



DR. GYURCSEK ISTVÁN

Exercises in Three-Phase Circuits

Sources and additional materials (recommended)

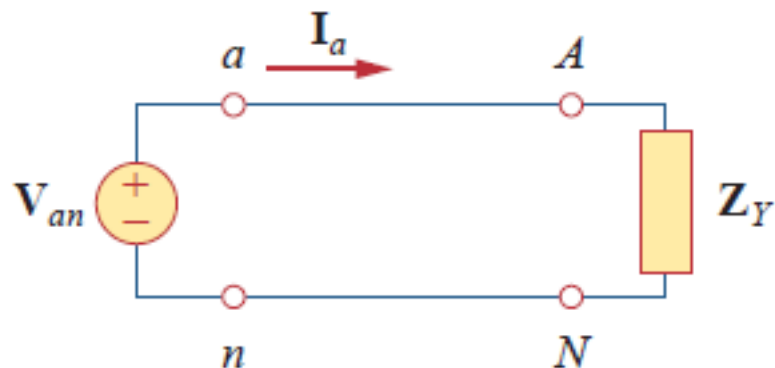
- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

Balanced Y-Y Circuits

THR.01 – Calculate the line currents in the three-wire Y-Y system.

Solution:

$$I_a = \frac{V_{an}}{Z_Y}$$

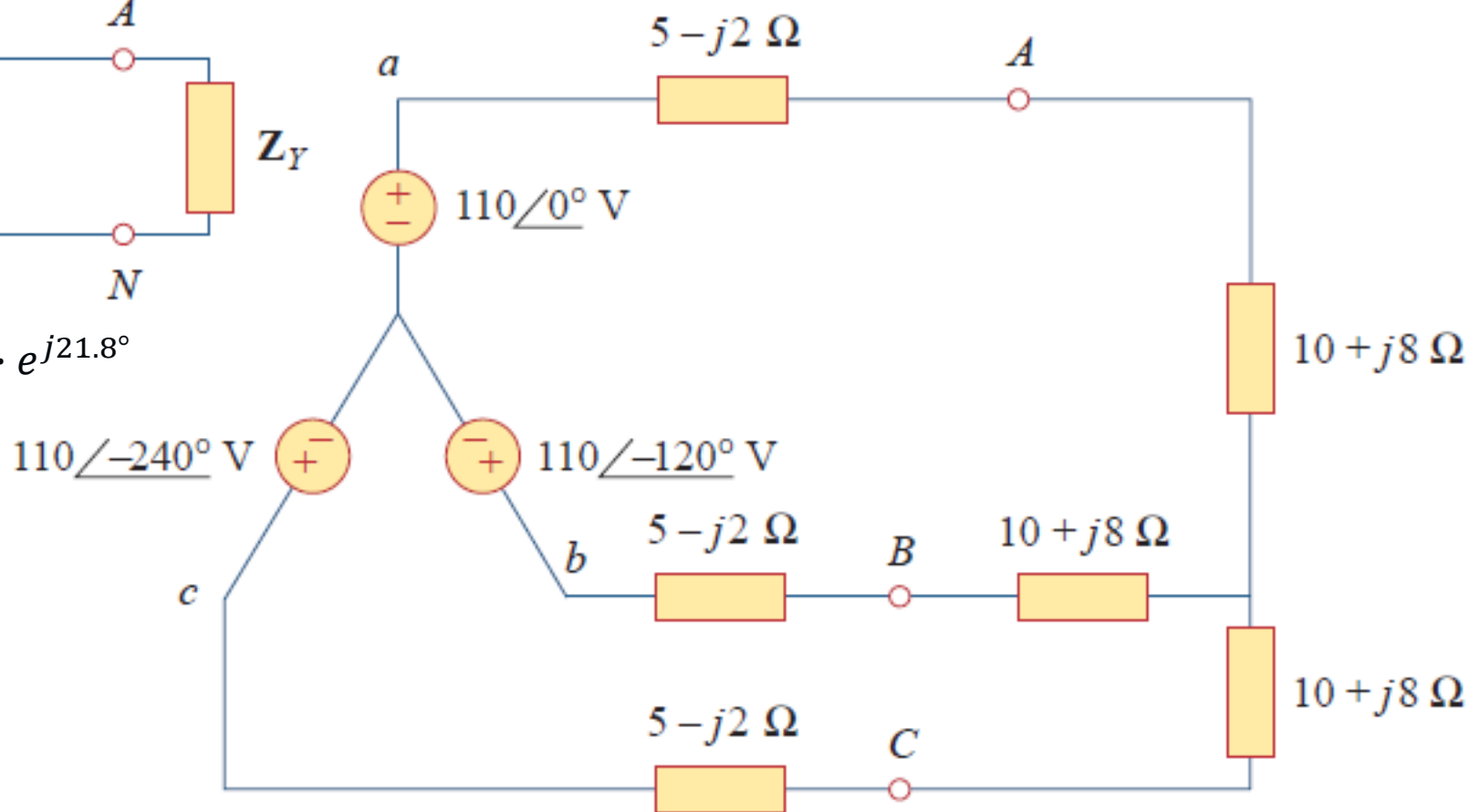


$$Z_Y = (5 - j2) + (10 + j8) = 16.16 \cdot e^{j21.8^\circ}$$

$$I_a = \frac{110 \cdot e^{j0^\circ}}{16.16 \cdot e^{j21.8^\circ}} = 6.81 \cdot e^{-j21.8^\circ}$$

$$I_b = I_a \cdot e^{-j120^\circ} = 6.81 \cdot e^{-j141.8^\circ}$$

$$I_c = I_a \cdot e^{-j240^\circ} = 6.81 \cdot e^{j98.2^\circ}$$





THR.02 – A Y-connected balanced three-phase generator with an impedance of $0.4 + j0.3 \Omega$ per phase is connected to a Y-connected balanced load with an impedance of $24 + j19 \Omega$ per phase. The line joining the generator and the load has an impedance of $0.6 + j0.7 \Omega$ per phase. Assuming a positive sequence for the source voltages and that $V_{an} = 120 e^{j30^\circ} V$, find: (a) the line voltages, (b) the line currents.

Solution: (a) $207.85 e^{j60^\circ} V$, $207.85 e^{-j60^\circ} V$, $207.85 e^{-j180^\circ} V$
(b) $3.75 e^{-j8.66^\circ} A$, $3.75 e^{-j128.66^\circ} A$, $3.75 e^{-j111.34^\circ} A$

Balanced Y-D Circuits

THR.03 – A balanced abc-sequence Y-connected source with $V_{an} = 100 e^{j10^\circ} V$ is connected to a Δ -connected balanced load $8 + j4 \Omega$ per phase. Calculate the phase and line currents.

Solution: $Z_{\Delta} = 8 + j4 = 8.944e^{j26.57^\circ} \Omega$

$$V_{an} = 100 e^{j10^\circ} \rightarrow V_{ab} = V_{an} \sqrt{3} e^{j30^\circ} = 100\sqrt{3} e^{j40^\circ} = V_{AB} = 173.2 e^{j40^\circ} V$$

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{173.2 e^{j40^\circ}}{8.944 e^{j26.57^\circ}} = 19.36 e^{j13.43^\circ} A$$

$$I_{BC} = I_{AB} e^{-j120^\circ} = 19.36 e^{-j106.57^\circ} A$$

$$I_{CA} = I_{AB} e^{j120^\circ} = 19.36 e^{j133.43^\circ} A$$

$$I_a = I_{AB} \sqrt{3} e^{-j30^\circ} = 19.36 \sqrt{3} e^{-j16.57^\circ} = 33.53 e^{-j16.57^\circ} A$$

... or (using single phase analysis)

$$I_b = I_a e^{-j120^\circ} = 33.53 e^{-j136.57^\circ} A$$

$$I_c = I_a e^{j120^\circ} = 33.53 e^{j103.43^\circ} A$$

$$I_a = \frac{V_{an}}{Z_{\Delta}/3} = \frac{100 e^{j10^\circ}}{2.981 e^{j26.57^\circ}} = 33.546 e^{-j16.57^\circ} A$$

Balanced D-D Circuits

THR.04 – A balanced Δ -connected load having an impedance $20 - j15 \Omega$ is connected to a Δ -connected, positive-sequence generator having $V_{ab} = 330 V$. Calculate the phase currents of the load and the line currents.

Solution: $Z_{\Delta} = 20 - j15 = 25e^{-j36.87^{\circ}} \Omega$

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{330}{25e^{-j36.87^{\circ}}} = 13.2e^{j36.87^{\circ}} A$$

$$I_{BC} = I_{AB}e^{-j120^{\circ}} = 13.2e^{-j83.13^{\circ}} A$$

$$I_{CA} = I_{AB}e^{j120^{\circ}} = 13.2e^{j156.87^{\circ}} A$$

$$I_a = I_{AB}\sqrt{3}e^{-j30^{\circ}} = 13.2\sqrt{3}e^{-j6.87^{\circ}} = 22.86e^{-j6.87^{\circ}} A$$

$$I_b = I_a e^{-j120^{\circ}} = 22.86e^{-j113.13^{\circ}} A$$

$$I_c = I_a e^{j120^{\circ}} = 22.86e^{-j126.87^{\circ}} A$$



THR.05 – A positive-sequence, balanced Δ -connected source supplies a balanced Δ -connected load. If the impedance per phase of the load is $18 + j12 \Omega$ and $I_a = 19.202e^{j35^\circ} A$ find I_{AB} and V_{AB} .

Solution: $I_{AB} = 11.094e^{j65^\circ} A$, $V_{AB} = 240e^{j98.69^\circ} V$

THR.06 – Balanced, Y-connected load is supplied by a balanced, positive sequence delta-connected source. Calculate the phase currents. $\mathbf{Z}_Y = (40 + j25) \Omega$, $\mathbf{V}_{ab} = 210 V$

Solution:
$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}} \cdot e^{-j30^\circ} = 121.2 \cdot e^{-j30^\circ}$$

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2 \cdot e^{-j30^\circ}}{47.12 \cdot e^{j32^\circ}} = 2.57 \cdot e^{-j62^\circ}$$

$$\mathbf{I}_b = \mathbf{I}_a \cdot e^{-j120^\circ} = 2.57 \cdot e^{-j182^\circ}$$

$$\mathbf{I}_c = \mathbf{I}_a \cdot e^{j120^\circ} = 2.57 \cdot e^{j58^\circ}$$

Power in Balanced System

THR.07 – Determine the total average power, reactive power, and complex power at the source and at the load.

Solution $V_a = 110 \text{ V}$

$$Z_Y = (5 - j2) + (10 + j8) = 16.16 \cdot e^{j21.8^\circ} \Omega$$

$$I_a = \frac{110}{16.16 \cdot e^{j21.8^\circ}} = 6.81 \cdot e^{-j21.8^\circ}$$

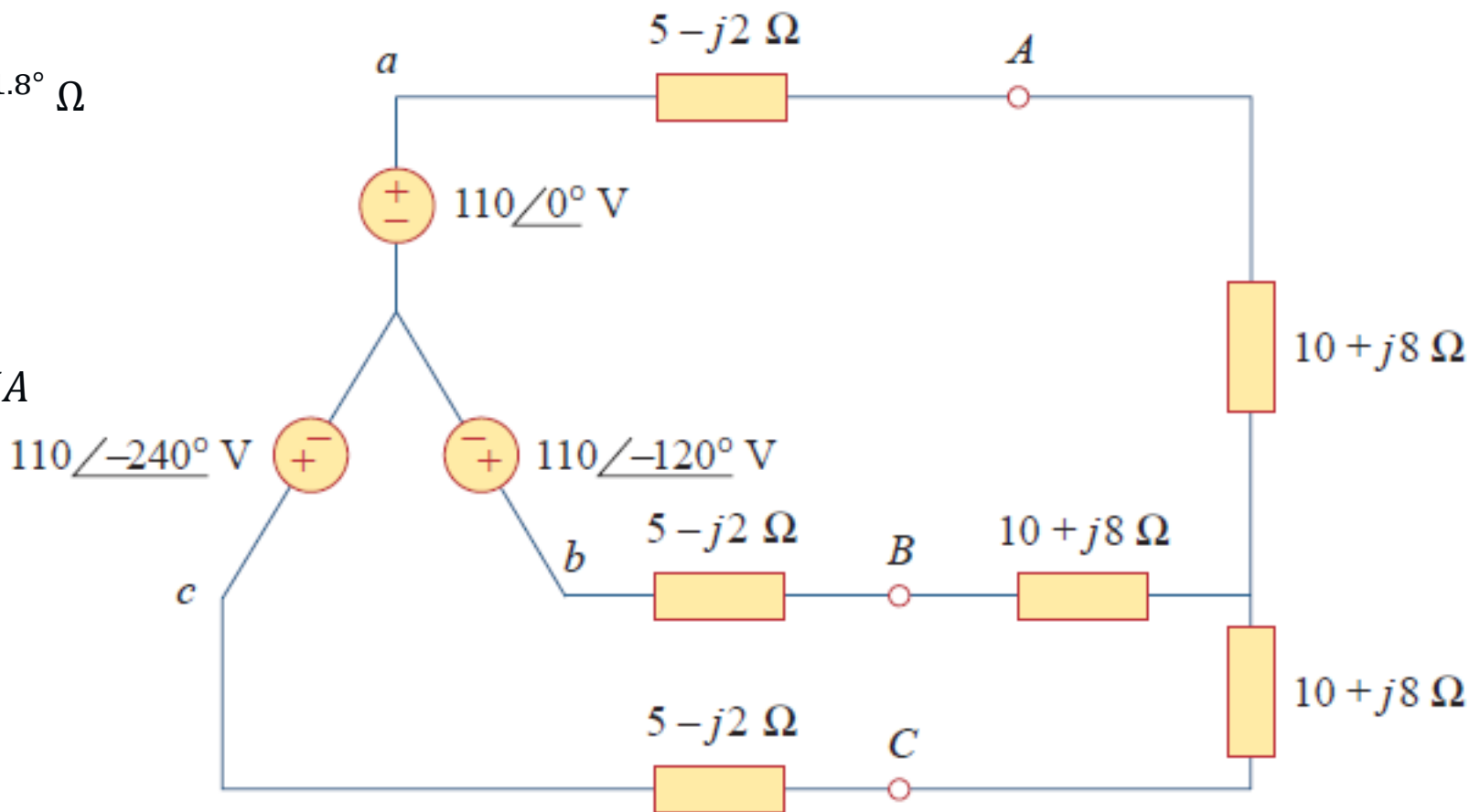
$$\begin{aligned} S_S &= -3V_a I_a^* = -3 \cdot 110 \cdot 6.81 \cdot e^{j21.8^\circ} \\ &= -2247 \cdot e^{j21.8^\circ} = -(2087 + j834.6) \text{ VA} \end{aligned}$$

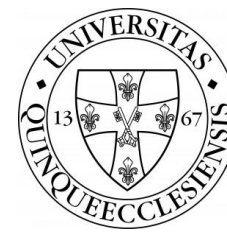
$$P_S = -2087 \text{ W} \quad Q_S = -834.6 \text{ VAR}$$

$$Z_a = 10 + j8 = 12.81 \cdot e^{j38.66^\circ} \Omega$$

$$\begin{aligned} S_L &= 3 I_a^2 Z_a = 3 \cdot 6.81^2 \cdot 12.81 \cdot e^{j38.66^\circ} \\ &= 1782 \cdot e^{j38.66^\circ} = (1392 + j1113) \text{ VA} \end{aligned}$$

$$P_L = 1392 \text{ W} \quad Q_L = 1113 \text{ VAR} \quad S_l = 3 I_a^2 Z_l = 3 \cdot 6.81^2 \cdot (5 - j2) = (695.6 - j278.3) \text{ VA} \quad S_S + S_L + S_l = 0$$





THR.08 – A three-phase motor can be regarded as a balanced Y-load. The motor draws 5.6 kW when the line voltage is 230 V and the line current is 17.5 A. Determine the power factor of the motor.

Solution $S = \sqrt{3}V_L I_L = \sqrt{3} \cdot 230 \cdot 17.5 = 6971.5 \text{ VA}$

$$P = S \cos \Theta = 5600 \text{ W} \rightarrow pf = \cos \Theta = \frac{P}{S} = \frac{5600}{6971.5} = 0.803$$

THR.09 – Calculate the line current required for a 30-kW three-phase motor having a power factor of 0.85 lagging if it is connected to a balanced source with a line voltage of 440 V. **Solution 46.31 A.**

Power in Balanced System

THR.10 – Balanced loads are connected to a 240-kV rms 60-Hz line. Load 1 draws 30 kW at a power factor of 0.6 lagging (IND), while load 2 draws 45 kVAR at a power factor of 0.8 lagging (IND). Assuming the abc sequence, determine: (a) the complex, real, and reactive powers absorbed by the combined load, (b) the line currents, and (c) the kVAR rating of the three capacitors Δ -connected in parallel with the load that will raise the power factor to 0.9 lagging and the capacitance of each capacitor.

Solution (a) $\cos^2 x + \sin^2 x = 1$

$$S_1 = \frac{P_1}{\cos \theta_1} = \frac{30,000}{0.6} = 50 \text{ kVA}$$

$$Q_1 = S_1 \sin \theta_1 = 50 \cdot 0.8 = 40 \text{ kVAR}$$

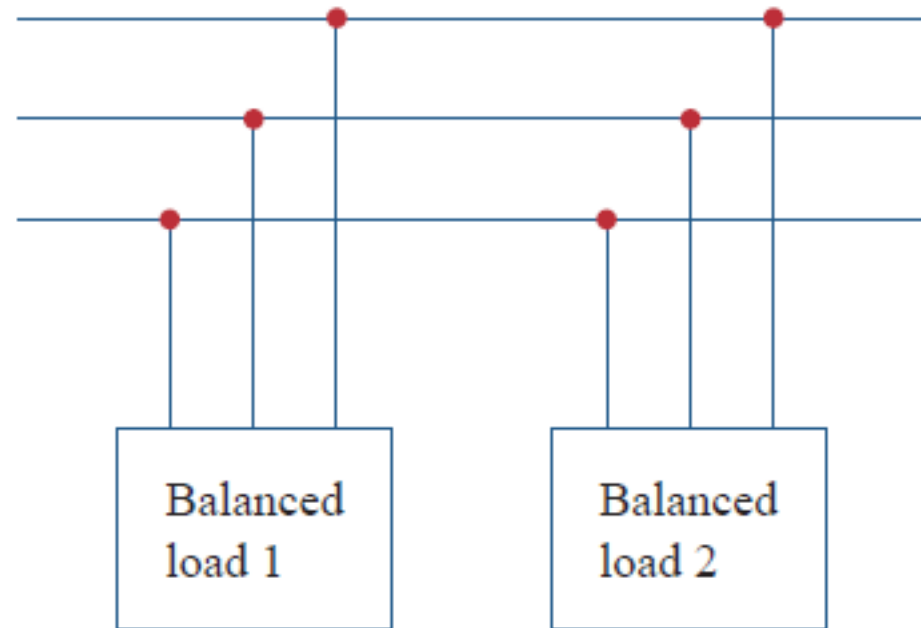
$$\mathbf{S}_1 = P_1 + jQ_1 = (30 + j40) \text{ kVA}$$

$$S_2 = \frac{Q_2}{\sin \theta_2} = \frac{45,000}{0.6} = 75 \text{ kVA}$$

$$P_2 = S_2 \cos \theta_2 = 75 \cdot 0.8 = 60 \text{ kW}$$

$$\mathbf{S}_2 = P_2 + jQ_2 = (60 + j45) \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = (90 + j85) = 123.8 e^{j43.36^\circ} \text{ kVA}$$



Power in Balanced System

Solution (b) - line currents $S = \sqrt{3}V_L I_L \rightarrow I_L = \frac{S}{\sqrt{3}V_L}$

$$I_{L1} = \frac{50,000}{\sqrt{3} 240,000} = 120.28 \text{ mA}$$

$$\Theta_1 = \cos^{-1}0.6 = 53.13^\circ \rightarrow I_{a1} = 120.28 e^{-j53.13^\circ} = 72.168 - j96.224$$

$$I_{L2} = \frac{75,000}{\sqrt{3} 240,000} = 180.42 \text{ mA}$$

$$\Theta_2 = \cos^{-1}0.8 = 36.87^\circ \rightarrow I_{a2} = 180.42 e^{-j36.87^\circ} = 144.336 - j108.252$$

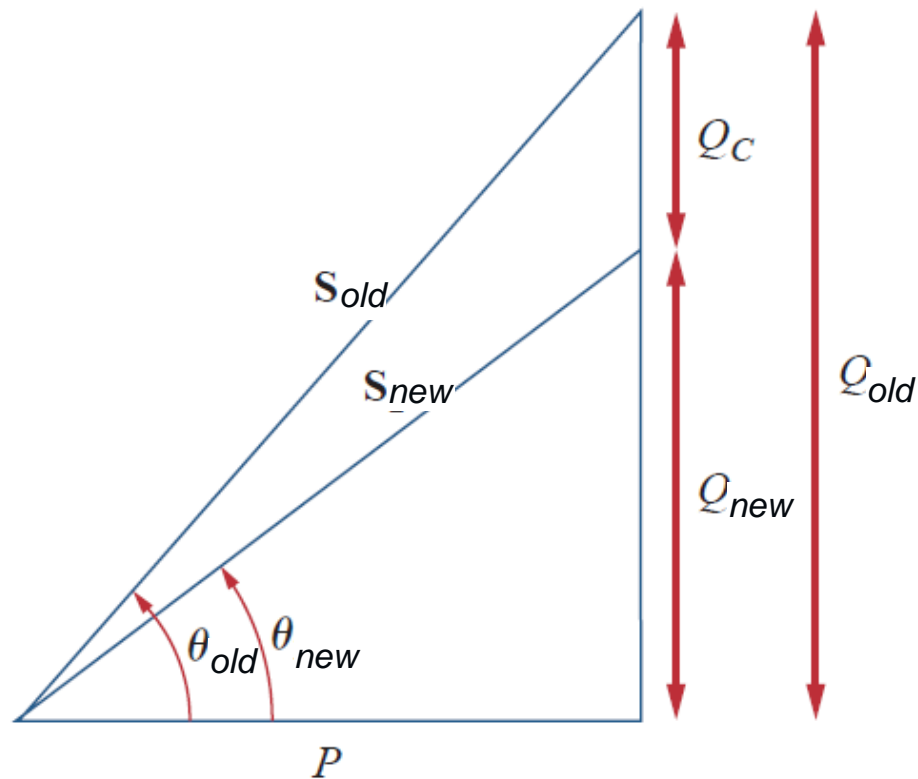
$$I_a = I_{a1} + I_{a2} = \dots = 216.5 - j204.472 = 297.8 e^{-j43.36^\circ} \text{ mA}$$

$$\text{or ... } S = S_1 + S_2 = 123.8 e^{j43.36^\circ} \text{ kVA} \rightarrow I_L = \frac{S}{\sqrt{3}V_L} = \frac{123,800}{\sqrt{3} 240,000} = 297.82 \text{ mA}, \quad I_a = 297.8 e^{-j43.36^\circ} \text{ mA}$$

$$\text{'abc' sequence} \rightarrow I_b = 297.8 e^{-j43.36^\circ} e^{-j120^\circ} = 297.8 e^{-j163.36^\circ} \text{ mA}, \quad I_c = 297.8 e^{j76.64^\circ} \text{ mA}$$

Power in Balanced System

Solution (c) – PFC (power factor correction)



$$Q_C = Q_{old} - Q_{new}$$

$$\frac{Q_C}{P} = \frac{Q_{old}}{P} - \frac{Q_{new}}{P}$$

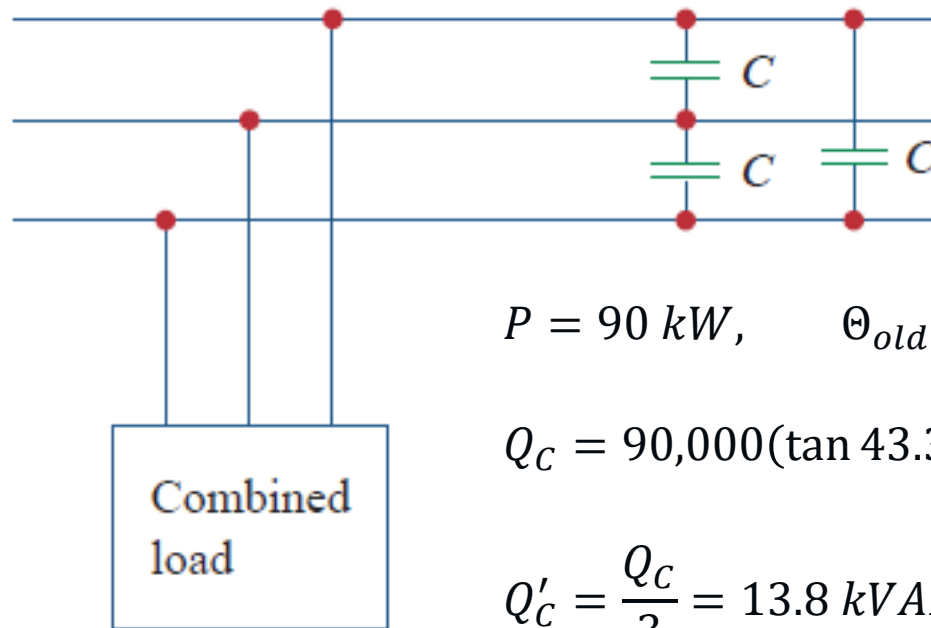
$$\frac{Q_C}{P} = \tan \Theta_{old} - \tan \Theta_{new}$$

$$Q_C = P(\tan \Theta_{old} - \tan \Theta_{new})$$

$$Q_C = \frac{V_{rms}^2}{X_C} = \omega C V_{rms}^2$$

$$C = \frac{Q_C}{\omega V_{rms}^2}$$

Power in Balanced System



$$Q_C = P(\tan \Theta_{old} - \tan \Theta_{new})$$

$$P = 90 \text{ kW}, \quad \Theta_{old} = 43.36^\circ, \quad \Theta_{new} = \cos^{-1} 0.9 = 25.84^\circ$$

$$Q_C = 90,000(\tan 43.36^\circ - \tan 25.84^\circ) = 41.4 \text{ kVAR}$$

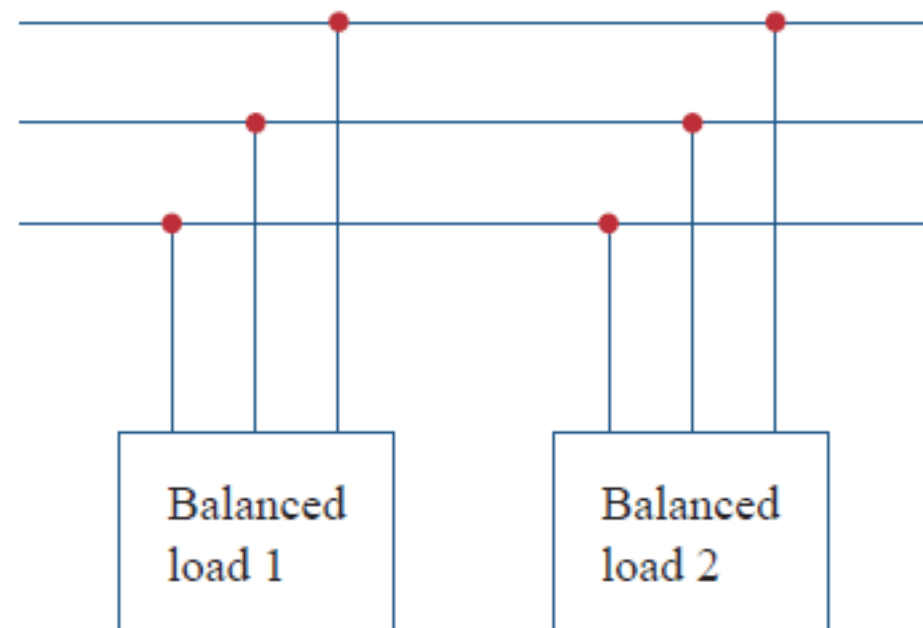
$$Q'_C = \frac{Q_C}{3} = 13.8 \text{ kVAR} \rightarrow C = \frac{Q'_C}{\omega V_{rms}^2} = \frac{13,800}{2\pi \cdot 60 \cdot 240,000^2} = 635.5 \text{ pF}$$

Power in Balanced System

THR.11

Two balanced loads are supplied by an 840-V rms 60-Hz line. Load 1 is Y-connected with $(30 + j40) \Omega$ per phase, while load 2 is a balanced three-phase motor drawing 48 kW at a power factor of 0.8 lagging. Assuming the abc sequence, calculate (a) the complex power absorbed by the combined load, (b) the kVAR rating of each of the three capacitors Δ -connected in parallel with the load to raise the power factor to unity ($pf = 1$), and (c) the current drawn from the supply at unity power factor condition.

Solution (a) $56.47 + j47.29$ kVA, (b) 15.76 kVAR, (c) 38.813 A.



Unbalanced Load

THR.12 – The unbalanced Y-load has been supplied by the balanced phase voltages of 230 V and the ,abc' sequence.

Calculate the line currents and the neutral current. $Z_A = 30 \Omega$, $Z_B = (15 + j10) \Omega$, $Z_C = (10 - j10) \Omega$

Solution: $V_{AN} = 230 \cdot e^{j0^\circ} V$, $V_{BN} = 230 \cdot e^{-j120^\circ} V$, $V_{CN} = 230 \cdot e^{j120^\circ} V$

$$I_a = \frac{V_{AN}}{Z_A} = \frac{230}{30} = 7.67 A$$

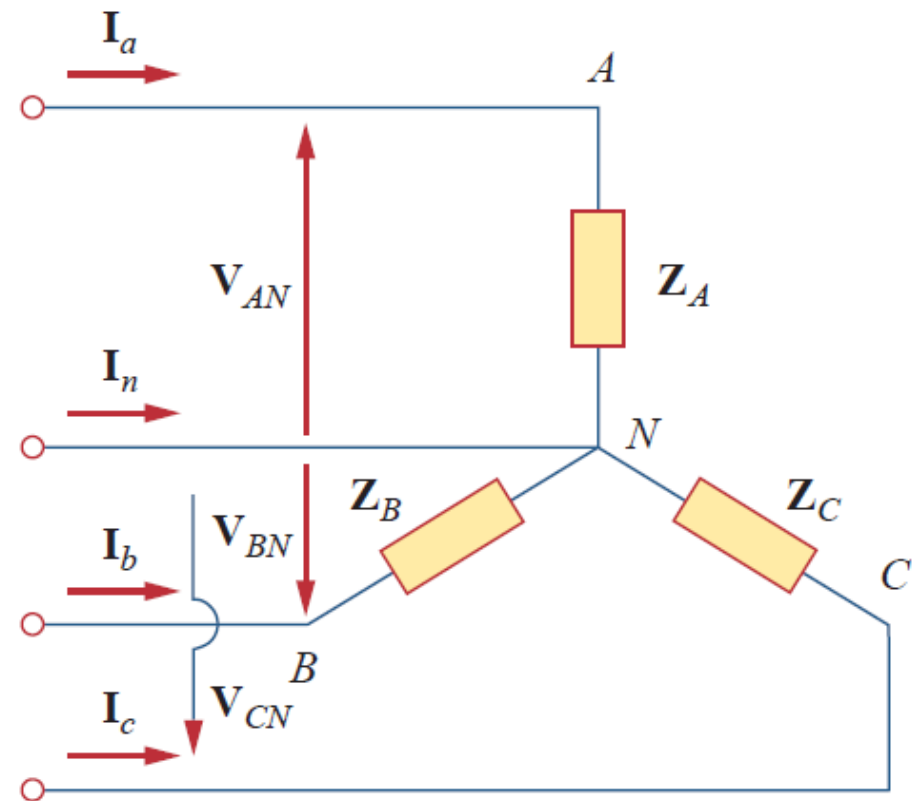
$$I_b = \frac{V_{BN}}{Z_B} = \frac{230 \cdot e^{-j120^\circ}}{15 + j10}$$

$$= \frac{230 \cdot e^{-j120^\circ}}{18.03 \cdot e^{j33.69^\circ}} = 12.75 \cdot e^{-j153.69^\circ} = (-11.42 - j5.65) A$$

$$I_c = \frac{V_{CN}}{Z_C} = \frac{230 \cdot e^{j120^\circ}}{10 - j10}$$

$$= \frac{230 \cdot e^{j120^\circ}}{14.14 \cdot e^{-j45^\circ}} = 16.27 \cdot e^{j165^\circ} = (-15.71 + j4.21) A$$

$$I_n = -(I_a + I_b + I_c) = (19.46 + j1.44) A$$

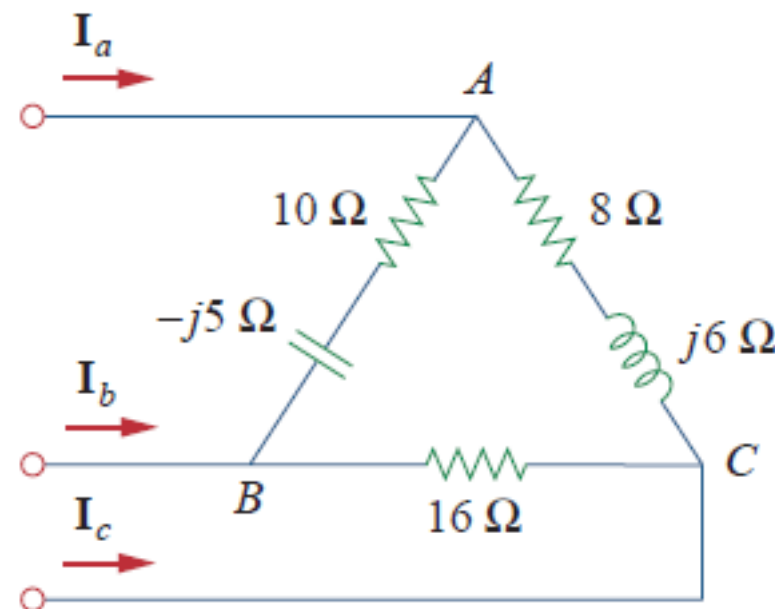


Unbalanced Load

THR.13

The unbalanced Δ -load is supplied by balanced line-to-line voltages of 240 V in the positive sequence. Find the line currents. Take \mathbf{V}_{ab} as reference.

Solution $21.66 e^{-j41.06^\circ} \text{ A}$, $34.98 e^{-j139.8^\circ} \text{ A}$, $38.24 e^{j74.27^\circ} \text{ A}$



Unbalanced Load

THR.14 – Find: (a) line currents, (b) total complex power absorbed by the load, (c) the total complex power supplied by the source.

Solution (a)

$$V_{an} = 120, V_{bn} = 120 e^{-j120^\circ}, V_{cn} = 120 e^{j120^\circ}$$

$$V_{bn} - V_{an} + (10 + j5)I_1 - 10I_2 = 0$$

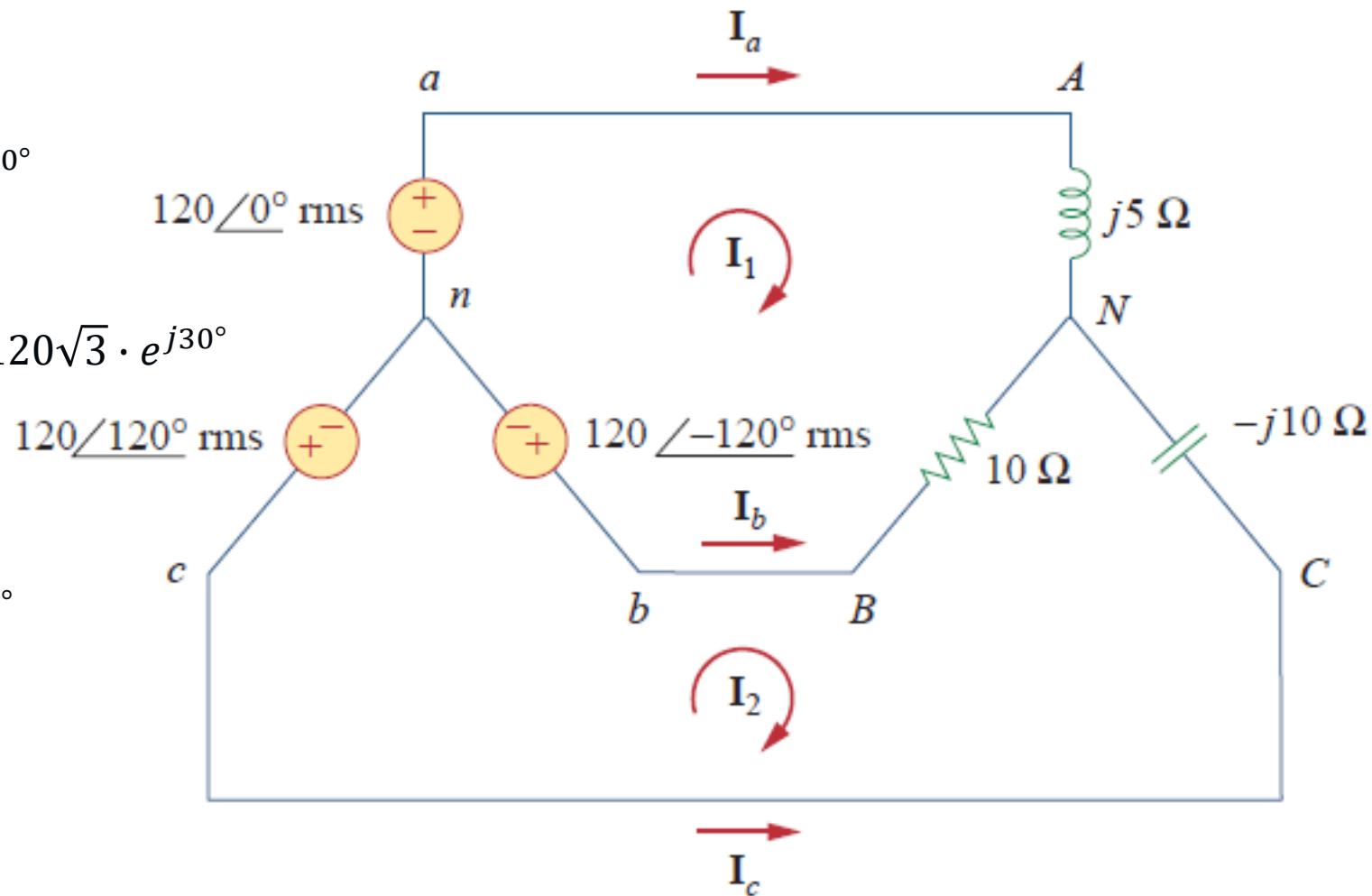
$$(10 + j5)I_1 - 10 \cdot I_2 = 120 - 120 e^{-j120^\circ} = 120\sqrt{3} \cdot e^{j30^\circ}$$

$$V_{cn} - V_{bn} + (10 - j10)I_2 - 10I_1 = 0$$

$$-10I_1 + (10 - j10)I_2 =$$

$$= 120 \cdot e^{-j120^\circ} - 120 \cdot e^{j120^\circ} = 120\sqrt{3} \cdot e^{-j90^\circ}$$

$$\begin{bmatrix} 10 + j5 & -10 \\ -10 & 10 - j10 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 120\sqrt{3} \cdot e^{j30^\circ} \\ 120\sqrt{3} \cdot e^{-j90^\circ} \end{bmatrix}$$



Unbalanced Load



$$\begin{bmatrix} 10 + j5 & -10 \\ -10 & 10 - j10 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 120\sqrt{3} \cdot e^{j30^\circ} \\ 120\sqrt{3} \cdot e^{-j90^\circ} \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 10 + j5 & -10 \\ -10 & 10 - j10 \end{vmatrix} = 50 - j50 = 70,71 \cdot e^{-j45^\circ}$$

$$\Delta_1 = \begin{vmatrix} 120\sqrt{3} \cdot e^{j30^\circ} & -10 \\ 120\sqrt{3} \cdot e^{-j90^\circ} & 10 - j10 \end{vmatrix} = 207.85 \cdot (13.66 - j13.66) = 4015.23 \cdot e^{-j45^\circ}$$

$$\Delta_2 = \begin{vmatrix} 10 + j5 & 120\sqrt{3} \cdot e^{j30^\circ} \\ -10 & 120\sqrt{3} \cdot e^{-j90^\circ} \end{vmatrix} = 207.85 \cdot (13.66 - j5) = 3023.4 \cdot e^{-j20.12^\circ}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{4015.23 \cdot e^{-j45^\circ}}{70,71 \cdot e^{-j45^\circ}} = 113.2 \cdot e^{j0^\circ} = 56.78 \text{ A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{3023.4 \cdot e^{-j20.12^\circ}}{70,71 \cdot e^{-j45^\circ}} = 42.75 \cdot e^{j24.88^\circ} = (38.18 + j18) \text{ A}$$

Unbalanced Load

$$I_a = I_1 = 56.78 \text{ A}$$

$$I_b = I_2 - I_1 =$$

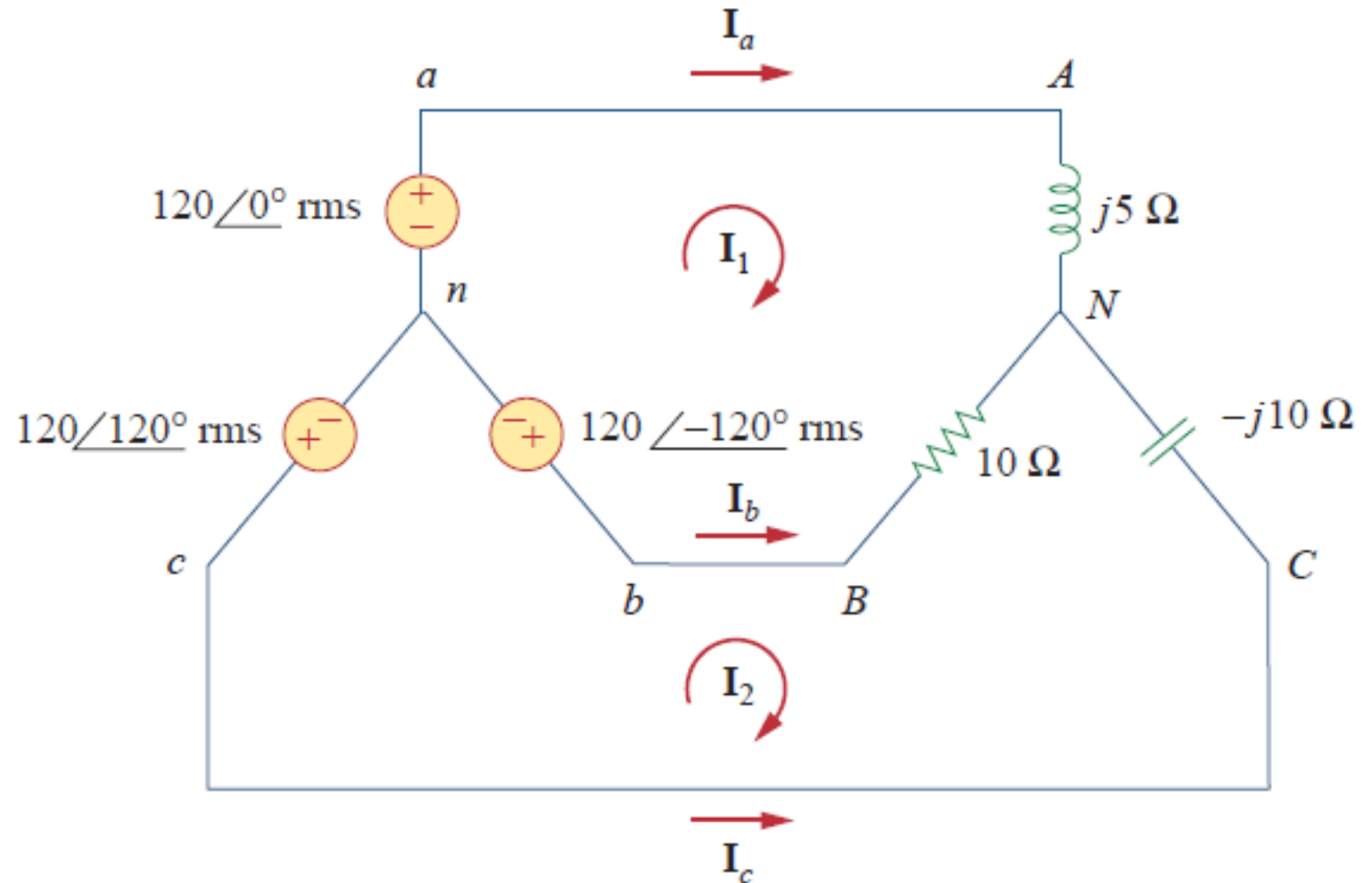
$$= (38.18 + j18) - 56.78 =$$

$$= 25.46 \cdot e^{j135^\circ} \text{ A}$$

$$I_c = -I_2 = 42.75 \cdot e^{-j155.12^\circ} \text{ A}$$

,Extremely' unbalanced load →

- voltage phase sequence is a-b-c
- current phase sequence is a-c-b



Unbalanced Load

Solution (b) complex power absorbed by the load

$$\mathbf{S}_A = I_a^2 \cdot \mathbf{Z}_A = 56.78^2 \cdot (j5) = j16.12 \text{ kVA} \rightarrow P_A = 0 \text{ W}, \quad Q_A = 16.12 \text{ kVAR}$$

$$\mathbf{S}_B = I_b^2 \cdot \mathbf{Z}_B = 25.46^2 \cdot 10 = 6.48 \text{ kVA} \rightarrow P_B = 6.48 \text{ kW}, \quad Q_B = 0$$

$$\mathbf{S}_C = I_c^2 \cdot \mathbf{Z}_C = 42.75^2 \cdot (-j10) = -j18.276 \text{ kVA} \rightarrow P_C = 0 \text{ W}, \quad Q_C = -18.276 \text{ kVAR}.$$

$$\mathbf{S}_L = \mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C = j16.12 + 6.48 - j18.276 = (6.48 - j2.156) \text{ kVA}$$

Solution (c) complex power supplied by the source

$$\mathbf{S}_a = -V_{an} \cdot \mathbf{I}_a^* = -120 \cdot 56.78 = -6.8136 \text{ kVA}$$

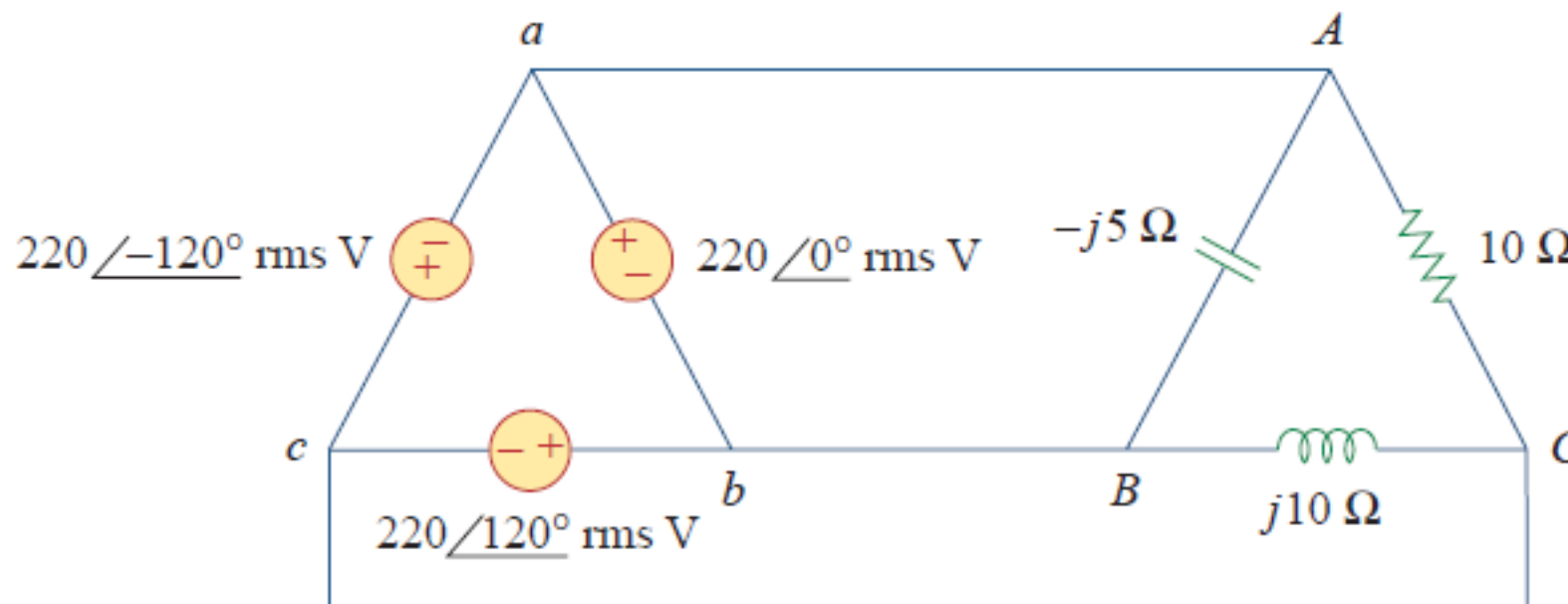
$$\mathbf{S}_b = -V_{bn} \cdot \mathbf{I}_b^* = -(120 \cdot e^{-j120^\circ}) \cdot 25.46 \cdot e^{-j135^\circ} = -3055.2 \cdot e^{j105^\circ} = (0.79 - j2.9511) \text{ kVA}$$

$$\mathbf{S}_c = -V_{cn} \cdot \mathbf{I}_c^* = -(120 \cdot e^{j120^\circ}) \cdot 42.75 \cdot e^{j155.12^\circ} = -5130 \cdot e^{j275.1^\circ} = (-456.03 + j5109.7) \text{ VA}$$

$$\mathbf{S}_s = \mathbf{S}_a + \mathbf{S}_b + \mathbf{S}_c = (-6.47963 + j2.1586) \text{ kVA} \quad \mathbf{S}_s + \mathbf{S}_L = 0$$

Unbalanced Load

THR.15 – Find the line currents and the real power absorbed by the load.



Solution $64 e^{j80.1^\circ} A$, $38.1 e^{-j60^\circ} A$, $42.5 e^{j225^\circ} A$

Three-Phase Circuits

THR.16 – Balanced voltage system supplies the unbalanced load. The phase voltage of the source system is 230 V with phase sequence of ,abc'. Take V_{an} as reference. Find the V_{Nn} voltage if $Z_A = 10 \Omega$, $Z_B = j20 \Omega$, $Z_C = -j10 \Omega$, $Z_n = 10 \Omega$.

Solution $Y_A = 0.1 \text{ S}, \quad Y_B = -j0.05 \text{ S}$

$Y_C = j0.1 \text{ S}, \quad Y_n = 0.1 \text{ S}$

$V_{an} = 230 \cdot e^{j0^\circ} \text{ V}$

$V_{bn} = 230 \cdot e^{-j120^\circ} \text{ V}$

$V_{cn} = 230 \cdot e^{j120^\circ} \text{ V}$

$$V_{Nn} = \frac{V_{an} \cdot Y_A + V_{bn} \cdot Y_B + V_{cn} \cdot Y_C}{Y_A + Y_B + Y_C + Y_n} = \dots$$

$= (-39.41 - j18.92) \text{ V} \rightarrow V_{Nn} = 43.71 \text{ V}$

