

DR. GYURCSEK ISTVÁN

Exercises in AC Solid State Analysis

Sources and additional materials (recommended)

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

AC Nodal Analysis

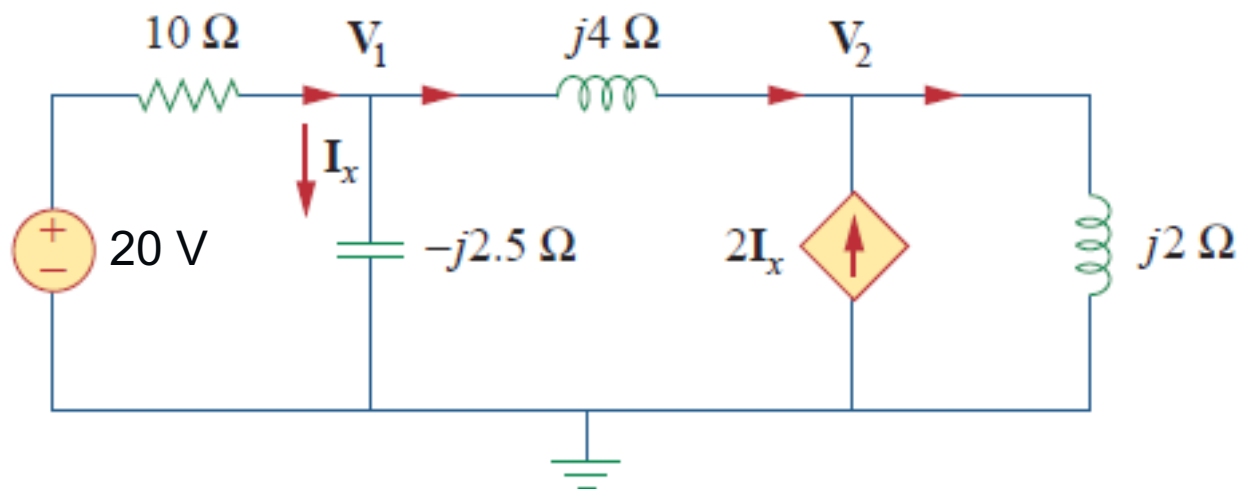
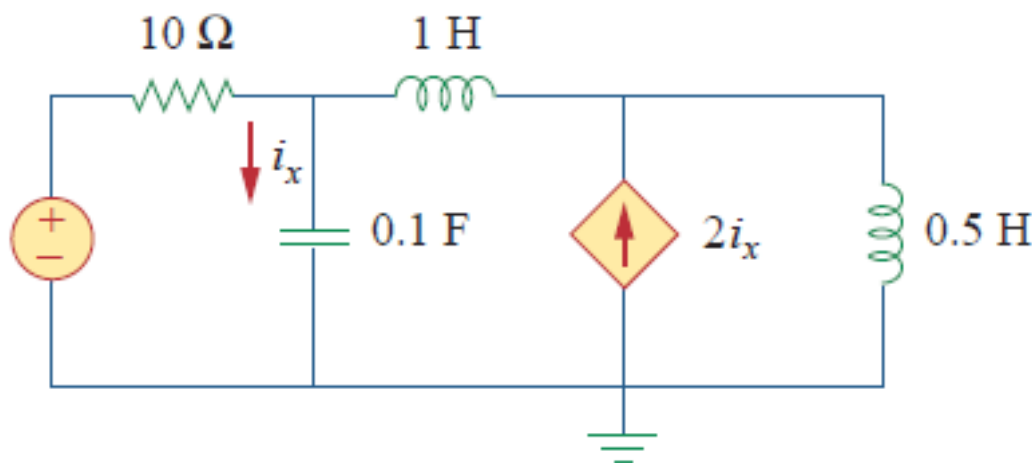
SSA.01 – Find i_x in the circuit.

Solution $Z_C = -j \frac{1}{\omega C} = -j2.5 \Omega$

$$Z_{L1} = j\omega L_1 = j4 \Omega$$

$$v_S = 20 \cos 4t V$$

$$Z_{L2} = j\omega L_2 = j2 \Omega$$



$$\frac{20 - V_1}{10} = \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4}$$

$$2I_x + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2} \quad (\&) \quad I_x = \frac{V_1}{-j2.5}$$

AC Nodal Analysis

$$\frac{20 - V_1}{10} = \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4} \rightarrow (1 + j1.5)V_1 + j2.5V_2 = 20$$

$$2I_x + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2} \quad (\&) \quad I_x = \frac{V_1}{-j2.5} \rightarrow 11V_1 + 15V_2 = 0$$

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$I_x = \frac{V_1}{-j2.5} = \frac{18.97 e^{j18.43^\circ}}{2.5 e^{-j90^\circ}} = 7.59 e^{j108.4^\circ}$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300$$

$$\Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}$$

AC Mesh Analysis

SSA.02 – Determine current I_o in the circuit.

Solution $(8 + j10 - j2)I_1 - (-j2)I_2 - j10I_3 = 0$

$$(4 - j2 - j2)I_2 - (-j2)I_1 - (-j2)I_3 + j20 = 0$$

$$I_3 = 5$$

$$(8 + j8)I_1 + j2I_2 = j50$$

$$j2I_1 + (4 - j4)I_2 = -j20 - j10$$

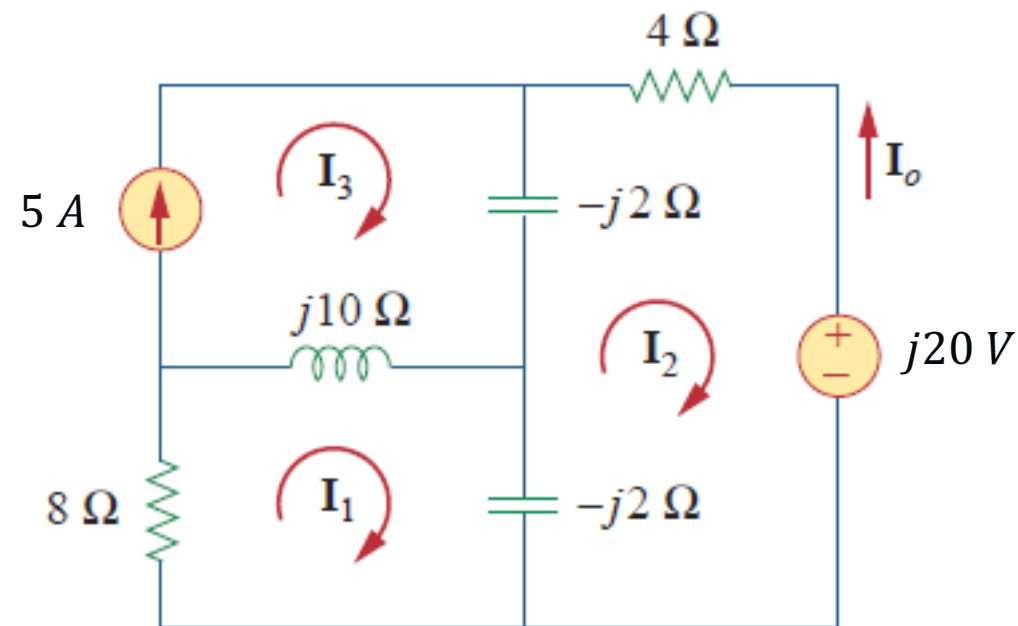
$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17 e^{-j35.22}$$

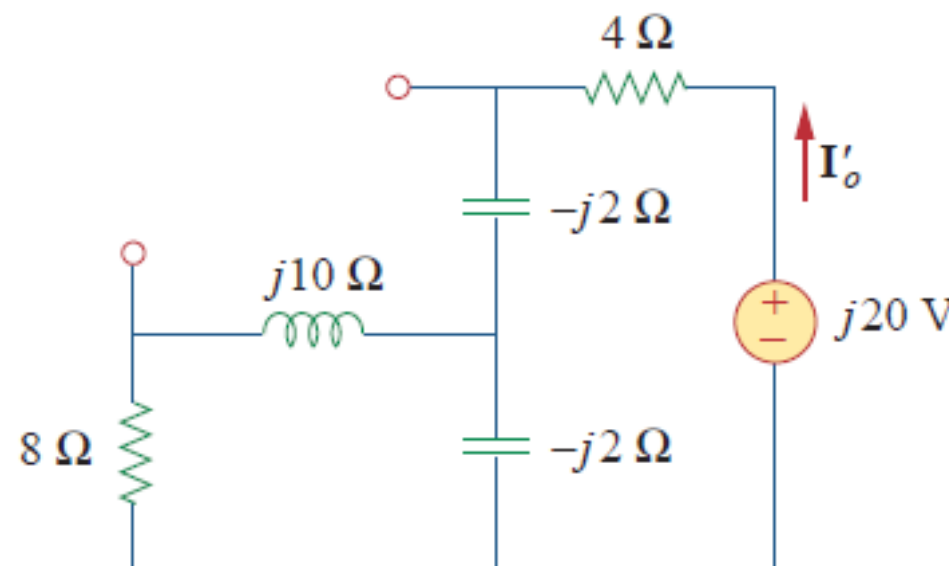
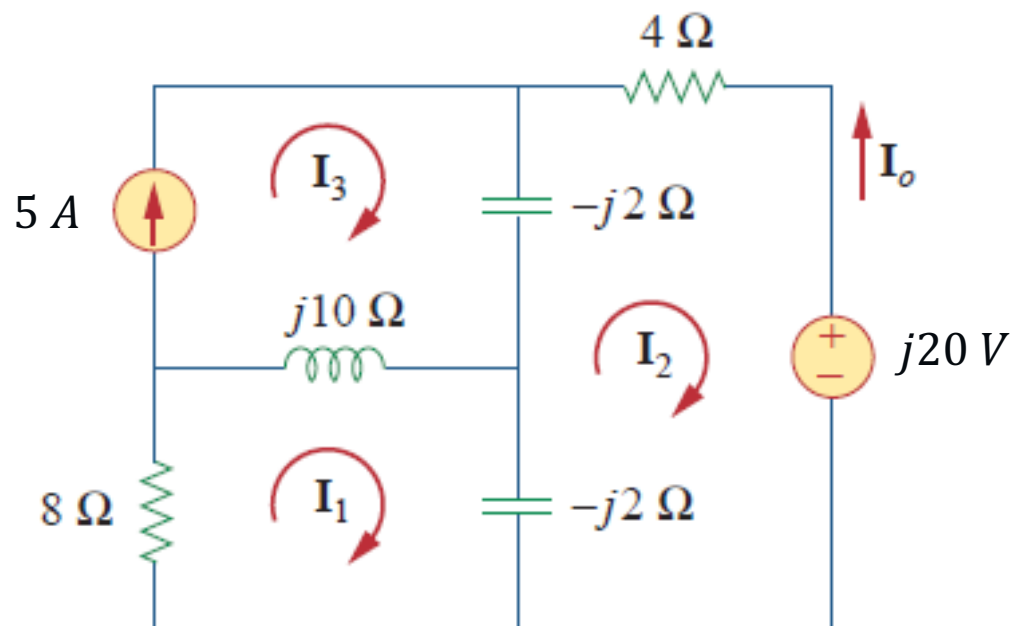
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{416.17 e^{-j35.22}}{68} = 6.12 e^{-j35.22}$$

$$I_o = -I_2 = 6.12 e^{j144.78}$$



Superposition Theorem in AC Circuits

SSA.03 – Find I_o in the circuit.

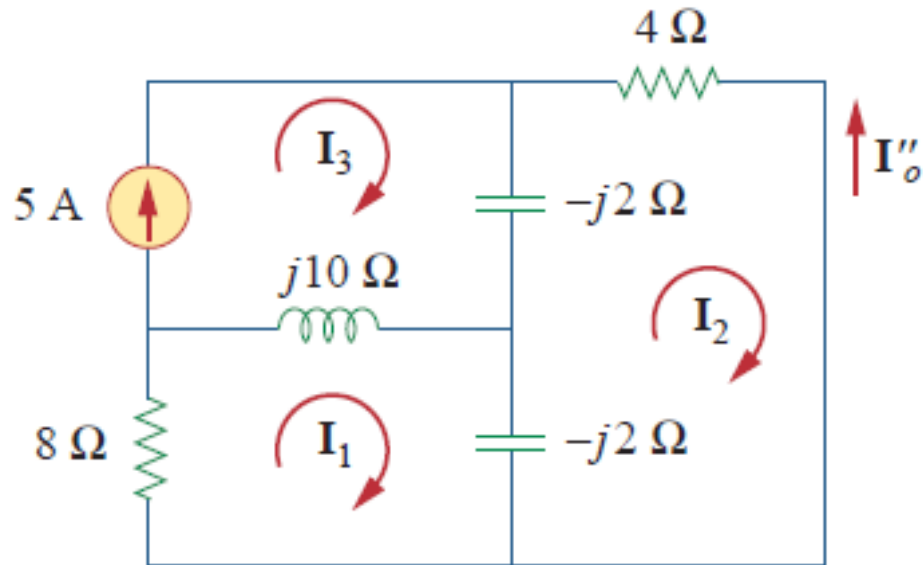


Solution $I_o = I'_o + I''_o$

$$\mathbf{Z} = \frac{-j2(8 + j10)}{-j2 + 8 + j10} = 0.25 - j2.25$$

$$I'_o = \frac{j20}{4 - j2 + \mathbf{Z}} = \frac{j20}{4.25 - j4.25} = -2.353 + j2.353$$

Superposition Theorem in AC Circuits



$$(8 + j8)I_1 - j10I_3 + j2I_2 = 0$$

$$(4 - j4)I_2 + j2I_1 + j2I_3 = 0$$

$$I_3 = 5$$

$$(4 - j4)I_2 + j2I_1 + j10 = 0 \rightarrow I_1 = (2 + j2)I_2 - 5$$

$$(8 + j8)[(2 + j2)I_2 - 5] - j50 + j2I_2 = 0$$

$$I_2 = \frac{90 - j40}{34} = 2.647 - j1.176$$

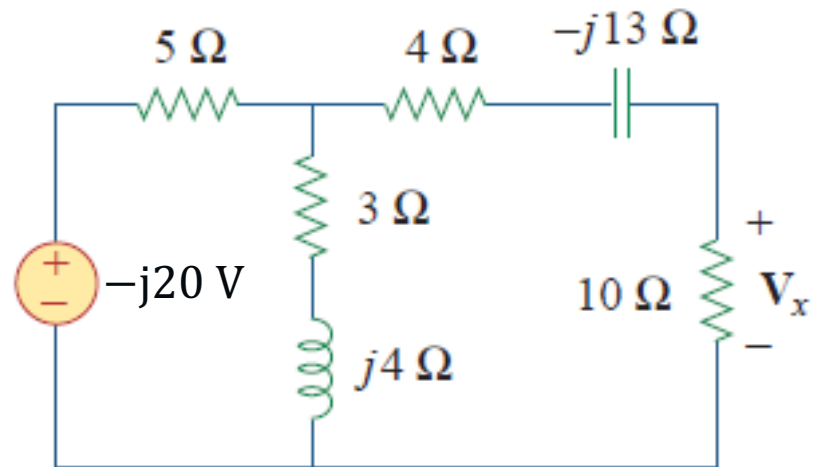
$$I''_0 = -I_2 = -2.647 + j1.176$$

$$I_0 = I'_0 + I''_0 = -2.353 + j2.353 - 2.647 + j1.176$$

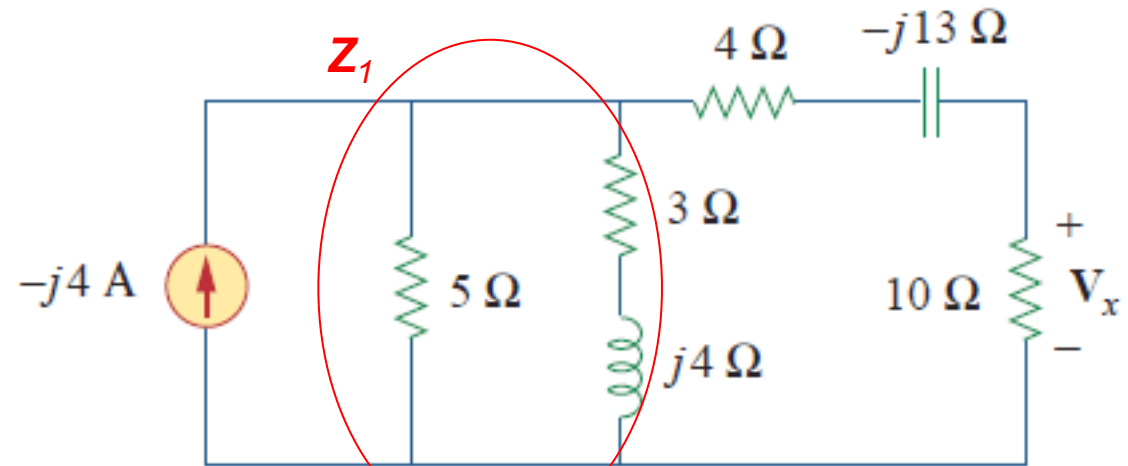
$$I_0 = -5 + j3.529 = 6.12 e^{j144.78^\circ} \text{ A}$$

AC Source Transformation

SSA.04 – Calculate V_x in the circuit.



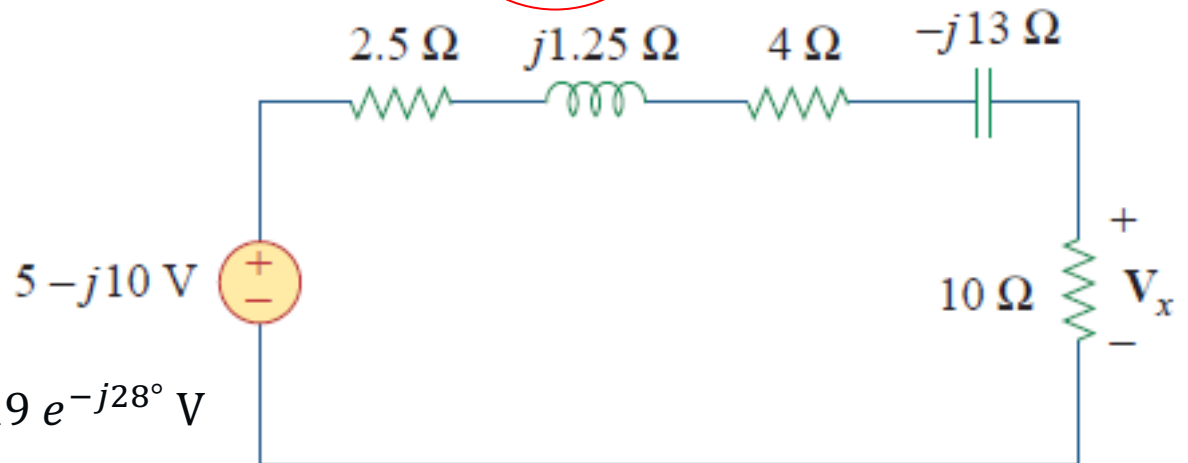
Solution



$$Z_1 = \frac{5(3 + j4)}{8 + j4} = 2.5 + j1.25 \Omega$$

$$V_S = I_S \cdot Z_1 = -j4(2.5 + j1.25) = 5 - j10 \text{ V}$$

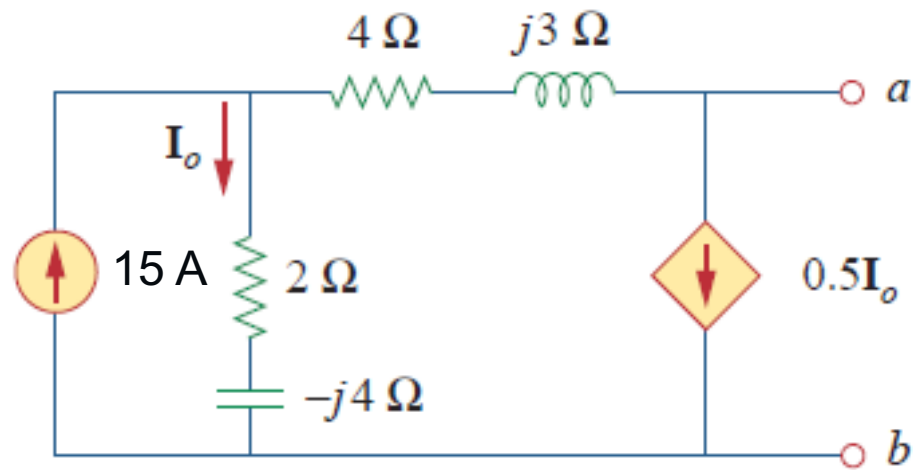
$$V_x = (5 - j10) \frac{10}{10 + 2.5 + j1.25 + 4 - j13} = 5.519 e^{-j28^\circ} \text{ V}$$



Thevenin Equivalent in AC Circuits

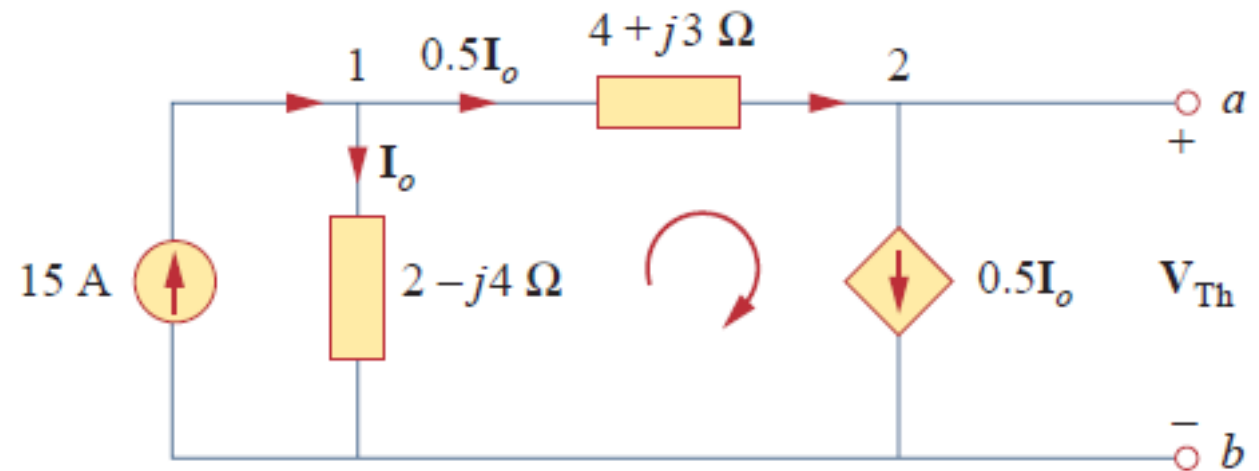
SSA.05

Find the Thevenin equivalent of the circuit in Figure as seen from terminals a-b.



Solution

To obtain V_{Th} ...



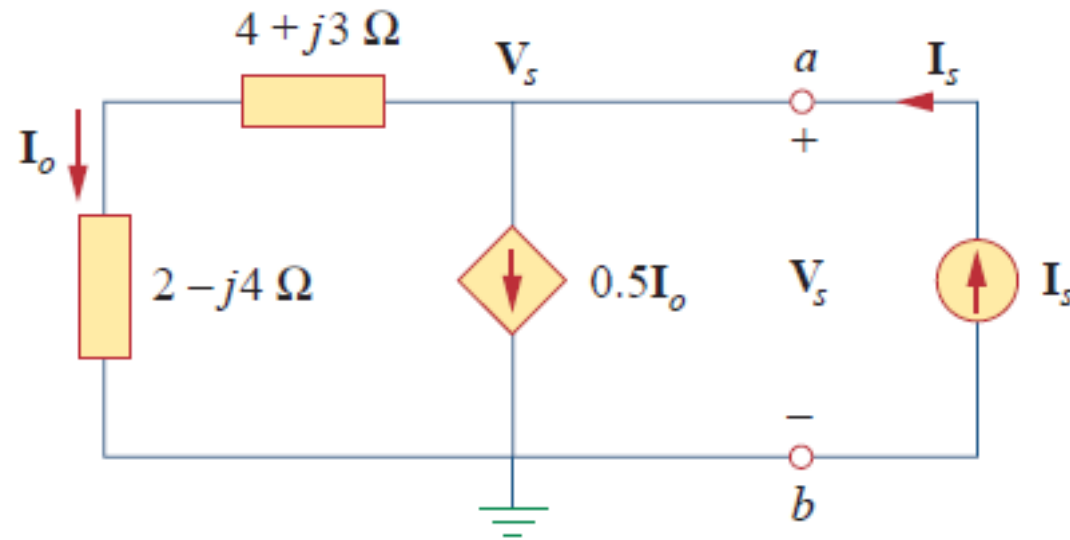
$$15 = I_o + 0.5I_o \rightarrow I_o = 10 \text{ A}$$

$$-I_o(2 - j4) + 0.5I_o(4 + j3) + V_{Th} = 0$$

$$V_{Th} = 10(2 - j4) - 5(4 + j3) = -j55 \text{ V}$$

Thevenin Equivalent in AC Circuits

To obtain Z_{Th} , we remove the **independent** source (only!).



$$I_S = I_0 + 0.5I_0 \rightarrow I_0 = \frac{I_S}{1.5}$$

$$I_S \triangleq 3 \text{ A (for convenience)} \rightarrow I_0 = 2 \text{ A}$$

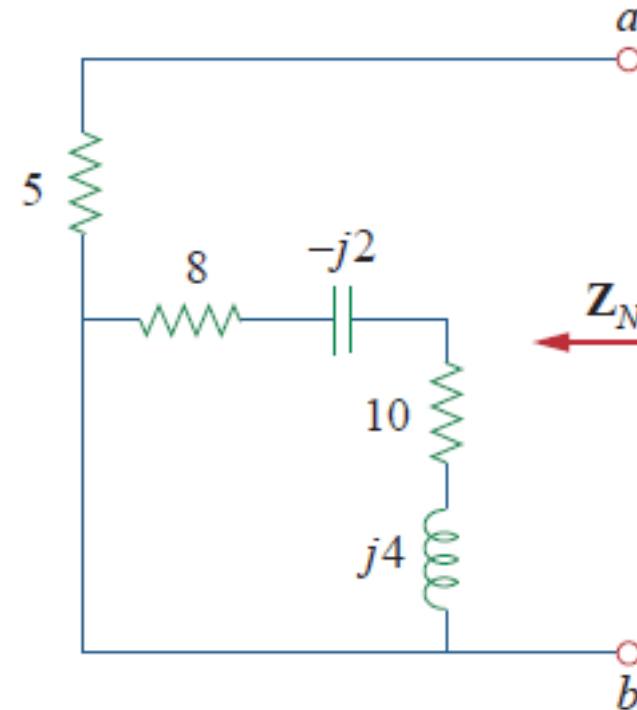
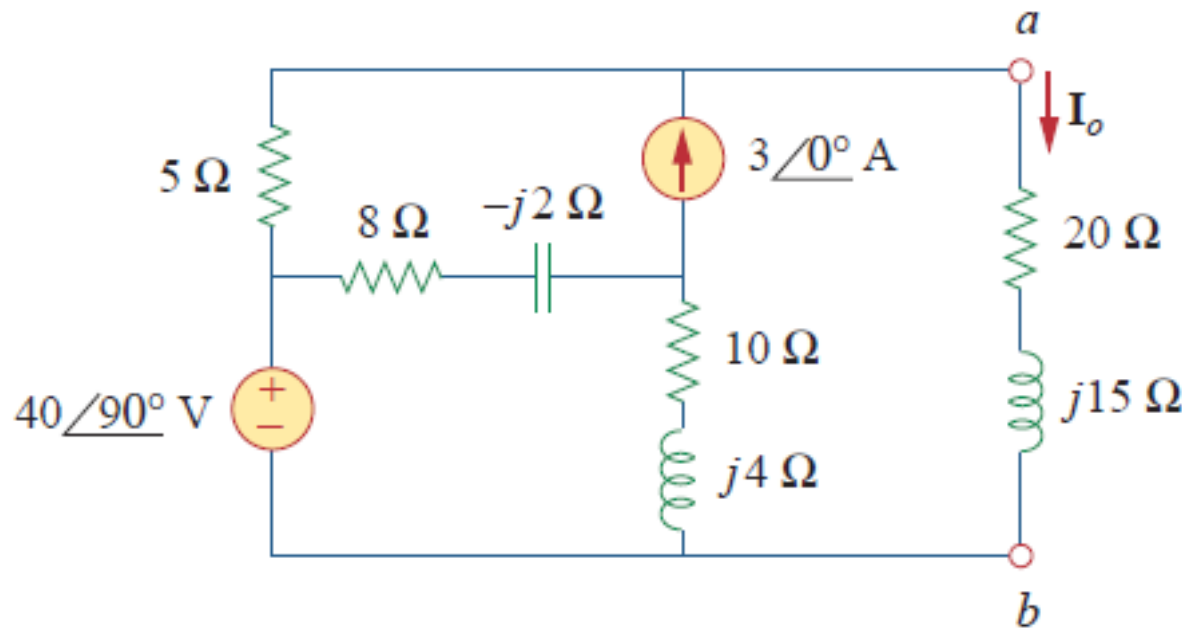
$$V_S = I_0(4 + j3 + 2 - j4) = 2(6 - j)$$

$$Z_{Th} = \frac{V_S}{I_S} = \frac{2(6 - j)}{3} = 4 - j0.67 \Omega$$

Norton Equivalent AC Circuits

SSA.06 – Obtain current I_o in Figure using Norton's theorem.

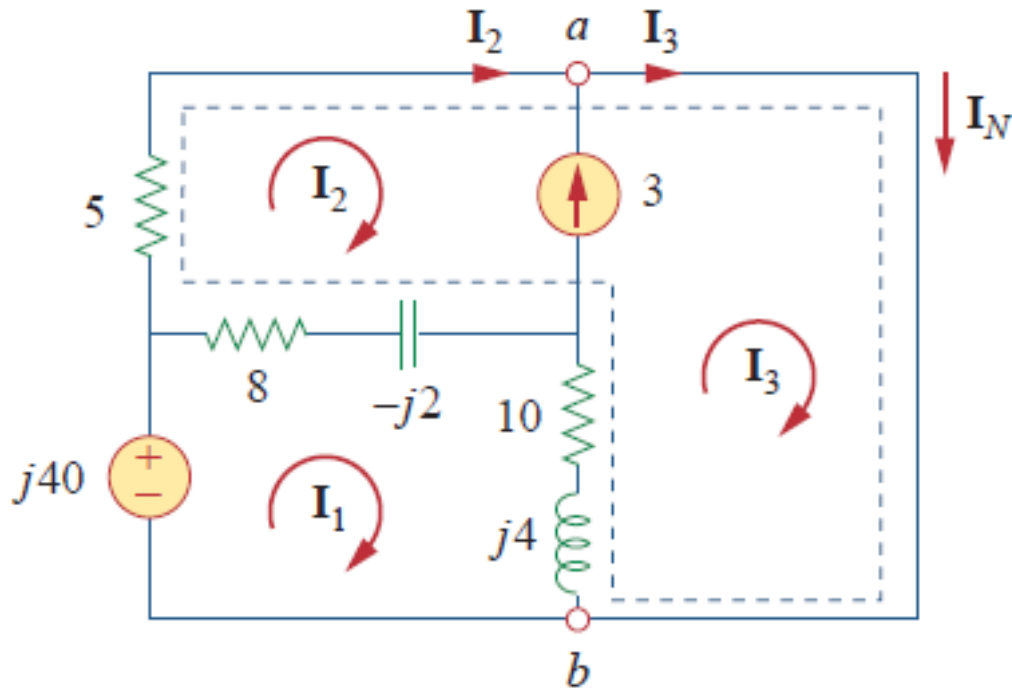
Solution – To obtain Z_N , ...



$$Z_N = 5 \Omega$$

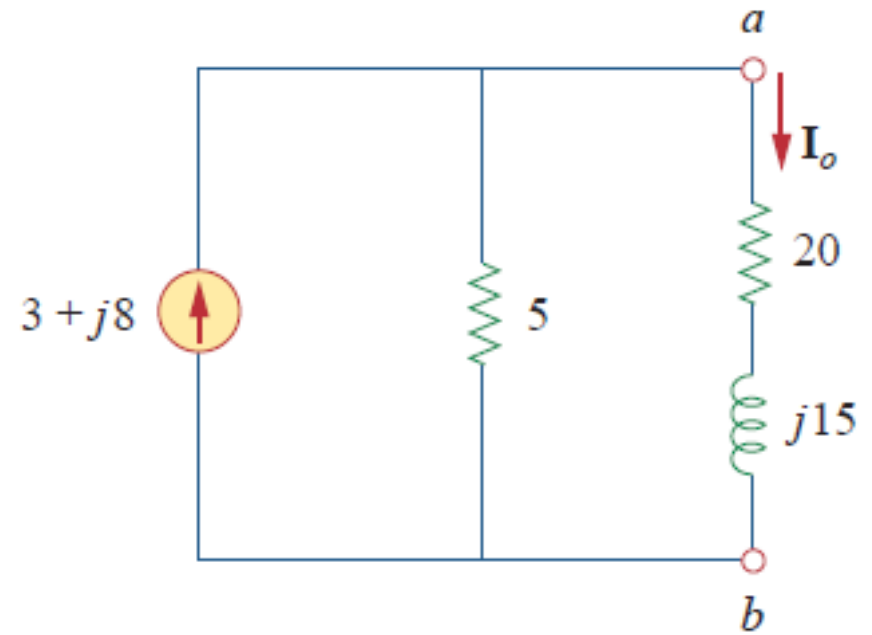
Norton Equivalent in AC Circuits

To get I_N ,



$$(1) + (2) \rightarrow -j40 + 5I_2 = 0 \rightarrow I_2 = j8$$

$$I_3 = I_2 + 3 \quad I_N = I_3 = I_2 + 3 = (3 + j8) A$$



Current division

$$I_0 = \frac{5}{5 + 20 + j15} I_N = \frac{3 + j8}{5 + j3} = 1.46 e^{j38.49^\circ} A$$

$$-j40 + (18 + j2)I_1 - (8 - j2)I_2 - (10 + j4)I_3 = 0 \quad (1)$$

$$(13 - j2)I_2 + (10 + j4)I_3 - (18 + j2)I_1 = 0 \quad (2)$$

Questions

