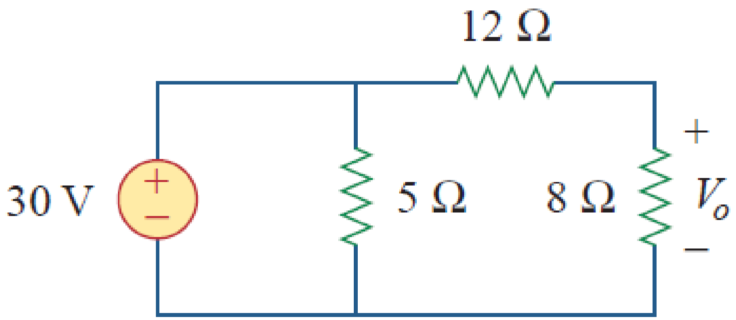
Solution

$$i_{01} = i_{S1}\frac{4}{4+20} = 2.5\ A \rightarrow v_{01} = 8i_{01} = 20\ V$$

$$i_{S2} = 2 \cdot i_{S1} \rightarrow v_{02} = 2 \cdot v_{01} = 40\ V$$

Linearity

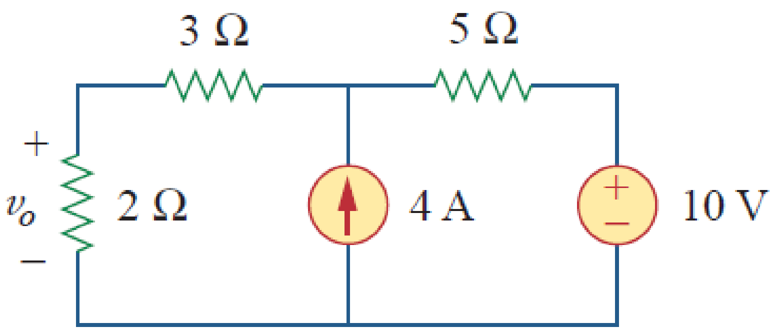
THO.03 – Assume v_0 = 1 V and use linearity to calculate actual value of v_0.



Solution $v_0 = 12\ V$

Superposition Principle

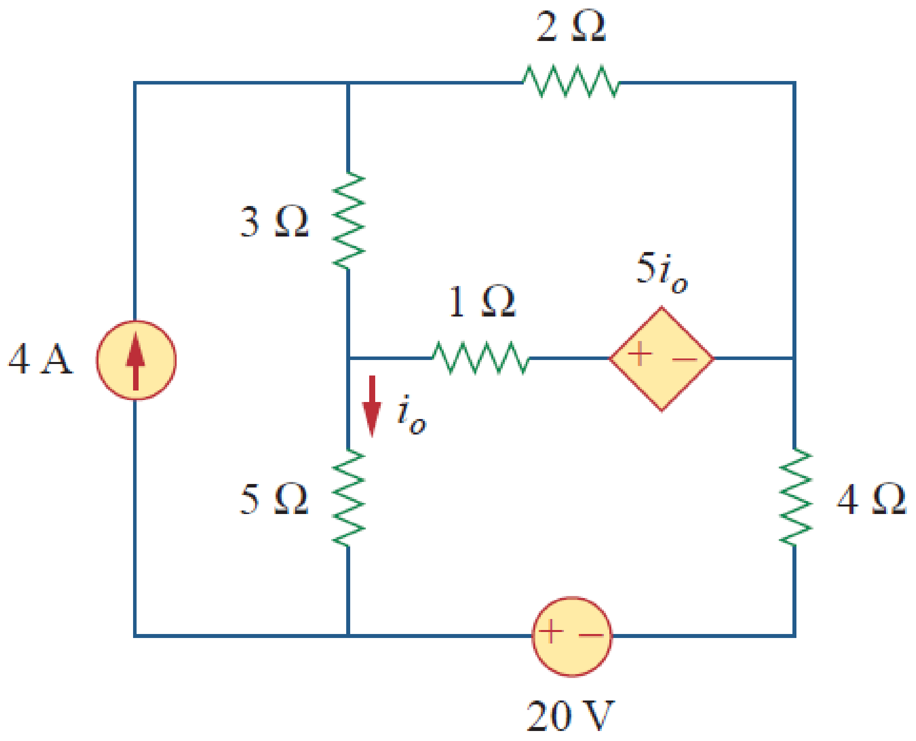
THO.04 – Using the superposition principle, find v_0 in the circuit.



Solution $v_0 = 6\ V$

Superposition Principle

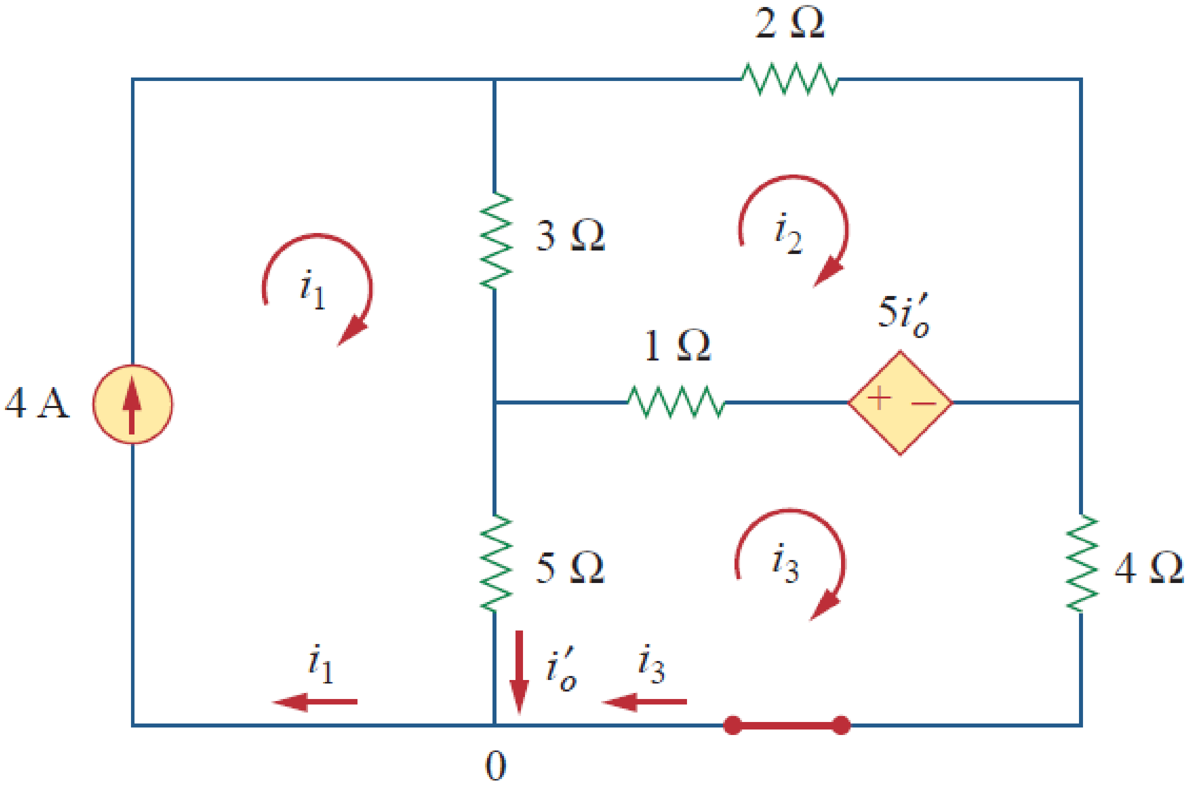
THO.05 – Using the superposition principle, find i_0 in the circuit.



Solution $$i_0 = i_0' + i_0''$$

Superposition Principle

$i_0' = ?$



$(\boldsymbol{loop}\ \mathbf{1})$: $i_1 = 4$

$(\boldsymbol{loop}\ \mathbf{2})$: $-3i_1 + 6i_2 - 1i_3 - 5i_0' = 0$

$(\boldsymbol{loop}\ \mathbf{3})$: $-5i_1 - 1i_2 + 10i_3 + 5i_0' = 0$

$(\boldsymbol{node}\ \mathbf{0})$: $i_3 = i_1 - i_0' = 4 - i_0'$

$(\boldsymbol{loop}\ \mathbf{1})(\boldsymbol{node}\ \mathbf{0}) \rightarrow (\boldsymbol{loop}\ \mathbf{2})$:
$3i_2 - 2i_0' = 8$

$(\boldsymbol{loop}\ \mathbf{1})(\boldsymbol{node}\ \mathbf{0}) \rightarrow (\boldsymbol{loop}\ \mathbf{3})$:
$i_2 + 5i_0' = 20$

$$i_0' = \frac{52}{17}\ A$$

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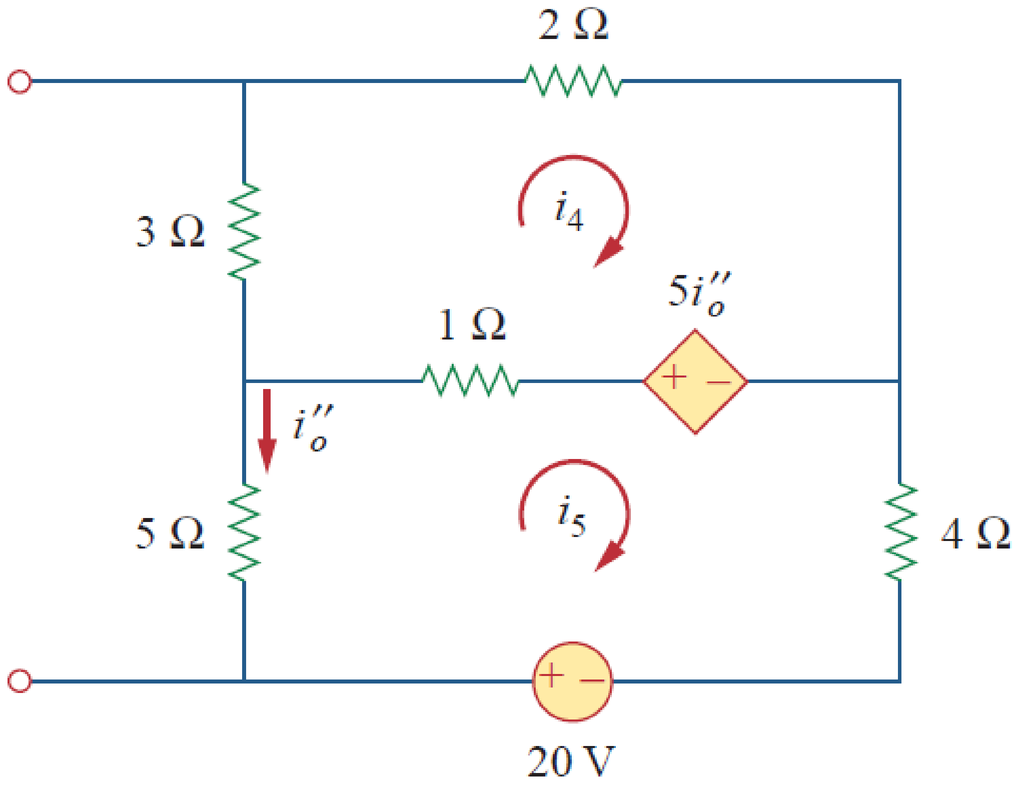
Superposition Principle

$i_0'' = ?$



$(\boldsymbol{loop\ 4})\text{: } 6i_4 - i_5 - 5i_0'' = 0$

$(\boldsymbol{loop\ 5})\text{: } -i_4 + 10i_5 - 20 + 5i_0'' = 0$

$(\boldsymbol{but})\text{: } i_5 = -i_0''$

$(\boldsymbol{but}) \rightarrow (\boldsymbol{loop\ 4})\text{: } 6i_4 - 4i_0'' = 0$

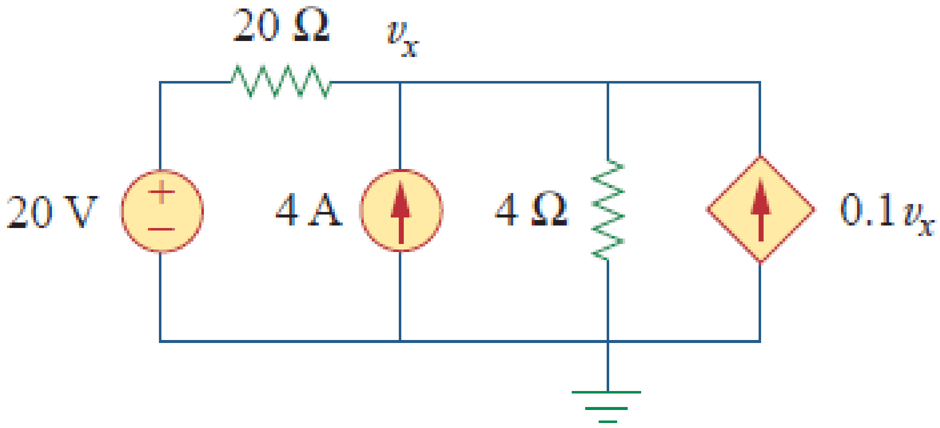
$(\boldsymbol{but}) \rightarrow (\boldsymbol{loop\ 5})\text{: } i_4 + 5i_0'' = -20$

$$i_0'' = -\frac{60}{17}\ A$$

$$i_0 = i_0' + i_0'' = \frac{52}{17} - \frac{60}{17} = -\frac{8}{17} = -0.47\ A$$

Superposition Principle

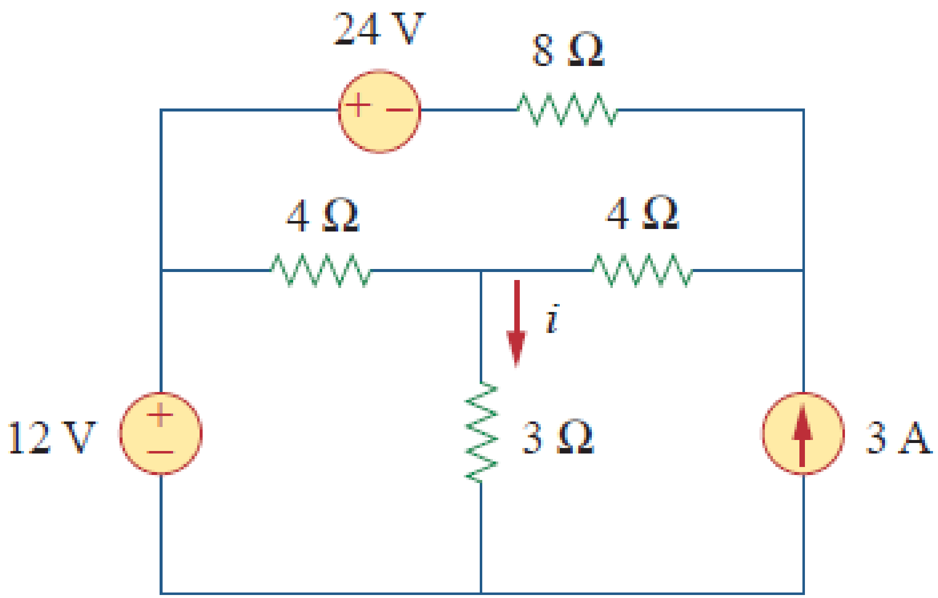
THO.06 – Using the superposition principle, find v_X in the circuit.



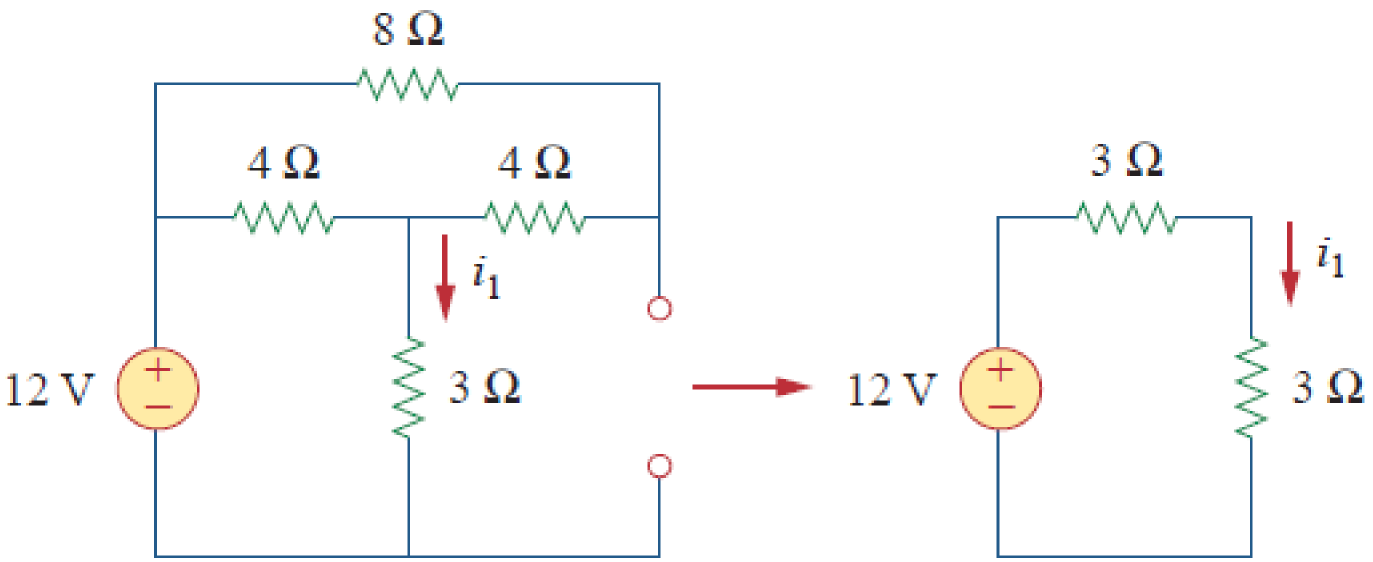
Solution $v_X = 25\ V$

Superposition Principle

THO.07 – Using the superposition principle, find i in the circuit.

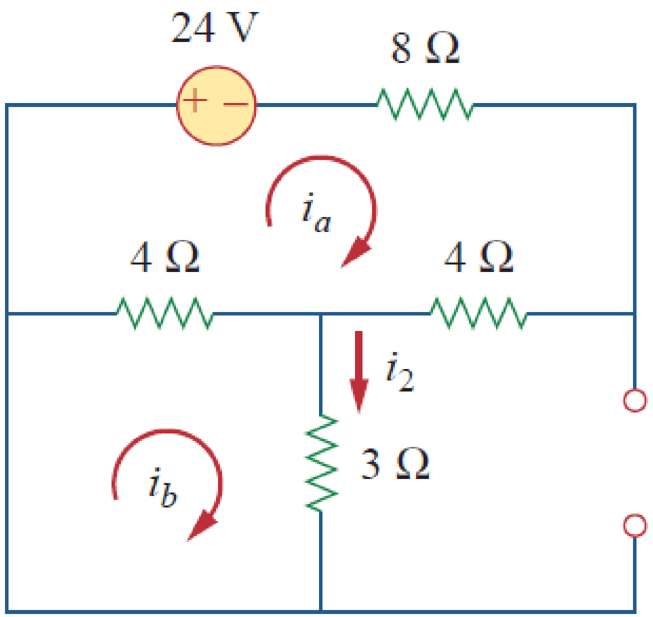


Solution $i = i_1 + i_2 + i_3$, $i_1 = ?$



$$i_1 = \frac{12}{6} = 2\ A$$

Superposition Principle



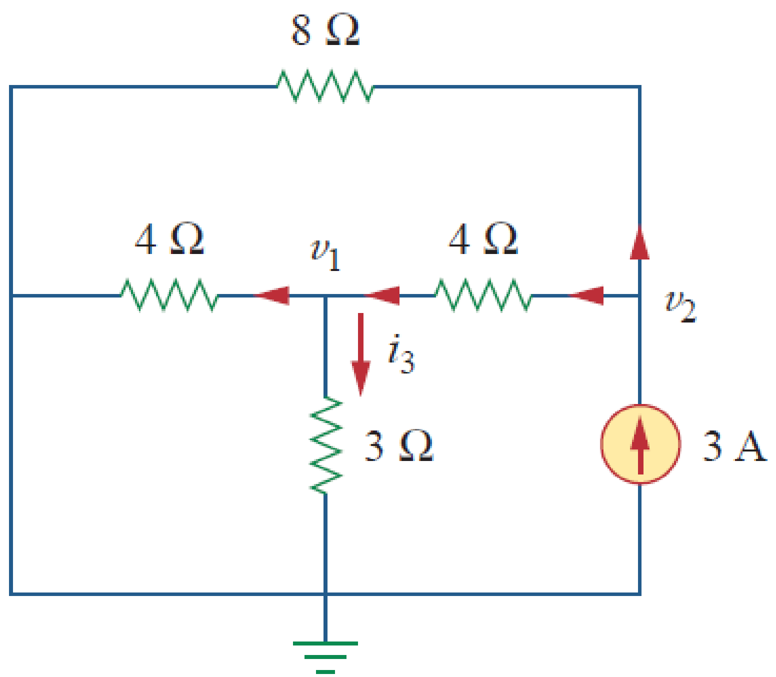
$i_2 = ?$

$16i_a - 4i_b + 24 = 0 \rightarrow (\mathbf{1}){:}\; 4i_a - i_b = -6$

$7i_b - 4i_a = 0 \rightarrow (\mathbf{2}){:}\; i_a = \frac{7}{4} i_b$

$(\mathbf{2}) \rightarrow (\mathbf{1}){:}\; i_2 = i_b = -1$

Superposition Principle



$i_3 = ?$

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \rightarrow (\mathbf{1}):\ 24 = 3v_2 - 2v_1$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \rightarrow (\mathbf{2}):\ v_2 = \frac{10}{3} v_1$$

$$(\mathbf{2}) \rightarrow (\mathbf{1}): v_1 = 3 \rightarrow i_3 = \frac{v_3}{3} = 1\ A$$

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2\ A$$

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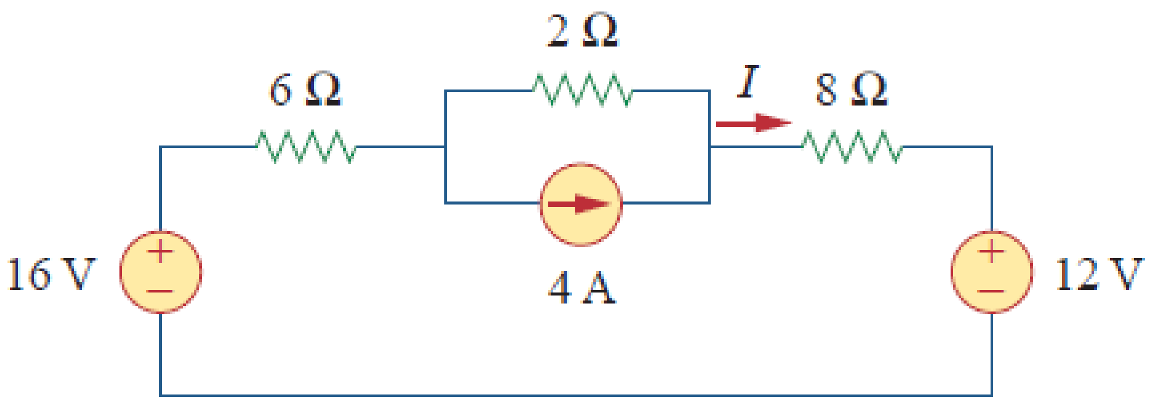
Superposition Principle

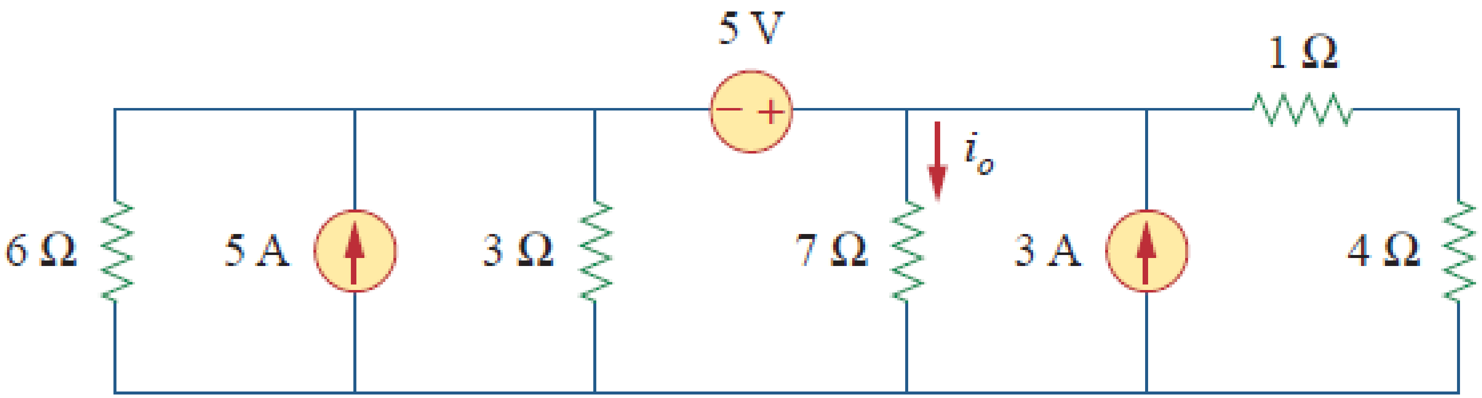
THO.08 – Using the superposition principle find *I* in the circuit.



Solution $I = 0.75\ A$

Source Transformation

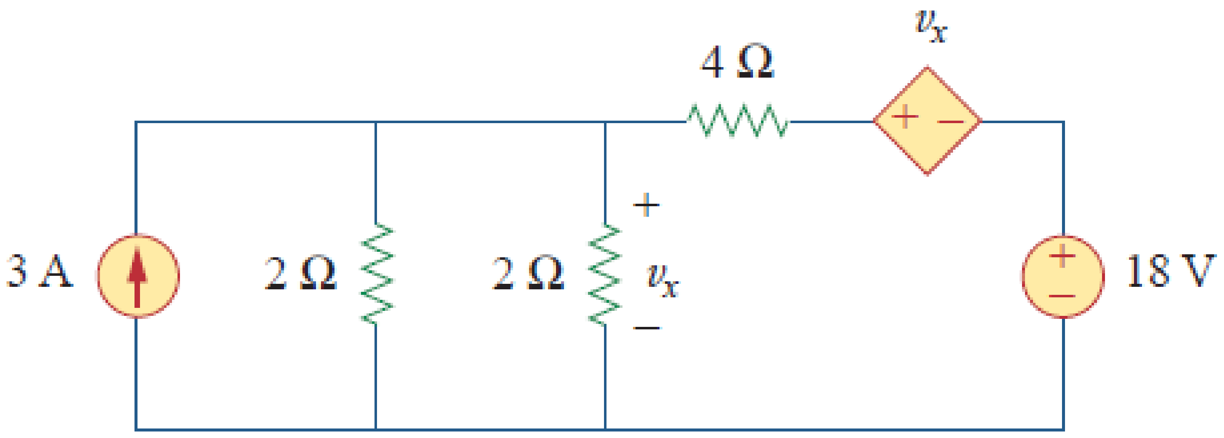
THO.09 – Using the source transformation find i_0 in the circuit.

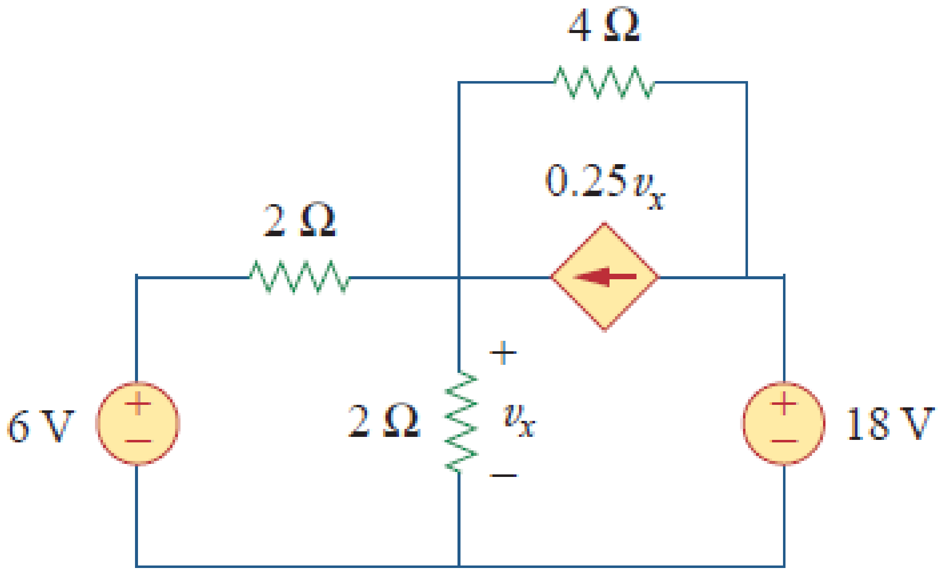


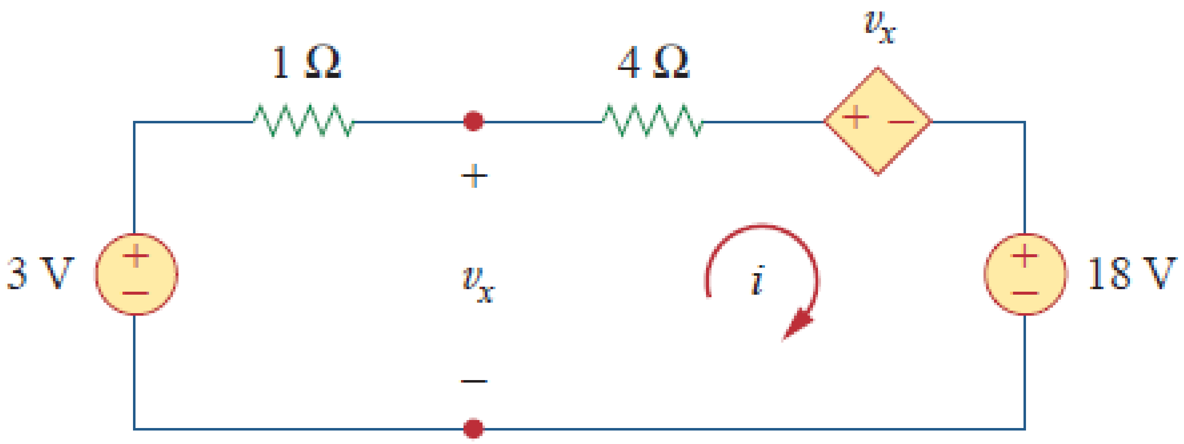
Solution $I = 1.78\ A$

Source Transformation

THO.10 – Using the source transformation find v_X in the circuit.







$(\boldsymbol{Loop\ 1})\text{: } -3 + 5i + v_x + 18 = 0$

$(\boldsymbol{Loop\ 2})\text{: } -3 + 1i + v_x = 0$

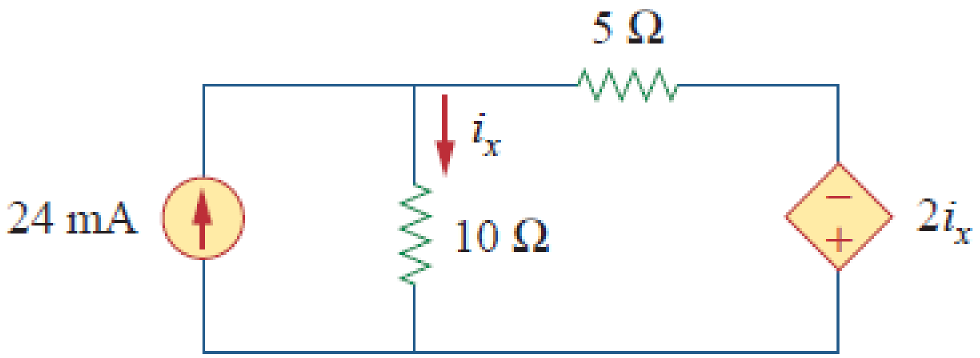
$(\boldsymbol{Loop\ 2}) \rightarrow (\boldsymbol{Loop\ 1})\text{: } 15 + 5i + 3 - i = 0 \rightarrow i = -4.5\ A$

$or\ \ldots\ (\boldsymbol{Loop\ 3})\text{: } -v_x + 4i + v_x + 18 = 0 \rightarrow i = -4.5\ A$

$v_x = 3 - i = 7.5\ V$

Source Transformation

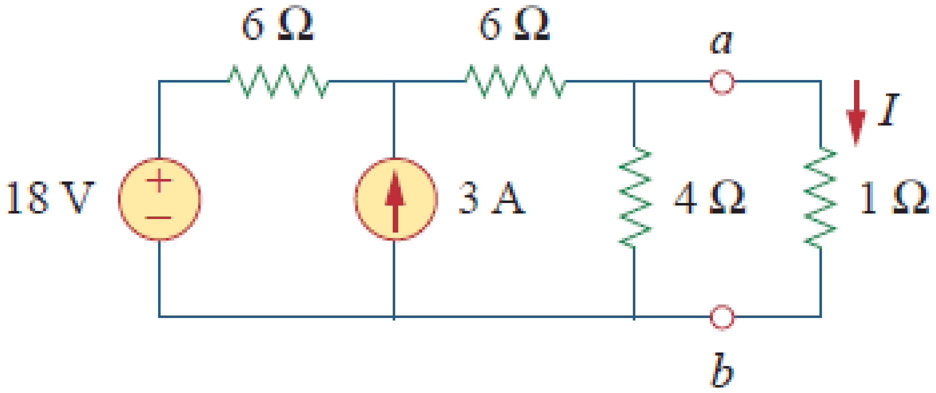
THO.11 – Using the source transformation find i_X in the circuit.



Solution $I = 7.056\ mA$

Thevenin's Theorem

THO.12 – Find the Thevenin equivalent circuit to the left of the terminals and find *I*.

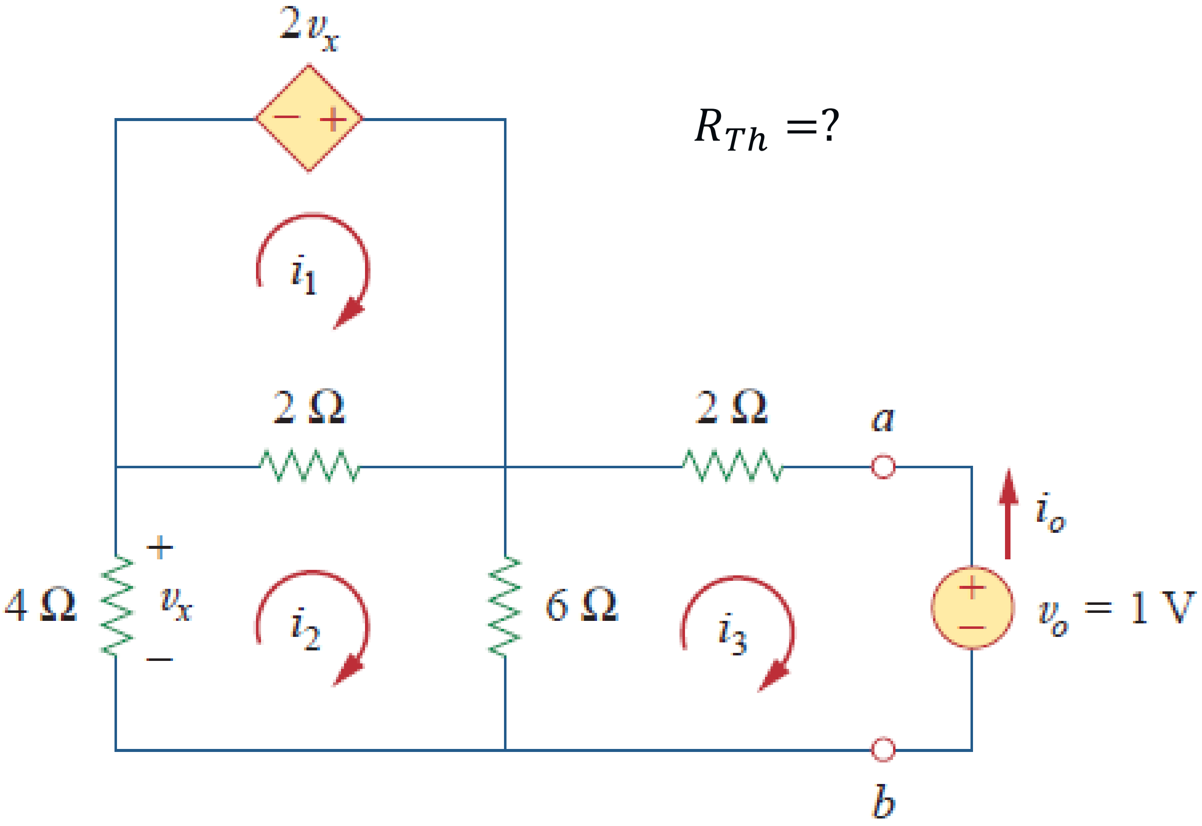


Solution $V_{Th} = 9\ V\,,\quad R_{Th} = 3\ \Omega\,,\quad I = 2.25\ A$

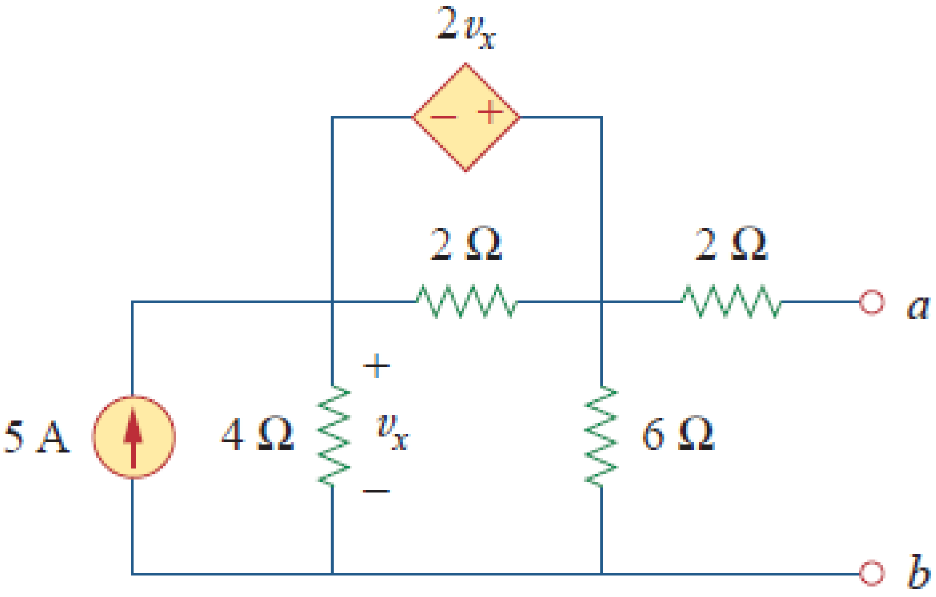
Thevenin’s Theorem

THO.13 – Find the Thevenin equivalent circuit at the terminals *a*-*b*.

Solution



R_{Th} =?



$$(\boldsymbol{Loop\ 1}):\ -2v_X + 2(i_1 - i_2) = 0 \rightarrow v_X = i_1 - i_2$$

$$(\boldsymbol{but}): -4i_2 = v_X = i_1 - i_2 \rightarrow i_1 = -3i_2$$

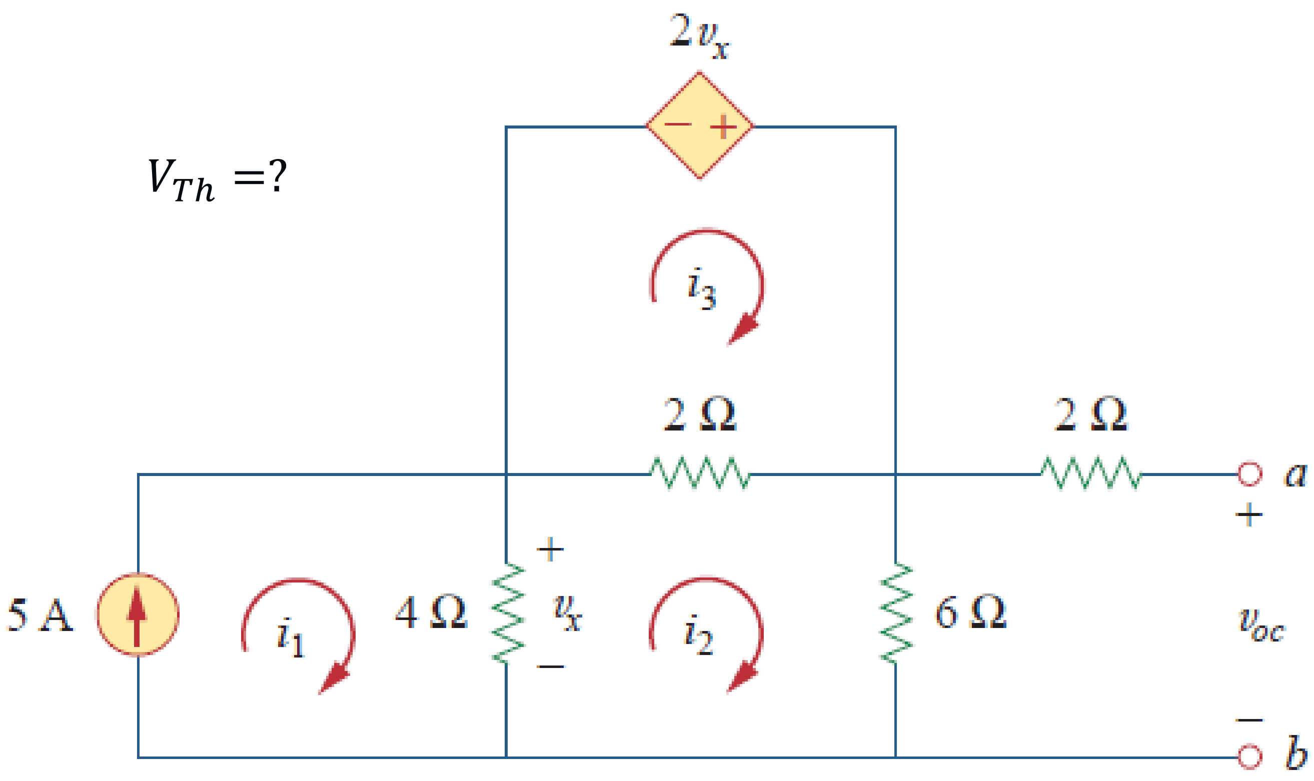
$$(\boldsymbol{Loop\ 2}):\ 4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

$$(\boldsymbol{Loop\ 3}):\ 6(i_3 - i_2) + 2i_3 + 1 = 0$$

$$i_3 = -\frac{1}{6}\ A \rightarrow i_0 = -i_3 = \frac{1}{6}\ A \rightarrow R_{Th} = \frac{1\ V}{i_0} = 6\ \Omega$$

Thevenin's Theorem

V_{Th} =?



$(\boldsymbol{Loop\ 1})\text{:}\ i_1 = 5\ A$

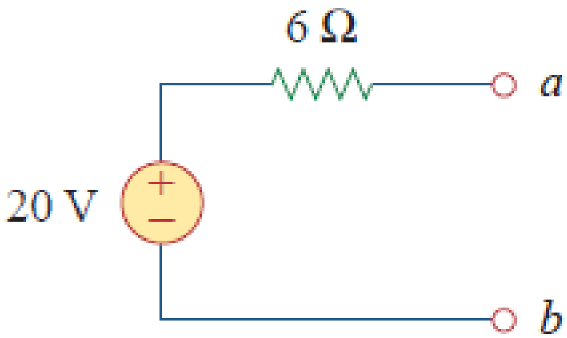
$(\boldsymbol{Loop\ 3})\text{:}\ -2v_x + 2(i_3 - i_2) = 0 \rightarrow v_X = i_3 - i_2$

$(\boldsymbol{Loop\ 2})\text{:}\ 4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$

$\rightarrow 12i_2 - 4i_1 - 2i_3 = 0$

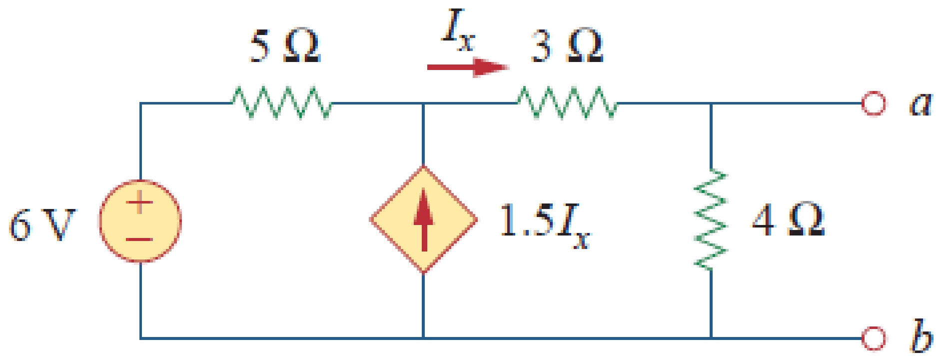
$(\boldsymbol{but})\text{:}\ 4(i_1 - i_2) = v_x$

$i_2 = \dfrac{10}{3}\ A \rightarrow V_{Th} = v_{OC} = 6i_2 = 20\ V$



Thevenin's Theorem

THO.14 – Find the Thevenin equivalent circuit at the terminals *a-b.*

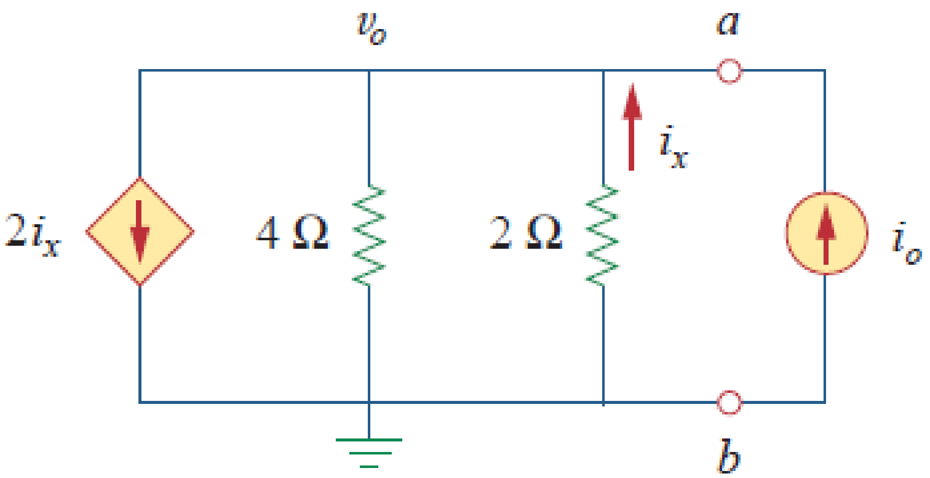


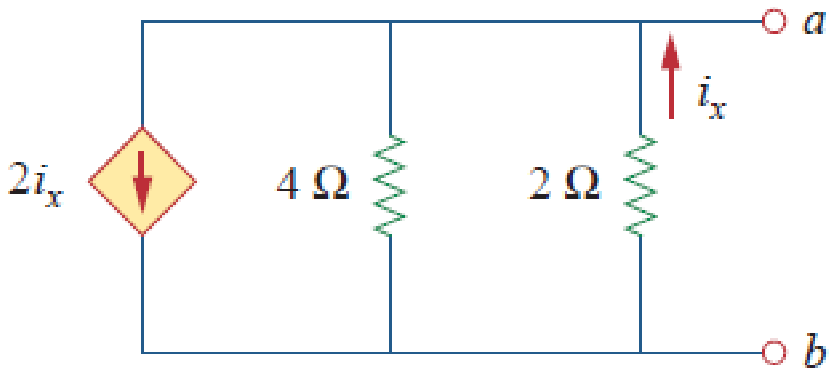
Solution $V_{Th} = 5.33\ V$, $R_{Th} = 0.44\ \Omega$

Thevenin's Theorem

THO.15 – Find the Thevenin equivalent circuit at the terminals *a-b. (Negative Resistance Simulation)*

Solution $i_0 = 1\ A$





$$(\mathbf{1}):\ 2i_x + \frac{(v_0 - 0)}{4} + \frac{(v_0 - 0)}{2} - 1 = 0$$

$$(\mathbf{2}):\ i_x = \frac{(0 - v_0)}{2}$$

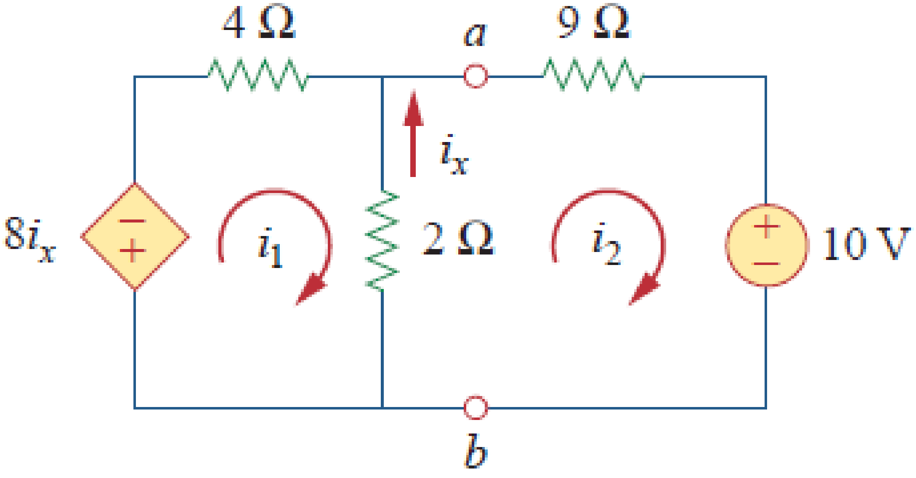
$$(\mathbf{2} \rightarrow \mathbf{1}): -\frac{2v_0}{2} + \frac{(v_0 - 0)}{4} + \frac{(v_0 - 0)}{2} - 1 = 0 \rightarrow v_0 = -4\ V$$

$$R_{Th} = \frac{v_0}{1} = -4\ \Omega\ (dependent\ source\ supplies\ the\ power)$$

Dependent source and resistors can be used to simulate negative resistance.

Thevenin's Theorem

Check... (supplying w. 10 V – 9 Ω)



$(\mathbf{1})$: $8i_x + 4i_1 + 2(i_1 - i_2) = 0$

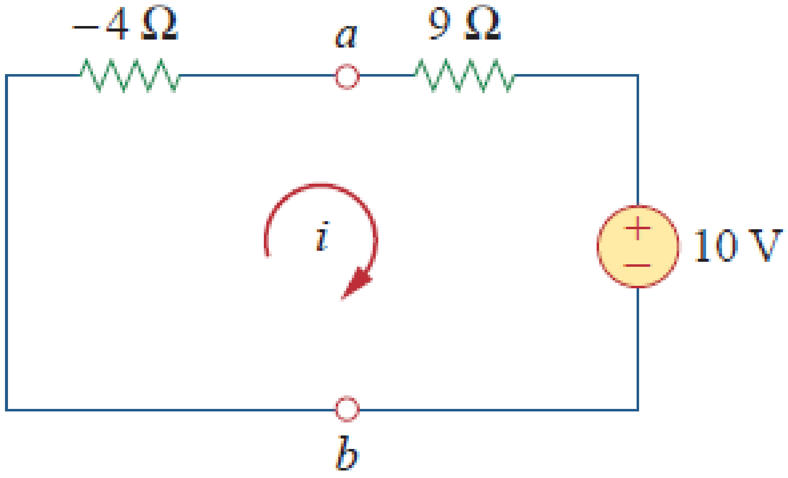
$(\mathbf{2})$: $2(i_2 - i_1) + 9i_2 + 10 = 0$

$(\boldsymbol{a})$: $i_x = i_2 - i_1$

$(\boldsymbol{a} \to \mathbf{1})$: $-2i_1 + 6i_2 = 0 \to i_1 = 3i_2$

$(\mathbf{2})$: $-2i_1 + 11i_2 = -10$

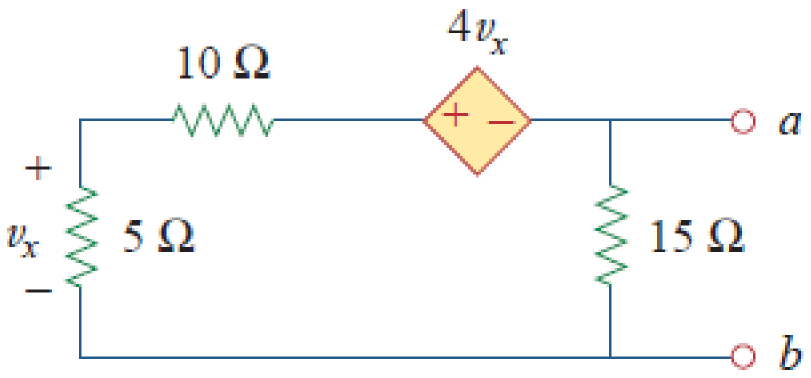
$-6i_2 + 11i_2 = -10 \to i_2 = -\dfrac{10}{5} = -2\ A$



$$i = -\frac{10\ V}{9 + (-4)} = -\frac{10}{5} = -2\ A \ \boxed{O.K.}$$

Thevenin's Theorem

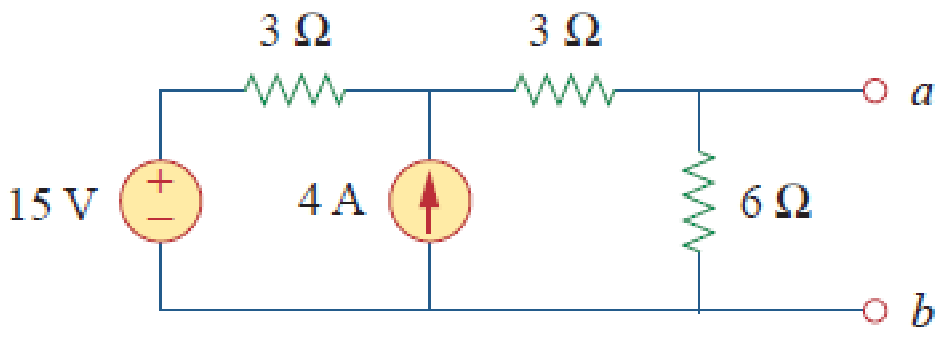
THO.16 – Obtain the Thevenin equivalent of the circuit.



Solution $V_{Th} = 0\ V$, $R_{Th} = -7.5\ \Omega$

Norton's Theorem

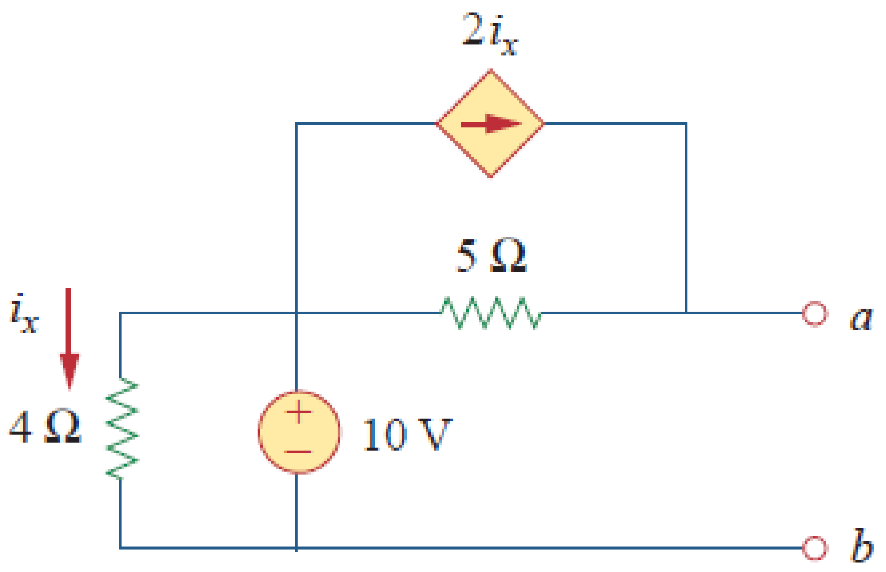
THO.17 – Find the Norton equivalent circuit.



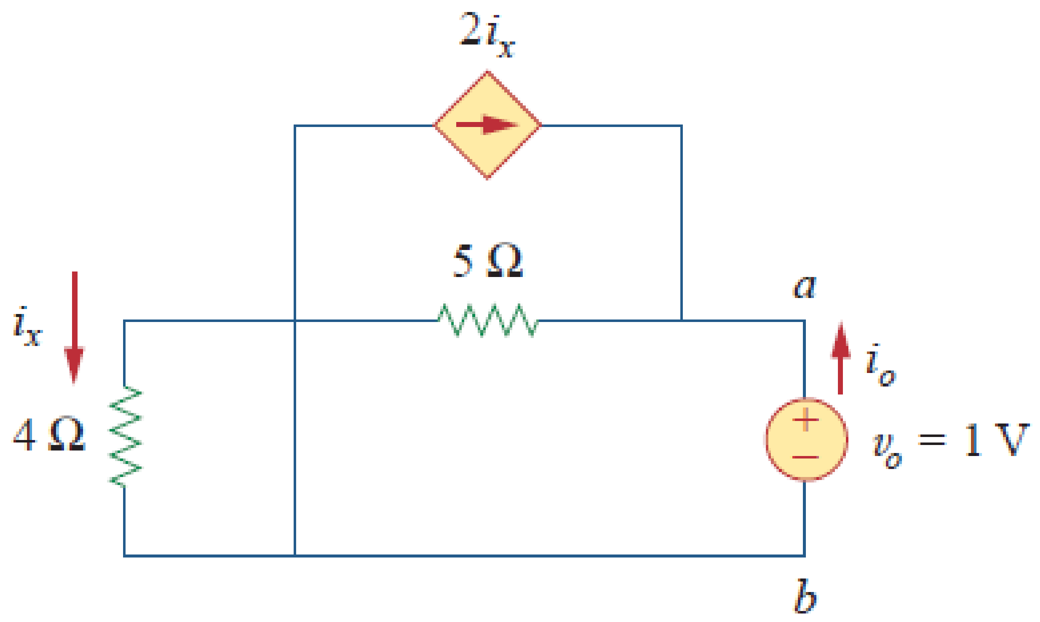
Solution $R_N = 3\ \Omega,\qquad I_N = 4.5\ A$

Norton’s Theorem

THO.18 – Find the Norton equivalent circuit.

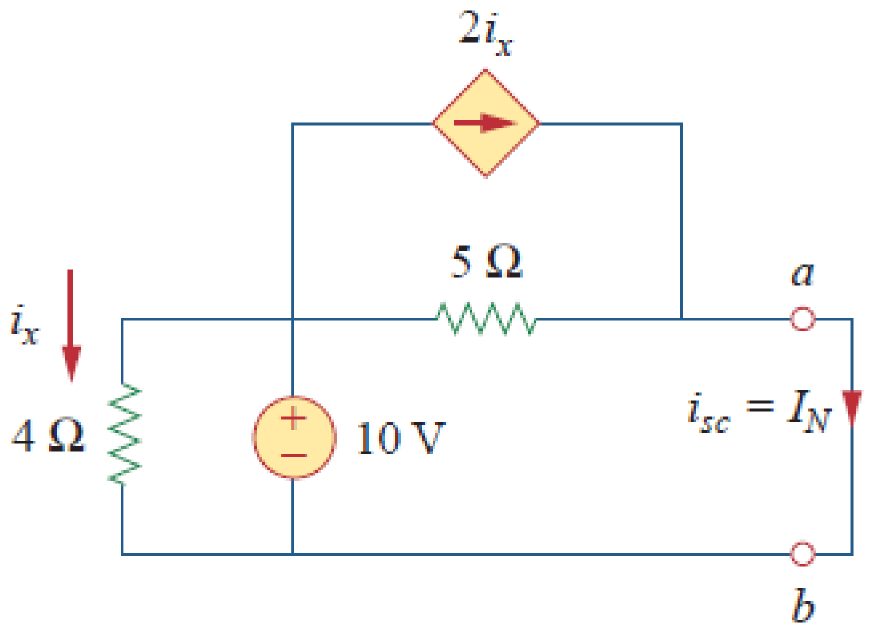


Solution



$$R_N = \frac{v_0}{i_0} = \frac{1}{0.2} = 5\ \Omega$$

Norton’s Theorem

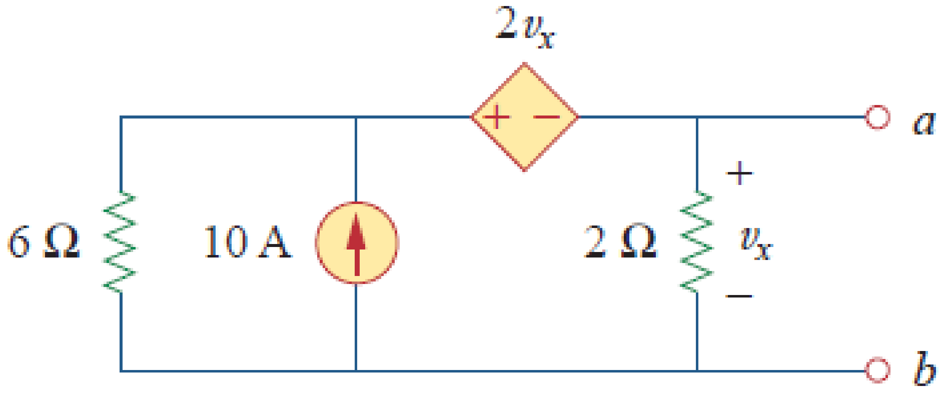


$$i_x = \frac{10}{4} = 2.5\ A$$

$$i_{SC} = i_N = \frac{10}{5} + 2i_x = 7\ A$$

Norton’s Theorem

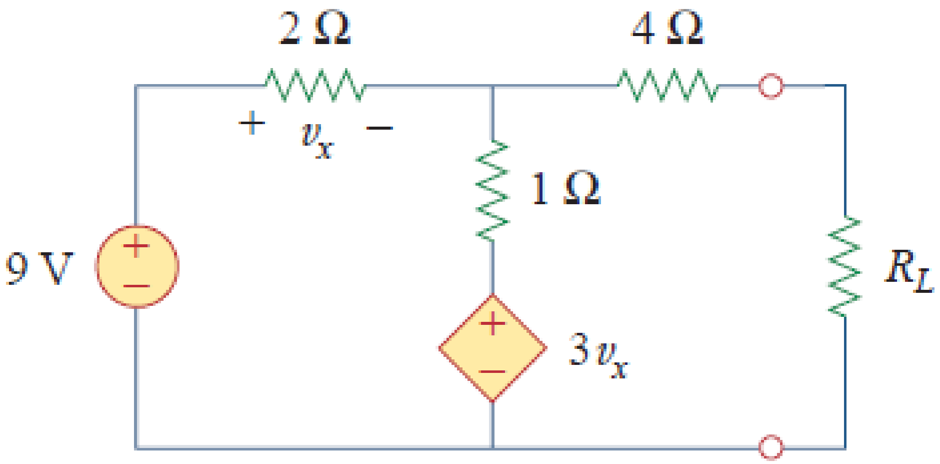
THO.19 – Find the Norton equivalent circuit.



Solution $R_N = 1\ \Omega\,,\qquad I_N = 10\ A$

Maximum Power Transfer

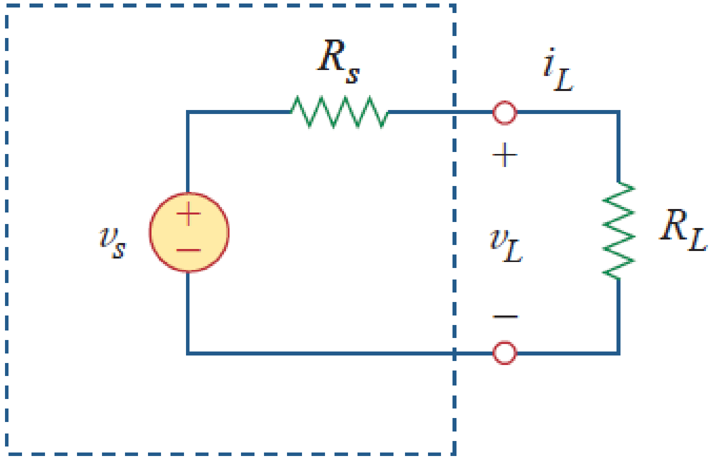
THO.20 – Find R_L for maximum power transfer. Find the maximum power.



Solution $R_L = 4.22\ \Omega\,,\qquad p_{max} = 2.9\ W$

App. - Real Sources

THO.21 – The terminal voltage of a voltage source is 12 V when connected to a 2-W load. When the load is disconnected, the terminal voltage rises to 12.4 V.
(a) Calculate the Thevenin equivalent source voltage v_S and internal resistance R_S
(b) Determine the voltage when an 8-Ω load is connected to the source.



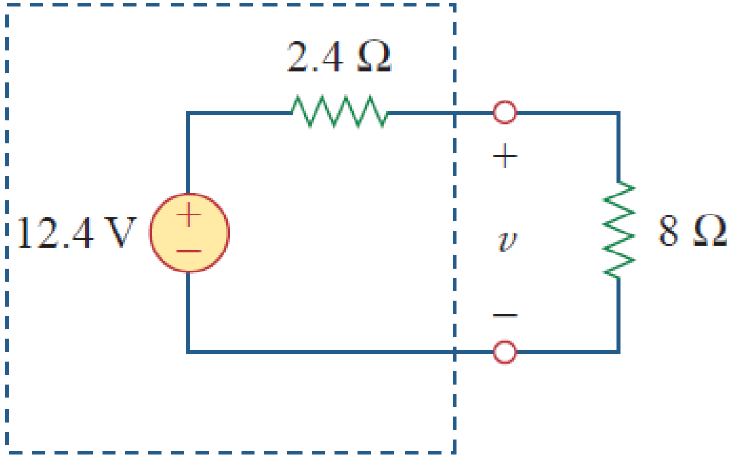
Solution $v_S = v_{OC} = 12.4\ V$

$$p_L = \frac{v_L^2}{R_L} \rightarrow R_L = \frac{v_L^2}{p_L} = \frac{144}{2} = 72\ \Omega$$

$$i_L = \frac{v_L}{R_L} = \frac{12}{72} = \frac{1}{6}\ A$$

$$12.4 - 12 = 0.4 = R_S \cdot i_L \rightarrow R_s = \frac{0.4}{i_L} = 2.4\ \Omega$$

$$v = \frac{8}{8 + 2.4} \cdot 12.4 = 9.538\ V$$



App. - Real Sources

THO.22
The measured open-circuit voltage across a certain amplifier is 9 V. The voltage drops to 8 V when a 20-Ω loudspeaker is connected to the amplifier. Calculate the voltage when a 10-Ω loudspeaker is used instead.

Solution 7.2 V

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Questions