DR. GYURCSEK ISTVÁN

Exercises with Sinusoids and Phasors

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*

Sinusoids and Phasors

PHA.01

Find the amplitude, phase, period, frequency and angular frequency of the sinusoid $v(t) = 12\,cos(314t + 12°)\ V$

PHA.02

Calculate the phase angle between $v_1(t) = -5\,cos(\omega t + 45°)\ V$ and $v_2(t) = 12\,sin(\omega t - 15°)\ V$
State which sinusoid is leading.

Solution: $v_1(t) = -5\,cos(\omega t + 45°) = 5\,cos(\omega t + 45° - 180°)\ V$

$$v_1(t) = 5\,cos(\omega t - 135°)\ V$$

$$v_2(t) = 12\,sin(\omega t - 15°) = 12\,cos(\omega t - 15° - 90°)\ V$$

$$v_2(t) = 12\,cos(\omega t - 105°)\ V$$

v_2 leads v_1 by 30°

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Sinusoids Example

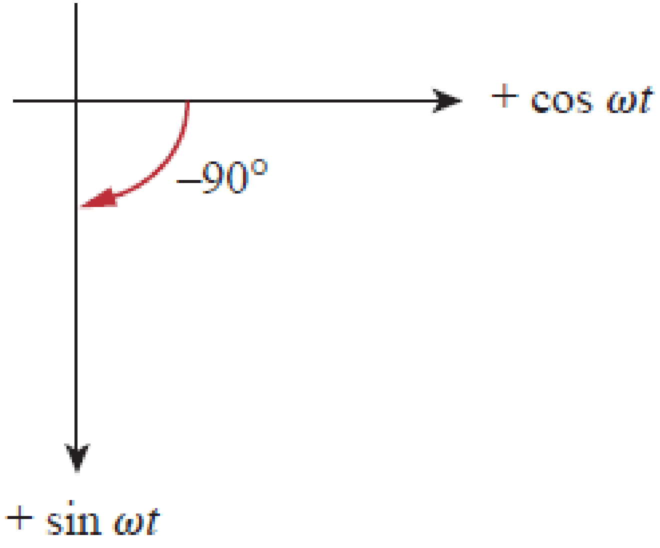
PHA.03 – Calculate the phase angle bw. v_1 and v_2. $v_1 = -10\cos(\omega t + 50°)$, $v_2 = 12\sin(\omega t - 10°)$

Solution 1 → cosine form + positive amplitudes

$$v_1 = -10\cos(\omega t + 50°) = 10\cos(\omega t + 50° - 180°) = 10\cos(\omega t - 130°)$$

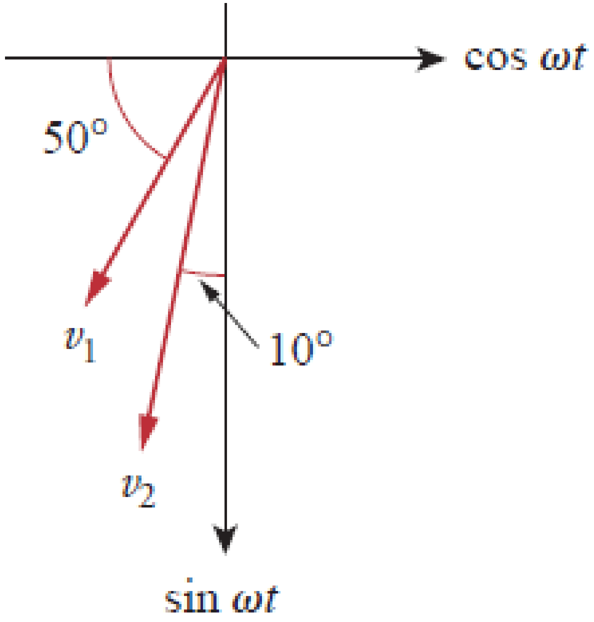
$$v_2 = 12\sin(\omega t - 10°) = 12\cos(\omega t - 10° - 90°) = 12\cos(\omega t - 100°)$$

$v_2\ leads\ v_1\ by\ 30°$



Solution 2 → simply drawing v_1 and v_2

→



$v_2\ leads\ v_1\ by\ 30°$

Sinusoids Example

PHA.04 – Calculate the phase angle bw. i_1 and i_2.

$$i_1 = -4\sin(377t + 25°)\ , \qquad i_2 = 5\cos(377t - 40°)$$

Solution $\quad i_1\ leads\ i_2\ by\ 155°$

PHA.05

Evaluate these complex numbers:

$$\sqrt[3]{25\ e^{-j45°} + 12\ e^{j30°}} \qquad \frac{10\ e^{-j30°} + (3 - j3)}{(2 + j4)(3 - j5)^*}$$

Solution:

$$25\ e^{-j45°} = 25(cos\,45 - j\ sin\,45) = 17.68 - j17.68$$

$$12\ e^{j30°} = 12(cos\,30 + j\ sin\,30) = 10.39 + j6$$

$$25\ e^{-j45°} + 12\ e^{j30°} = 17.68 - j17.68 + 10.39 + j6 = 28.07 - j11.68 = 30.40\ e^{-j22.59°}$$

$$\sqrt[3]{25\ e^{-j45°} + 12\ e^{j30°}} = \sqrt[3]{30.40\ e^{-j22.59°}} = 3.12\ e^{-j7.53°} = 3.09 - j0.41$$

$$\frac{10\ e^{-j30°} + (3 - j3)}{(2 + j4)(3 - j5)^*} = \frac{(8.66 - j5) + (3 - j3)}{(2 + j4)(3 + j5)} = \frac{11.66 - j8}{-14 + j22} = \frac{14.14\ e^{-j34.45°}}{26.07\ e^{j122.47°}} = 0.542\ e^{-j156.92°}$$

Phasor Examples

PHA.06 – Evaluate the folowing complex numbers.

- ❑ $\left[(5-j2)(-1+j4)-5e^{j60°}\right]^{*}=?$
- ❑ $\dfrac{10+j5+3e^{j40°}}{-3+j4}+10e^{j30°}+j5=?$

Solution

- ❑ $-15.5-j13.67$
- ❑ $8.293+j7.2$

PHA.07

(1). Transform these sinusoids to phasors: $v = -24\,sin(314t + 63°)\ V$, $i = 0.6\,cos(314t - 15°)\ A$

(2). Find the sinusoids represented by these phasors: $\boldsymbol{V} = j3\ e^{-j35°}$, $\boldsymbol{I} = -3 + j4$

Solution:

$$v = -24\,sin(314t + 63°) = 24\,cos(314t + 63° - 90° + 180°) = 24\,cos(314t + 153°)\ V\ \rightarrow \boldsymbol{V} = 24\ e^{j153°}\ V$$

$$i = 0.6\,cos(314t - 15°)\ A \rightarrow \boldsymbol{I} = 0.6\ e^{-j15°}\ A$$

$$\boldsymbol{V} = j3\ e^{-j35°} = 3\ e^{j90°}e^{-j35°} = 3\ e^{j55°}\ V \qquad \rightarrow v(t) = 3\,cos(\omega t + 55°)\ V$$

$$\boldsymbol{I} = -3 + j4 = 5\ e^{j126.87°}\ A\ \rightarrow i(t) = 5\,cos(\omega t + 126.87°)\,A$$

PHA.08

Using the phasor approach, determine the current i(t) in a circuit described by the integro-differential equation.

$$i + 2\int i\, dt - 3\frac{di}{dt} = 4\, cos(5t + 75°)\ \mathrm{A}$$

Solution:

Transforming each term in the equation from time domain to phasor domain:

$$\boldsymbol{I} + 2\frac{\boldsymbol{I}}{j\omega} - 3j\omega\boldsymbol{I} = 4\, e^{j75°}\ A$$

$$\omega = 5\frac{rad}{s} \rightarrow \boldsymbol{I} - j0.4\boldsymbol{I} - j15\boldsymbol{I} = (1 - j15.4)\boldsymbol{I} = 4\, e^{j75°}$$

$$\boldsymbol{I} = \frac{4\, e^{j75°}}{1 - j15.4} = \frac{4\, e^{j75°}}{15.43\, e^{-j86.28°}} = 0.258\, e^{j161.28°}\ \ A$$

$$i(t) = 0.258\, cos\, (5t + 161.28°)$$

PHA.09 – Find the sum of the following currents. $i_1 = 4\cos(\omega t + 30°)\ A$, $i_2 = 5\sin(\omega t - 20°)\ A$

Solution 1

$$\boldsymbol{I}_1 = 4\, e^{j30°} = 3.464 + j2\ A$$

$$i_2 = 5\sin(\omega t - 20°) = 5\cos(\omega t - 20° - 90°) \rightarrow \boldsymbol{I}_2 = 5\, e^{-j110°} = -1.71 - j4.698\ A$$

$$\boldsymbol{I} = \boldsymbol{I}_1 + \boldsymbol{I}_2 = 3.464 + j2 - 1.71 - j4.698 = 1.754 - j2.698 = 3.218\, e^{-j56.97°}$$

$$i = 3.218\cos(\omega t - 56.97°)\ A$$

Solution 2 *(hard way)* $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Phasor Relationships for RLC Examples

PHA.10

The voltage $v = 24\,cos(314t + 45°)\ V$ is applied to a 100 mH inductor. Find the steady-state current through the inductor.

Solution: $\boldsymbol{V} = j\omega L \cdot \boldsymbol{I}\ where\ \omega = 314\dfrac{rad}{s}\ (f = 50Hz) \rightarrow \boldsymbol{V} = 24\,e^{j45°}\,V$

$$\boldsymbol{I} = \frac{24\,e^{j45°}}{j31.4} = \frac{24\,e^{j45°}}{31.4\,e^{j90°}} = 0.764\,e^{-j45°}\,A \qquad i(t) = 764\,cos(314t - 45°)\ mA$$

Phasor Relationships for RLC Examples

PHA.11

The voltage $v = 6\,cos(100t - 30°)\ V$ is applied to a 50 μF capacitor. Calculate the current through the capacitor.

Solution: $\boldsymbol{V} = 6 \cdot e^{-j30°}$

$$\boldsymbol{I} = 100 \cdot e^{j90°} \cdot 50 \cdot 10^{-6} \cdot 6 \cdot e^{-j30°} = 30 \cdot 10^{-3} \cdot e^{j60°}\ A$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{-jX_C} = j\omega C\ \boldsymbol{V}$$

$$i(t) = 30\,cos(100t + 60°)\ mA$$

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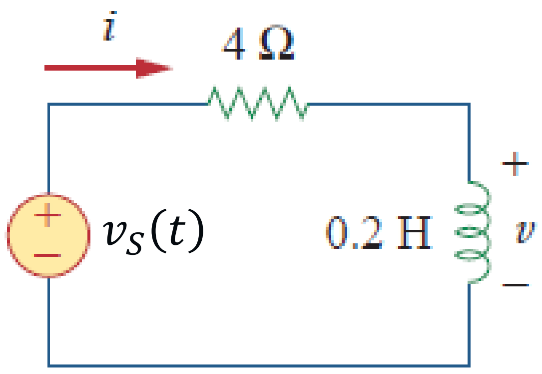
PHA.12 $v_S(t) = 5\,sin(10t)$

$v(t) = ?, \qquad i(t) = ?$



Solution: $\boldsymbol{V}_S = 5\,V \qquad X_L = \omega L = 10 \cdot 0.2 = 2\,\Omega$

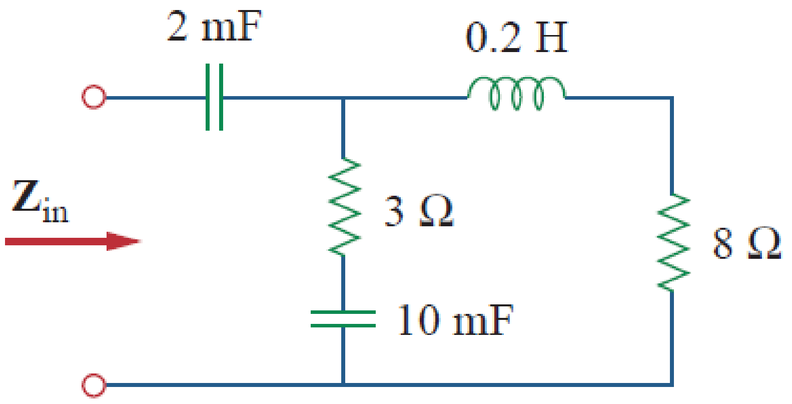
$$\boldsymbol{Z} = R + jX_L = (4 + j2)\,\Omega$$

$$\boldsymbol{V} = \boldsymbol{V}_S \frac{jX_L}{R + jX_L} = 5\frac{j2}{4 + j2} = \frac{j10}{4 + j2} = \frac{j5}{2 + j} \cdot \frac{2 - j}{2 - j} = \frac{j10 + 5}{5} = (1 + j2) = 2.24\,e^{j63.43°}\,V$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{jX_L} = \frac{2.24\,e^{j63.43°}}{2\,e^{j90°}} = 1.12\,e^{-j26.57°}\,A$$

$$v(t) = 2.24\sin(10t + 63.43°)\,V, \qquad i(t) = 1.12\sin(10t - 26.57°)\,A$$

PHA.13 $\omega = 50\frac{rad}{s}$, $\boldsymbol{Z}_{in} = ?$



Solution: $\boldsymbol{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j \cdot 50 \cdot 2 \cdot 10^{-3}} = -j10\ \Omega$

$$\boldsymbol{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j \cdot 50 \cdot 10 \cdot 10^{-3}} = (3 - j2)\ \Omega$$

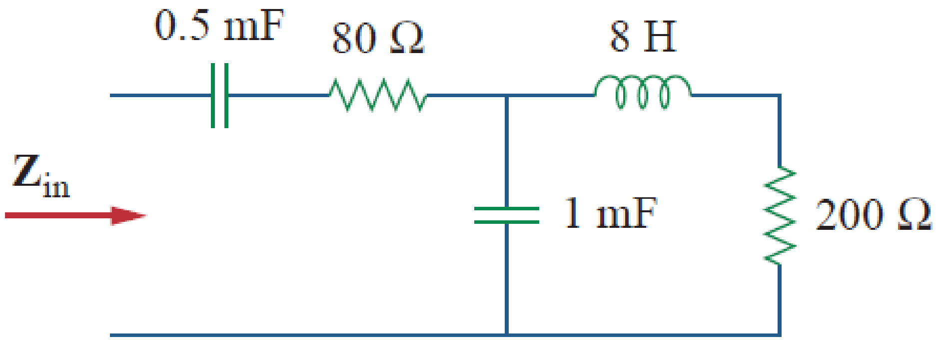
$$\boldsymbol{Z}_3 = 8 + j\omega L = 8 + j \cdot 50 \cdot 0.2 = (8 + j10)\ \Omega$$

$$\boldsymbol{Z}_{in} = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 \times \boldsymbol{Z}_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8} = -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2}$$

$$= -j10 + 3.22 - j1.07 = (3.22 - j11.07)\ \Omega$$

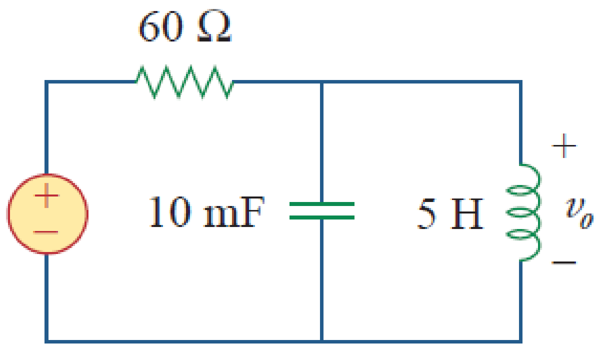
Impedance and Admittance Examples

PHA.14 $\omega = 10\dfrac{rad}{s}$, $\boldsymbol{Z}_{in} =?$



Solution: $\boldsymbol{Z}_{in} = (129.52 - j295)\ \Omega$

PHA.15 – $v_S(t) = 20\cos(4t - 15°)\ V$, $v_0(t) = ?$



Solution: $v_S(t) = 20\cos(4t - 15°) \rightarrow \boldsymbol{V}_S = 20\ e^{-j15°}, (\omega = 4)$

$$X_C = \frac{1}{\omega C} = 25\ \Omega \rightarrow \boldsymbol{Z}_C = -j25\ \Omega$$

$$X_L = \omega L = 20\ \Omega \rightarrow \boldsymbol{Z}_L = j20\ \Omega$$

$$\boldsymbol{Z}_{LC} = \boldsymbol{Z}_L \times \boldsymbol{Z}_C = \frac{j20 \cdot (-j25)}{-j5} = j100\ \Omega$$

$$\boldsymbol{V}_o = \boldsymbol{V}_S \frac{\boldsymbol{Z}_{LC}}{60 + \boldsymbol{Z}_{LC}} = 20\ e^{-j15°} \frac{j100}{60 + j100} = \cdots = 17.15\ e^{j15.96°} \rightarrow v_0(t) = 17.15\cos(4t + 15.96°)\ V$$

Questions