DR. GYURCSEK ISTVÁN

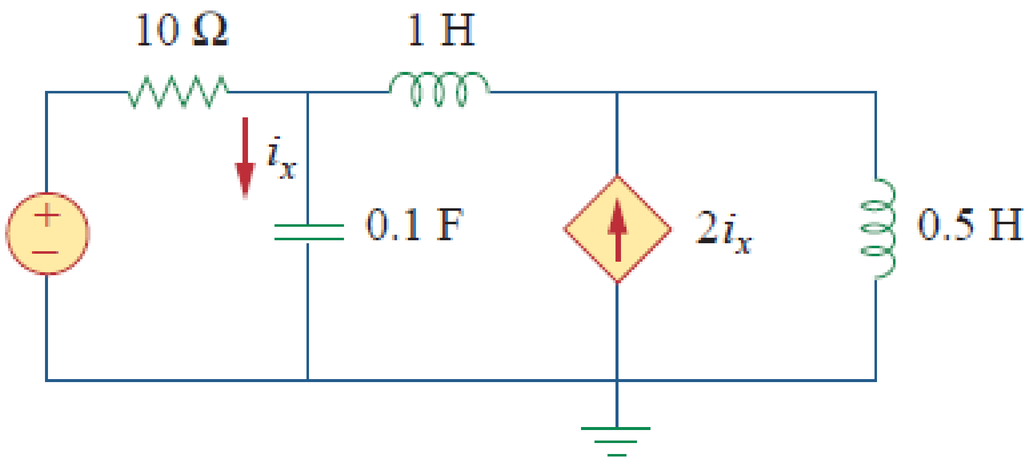
Exercises in AC Solid State Analysis

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

AC Nodal Analysis

SSA.01 – Find i_x in the circuit.

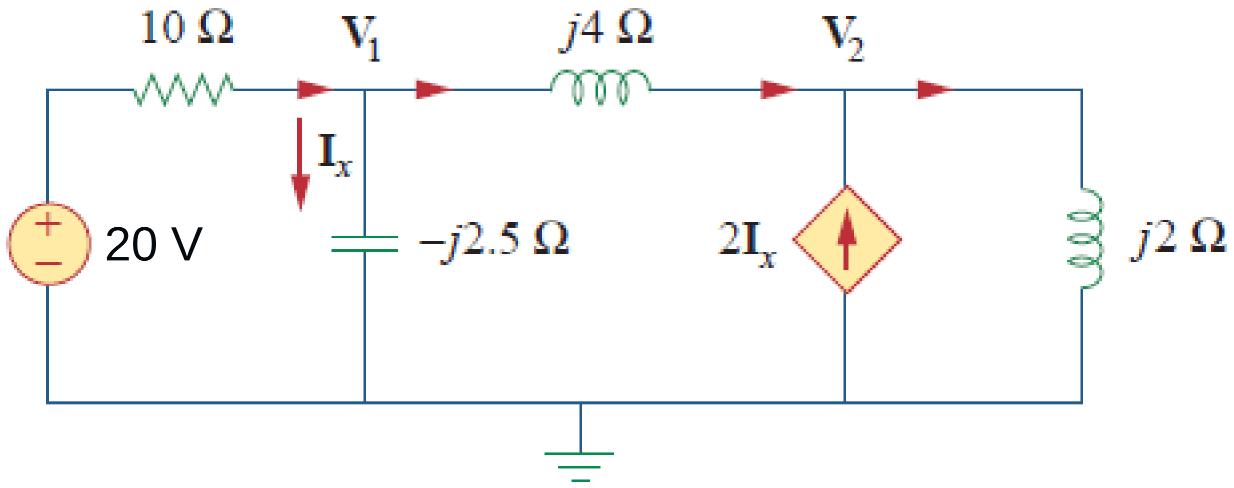


Solution

$$Z_C = -j\frac{1}{\omega C} = -j2.5\ \Omega$$

$$Z_{L1} = j\omega L_1 = j4\ \Omega \qquad v_S = 20\cos 4t\ V$$

$$Z_{L2} = j\omega L_2 = j2\ \Omega$$



$$\frac{20 - V_1}{10} = \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4}$$

$$2I_x + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2} \quad (\&) \quad I_x = \frac{V_1}{-j2.5}$$

AC Nodal Analysis

$$\frac{20 - \boldsymbol{V}_1}{10} = \frac{\boldsymbol{V}_1}{-j2.5} + \frac{\boldsymbol{V}_1 - \boldsymbol{V}_2}{j4} \rightarrow (1 + j1.5)\boldsymbol{V}_1 + j2.5\boldsymbol{V}_2 = 20$$

$$2\boldsymbol{I}_x + \frac{\boldsymbol{V}_1 - \boldsymbol{V}_2}{j4} = \frac{\boldsymbol{V}_2}{j2} \quad \text{(\&)} \quad \boldsymbol{I}_x = \frac{\boldsymbol{V}_1}{-j2.5} \rightarrow 11\boldsymbol{V}_1 + 15\boldsymbol{V}_2 = 0$$

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_1 \\ \boldsymbol{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300$$

$$\Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\boldsymbol{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\ e^{j18.43°}$$

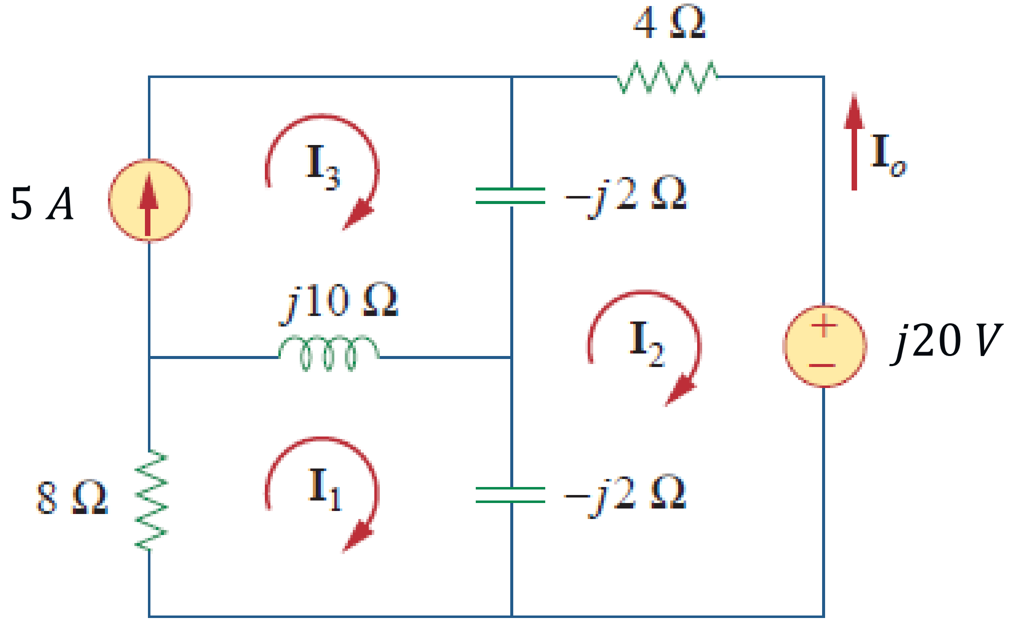
$$\boldsymbol{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\ e^{j198.3°}$$

$$\boldsymbol{I}_x = \frac{\boldsymbol{V}_1}{-j2.5} = \frac{18.97\ e^{j18.43°}}{2.5\ e^{-j90°}} = 7.59\ e^{j108.4°}$$

$$i_x = 7.59 \cos(4t + 108.4°)\ A$$

AC Mesh Analysis

SSA.02 – Determine current $\mathbf{I}_o$ in the circuit.



Solution

$$(8 + j10 - j2)\boldsymbol{I}_1 - (-j2)\boldsymbol{I}_2 - j10\boldsymbol{I}_3 = 0$$

$$(4 - j2 - j2)\boldsymbol{I}_2 - (-j2)\boldsymbol{I}_1 - (-j2)\boldsymbol{I}_3 + j20 = 0$$

$$\boldsymbol{I}_3 = 5$$

$$(8 + j8)\boldsymbol{I}_1 + j2\boldsymbol{I}_2 = j50$$

$$j2\boldsymbol{I}_1 + (4 - j4)\,\boldsymbol{I}_2 = -j20 - j10$$

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

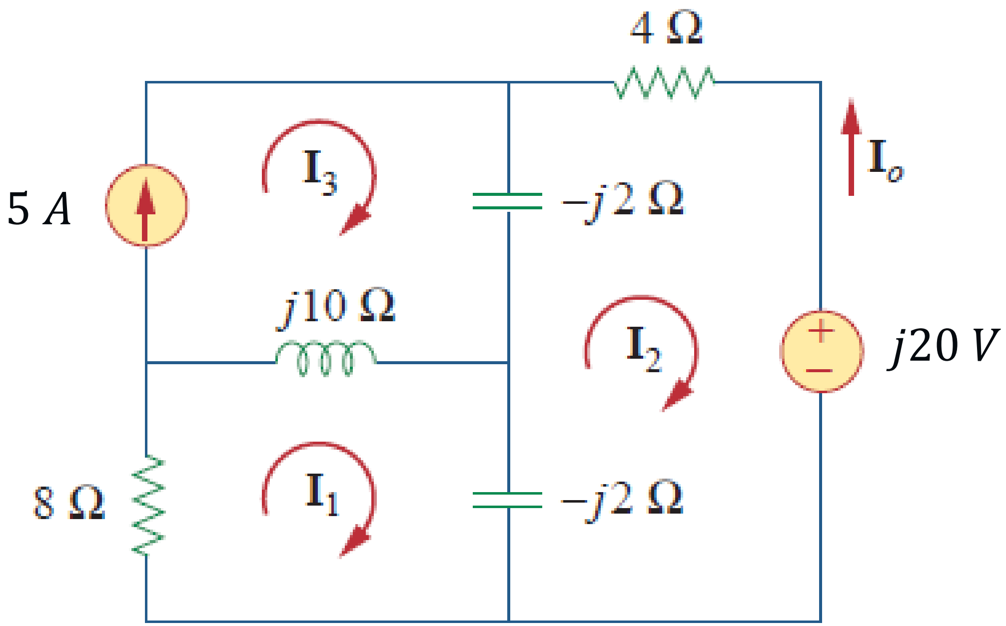
$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\, e^{-j35.22}$$

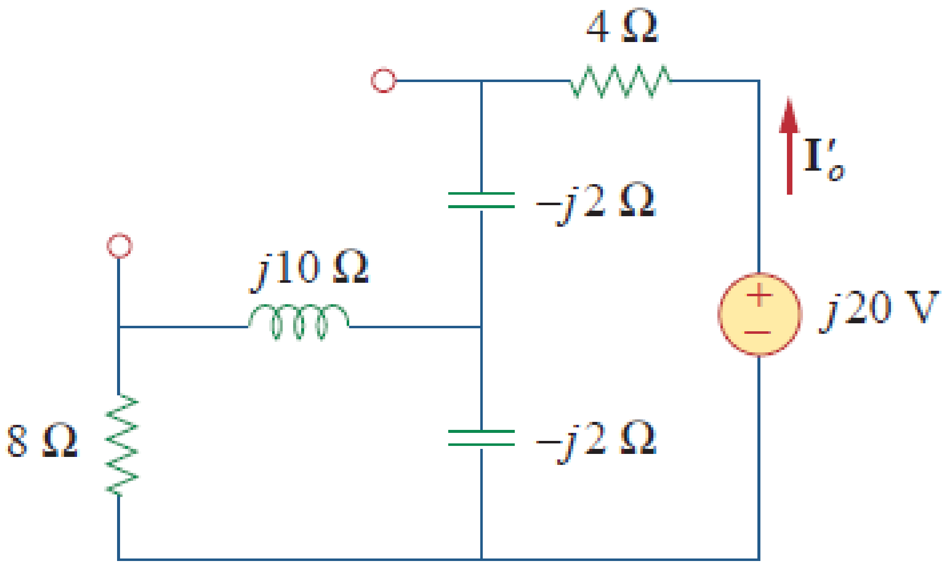
$$\boldsymbol{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\, e^{-j35.22}}{68} = 6.12\, e^{-j35.22}$$

$$\boldsymbol{I}_0 = -\boldsymbol{I}_2 = 6.12\, e^{j144.78}$$

Superposition Theorem

SSA.03 – Find I_o in the circuit.

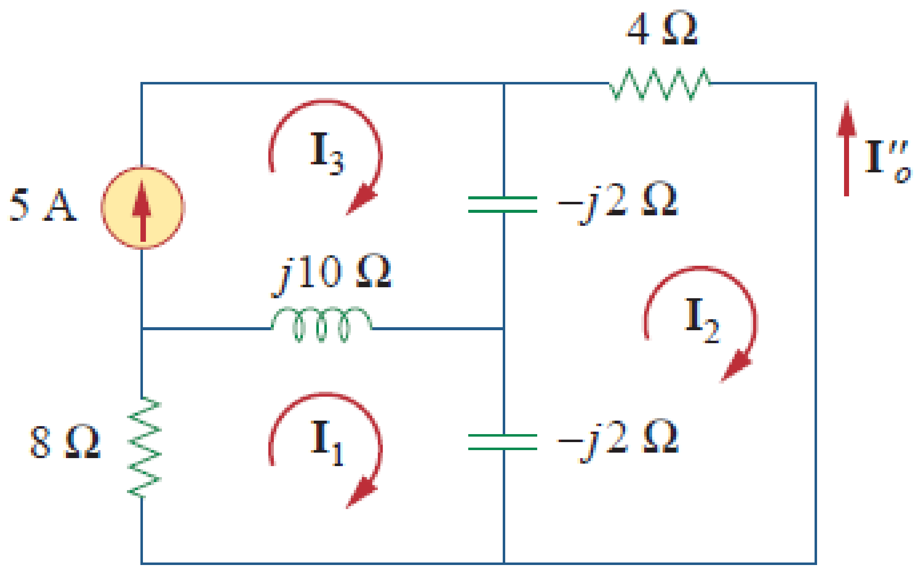




Solution $\boldsymbol{I}_0 = \boldsymbol{I}'_0 + \boldsymbol{I}_0''$

$$\boldsymbol{Z} = \frac{-j2(8+j10)}{-j2+8+j10} = 0.25 - \mathrm{j}2.25 \qquad \boldsymbol{I}'_0 = \frac{j20}{4-j2+\boldsymbol{Z}} = \frac{j20}{4.25-j4.25} = -2.353 + j2.353$$

Superposition Theorem



$$(8+j8)\boldsymbol{I}_1 - j10\boldsymbol{I}_3 + j2\boldsymbol{I}_2 = 0$$

$$(4-j4)\boldsymbol{I}_2 + j2\boldsymbol{I}_1 + j2\boldsymbol{I}_3 = 0$$

$$\boldsymbol{I}_3 = 5$$

$$(4-j4)\boldsymbol{I}_2 + j2\boldsymbol{I}_1 + j10 = 0 \rightarrow \boldsymbol{I}_1 = (2+j2)\boldsymbol{I}_2 - 5$$

$$(8+j8)[(2+j2)\boldsymbol{I}_2 - 5] - j50 + j2\boldsymbol{I}_2 = 0$$

$$\boldsymbol{I}_2 = \frac{90-j40}{34} = 2.647 - j1.176$$

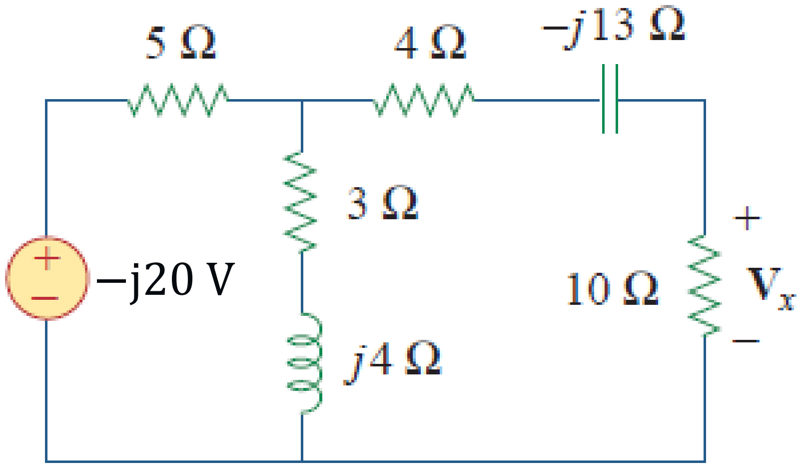
$$\boldsymbol{I}_0'' = -\boldsymbol{I}_2 = -2.647 + j1.176$$

$$\boldsymbol{I}_0 = \boldsymbol{I}_0' + \boldsymbol{I}_0'' = -2.353 + j2.353 - 2.647 + j1.176$$

$$\boldsymbol{I}_0 = -5 + \mathrm{j}3.529 = 6.12\ e^{j144.78°}\ A$$

AC Source Transform

SSA.04 – Calculate V_x in the circuit.

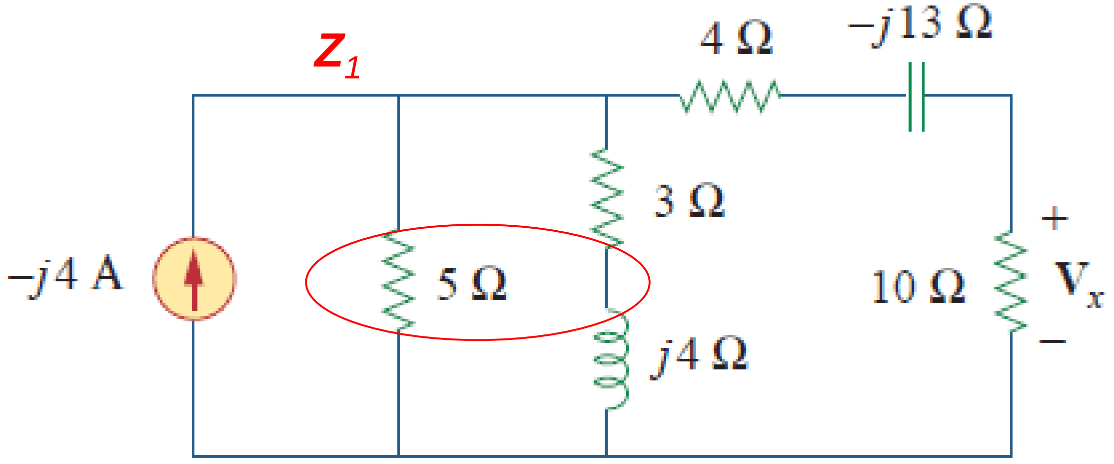


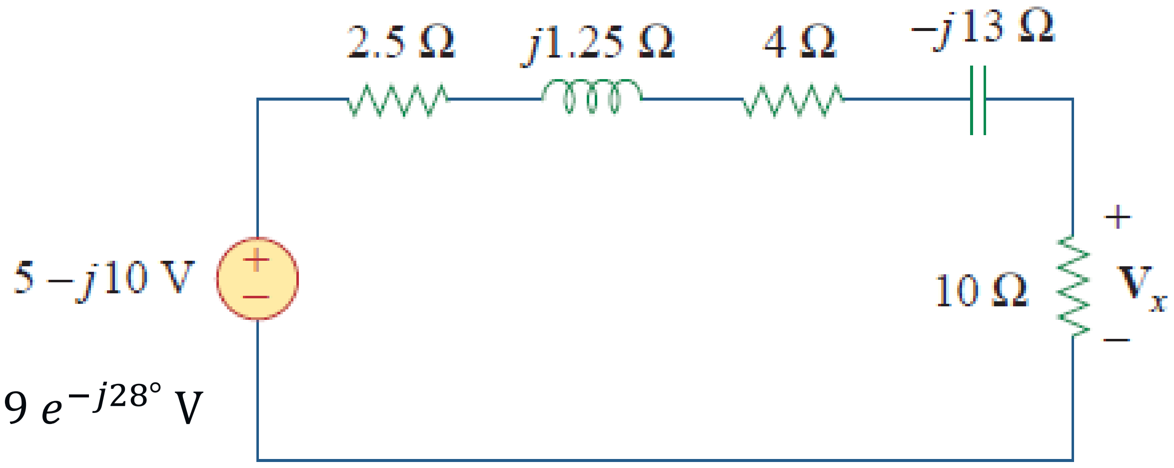
$$Z_1 = \frac{5(3+j4)}{8+j4} = 2.5 + \mathrm{j}1.25\ \Omega$$

$$V_S = I_S \cdot Z_1 = -j4(2.5 + \mathrm{j}1.25) = 5 - j10\ \mathrm{V}$$

$$V_x = (5 - j10)\frac{10}{10 + 2.5 + j1.25 + 4 - j13} = 5.519\ e^{-j28°}\ \mathrm{V}$$

Solution

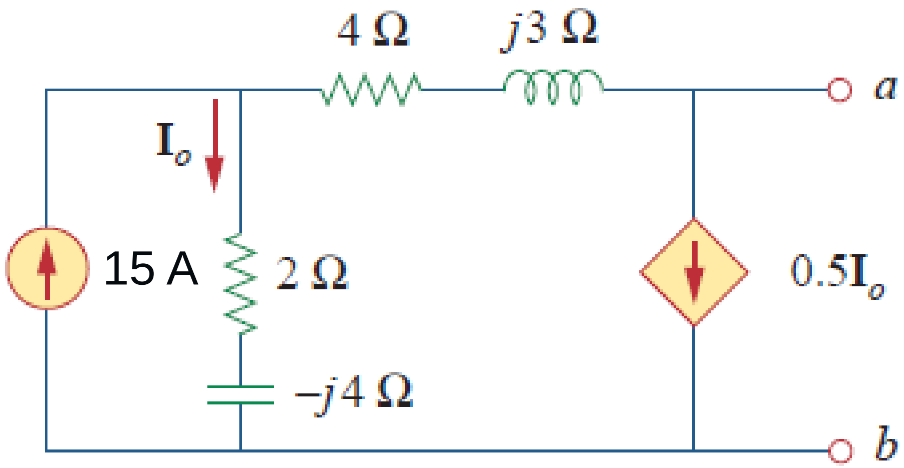




AC Thevenin Equivalent

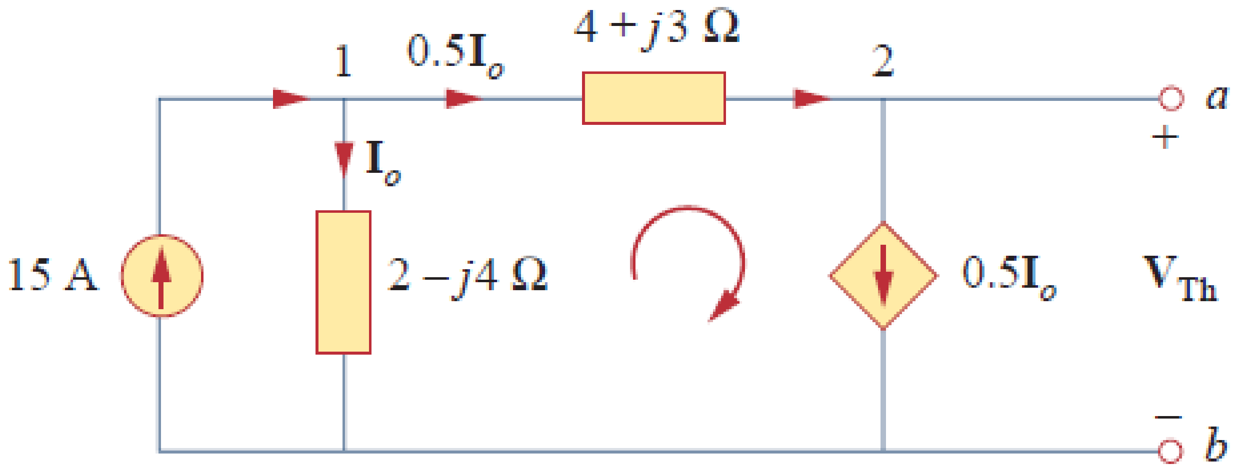
SSA.05

Find the Thevenin equivalent of the circuit in Figure as seen from terminals a-b.



Solution

To obtain $\mathbf{V}_{Th}$ …



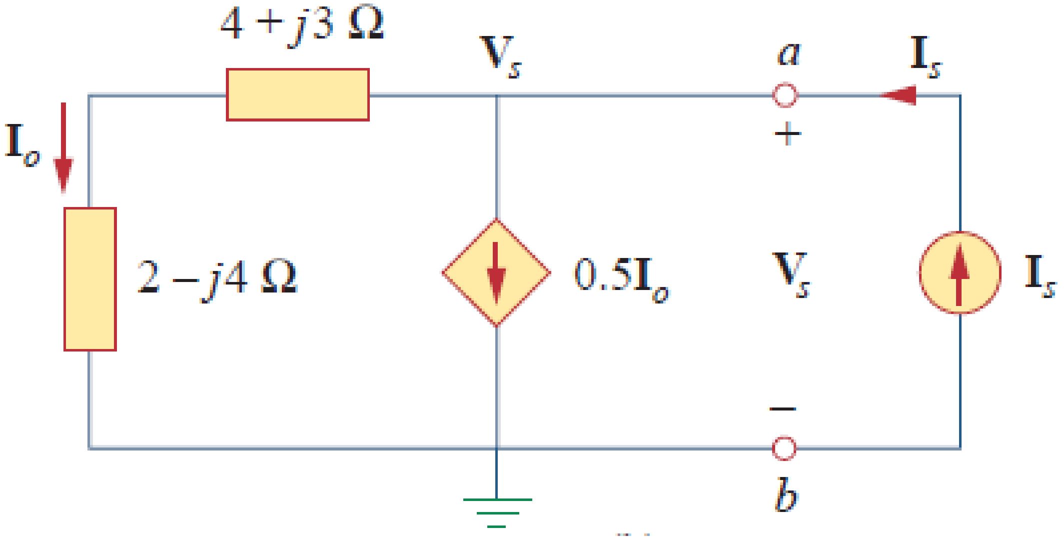
$$15 = \boldsymbol{I}_0 + 0.5\boldsymbol{I}_0 \rightarrow \boldsymbol{I}_0 = 10 \text{ A}$$

$$-\boldsymbol{I}_0(2 - j4) + 0.5\boldsymbol{I}_0(4 + j3) + V_{Th} = 0$$

$$\boldsymbol{V}_{Th} = 10(2 - j4) - 5(4 + j3) = -j55 \text{ V}$$

AC Thevenin Equivalent

To obtain $\mathbf{Z}_{Th}$, we remove the **independent** source (only!).



$$\boldsymbol{I}_S = \boldsymbol{I}_0 + 0.5\boldsymbol{I}_0 \rightarrow \boldsymbol{I}_0 = \frac{\boldsymbol{I}_S}{1.5}$$

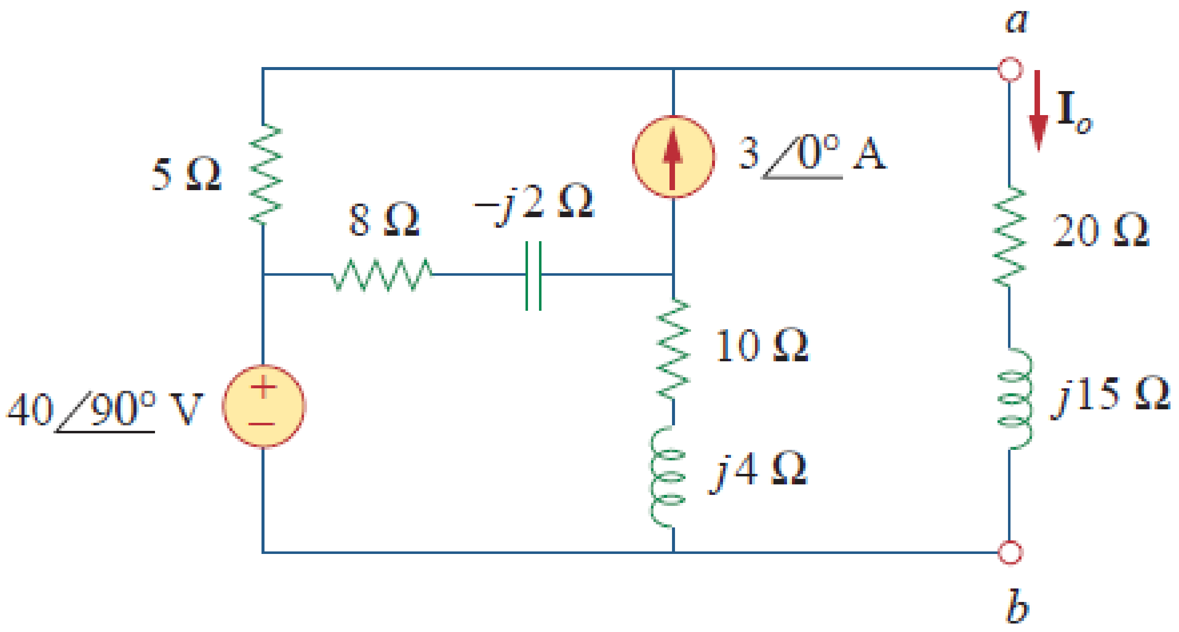
$$\boldsymbol{I}_S \triangleq 3\ A\ (for\ convenience) \rightarrow \boldsymbol{I}_0 = 2\ A$$

$$\boldsymbol{V}_S = \boldsymbol{I}_0(4 + j3 + 2 - j4) = 2(6 - j)$$

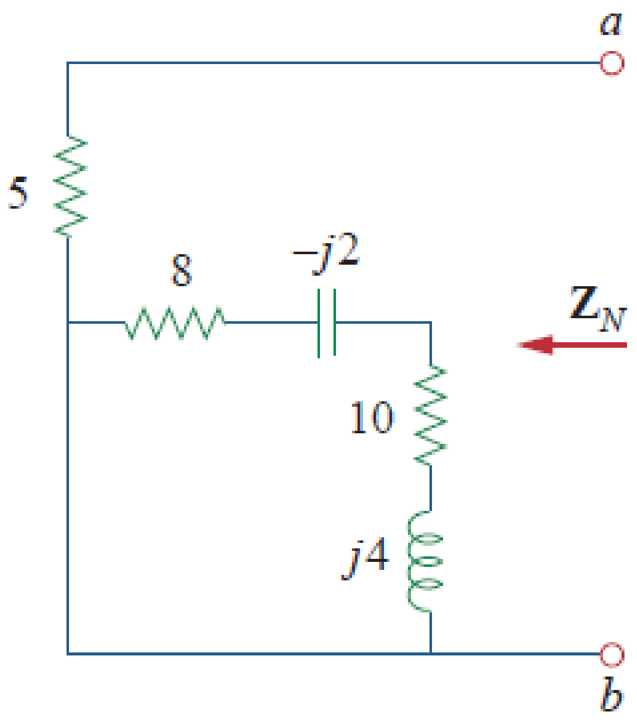
$$\boldsymbol{Z}_{Th} = \frac{\boldsymbol{V}_S}{\boldsymbol{I}_S} = \frac{2(6 - j)}{3} = 4 - j0.67\ \Omega$$

AC Norton Equivalent

SSA.06 – Obtain current $\mathbf{I}_o$ in Figure using Norton's theorem.



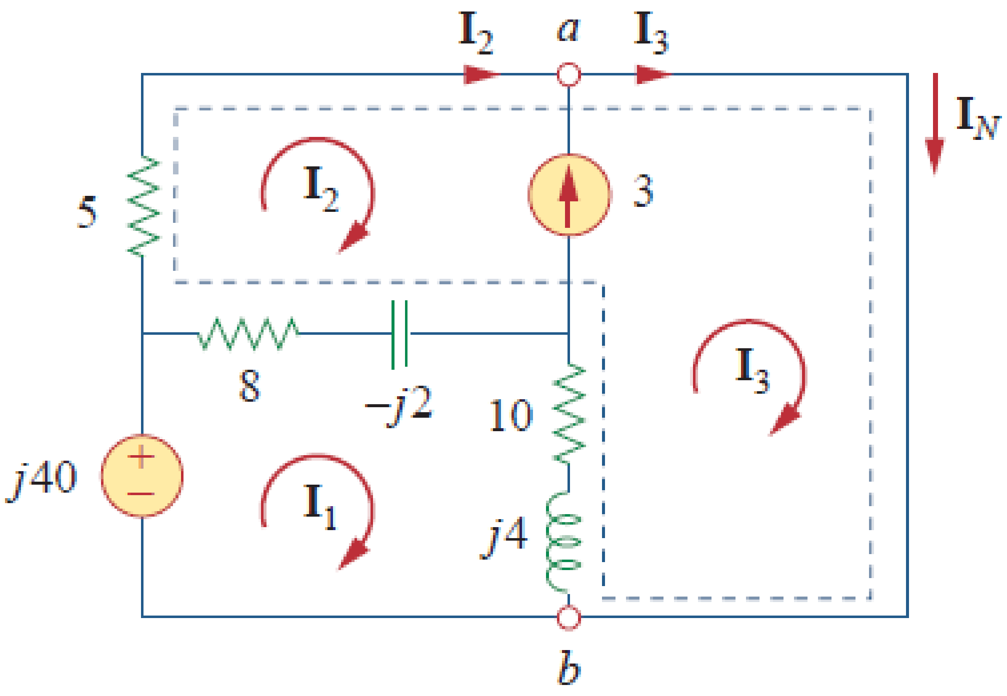
Solution – To obtain $\mathbf{Z}_N$, …



$$Z_N = 5\ \Omega$$

AC Norton Equivalent

To get $\mathbf{I}_N$, ….

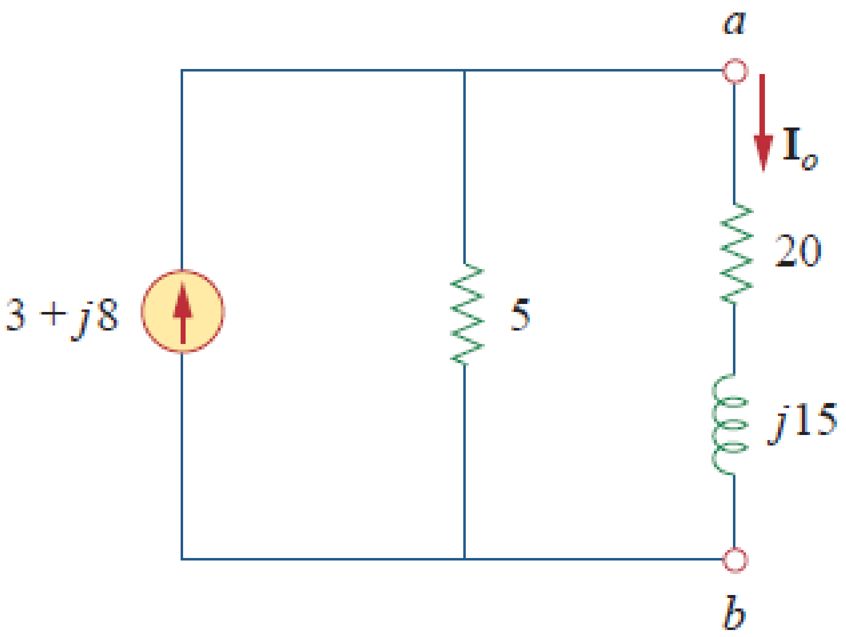


$$-j40 + (18 + j2)\boldsymbol{I}_1 - (8 - j2)\boldsymbol{I}_2 - (10 + j4)\boldsymbol{I}_3 = 0 \ (1)$$

$$(13 - j2)\boldsymbol{I}_2 + (10 + j4)\boldsymbol{I}_3 - (18 + j2)\boldsymbol{I}_1 = 0 \ (2)$$

$$(1) + (2) \rightarrow -j40 + 5\boldsymbol{I}_2 = 0 \rightarrow \boldsymbol{I}_2 = j8$$

$$\boldsymbol{I}_3 = \boldsymbol{I}_2 + 3 \qquad \boldsymbol{I}_N = \boldsymbol{I}_3 = \boldsymbol{I}_2 + 3 = (3 + j8)\ A$$



Current division

$$\boldsymbol{I}_0 = \frac{5}{5 + 20 + j15}\boldsymbol{I}_N = \frac{3 + j8}{5 + j3} = 1.46\ e^{j38.49°}\ A$$

Questions