



DR. GYURCSEK ISTVÁN

# Exercises in AC Power Analysis

*Sources and additional materials (recommended)*

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/1*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

# Effective Values



SPQ.01 – Determine the mean values of a sinusoidal voltage.

**Solution:**  $v(t) = V_p \cos \omega t$        $V_{avg} = \frac{1}{T} \int_0^T v(t) dt = \frac{V_p}{T} \int_0^T \cos \omega t dt = 0$

$$V_{abs} = \frac{1}{T} \int_0^T |v(t)| dt = \frac{V_p}{T/4} \int_0^{T/4} \cos \omega t dt = \frac{4V_p}{T} \left| \frac{\sin \omega t}{\omega} \right|_0^{T/4} = \frac{4V_p}{T} \frac{T}{2\pi} = \frac{2}{\pi} V_p$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt} = \sqrt{\frac{4V_p^2}{T} \int_0^{T/4} \cos^2 \omega t dt} = \sqrt{\frac{4V_p^2}{T} \int_0^{T/4} \frac{1 + \cos 2\omega t}{2} dt} = \sqrt{\frac{4V_p^2}{T} \left[ \frac{1}{2} t + \frac{\sin 2\omega t}{4\omega} \right]_0^{T/4}} = \sqrt{\frac{V_p^2}{2}} = \frac{V_p}{\sqrt{2}}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha, \quad \cos^2 \alpha + \sin^2 \alpha = 1 \quad \rightarrow \quad \cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

$$\frac{V_{rms}}{V_{abs}} = \frac{\pi}{2\sqrt{2}} \cong 1.11$$

# Effective Values

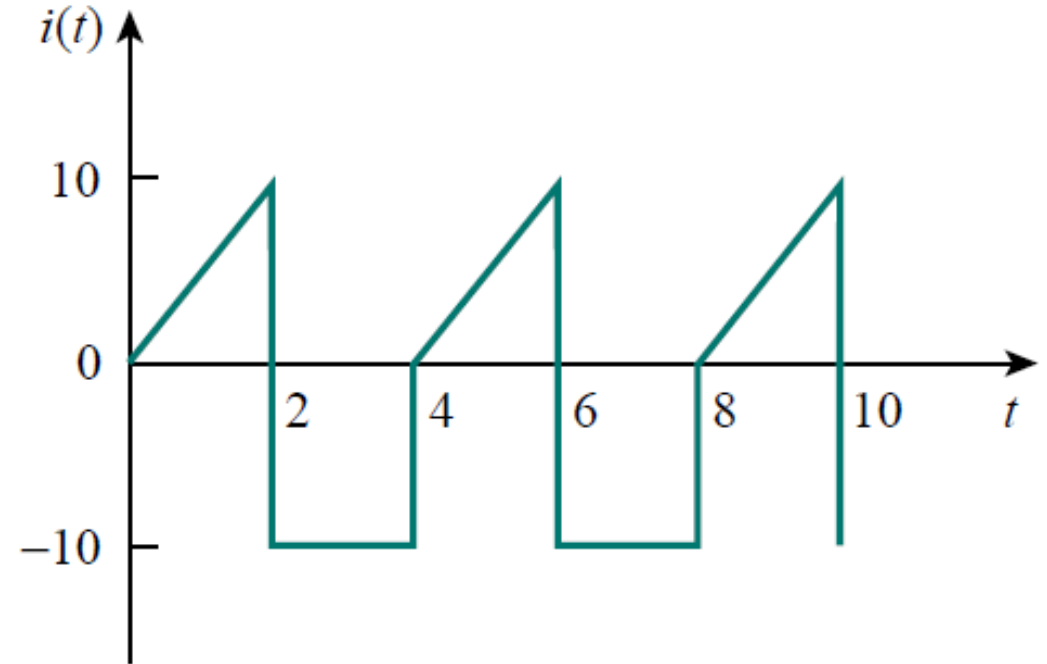


## SPQ.02

Determine the RMS value of the current waveform in figure.

Solution:

$$i(t) = \begin{cases} 5t, & 0 \leq t < 2 \\ -10, & 2 \leq t < 4 \end{cases}$$



$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2 dt} = \sqrt{\frac{1}{4} \left[ \int_0^2 (5t)^2 dt + \int_2^4 (-10)^2 dt \right]} = \sqrt{\frac{1}{4} \left[ 25 \frac{t^3}{3} \Big|_0^2 + 100t \Big|_2^4 \right]} = \sqrt{\frac{1}{4} \left( \frac{200}{3} + 200 \right)} = 8.16 \text{ A}$$

# Average Power



**SPQ.03** – Find the instantaneous power and the average power when

$$v(t) = 230 \cdot \sqrt{2} \cdot \cos(314t + 50^\circ) \text{ V}$$

$$i(t) = 10 \cdot \sqrt{2} \cdot \cos(314t - 10^\circ) \text{ A}$$

**Solution:**

$$p(t) = 230 \cdot 10 \cdot \cos 60^\circ + 230 \cdot 10 \cdot \cos(628t + 40^\circ) = 1150 + 2300 \cdot \cos(628t + 40^\circ)$$

$$P = 1.15 \text{ kW}$$

# Average Power



## SPQ.04

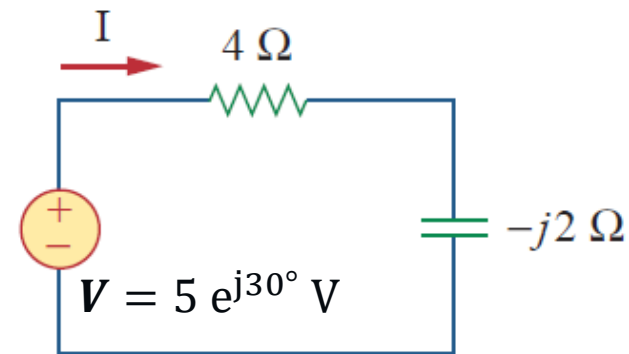
Find the average power supplied by the source and the average power absorbed by the resistor.  $\mathbf{V}_{(rms)} = 5 e^{j30^\circ} \text{ V}$

**Solution:**

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{5 e^{j30^\circ}}{4 - j2} = \frac{5 e^{j30^\circ}}{4.472 e^{-j26.57^\circ}} = 1.118 e^{j56.57^\circ} \text{ A}$$

$$P = V \cdot I \cdot \cos(\varphi_U - \varphi_I) = 5 \cdot 1.118 \cdot \cos(30^\circ - 56.57^\circ) = 5 \text{ W}$$

$$\mathbf{V}_R = 4 \cdot \mathbf{I} = 4.472 e^{j56.57^\circ} \text{ V} \rightarrow P_R = \mathbf{V}_R \cdot \mathbf{I} = 4.472 \cdot 1.118 = 5 \text{ W} \quad P = \text{Re}\{\mathbf{S}\} = \text{Re}\{\mathbf{V} \cdot \mathbf{I}^*\} = \dots = 5 \text{ W}$$



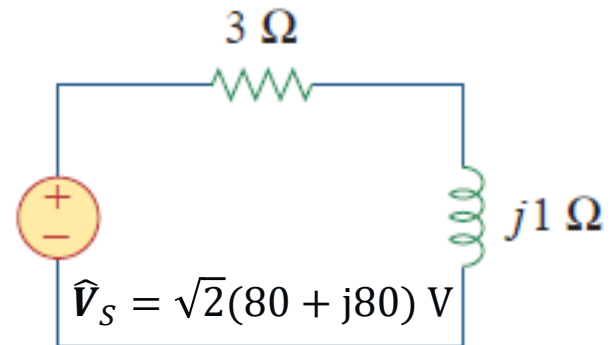
# Average Power



## SPQ.05

Find the average power supplied by the source and the average power absorbed by the resistor and inductor.

**Solution:**

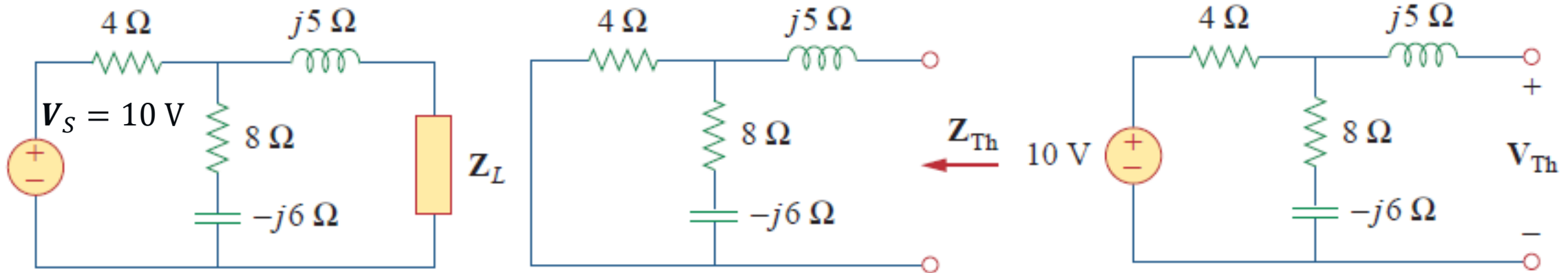


$$\hat{V}_S = 160 e^{j45^\circ}, \quad P_R = 3.34 \text{ kW}, \quad P_L = 0, \quad P_S = -3.34 \text{ kW}$$

# Maximum Power Transfer



**SPQ.06** – Determine  $Z_L$  to maximize the average power. What is the maximum average power?



**Solution:** *Comment:*  $Z_1 \times Z_2 = Z_1 \parallel Z_2$

$$Z_{Th} = j5 + 4 \times (8 - j6) = j5 + \frac{4(8 - j6)}{12 - j6} = (2.933 + j4.467) \Omega \quad \text{max power} \rightarrow Z_L = Z_{Th}^* = (2.933 - j4.467) \Omega$$

$$V_{Th} = 10 \frac{8 - j6}{4 + 8 - j6} = 7.454 e^{-j10.3^\circ} \text{ V}$$

$$P_{max} = \frac{|V_{Th}|^2}{8R_{Th}} = \frac{(7.454)^2}{8 \cdot 2.933} = 2.368 \text{ W}$$

# Maximum Power Transfer



SPQ.07 – Find the maximum average power.

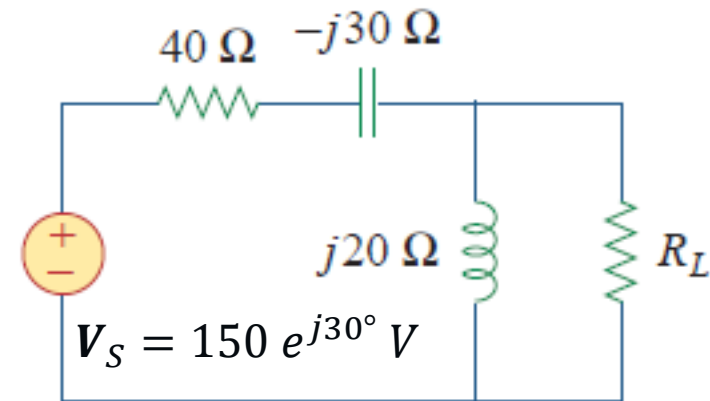
Solution:

$$\mathbf{Z}_{Th} = j20 \times (40 - j30) = \frac{j20(40 - j30)}{j20 + 40 - j30} = (9.41233 + j22.35) \Omega$$

$$\mathbf{V}_{Th} = 150 e^{j30^\circ} \frac{j20}{j20 + 40 - j30} = 72.76 e^{j134^\circ} \text{ V}$$

$$R_L = |\mathbf{Z}_{Th}| = \sqrt{9.41233^2 + 22.35^2} = 24.25 \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + R_L} = \frac{72.76 e^{j134^\circ}}{33.66 + j22.35} = 1.8 e^{j100.42^\circ} \text{ A}$$



$$P_{max} = \frac{1}{2} |\mathbf{I}|^2 R_L = \frac{1.8^2}{2} 24.25 = 39.29 \text{ W}$$



# Apparent Power and ,pf'



**SPQ.08** – Find the apparent power and the power factor of a series-connected load if  $v(t) = 120 \cos(100\pi t - 20^\circ) \text{ V}$ ,  $i(t) = 4 \cos(100\pi t + 10^\circ) \text{ A}$   
Determine the element values that form the series-connected load.

**Solution**  $S = V_{rms} I_{rms} = \frac{120}{\sqrt{2}} \cdot \frac{4}{\sqrt{2}} = 240 \text{ VA}$

$$pf = \cos(\Theta_v - \Theta_i) = \cos(-20^\circ - 10^\circ) = 0.866 \text{ (leading = CAP)}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120 e^{-j20^\circ}}{4 e^{j10^\circ}} = 30 e^{-j30^\circ} = R - jX_C = 25.98 - j15 \Omega$$

$$R = 25.98 \Omega, \quad X_C = \frac{1}{\omega C} \rightarrow C = \frac{1}{100\pi \cdot 15} = 212.2 \mu\text{F}$$

# Apparent Power and ,pf'



**SPQ.09** – Determine the power factor of the entire circuit and the average power delivered by the source.

**Solution**

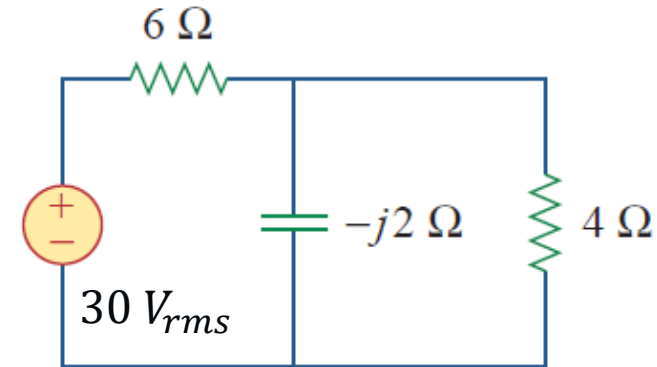
$$\mathbf{Z} = 6 + 4 \times (-j2) = 6 + \frac{-j2 \cdot 4}{4 - j2} = 6.8 - j1.6 = 7 e^{-j13.24^\circ} \Omega$$

$$pf = \cos(-13.24^\circ) = 0.9734 \text{ (leading = CAP)}$$

$$\mathbf{I}_{rms} = \frac{\mathbf{V}_{rms}}{\mathbf{Z}} = \frac{30}{7 e^{-j13.24^\circ}} = 4.286 e^{j13.24^\circ}$$

$$P = V_{rms} \cdot I_{rms} \cdot pf = 30 \cdot 4.286 \cdot 0.9734 = 125 \text{ W}$$

$$\text{or ... } P = I_{rms}^2 \cdot R = 4.286^2 \cdot 6.8 = 125 \text{ W}$$



# Complex Power



**SPQ.10** – Find the complex and apparent powers, the real and reactive powers, the power factor and load impedance if

$$v(t) = 60 \cos(\omega t - 10^\circ) \text{ V}, \quad i(t) = 1.5 \cos(\omega t + 50^\circ) \text{ A}$$

**Solution**

$$\mathbf{V}_{rms} = \frac{60}{\sqrt{2}} e^{-j10^\circ}, \quad \mathbf{I}_{rms} = \frac{1.5}{\sqrt{2}} e^{j50^\circ}$$

$$\mathbf{S} = \mathbf{V}_{rms} \mathbf{I}_{rms}^* = \frac{60}{\sqrt{2}} e^{-j10^\circ} \cdot \frac{1.5}{\sqrt{2}} e^{-j50^\circ} = 45 e^{-j60^\circ} \text{ VA} \rightarrow S = 45 \text{ VA}$$

$$\mathbf{S} = 45 e^{-j60^\circ} = 45 \cos(-60^\circ) + j45 \sin(-60^\circ) = (22.5 - j38.97) \text{ VA} \rightarrow P = 22.5 \text{ W}, \quad Q = -38.97 \text{ VAR}$$

$$pf = \cos(-60^\circ) = 0.5 \text{ (leading = CAP)}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{60 e^{-j10^\circ}}{1.5 e^{j50^\circ}} = 40 e^{-j60^\circ} \Omega$$

# Complex Power



## SPQ.11

A load  $\mathbf{Z}$  draws 12 kVA at a power factor of 0.856 lagging from a 230 V<sub>RMS</sub> sinusoidal source. Calculate:

- (a) the average and reactive powers delivered to the load
- (b) the peak current
- (c) the load impedance

## Solution

$$P = S \cos \varphi = 10.272 \text{ kW} \quad Q = S \sin \varphi = 6.204 \text{ kVAr}$$

$$\mathbf{S} = P + jQ = (10.272 + j6.204) \text{ kVA}$$

$$\mathbf{I}^* = \frac{\mathbf{S}}{\mathbf{V}} = \frac{10,272 + j6,204}{230 e^{j0^\circ}} = 44.66 + j26.97 = 52.17 e^{j31.13^\circ} \text{ A}$$

$$I_m = I\sqrt{2} = 52.17 \cdot \sqrt{2} = 73.78 \text{ A} \quad \mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{230}{52.17 e^{-j31.13^\circ}} = 4.41 e^{j31.13^\circ} \Omega$$

# Conservation of Power



**SPQ.12** – A load being fed by a voltage source through a transmission line. The impedance of the line is represented by the  $(4+j2)$  ohm impedance and a return path. Find the real power and reactive power absorbed by: (a) the source, (b) the line, and (c) the load.

## Solution

$$\mathbf{Z} = (4 + j2) + (15 - j10) = 19 - j8 = 20.62 e^{-j22.83^\circ} \Omega$$

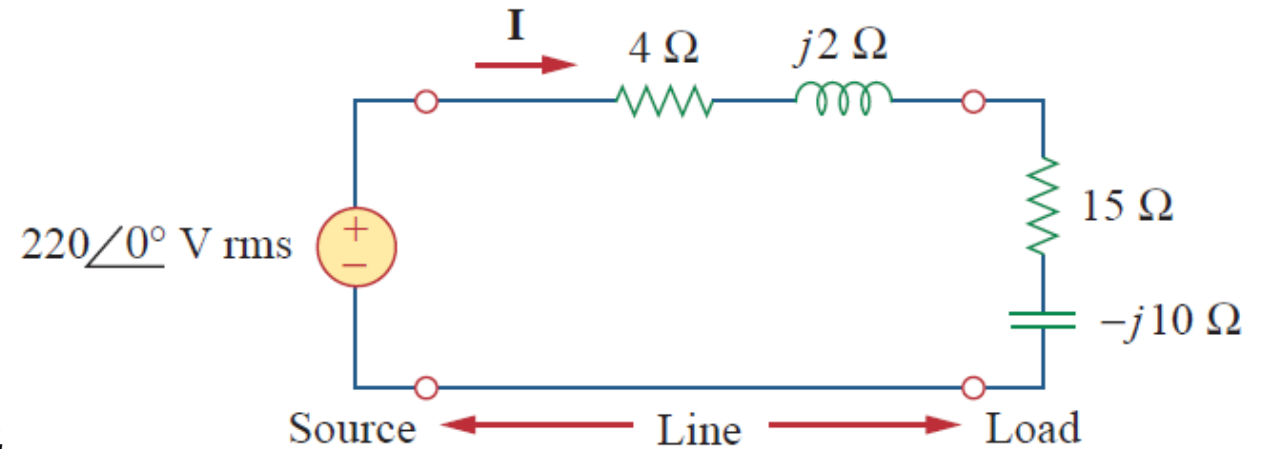
$$\mathbf{I} = \frac{\mathbf{V}_S}{\mathbf{Z}} = \frac{220}{20.62 e^{-j22.83^\circ}} = 10.67 e^{j22.83^\circ} \text{ A}$$

$$\mathbf{S}_{source} = \mathbf{V}_S \mathbf{I}^* = 220 \cdot 10.67 e^{-j22.83^\circ} = 2347.4 e^{-j22.83^\circ} = (2163.4 - j910.8) \text{ VA} \rightarrow P = 2163.4 \text{ W}, Q = -910.8 \text{ VAR}$$

$$\mathbf{V}_{line} = (4 + j2)\mathbf{I} = 4.472 e^{j26.57^\circ} \cdot 10.67 e^{j22.83^\circ} = 47.72 e^{j49.4^\circ} \text{ V}_{rms}$$

$$\mathbf{S}_{line} = \mathbf{V}_{line} \mathbf{I}^* = 47.72 e^{j49.4^\circ} 10.67 e^{-j22.83^\circ} = 509.2 e^{j26.57^\circ} = (455.4 + j227.7) \text{ VA} \rightarrow P = 455.4 \text{ W}, Q = 227.7 \text{ VAR}$$

$$\text{or ... } \mathbf{S}_{line} = I^2 \mathbf{Z}_{line} = 10.67^2 (4 + j2) = (455.4 + j227.7) \text{ VA} \rightarrow P = 455.4 \text{ W}, Q = 227.7 \text{ VAR}$$



# Conservation of Power



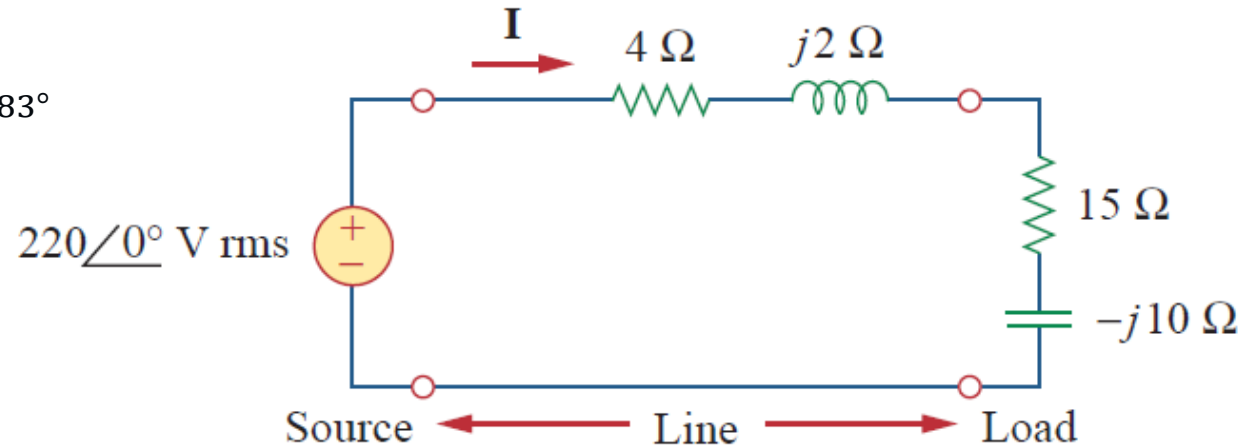
$$V_{load} = (15 - j10)I = 18.03 e^{-j33.7^\circ} \cdot 10.67 e^{j22.83^\circ}$$

$$= 192.38 e^{-j10.87^\circ} V_{rms}$$

$$S_{load} = V_{load}I^* = 192.38 e^{-j10.87^\circ} 10.67 e^{-j22.83^\circ}$$

$$= 2053 e^{-j33.7^\circ} = 1708 - j1139 VA \rightarrow P = 1708 W, Q = -1139 VAR$$

$$\text{Thanks to Mr. Tellegen} \rightarrow \left\{ \begin{array}{l} S_{source} = (2163.4 - j910.8) VA \\ S_{line} = (455.4 + j227.7) VA \\ S_{load} = 1708 - j1139 VA \end{array} \right\} \rightarrow S_{source} = S_{line} + S_{load}$$



# Power Factor Correction



## SPQ.13

Connected to a 230 V<sub>RMS</sub>, 50 Hz power line, a load absorbs 4 kW at a lagging (*IND*) power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.9.

### Solution

As the  $\cos \varphi_1 = 0.8 \rightarrow \varphi_1 = 36.87^\circ$  The apparent power  $S_1 = \frac{P}{\cos \varphi_1} = \frac{4000}{0.8} = 5000 \text{ VA}$

The reactive power  $Q_1 = S_1 \sin \varphi_1 = 5000 \sin 36.87^\circ = 3000 \text{ VAr}$

$\cos \varphi_2 = 0.9 \rightarrow \varphi_2 = 25.84^\circ$   $S_2 = \frac{P}{\cos \varphi_2} = \frac{4000}{0.9} = 4444.4 \text{ VA}$

$Q_2 = S_2 \sin \varphi_2 = 4444.44 \sin 25.84^\circ = 1937.15 \text{ VAr}$

$Q_C = Q_1 - Q_2 = 3000 - 1937.15 = 1062.85 \text{ VAr}$   $Q_C = \frac{V_{RMS}^2}{X_C} \rightarrow C = \frac{Q_C}{\omega V_{RMS}^2} = \frac{1062.85}{2\pi \cdot 50 \cdot 230^2} = 63.98 \mu\text{F}$

# Power Factor Correction



**SPQ.14** – Find the value of parallel capacitance needed to correct a load of 140 kVAR at 0.85 lagging (IND) pf to unity pf (i.e.  $pf = 1$ ). Assume that the load is supplied by a 110-V (rms), 60-Hz line.

**Solution**  $C = 30.69 \text{ mF}$ .



# Questions

