DR. GYURCSEK ISTVÁN

Exercises with Capacitors and Inductors

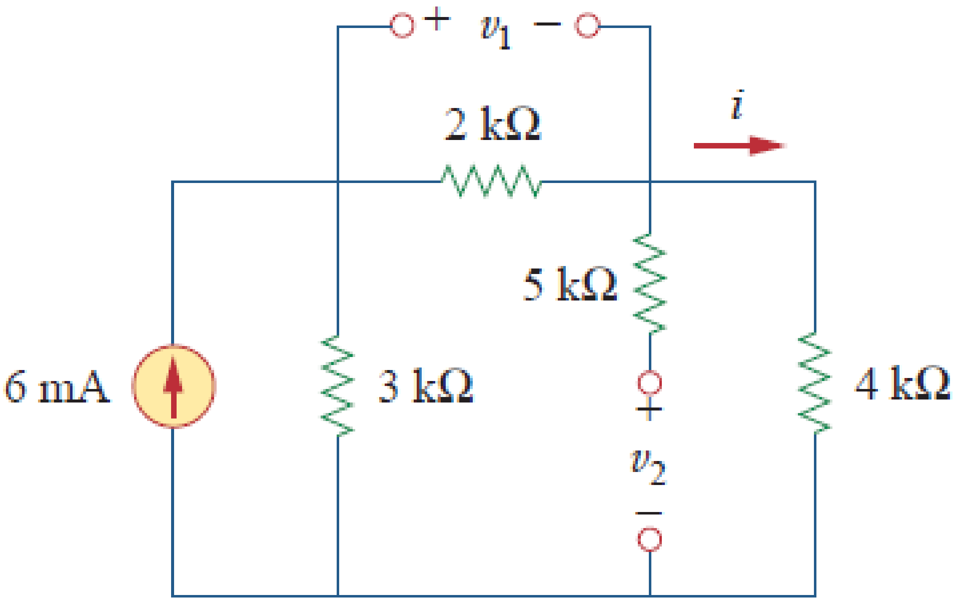
Sources and additional materials (recommended)

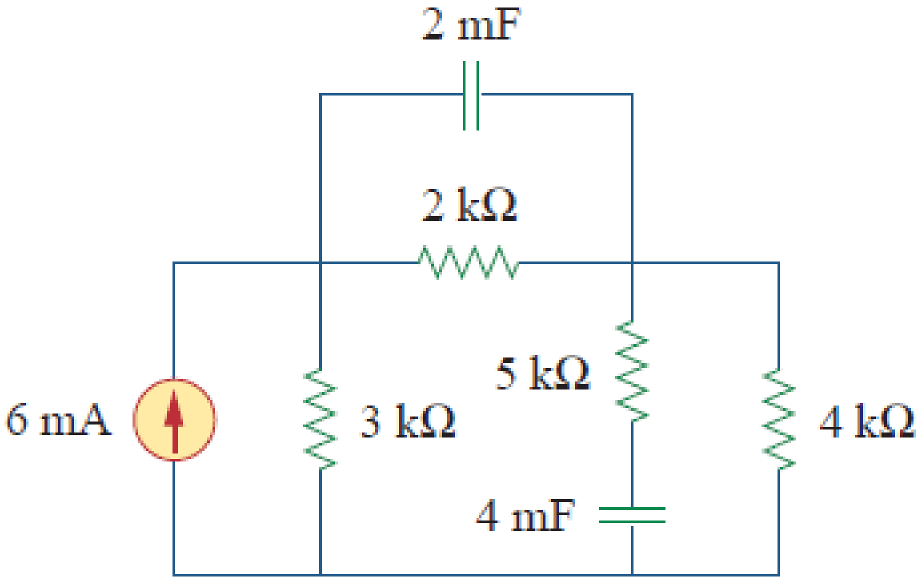
- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

Stored Energy in Capacitors

LCE.01 – Calculate energy stored in each capoacitor.

Solution





$$i = 6\frac{3}{3+2+4} = 2\ mA$$

$$v_1 = 2000i = 4\ V\ , \qquad v_2 = 4000i = 8\ V$$

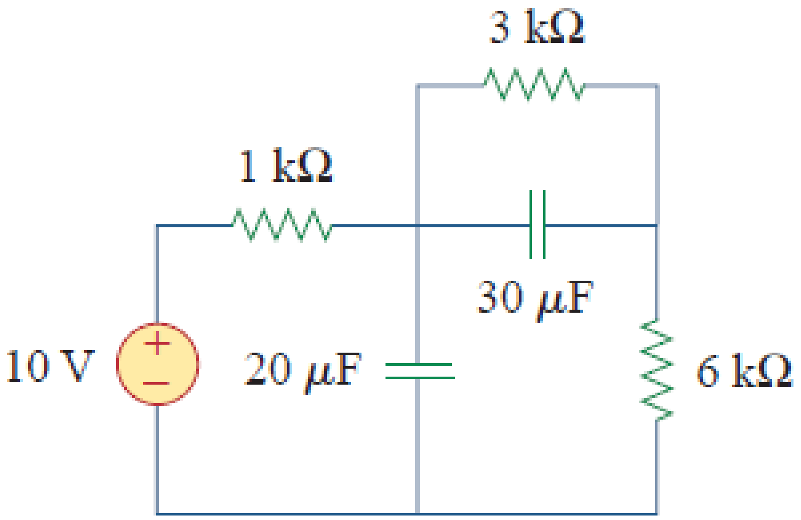
$$w_1 = \frac{1}{2}C_1 v_1^2 = \frac{1}{2}\cdot 2\cdot 10^{-3}\cdot 16 = 16\ mJ$$

$$w_2 = \frac{1}{2}C_2 v_2^2 = \frac{1}{2}\cdot 4\cdot 10^{-3}\cdot 64 = 128\ mJ$$

gyurcsek.istvan@mik.pte.hu

Stored Energy in Capacitors

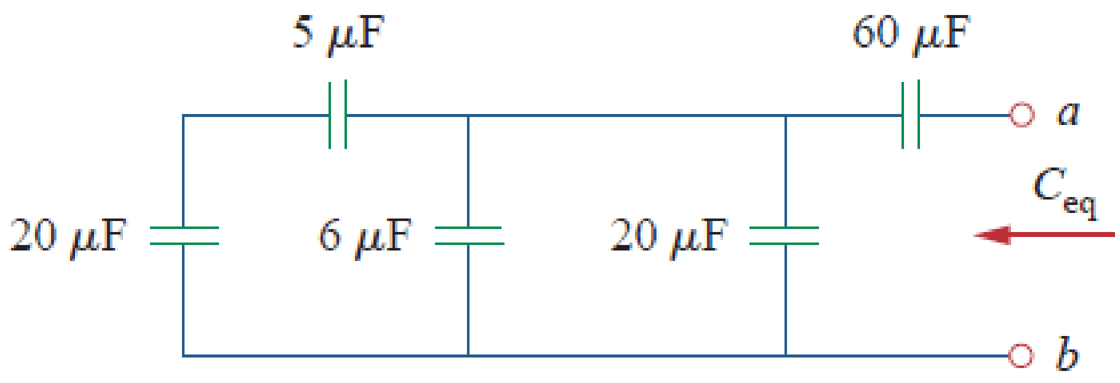
LCE.02 – Calculate energy stored in each capoacitor.



Solution $w_1 = 810\ \mu J$, $w_2 = 135\ \mu J$

Equivalent Capacitance

LCE.03 – Find the equivalent capacitance between *a*-*b*

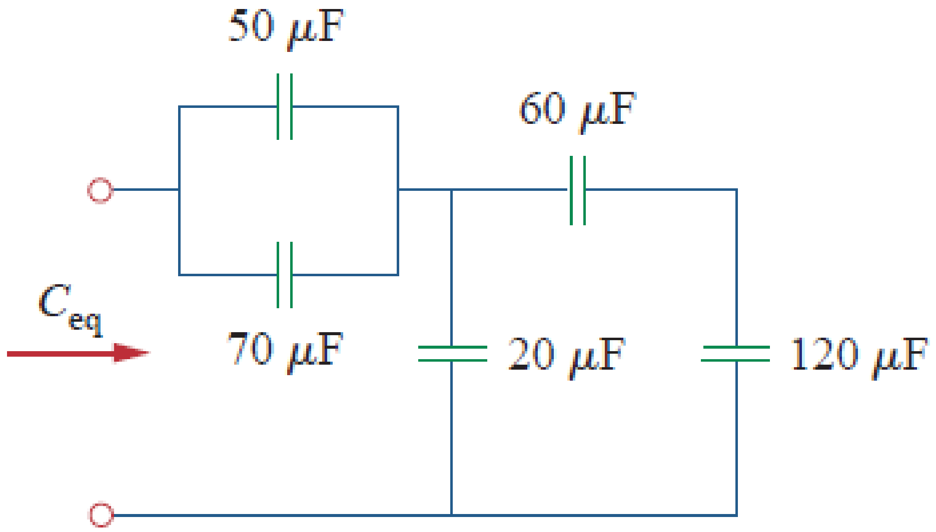


Solution

$$\frac{20 \cdot 5}{20+5} = 4\ \mu F \rightarrow 4+6+20 = 30\ \mu F \rightarrow \frac{30 \cdot 60}{30+60} = 20\ \mu F$$

Equivalent Capacitance

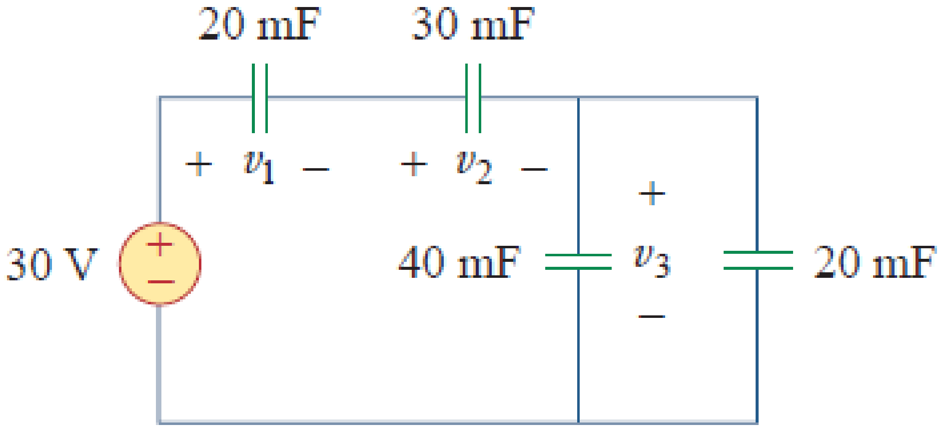
LCE.04 – Find the equivalent capacitance.



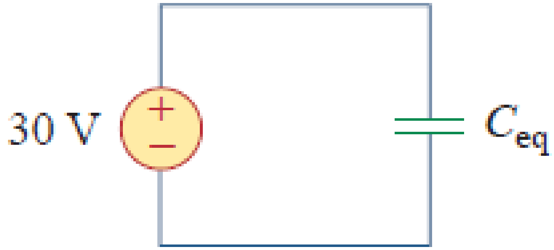
Solution $40\ \mu F$

Equivalent Capacitance

LCE.05 – Find the voltage across each capacitor.



Solution



$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} = 10\ mF$$

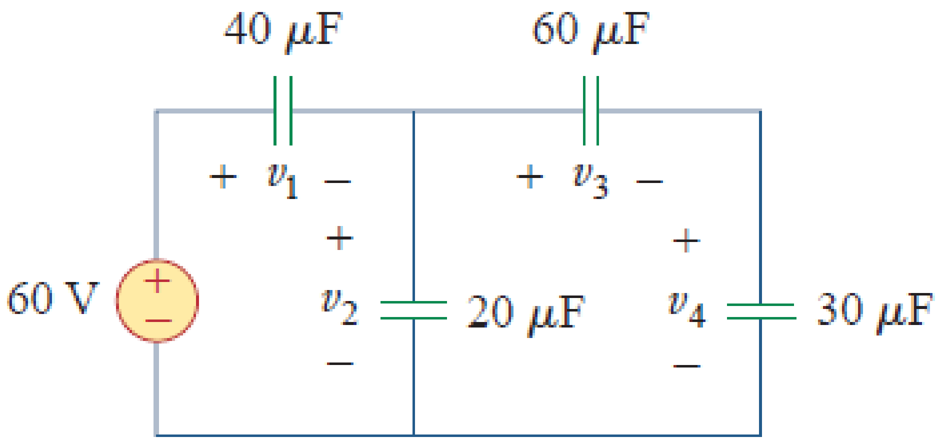
$$q = C_{eq} v = 10m \cdot 30 = 300\ mC$$

$$v_1 = \frac{q}{C_1} = \frac{300m}{20m} = 15\ V \qquad v_2 = \frac{q}{C_2} = \frac{300m}{30m} = 10\ V$$

$$v_3 = \frac{q}{C_{34}} = \frac{300m}{(40 + 20)m} = 5\ V\ or \ldots v_3 = 30 - v_1 - v_2 = 5\ V$$

Equivalent Capacitance

LCE.06 – Find the voltage across each capacitor.



Solution $v_1 = 30\ V\ ,\ \ v_2 = 30\ V\ ,\ \ v_3 = 10\ V\ ,\ \ v_4 = 20\ V$

Inductor Circuits

LCE.07 – $v_C = ?$, $i_L = ?$, $w_C = ?$ $w_L = ?$

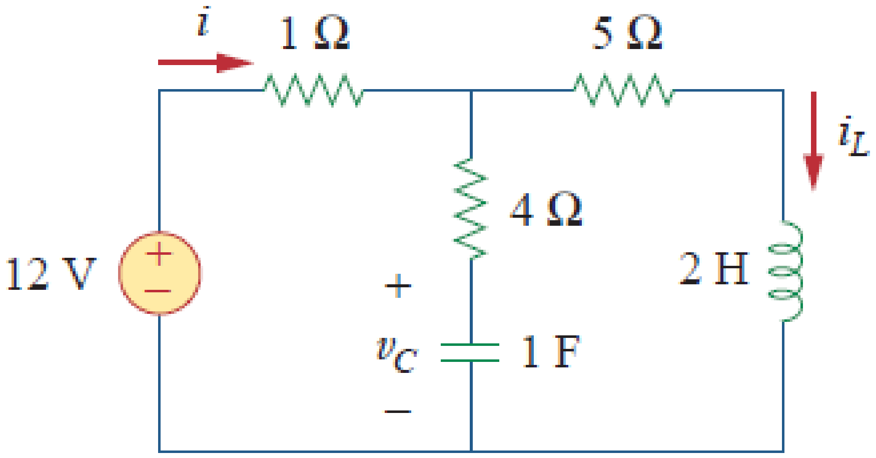
Solution

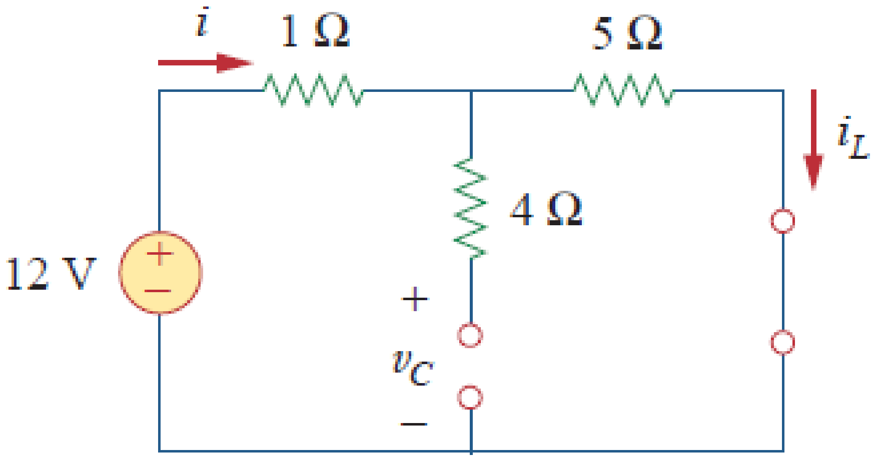
$$i = i_L = \frac{12}{1+5} = 2\ A$$

$$v_C = 5i = 10\ V$$

$$w_C = \frac{1}{2} C v_C^2 = \frac{1}{2} \cdot 1 \cdot 100 = 50\ J$$

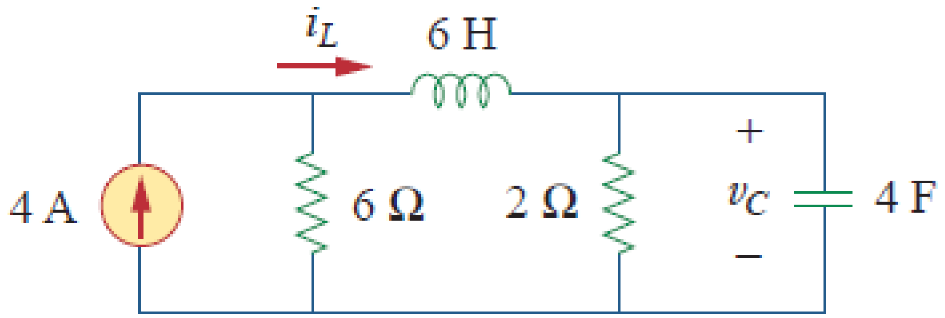
$$w_L = \frac{1}{2} L i_L^2 = \frac{1}{2} \cdot 2 \cdot 4 = 4\ J$$





Inductor Circuits

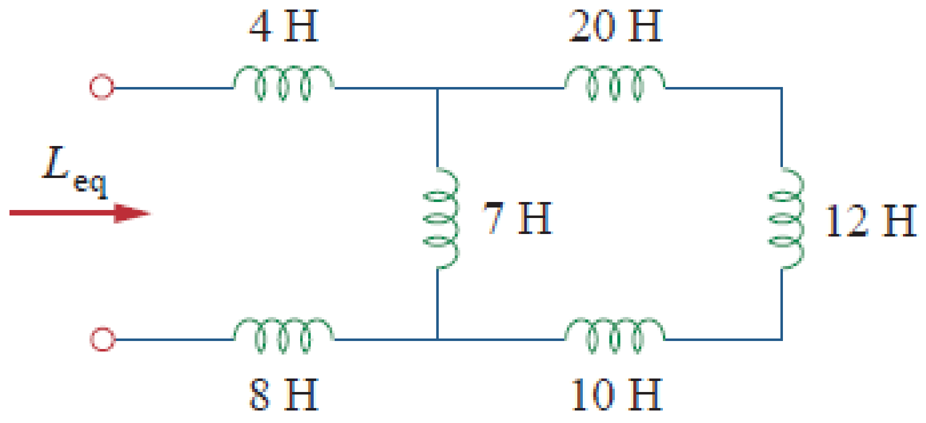
LCE.08 – v_C = ?, i_L = ?, w_C = ? w_L = ?



Solution $v_c = 6\ V\ ,\qquad i_L = 3\ \text{A}\ ,\qquad w_C = 72\ J\ ,\qquad w_L = 27\ J$

Equivalent Inductance

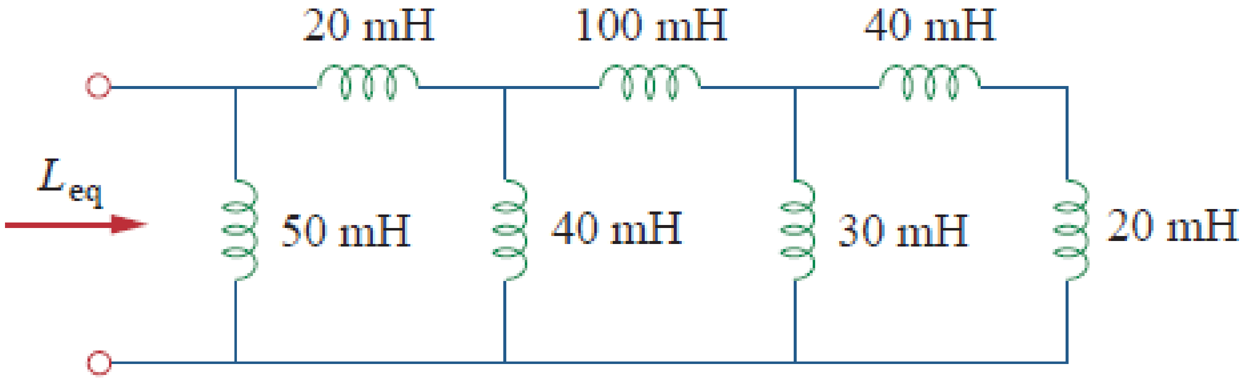
LCE.09 – Find the equivalent inductance.



Solution $L_{eq} = 4 + \dfrac{7 \times 42}{7 + 42} + 8 = 4 + 6 + 8 = 18\ H$

Equivalent Inductance

LCE.10 – Find the equivalent inductance.



Solution 25 mH

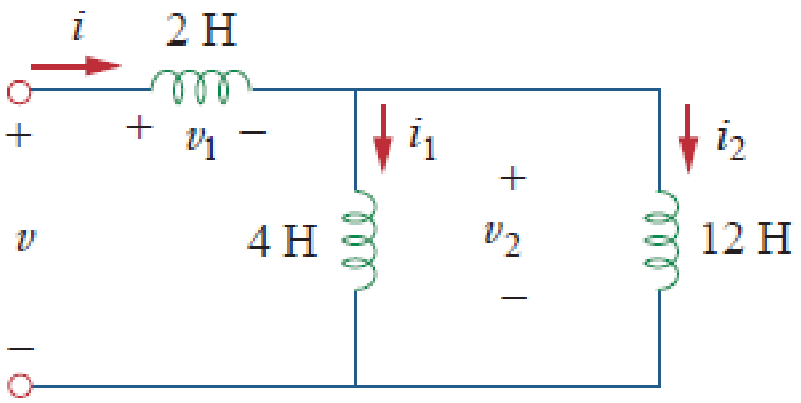
Equivalent Inductance

LCE.11 – $i(t) = 4(2 - e^{-10t})\ mA\ ,\qquad i_2(0) = -1\ mA$

$a)\ i_1(0) = ?$

$b)\ v(t) = ?,\ \ v_1(t) = ?\ ,\ v_2(t) = ?$

$c)\ i_1(t) = ?,\qquad i_2(t) = ?$



Solution

$$a)\ i(0) = 4(2 - e^0) = 4\ mA \rightarrow i_1(0) = i(0) - i_2(0) = 4 - (-1) = 5\ mA$$

$$b)\ L_{eq} = 2 + 4 \times 12 = 2 + 3 = 5\ H \qquad v(t) = L_{eq}\frac{di}{dt} = 5 \cdot 4 \cdot (-1)(-10)e^{-10t} = 200\ e^{-10t}\ mV$$

$$v_1(t) = 2\frac{di}{dt} = 2 \cdot (-4)(-10)e^{-10t} = 80\ e^{-10t}\ mV$$

$$v(t) = v_1(t) + v_2(t) \rightarrow v_2(t) = v(t) - v_1(t) = 120\ e^{-10t}\ mV$$

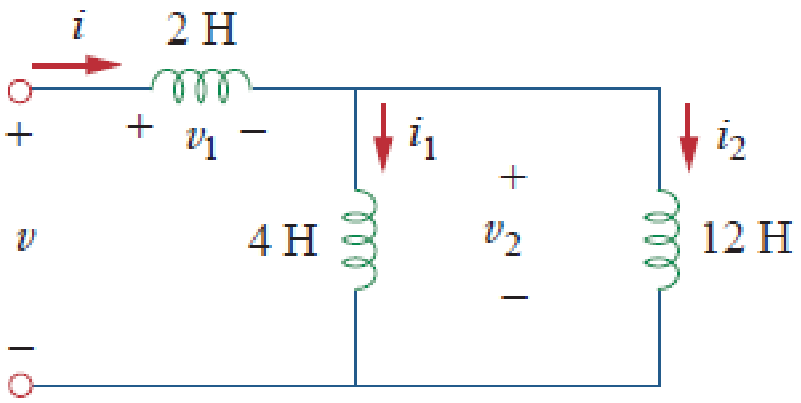
gyurcsek.istvan@mik.pte.hu

Equivalent Inductance

$c)\ i_1(t) = ?,\qquad i_2(t) = ?$

$$i_1(t) = \frac{1}{4}\int_0^t v_2 dt + i_1(0) = \frac{120}{4}\int_0^t e^{-10t} dt + 5\ mA$$

$$= -3e^{-10t}\Big|_0^t + 5\ mA = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t}\ mA$$



$$i_2(t) = \frac{1}{12}\int_0^t v_2 dt + i_2(0) = \frac{120}{12}\int_0^t e^{-10t} dt - 1\ mA$$

$$= -e^{-10t}\Big|_0^t - 1\ mA = -e^{-10t} + 1 - 1 = -e^{-10t}\ mA$$

$Check \rightarrow i(t) = i_1(t) + i_2(t)$

gyurcsek.istvan@mik.pte.hu

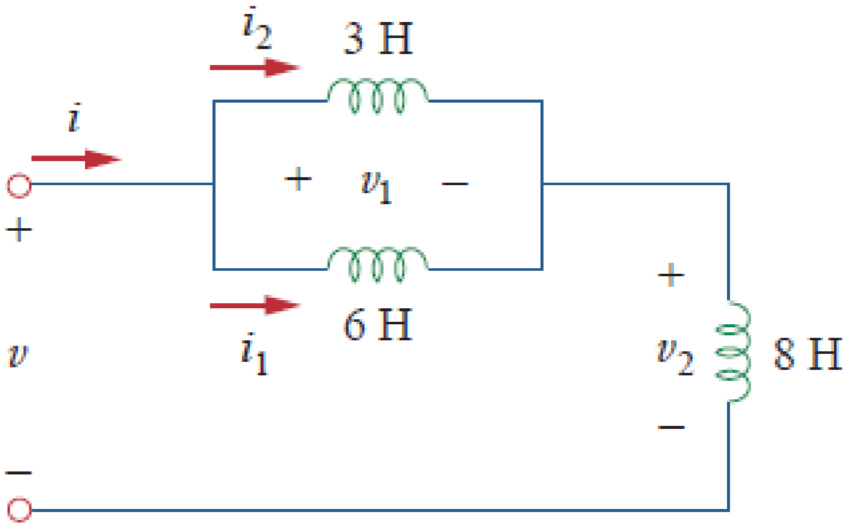
Equivalent Inductance

LCE.12 – $i_1(t) = 0.6e^{-2t}\ A\,, \qquad i(0) = 1.4\ A$

$a)\ i_2(0) = ?$

$b)\ i_2(t) = ?,\ \ i(t) = ?$

$c)\ v_1(t) = ?, \qquad v_2(t) = ?, \qquad v(t) = ?$



Solution $a)\ i_2(0) = 0.8\ A$

$b)\ i_2(t) = (-0.4 + 1.2e^{-2t})\ A,\ \ i(t) = (-0.4 + 1.8e^{-2t})\ A$

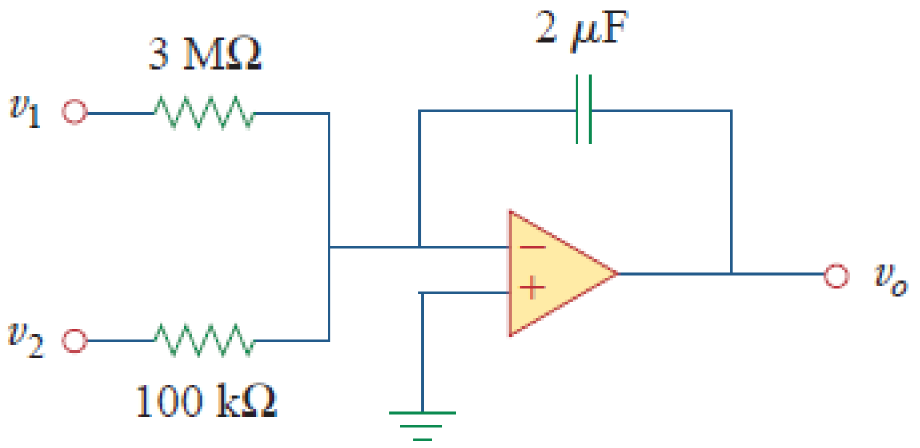
$c)\ v_1(t) = -36e^{-2t}\ V, \qquad v_2(t) = -7.2e^{-2t}\ V, \qquad v(t) = -28.8e^{-2t}\ V$

Integrator Example

LCE.13

$v_1 = 10\cos 2t \ mV, v_2 = 0.5t \ mV, v_C(0) = 0 \ V$

$v_o = ?$



Solution *(summing intergaror)*

$$v_0 = -\frac{1}{R_1C}\int v_1 dt - \frac{1}{R_2C}\int v_2 dt = -\frac{1}{3\cdot 10^6 \cdot 2\cdot 10^{-6}}\int_0^t 10\cos 2t \, dt - \frac{1}{100\cdot 10^3\cdot 2\cdot 10^{-6}}\int_0^t 0.5t \, dt$$

$$= -\frac{1}{6}\frac{10}{2}\sin 2t - \frac{1}{0.2}\frac{0.5t^2}{2} = -0.833\sin 2t - 1.25t^2 \ mV$$

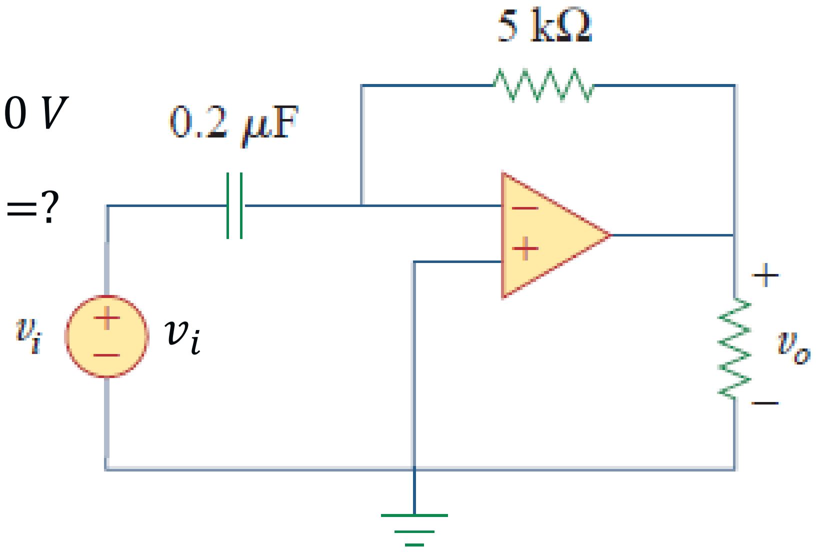
gyurcsek.istvan@mik.pte.hu

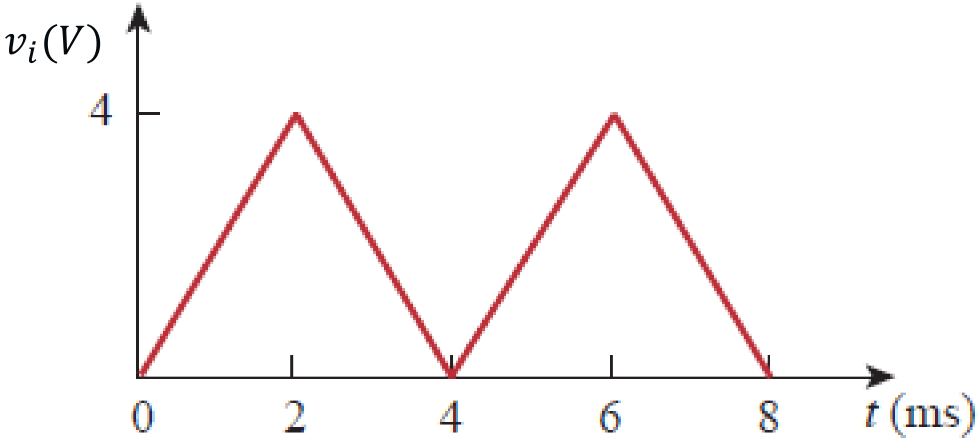
Differentiator Example

LCE.14

$v_o(0) = 0\ V$

$v_o(t) = ?$



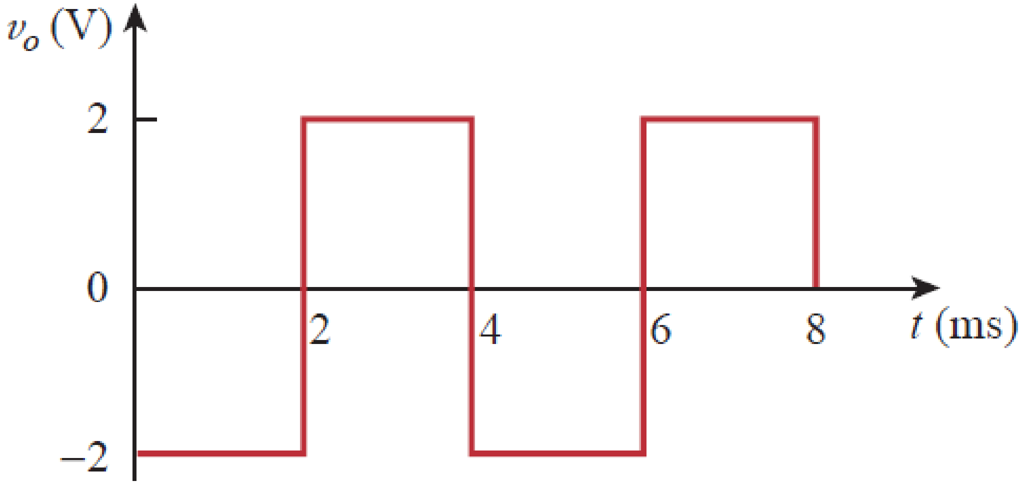


Solution *(differentiator circuit)*

$$RC = 5 \cdot 10^{-3} \cdot 2 \cdot 10^{-6} = 1\ ms$$

$$v_i = \begin{cases} 2000t & 0 < t < 2\ ms \\ 8 - 2000t & 2 < t < 4\ ms \end{cases}$$

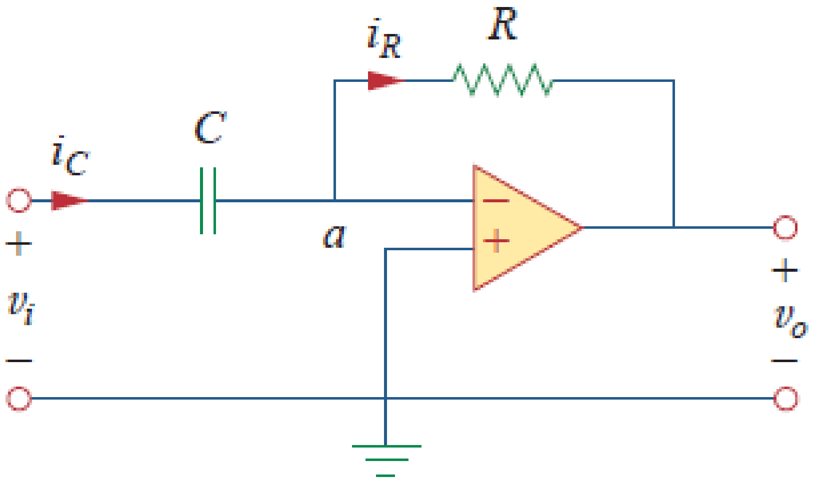
$$v_o = -RC\frac{dv_i}{dt} = \begin{cases} -2\ V & 0 < t < 2\ ms \\ +2\ V & 2 < t < 4\ ms \end{cases}$$



Differentiator Example

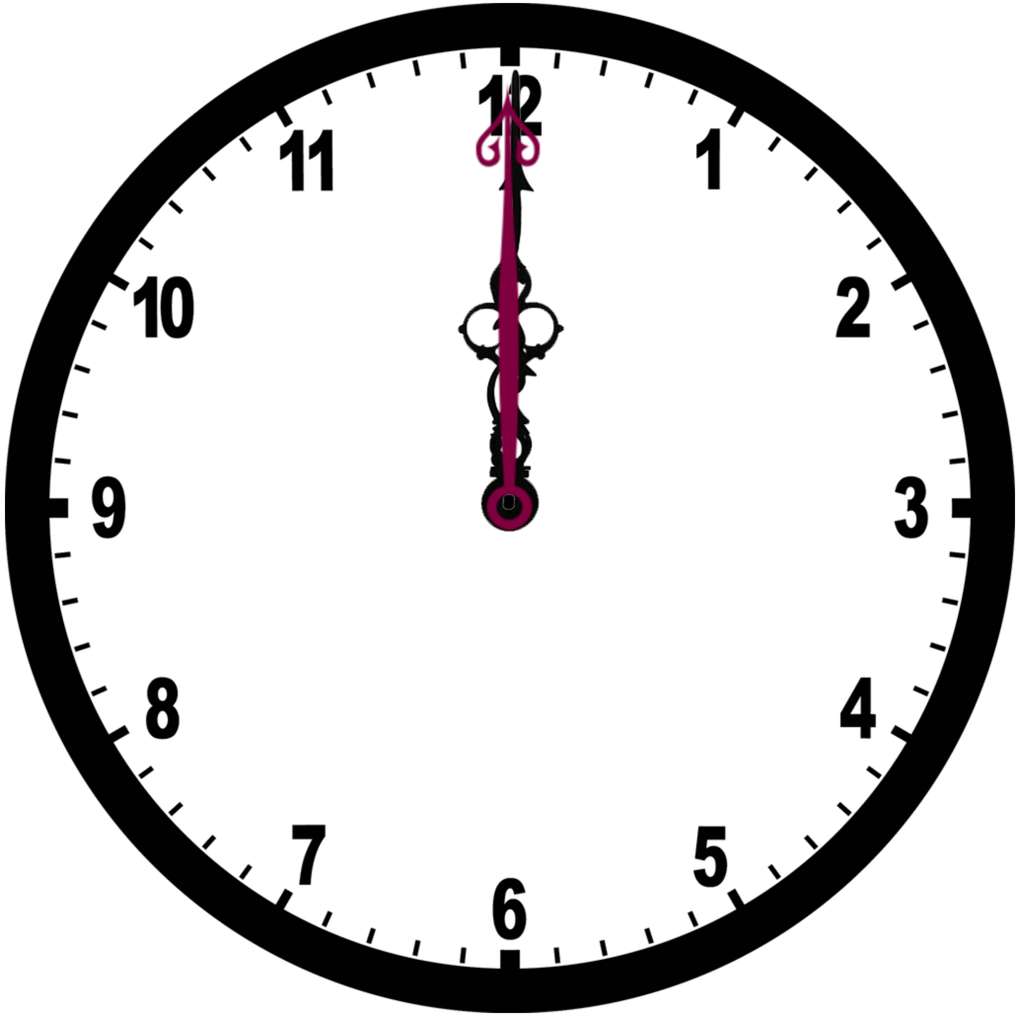
LCE.15

$R = 100\ k\Omega, C = 100\ nF, v_i(t) = 3t\ V, v_o(t) = ?$



Solution *(-30 mV)*

Questions



gyurcsek.istvan@mik.pte.hu