DR. GYURCSEK ISTVÁN

Exercises in AC Power Analysis

Sources and additional materials (recommended)

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

Effective Values

SPQ.01 – Determine the mean values of a sinusoidal voltage.

Solution: $v(t) = V_p \cos \omega t$ $\qquad V_{avg} = \frac{1}{T}\int_0^T v(t)dt = \frac{V_p}{T}\int_0^T \cos \omega t \ \ dt = 0$

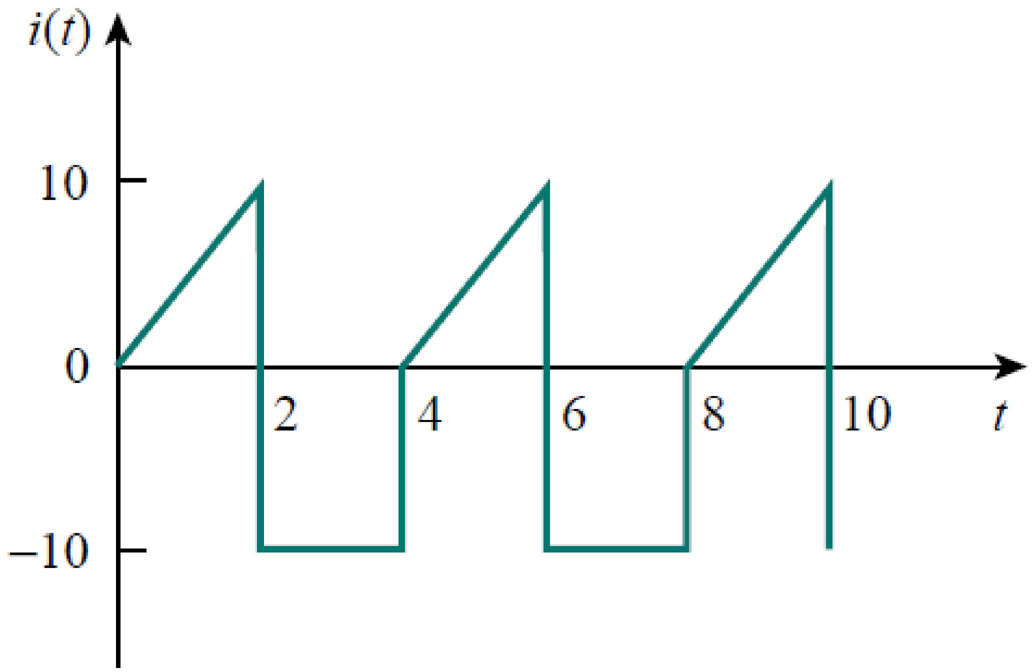
$$V_{abs} = \frac{1}{T}\int_0^T |v(t)|dt = \frac{V_p}{T/4}\int_0^{T/4} \cos \omega t \ \ dt = \frac{4V_p}{T}\left|\frac{\sin \omega t}{\omega}\right|_0^{T/4} = \frac{4V_p}{T}\frac{T}{2\pi} = \frac{2}{\pi}V_p$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \sqrt{\frac{4V_p^2}{T}\int_0^T cos^2\omega tdt} = \sqrt{\frac{4V_p^2}{T}\int_0^{T/4} \frac{1+cos2\omega t}{2}dt} = \sqrt{\frac{4V_p^2}{T}\left[\frac{1}{2}t + \frac{sin2\omega t}{4\omega}\right]_0^{T/4}} = \sqrt{\frac{V_p^2}{2}} = \frac{V_p}{\sqrt{2}}$$

$cos\,2 \propto = cos^2 \propto - sin^2 \propto,$ $\qquad cos^2 \propto + sin^2 \propto = 1$ $\quad \rightarrow \quad$ $cos^2 \propto = \frac{1+cos2\omega t}{2}$ $\qquad \frac{V_{rms}}{V_{abs}} = \frac{\pi}{2\sqrt{2}} \cong 1.11$

SPQ.02

Determine the RMS value of the current waveform in figure.



Solution:

$$\mathrm{i}(t)=\begin{cases}5t, & 0\le t<2\\ -10, & 2\le t<4\end{cases}$$

$$I_{rms}=\sqrt{\frac{1}{T}\int_0^T i^2 dt}=\sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\,dt+\int_2^4 (-10)^2\,dt\right]}=\sqrt{\frac{1}{4}\left[25\frac{t^3}{3}\bigg|_0^2+100t\bigg|_2^4\right]}=\sqrt{\frac{1}{4}\left(\frac{200}{3}+200\right)}=8.16\ A$$

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Average Power

SPQ.03 – Find the instantaneous power and the average power when

$$v(t) = 230 \cdot \sqrt{2} \cdot cos(314t + 50°)\ V$$

$$i(t) = 10 \cdot \sqrt{2} \cdot cos(314t - 10°)\ A$$

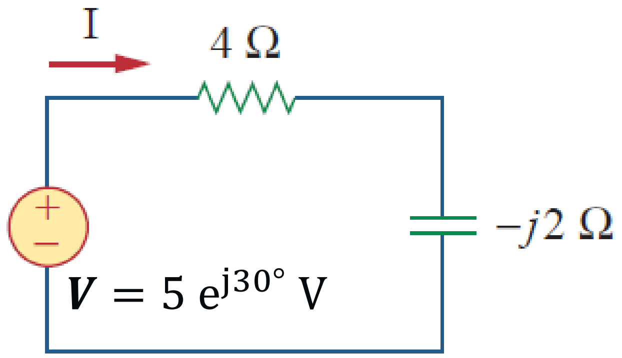
Solution:

$$p(t) = 230 \cdot 10 \cdot cos\,60° + 230 \cdot 10 \cdot cos(628t + 40°) = 1150 + 2300 \cdot cos(628t + 40°)$$

$$P = 1.15\ kW$$

SPQ.04

Find the average power supplied by the source and the average power absorbed by the resistor. $\boldsymbol{V} = 5\ \mathrm{e}^{\mathrm{j}30°}\ \mathrm{V}$



Solution:

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}} = \frac{5\ e^{j30°}}{4 - j2} = \frac{5\ e^{j30°}}{4.472\ e^{-j26.57°}} = 1.118\ e^{j56.57°}\ A$$

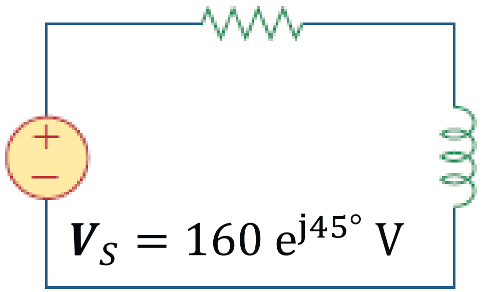
$$P = V \cdot I \cdot cos(\varphi_U - \varphi_I) = 5 \cdot 1.118 \cdot cos(30° - 56.57°) = 5\ W$$

$$\boldsymbol{V}_R = 4 \cdot \boldsymbol{I} = 4.472\ e^{j56.57°}\ V \rightarrow P_R = V_R \cdot I = 4.472 \cdot 1.118 = 5\ W$$

$$P = Re\{\boldsymbol{S}\} = Re\{\boldsymbol{V} \cdot \boldsymbol{I}^*\} = \cdots = 5\ W$$

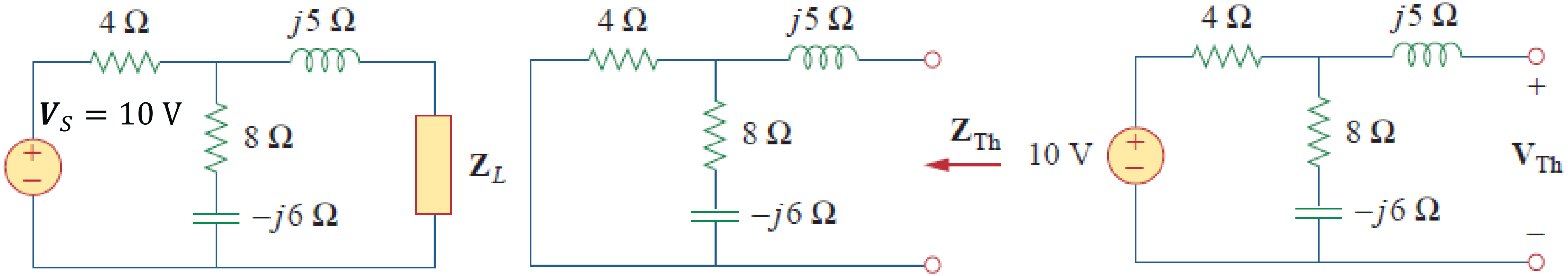
SPQ.05

Find the average power supplied by the source and the average power absorbed by the resistor and inductor. $\boldsymbol{V}_S = 160\ \mathrm{e}^{\mathrm{j}45°}\ \mathrm{V}$



Solution: $P_R = 3.84\ \mathrm{kW},\quad P_L = 0,\quad P_S = 3.84\ \mathrm{kW}$

SPQ.06 – Determine Z_L to maximize the average power. What is the maximum average power?



Solution: $\mathit{Comment}\text{:}\ \boldsymbol{Z}_1 \times \boldsymbol{Z}_2 = \boldsymbol{Z}_1 \parallel \boldsymbol{Z}_2$

$$\boldsymbol{Z}_{Th} = \mathrm{j}5 + 4 \times (8 - j6) = j5 + \frac{4(8 - j6)}{12 - j6} = (2.933 + j4.467)\ \Omega$$

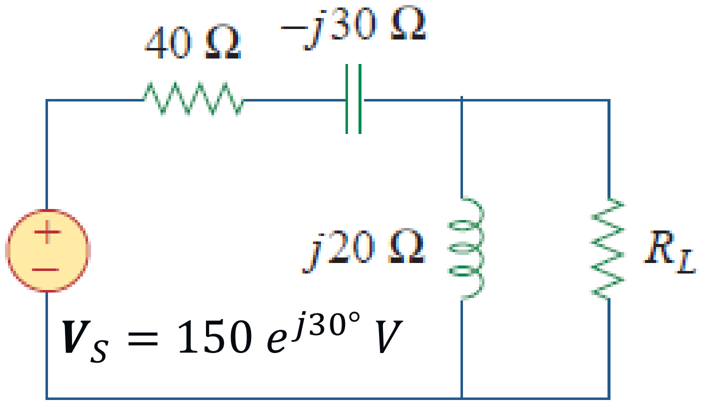
$$max\ power \rightarrow \boldsymbol{Z}_{\boldsymbol{L}} = \boldsymbol{Z}_{Th}^{*} = (2.933 - j4.467)\ \Omega$$

$$\boldsymbol{V}_{Th} = 10\,\frac{8 - j6}{4 + 8 - j6} = 7.454\ e^{-j10.3^\circ}\ V$$

$$P_{max} = \frac{|\boldsymbol{V}_{Th}|^2}{8R_{Th}} = \frac{(7.454)^2}{8 \cdot 2.933} = 2.368\ \mathrm{W}$$

SPQ.07 – Find the maximum average power.

Solution:



$$\boldsymbol{Z}_{Th} = \mathrm{j}20 \times (40 - j30) = \frac{J20(40 - J30)}{J20 + 40 - J30} = (9.41233 + j22.35)\ \Omega$$

$$\boldsymbol{V}_{Th} = 150\ e^{j30°} \frac{J20}{j20 + 40 - j30} = 72.76\ e^{j134°}\ V$$

$$R_L = |\boldsymbol{Z}_{Th}| = \sqrt{9.41233^2 + 22.35^2} = 24.25\ \Omega$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}_{Th}}{\boldsymbol{Z}_{Th} + R_L} = \frac{72.76\ e^{j134°}}{33.66 + j22.35} = 1.8\ e^{j100.42°}\ A$$

$$P_{max} = \frac{1}{2}|\boldsymbol{I}|^2 R_L = \frac{1.8^2}{2} 24.25 = 39.29\ W$$

Apparent Power and Power Factor

SPQ.08 – Find the apparent power and the power factor of a series-connected load if

$$v(t) = 120\cos(100\pi t - 20°)\ V,\ i(t) = 4\cos(100\pi t + 10°)\ A$$

Determine the element values that form the series-connected load.

Solution $S = V_{rms} I_{rms} = \dfrac{120}{\sqrt{2}} \cdot \dfrac{4}{\sqrt{2}} = 240\ VA$

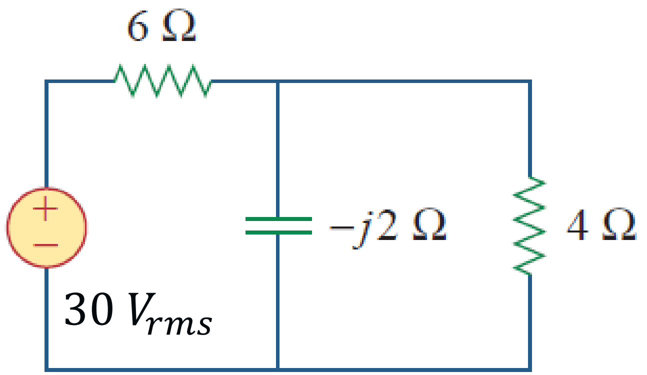
$$pf = \cos(\Theta_v - \Theta_i) = \cos(-20° - 10°) = 0.866\ (leading = CAP)$$

$$\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{120\ e^{-j20°}}{4\ e^{j10°}} = 30\ e^{-j30°} = R - jX_C = 25.98 - j15\ \Omega$$

$$R = 25.98\ \Omega, \qquad X_c = \frac{1}{\omega C} \to C = \frac{1}{100\pi \cdot 15} = 212.2\ \mu F$$

SPQ.09 – Determine the power factor of the entire circuitand the average power delivered by the source.

Solution



$$\boldsymbol{Z} = 6 + 4\times(-j2) = 6 + \frac{-j2\cdot 4}{4 - j2} = 6.8 - j1.6 = 7\ e^{-j13.24°}\ \Omega$$

$$pf\ = \cos(-13.24°) = 0.9734\ (leading = CAP)$$

$$\boldsymbol{I}_{rms} = \frac{\boldsymbol{V}_{rms}}{\boldsymbol{Z}} = \frac{30}{7\ e^{-j13.24°}} = 4.286e^{j13.24°}$$

$$P = V_{rms}\cdot I_{rms}\cdot pf\ = 30\cdot 4.286\cdot 0.9734 = 125\ W$$

$$or\ \ldots\ P = I_{rms}^2\cdot R = 4.286^2\cdot 6.8 = 125\ W$$

SPQ.10 – Find the complex and apparent powers, the real and reactive powers, the power factor and load impedance if

$$v(t) = 60\,cos(\omega t - 10°)\ V, \qquad i(t) = 1.5\cos(\omega t + 50°)\ A$$

Solution

$$\boldsymbol{V}_{rms} = \frac{60}{\sqrt{2}}\ e^{-j10°}\,, \qquad \boldsymbol{I}_{rms} = \frac{1.5}{\sqrt{2}}\ e^{j50°}$$

$$\boldsymbol{S} = \boldsymbol{V}_{rms}\boldsymbol{I}^{*}_{rms} = \frac{60}{\sqrt{2}}\ e^{-j10°} \cdot \frac{1.5}{\sqrt{2}}\ e^{-j50°} = 45\ e^{-j60°}\ VA \rightarrow S = 45\ VA$$

$$\boldsymbol{S} = 45\ e^{-j60°} = 45\cos(-60°) + j\sin(-60°) = (22.5 - j38.97)\ VA \rightarrow P = 22.5\ W\,, \qquad Q = -38.97\ VAR$$

$$pf = \cos(-60°) = 0.5\ (leading = CAP) \qquad\qquad \boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{60\ e^{-j10°}}{1.5\ e^{j50°}} = 40\ e^{-j60°}\ \Omega$$

Complex Power

SPQ.11

A load $\boldsymbol{Z}$ draws 12 kVA at a power factor of 0.856 lagging from a 230 V_{RMS} sinusoidal source. Calculate:

(a) the average and reactive powers delivered to the load

(b) the peak current

(c) the load impedance

Solution

$$P = S\cos\varphi = 10.272\ kW \qquad Q = S\sin\varphi = 6.204\ kVAr$$

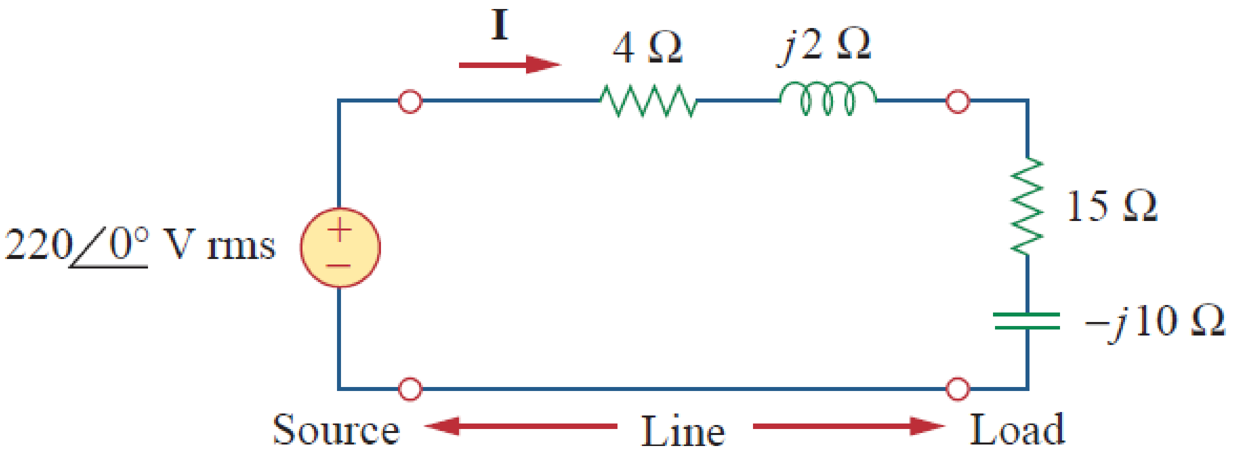
$$\boldsymbol{S} = P + jQ = (10.272 + j6.204)\ kVA$$

$$\boldsymbol{I}^* = \frac{\boldsymbol{S}}{\boldsymbol{V}} = \frac{10{,}272 + j6{,}204}{230\ e^{j0^\circ}} = 44.66 + j26.97 = 52.17\ e^{j31.13^\circ}\ A$$

$$I_m = I\sqrt{2} = 52.17 \cdot \sqrt{2} = 73.78\ A \qquad \boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{230}{52.17\ e^{-j31.13^\circ}} = 4.41\ e^{j31.13^\circ}\ \Omega$$

Conservation of Power

SPQ.12 – A load being fed by a voltage source through a transmission line. The impedance of the line is represented by the (4+j2) ohm impedance and a return path. Find the real power and reactive power absorbed by: (a) the source, (b) the line, and (c) the load.



Solution

$$\boldsymbol{Z} = (4 + j2) + (15 - j10) = 19 - j8 = 20.62\ e^{-j22.83^\circ}\ \Omega$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}_S}{\boldsymbol{Z}} = \frac{220}{20.62\ e^{-j22.83^\circ}} = 10.67\ e^{j22.83^\circ}\ A$$

$$\boldsymbol{S}_{source} = \boldsymbol{V}_S \boldsymbol{I}^* = 220 \cdot 10.67\ e^{-j22.83^\circ} = 2347.4\ e^{-j22.83^\circ} = (2163.4 - j910.8)\ VA \rightarrow P = 2163.4\ W, Q = -910.8\ VAR$$

$$\boldsymbol{V}_{line} = (4 + j2)\boldsymbol{I} = 4.472\ e^{j26.57^\circ} \cdot 10.67\ e^{j22.83^\circ} = 47.72\ e^{j49.4^\circ}\ V_{rms}$$

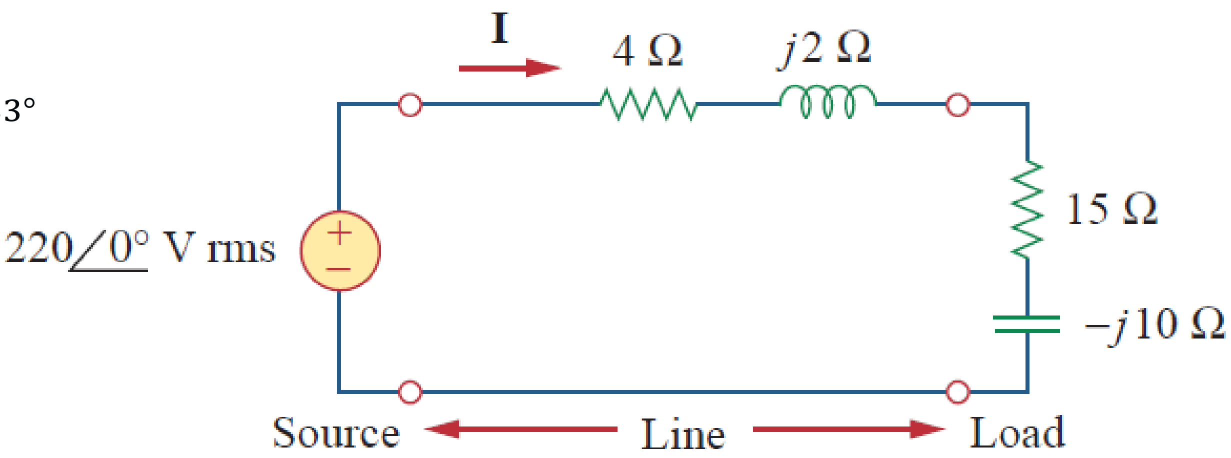
$$\boldsymbol{S}_{line} = \boldsymbol{V}_{line} \boldsymbol{I}^* = 47.72\ e^{j49.4^\circ} 10.67\ e^{-j22.83^\circ} = 509.2\ e^{j26.57^\circ} = (455.4 + j227.7)\ VA \rightarrow P = 455.4\ W, Q = 227.7\ VAR$$

$$or\ \dots\ \boldsymbol{S}_{line} = I^2 \boldsymbol{Z}_{line} = 10.67^2 (4 + j2) = (455.4 + j227.7)\ VA \rightarrow P = 455.4\ W, Q = 227.7\ VAR$$

Conservation of Power

$$\boldsymbol{V}_{load} = (15 - j10)\boldsymbol{I} = 18.03\, e^{-j33.7^\circ} \cdot 10.67\, e^{j22.83^\circ}$$

$$= 192.38\, e^{-j10.87^\circ}\, V_{rms}$$



$$\boldsymbol{S}_{load} = \boldsymbol{V}_{load}\boldsymbol{I}^* = 192.38\, e^{-j10.87^\circ} 10.67\, e^{-j22.83^\circ}$$

$$= 2053\, e^{-j33.7^\circ} = 1708 - j1139\, VA \rightarrow P = 1708\, W, Q = -1139\, VAR$$

$$Thanks\ to\ Mr.Tellegen \rightarrow \left\{ \begin{matrix} \boldsymbol{S}_{source} = (2163.4 - j910.8)\, VA \\ \boldsymbol{S}_{line} = (455.4 + j227.7)\, VA \\ \boldsymbol{S}_{load} = 1708 - j1139\, VA \end{matrix} \right\} \rightarrow \boldsymbol{S}_{source} = \boldsymbol{S}_{line} + \boldsymbol{S}_{load}$$

Power Factor Correction

SPQ.13

Connected to a 230 V_{RMS}, 50 Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8.
Find the value of capacitance necessary to raise the pf to 0.9.

Solution

As the $\cos\varphi_1 = 0.8 \rightarrow \varphi_1 = 36.87°$ The apparent power $S_1 = \dfrac{P}{\cos\varphi_1} = \dfrac{4000}{0.8} = 5000\,VA$

The reactive power $Q_1 = S_1 \sin\varphi_1 = 5000 \sin 36.87° = 3000\,VAr$

$\cos\varphi_2 = 0.9 \rightarrow \varphi_2 = 25.84°$ $S_2 = \dfrac{P}{\cos\varphi_2} = \dfrac{4000}{0.9} = 4444.4\,VA$

$Q_2 = S_2 \sin\varphi_2 = 4444.44 \sin 25.84° = 1937.15\,VAr$

$Q_C = Q_1 - Q_2 = 3000 - 1937.15 = 1062.85\,VAr$ $Q_C = \dfrac{V_{RMS}^2}{X_C} \rightarrow C = \dfrac{Q_C}{\omega V_{RMS}^2} = \dfrac{1062.85}{2\pi \cdot 50 \cdot 230^2} = 63.98\,\mu F$

Power Factor Correction

SPQ.14 – Find the value of parallel capacitance needed to correct a load of 140 kVAR at 0.85 lagging (IND) pf to unity pf (i.e. *pf* = *1*). Assume that the load is supplied by a 110-V (rms), 60-Hz line. **Solution** C = 30.69 mF.

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Questions

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