



DR. GYURCSEK ISTVÁN

Exercises with Sinusoids and Phasors

Sources and additional materials (recommended)

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- ❑ *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*

Sinusoids and Phasors



PHA.01

Find the amplitude, phase, period, frequency and angular frequency of the sinusoid $v(t) = 12 \cos(314t + 12^\circ) V$

PHA.02

Calculate the phase angle between $v_1(t) = -5 \cos(\omega t + 45^\circ) V$ and $v_2(t) = 12 \sin(\omega t - 15^\circ) V$
State which sinusoid is leading.

Solution: $v_1(t) = -5 \cos(\omega t + 45^\circ) = 5 \cos(\omega t + 45^\circ - 180^\circ) V$

$$v_1(t) = 5 \cos(\omega t - 135^\circ) V$$

$$v_2(t) = 12 \sin(\omega t - 15^\circ) = 12 \cos(\omega t - 15^\circ - 90^\circ) V$$

$$v_2(t) = 12 \cos(\omega t - 105^\circ) V$$

v_2 leads v_1 by 30°

Sinusoids Example



PHA.03 – Calculate the phase angle bw. v_1 and v_2 . $v_1 = -10 \cos(\omega t + 50^\circ)$, $v_2 = 12 \sin(\omega t - 10^\circ)$

Solution 1 → cosine form + positive amplitudes

$$v_1 = -10 \cos(\omega t + 50^\circ) = 10 \cos(\omega t + 50^\circ - 180^\circ) = 10 \cos(\omega t - 130^\circ)$$

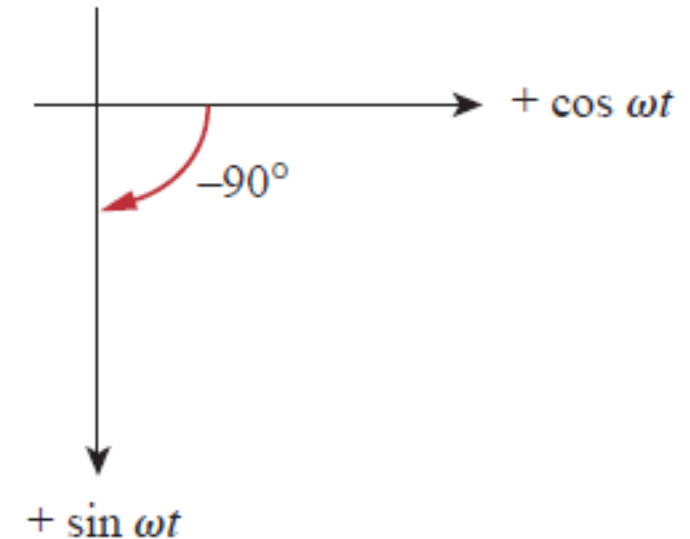
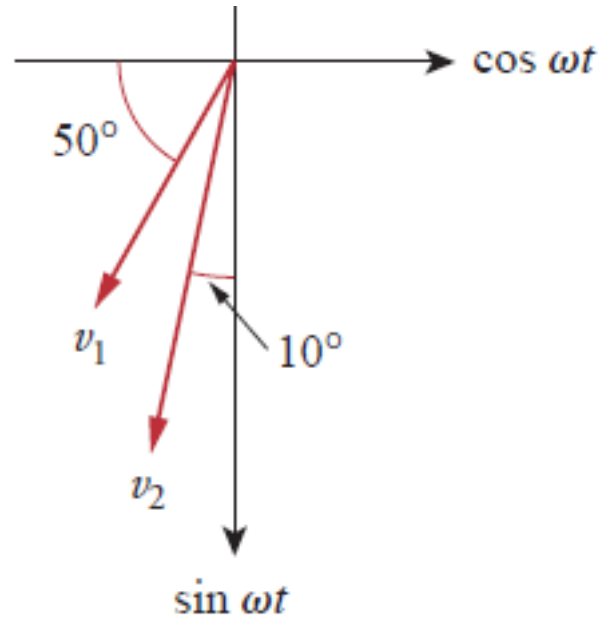
$$v_2 = 12 \sin(\omega t - 10^\circ) = 12 \cos(\omega t - 10^\circ - 90^\circ) = 12 \cos(\omega t - 100^\circ)$$

v_2 leads v_1 by 30°

Solution 2 → simply drawing v_1 and v_2

→

v_2 leads v_1 by 30°



Sinusoids Example



PHA.04 – Calculate the phase angle bw. i_1 and i_2 .

$$i_1 = -4 \sin(377t + 25^\circ) , \quad i_2 = 5 \cos(377t - 40^\circ)$$

Solution i_1 leads i_2 by 155°

Sinusoids and Phasors



PHA.05

Evaluate these complex numbers:

$$\sqrt[3]{25 e^{-j45^\circ} + 12 e^{j30^\circ}}$$

$$\frac{10 e^{-j30^\circ} + (3 - j3)}{(2 + j4)(3 - j5)^*}$$

Solution:

$$25 e^{-j45^\circ} = 25(\cos 45 - j \sin 45) = 17.68 - j17.68$$

$$12 e^{j30^\circ} = 12(\cos 30 + j \sin 30) = 10.39 + j6$$

$$25 e^{-j45^\circ} + 12 e^{j30^\circ} = 17.68 - j17.68 + 10.39 + j6 = 28.07 - j11.68 = 30.40 e^{-j22.59^\circ}$$

$$\sqrt[3]{25 e^{-j45^\circ} + 12 e^{j30^\circ}} = \sqrt[3]{30.40 e^{-j22.59^\circ}} = 3.12 e^{-j7.53^\circ} = 3.09 - j0.41$$

$$\frac{10 e^{-j30^\circ} + (3 - j3)}{(2 + j4)(3 - j5)^*} = \frac{(8.66 - j5) + (3 - j3)}{(2 + j4)(3 + j5)} = \frac{11.66 - j8}{-14 + j22} = \frac{14.14 e^{-j34.45^\circ}}{26.07 e^{j122.47^\circ}} = 0.542 e^{-j156.92^\circ}$$

Phasor Examples



PHA.06 – Evaluate the following complex numbers.

$$\square \quad [(5 + j2)(-1 + j4) - 5e^{j60^\circ}]^* = ?$$

$$\square \quad \frac{10 + j5 + 3e^{j40^\circ}}{-3 + j4} + 10e^{j30^\circ} + j5 = ?$$

Solution

$$\square \quad -15.5 - j13.67$$

$$\square \quad 8.293 + j7.2$$

Sinusoids and Phasors



PHA.07

(1). Transform these sinusoids to phasors: $v = -24 \sin(314t + 63^\circ) \text{ V}$, $i = 0.6 \cos(314t - 15^\circ) \text{ A}$

(2). Find the sinusoids represented by these phasors: $V = j3 e^{-j35^\circ}$, $I = -3 + j4$

Solution:

$$v = -24 \sin(314t + 63^\circ) = 24 \cos(314t + 63^\circ - 90^\circ + 180^\circ) = 24 \cos(314t + 153^\circ) \text{ V} \rightarrow V = 24 e^{j153^\circ} \text{ V}$$

$$i = 0.6 \cos(314t - 15^\circ) \text{ A} \rightarrow I = 0.6 e^{-j15^\circ} \text{ A}$$

$$V = j3 e^{-j35^\circ} = 3 e^{j90^\circ} e^{-j35^\circ} = 3 e^{j55^\circ} \text{ V} \rightarrow v(t) = 3 \cos(\omega t + 55^\circ) \text{ V}$$

$$I = -3 + j4 = 5 e^{j126.87^\circ} \text{ A} \rightarrow i(t) = 5 \cos(\omega t + 126.87^\circ) \text{ A}$$

Sinusoids and Phasors



PHA.08

Using the phasor approach, determine the current $i(t)$ in a circuit described by the integro-differential equation.

$$i + 2 \int i dt - 3 \frac{di}{dt} = 4 \cos(5t + 75^\circ) \text{ A}$$

Solution:

Transforming each term in the equation from time domain to phasor domain:

$$\mathbf{I} + 2 \frac{\mathbf{I}}{j\omega} - 3j\omega\mathbf{I} = 4 e^{j75^\circ} \text{ A}$$

$$\omega = 5 \frac{\text{rad}}{\text{s}} \rightarrow \mathbf{I} - j0.4\mathbf{I} - j15\mathbf{I} = (1 - j15.4)\mathbf{I} = 4 e^{j75^\circ}$$

$$\mathbf{I} = \frac{4 e^{j75^\circ}}{1 - j15.4} = \frac{4 e^{j75^\circ}}{15.43 e^{-j86.28^\circ}} = 0.258 e^{j161.28^\circ} \text{ A}$$

$$i(t) = 0.258 \cos(5t + 161.28^\circ)$$

Phasor Examples



PHA.09 – Find the sum of the following currents. $i_1 = 4 \cos(\omega t + 30^\circ) \text{ A}$, $i_2 = 5 \sin(\omega t - 20^\circ) \text{ A}$

Solution 1

$$I_1 = 4 e^{j30^\circ} = 3.464 + j2 \text{ A}$$

$$i_2 = 5 \sin(\omega t - 20^\circ) = 5 \cos(\omega t - 20^\circ - 90^\circ) \rightarrow I_2 = 5 e^{-j110^\circ} = -1.71 - j4.698 \text{ A}$$

$$I = I_1 + I_2 = 3.464 + j2 - 1.71 - j4.698 = 1.754 - j2.698 = 3.218 e^{-j56.97^\circ}$$

$$i = 3.218 \cos(\omega t - 56.97^\circ) \text{ A}$$

Solution 2 (hard way) $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

Phasor Relationships



PHA.10

The voltage $v = 24 \cos(314t + 45^\circ)$ V is applied to a 100 mH inductor. Find the steady-state current through the inductor.

Solution: $V = j\omega L \cdot I$ where $\omega = 314 \frac{\text{rad}}{\text{s}}$ ($f = 50\text{Hz}$) $\rightarrow V = 24 e^{j45^\circ}$ V

$$I = \frac{24 e^{j45^\circ}}{j31.4} = \frac{24 e^{j45^\circ}}{31.4 e^{j90^\circ}} = 0.764 e^{-j45^\circ} \text{ A} \qquad i(t) = 764 \cos(314t - 45^\circ) \text{ mA}$$

Phasor Relationships



PHA.11

The voltage $v = 6 \cos(100t - 30^\circ)$ V is applied to a $50 \mu\text{F}$ capacitor. Calculate the current through the capacitor.

Solution: $V = 6 \cdot e^{-j30^\circ}$

$$I = 100 \cdot e^{j90^\circ} \cdot 50 \cdot 10^{-6} \cdot 6 \cdot e^{-j30^\circ} = 30 \cdot 10^{-3} \cdot e^{j60^\circ} \text{ A}$$

$$I = \frac{V}{-jX_C} = j\omega C V$$

$$i(t) = 30 \cos(100t + 60^\circ) \text{ mA}$$

Impedance and Admittance



PHA.12

$$v_S(t) = 5 \sin(10t)$$

$$v(t) = ?, \quad i(t) = ?$$

Solution:

$$V_S = 5 \text{ V}$$

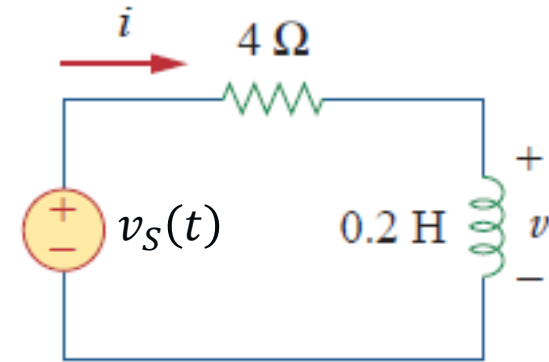
$$X_L = \omega L = 10 \cdot 0.2 = 2 \Omega$$

$$Z = R + jX_L = (4 + j2) \Omega$$

$$V = V_S \frac{jX_L}{R + jX_L} = 5 \frac{j2}{4 + j2} = \frac{j10}{4 + j2} = \frac{j5}{2 + j} \cdot \frac{2 - j}{2 - j} = \frac{j10 + 5}{5} = (1 + j2) = 2.24 e^{j63.43^\circ} \text{ V}$$

$$I = \frac{V}{jX_L} = \frac{2.24 e^{j63.43^\circ}}{2 e^{j90^\circ}} = 1.12 e^{-j26.57^\circ} \text{ A}$$

$$v(t) = 2.24 \sin(10t + 63.43^\circ) \text{ V}, \quad i(t) = 1.12 \sin(10t - 26.57^\circ) \text{ A}$$



Impedance and Admittance



PHA.13 $\omega = 50 \frac{\text{rad}}{\text{s}}$, $Z_{in} = ?$

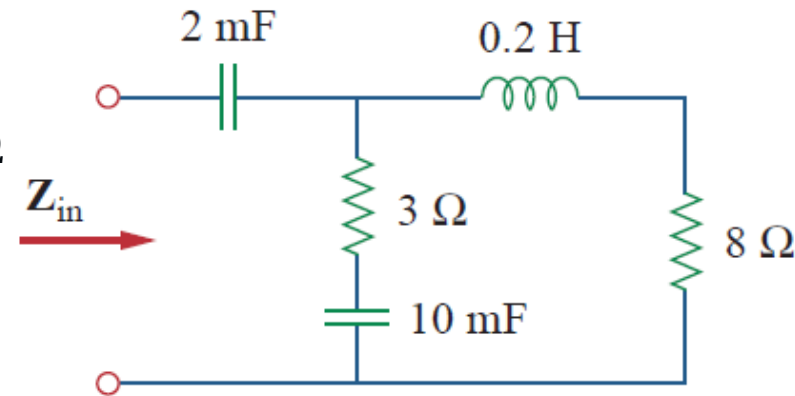
Solution: $Z_1 = \frac{1}{j\omega C} = \frac{1}{j \cdot 50 \cdot 2 \cdot 10^{-3}} = -j10 \Omega$

$$Z_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j \cdot 50 \cdot 10 \cdot 10^{-3}} = (3 - j2) \Omega$$

$$Z_3 = 8 + j\omega L = 8 + j \cdot 50 \cdot 0.2 = (8 + j10) \Omega$$

$$Z_{in} = Z_1 + Z_2 \times Z_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8} = -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2}$$

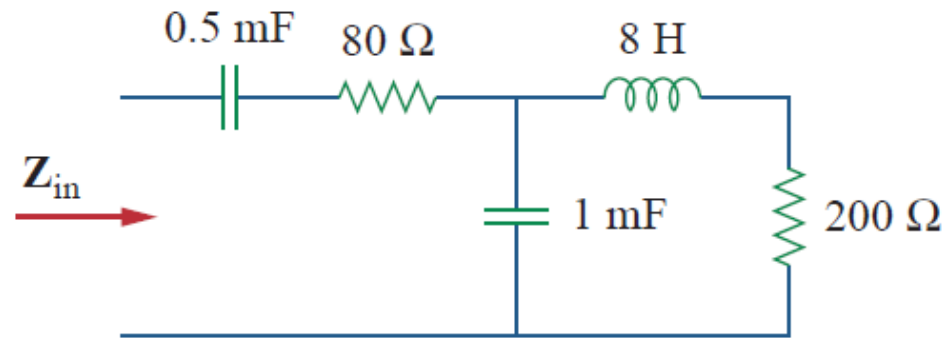
$$= -j10 + 3.22 - j1.07 = (3.22 - j11.07) \Omega$$



Impedance and Admittance



PHA.14 $\omega = 10 \frac{\text{rad}}{\text{s}}$, $Z_{in} = ?$



Solution: $Z_{in} = (129.52 - j295) \Omega$

Impedance and Admittance



PHA.15 – $v_S(t) = 20 \cos(4t - 15^\circ) \text{ V}$, $v_0(t) = ?$

Solution: $v_S(t) = 20 \cos(4t - 15^\circ) \rightarrow \mathbf{V}_S = 20 e^{-j15^\circ}$, ($\omega = 4$)

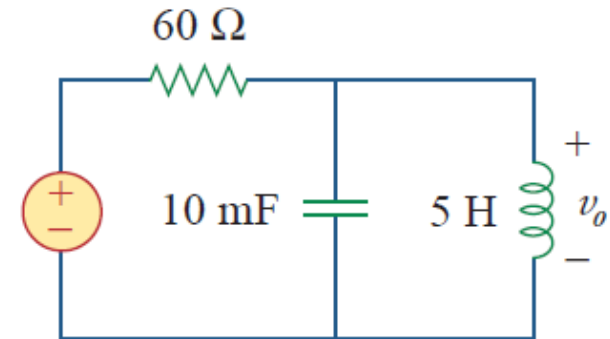
$$X_C = \frac{1}{\omega C} = 25 \Omega \rightarrow \mathbf{Z}_C = -j25 \Omega$$

$$X_L = \omega L = 20 \Omega \rightarrow \mathbf{Z}_L = j20 \Omega$$

$$\mathbf{Z}_{LC} = \mathbf{Z}_L \times \mathbf{Z}_C = \frac{j20 \cdot (-j25)}{-j5} = j100 \Omega$$

$$\mathbf{V}_0 = \mathbf{V}_S \frac{\mathbf{Z}_{LC}}{60 + \mathbf{Z}_{LC}} = 20 e^{-j15^\circ} \frac{j100}{60 + j100} = \dots = 17.15 e^{j15.96^\circ} \rightarrow v_0(t) = 17.15 \cos(4t + 15.96^\circ) \text{ V}$$

$$\text{or ... nodal analysis} \rightarrow \frac{V_S - V_0}{60} = \frac{V_0 - 0}{-j25} + \frac{V_0 - 0}{j20} \rightarrow \dots \rightarrow \mathbf{V}_0 = 17.15 e^{j15.96^\circ}$$



Questions

