

DR. GYURCSEK ISTVÁN

# Exercises in Resonance Circuits

*Sources and additional materials (recommended)*

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

# AC Circuits



**RES.01** - Calculate the equivalent impedance, the circuit current, power factor and draw the voltage phasor diagram.

## Solution

$$X_L = \omega \cdot L = 2\pi \cdot 50 \cdot 0.15 = 47.13\Omega$$

$$X_C = \frac{1}{\omega \cdot C} = \frac{1}{2\pi \cdot 50 \cdot 100 \cdot 10^{-6}} = 31.83\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 19.4\Omega$$

$$I = \frac{V_S}{Z} = \frac{100}{19.4} = 5.15 \text{ A}$$

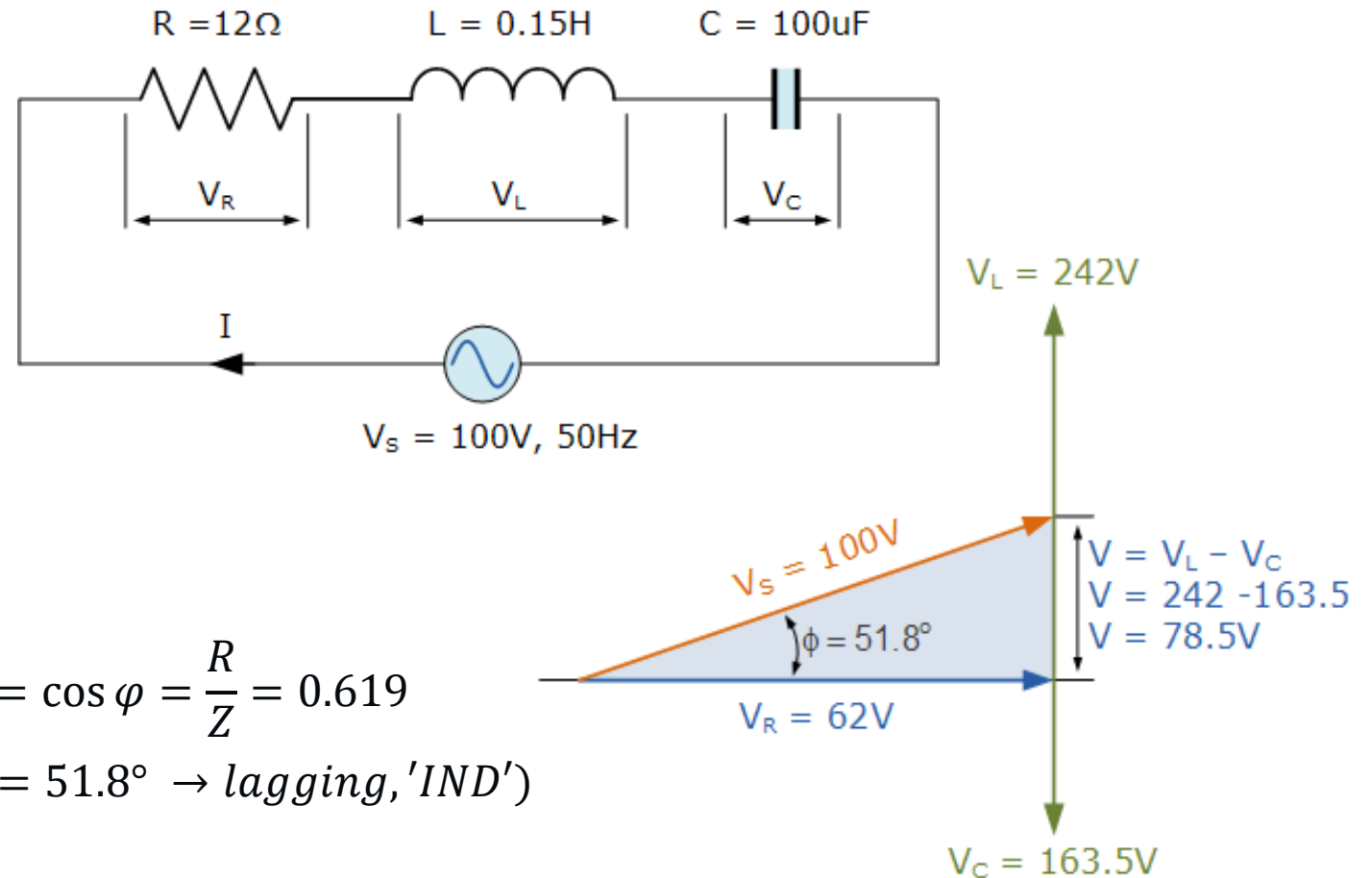
$$V_R = I \cdot R = 5.15 \cdot 12 = 61.8 \text{ V}$$

$$V_L = I \cdot X_L = 5.15 \cdot 47.13 = 242.4 \text{ V}$$

$$V_C = I \cdot X_C = 5.15 \cdot 31.83 = 163.5 \text{ V}$$

$$pf = \cos \varphi = \frac{R}{Z} = 0.619$$

( $\varphi = 51.8^\circ \rightarrow$  lagging, 'IND')



# AC Circuits



**RES.02** - Calculate the impedance of the parallel RLC circuit and the current drawn from the supply.

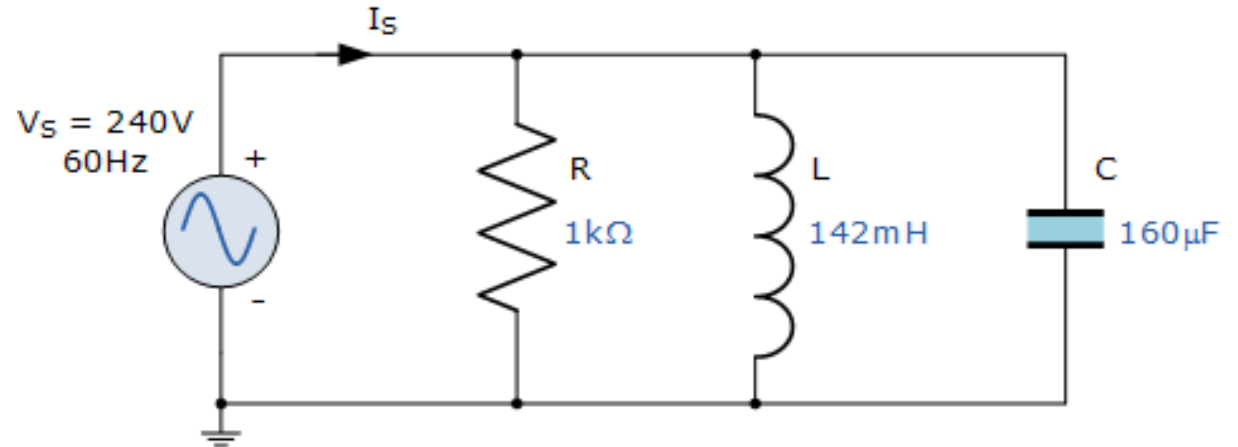
**Solution**

$$X_L = \omega \cdot L = 2\pi \cdot 60 \cdot 142 \cdot 10^{-3} = 53.54 \Omega$$

$$X_C = \frac{1}{\omega \cdot C} = \frac{1}{2\pi \cdot 60 \cdot 160 \cdot 10^{-6}} = 16.58 \Omega$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{1000}\right)^2 + \left(\frac{1}{53.54} - \frac{1}{16.58}\right)^2}} = \dots = 24 \Omega$$

$$I_S = \frac{V_S}{Z} = \frac{240}{24} = 10 \text{ A}$$

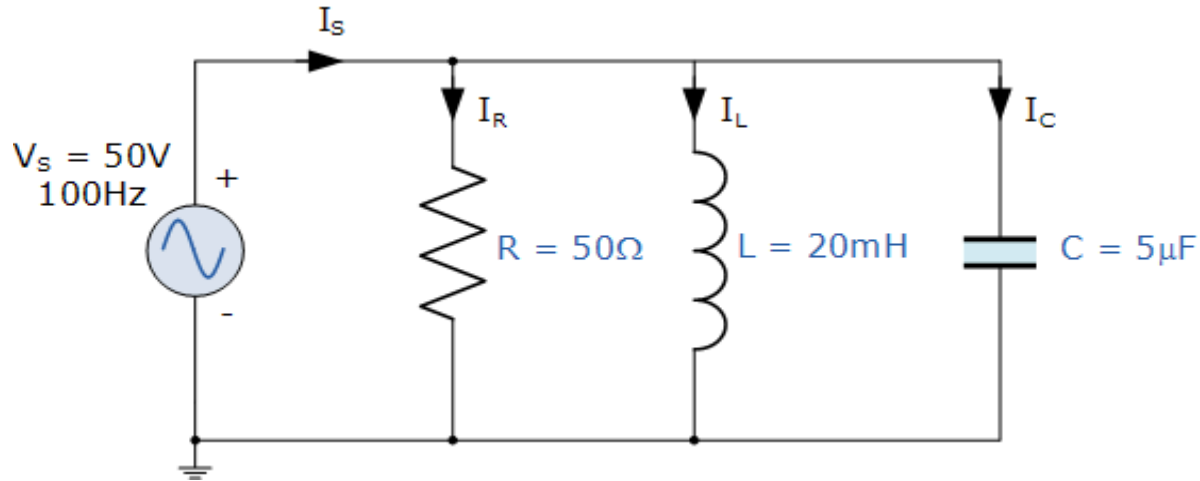


# AC Circuits



## RES.03

Calculate the  $I_S$ ,  $I_R$ ,  $I_L$ ,  $I_C$ ,  $Z$ , and the phase angle. Construct the current and admittance triangles representing the circuit.



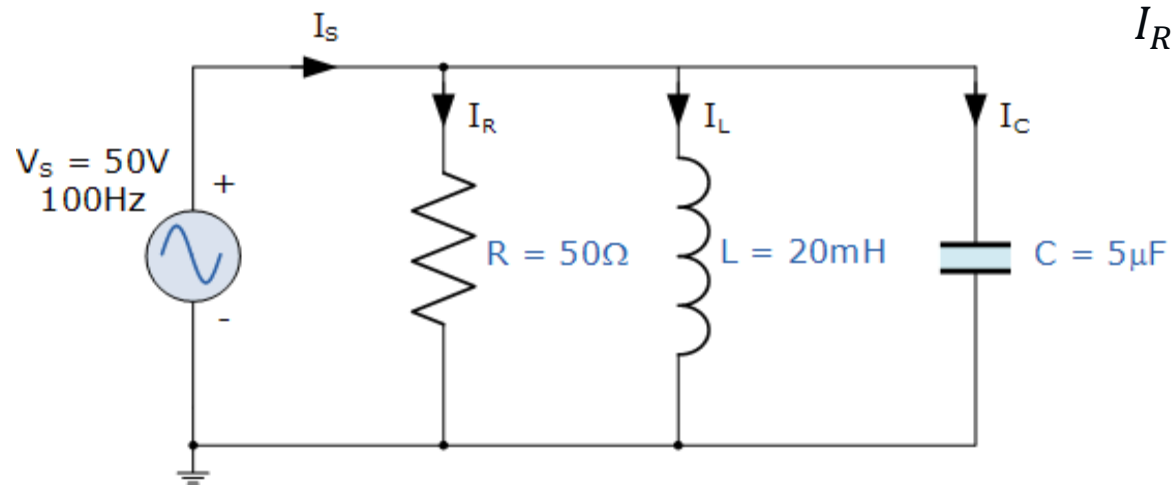
### Solution

$$X_L = \omega \cdot L = 2\pi \cdot 100 \cdot 20 \cdot 10^{-3} = 12.6 \Omega$$

$$X_C = \frac{1}{\omega \cdot C} = \frac{1}{2\pi \cdot 100 \cdot 5 \cdot 10^{-6}} = 318.3 \Omega$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{50}\right)^2 + \left(\frac{1}{12.6} - \frac{1}{318.3}\right)^2}} = \dots = 12.7 \Omega$$

# AC Circuits



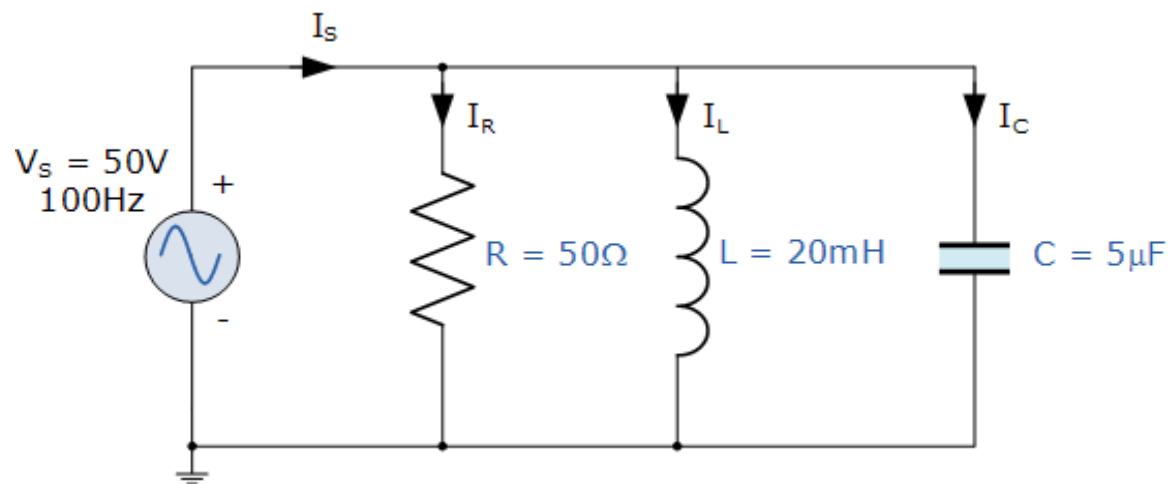
$$I_R = \frac{V_S}{R} = \frac{50}{50} = 1 A$$

$$I_L = \frac{V_S}{X_L} = \frac{50}{12.6} = 3.9 A$$

$$I_C = \frac{V_S}{X_C} = \frac{50}{318.3} = 160 mA$$

$$I_S = \sqrt{I_R^2 + (I_L - I_C)^2} = \dots = 3.87 A$$

# AC Circuits



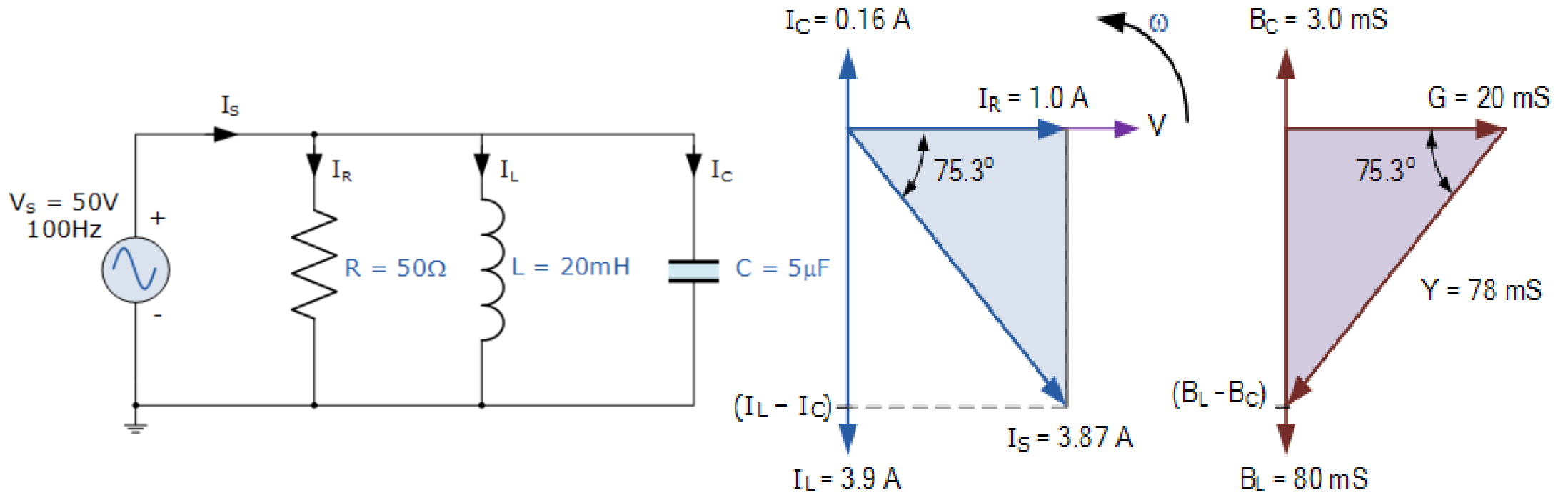
$$G = \frac{1}{R} = \frac{1}{50} = 20 \text{ mS}$$

$$B_L = \frac{1}{X_L} = \frac{1}{12.6} = 80 \text{ mS}$$

$$B_C = \frac{1}{X_C} = \frac{1}{318.3} = 3 \text{ mS}$$

$$Y = \frac{1}{Z} = \frac{1}{12.7} = 78 \text{ mS}$$

# AC Circuits



$$\cos \varphi = \frac{G}{Y} = \frac{20}{78} = 0.256 \rightarrow \varphi = 75.3^\circ \text{ (lag. 'IND')}$$

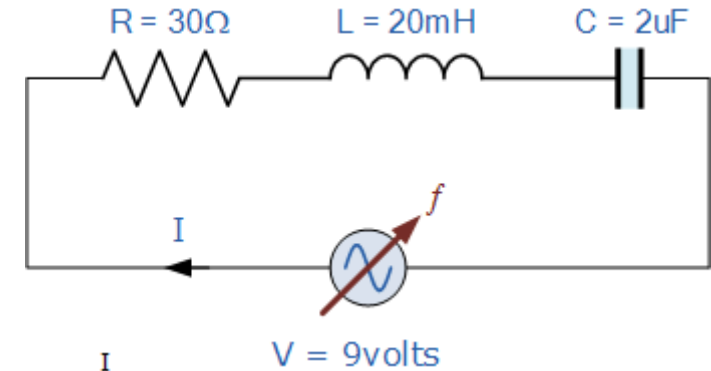
Current Triangle

Admittance Triangle

# Resonance Circuits



**RES.04** - Calculate, the resonant frequency, the current at resonance, the voltage across the inductor and capacitor at resonance, the quality factor and the bandwidth of the circuit. Also sketch the corresponding current waveform for all frequencies.



$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.02 \cdot 2 \cdot 10^{-6}}} = 796 \text{ Hz}$$

$$Q = \frac{X_L}{R} = \frac{100}{30} = 3.33$$

$$I = \frac{V}{R} = \frac{9}{30} = 300 \text{ mA}$$

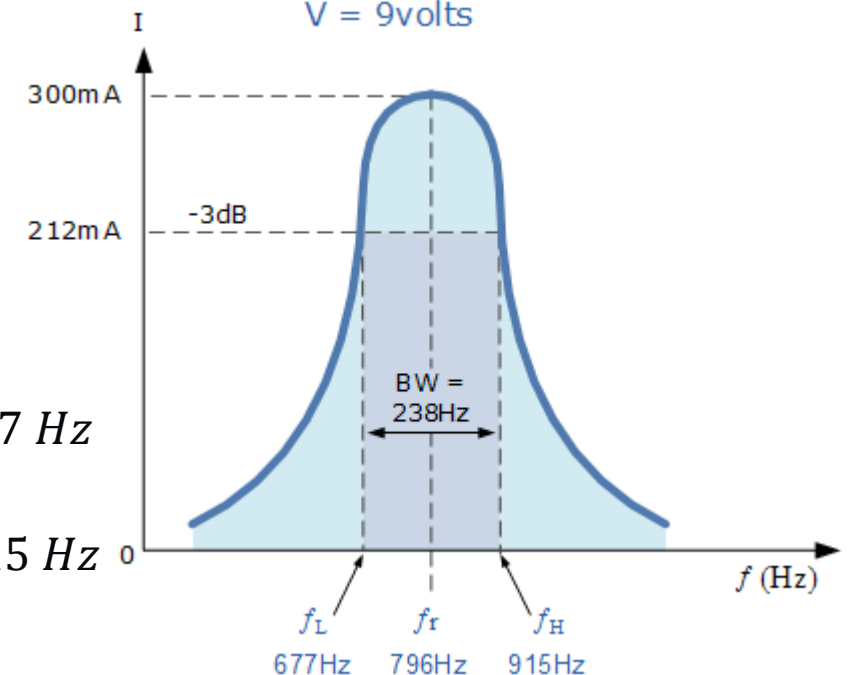
$$BW = \frac{f_r}{Q} = \frac{796}{3.33} = 238 \text{ Hz}$$

$$X_L = \omega \cdot L = 2\pi \cdot 796 \cdot 0.02 = 100 \Omega$$

$$f_L = f_r - \frac{1}{2} BW = 796 - \frac{238}{2} = 677 \text{ Hz}$$

$$V_L (= V_C) = I \cdot X_L = 0.3 \cdot 100 = 30 \text{ V}$$

$$f_H = f_r + \frac{1}{2} BW = 796 + \frac{238}{2} = 915 \text{ Hz}$$





# Resonance Circuits



## RES.05

A series circuit consists of a ,R' of 4Ω, an ,L' of 500mH and a variable capacitance connected across a 100V, 50Hz supply. Find the capacitance required to give series resonance and the voltages generated across both the inductor and the capacitor.

### Solution

$$X_L = \omega \cdot L = 2\pi \cdot 50 \cdot 0.5 = 157.1 \Omega$$

$$\text{At resonance } X_C = X_L = 157.1 \Omega \rightarrow C = \frac{1}{\omega \cdot X_C} = \frac{1}{2\pi \cdot 50 \cdot 157.1} = 20.3 \mu F$$

$$I_S = \frac{V}{R} = \frac{100}{4} = 25 A$$

$$V_L = V_C = I \cdot X_L = 25 \cdot 157.1 = 3.927 kV$$

$$\text{(See that } \rightarrow V_L = V_C = I \cdot X_L = \frac{V}{R} \cdot X_L = V \cdot \frac{X_L}{R} = V \cdot Q)$$

# Resonance Circuits



**RES.06** - Find  $f_r$ ,  $Q$ , BW, the circuit current at resonance and current magnification.

**Solution**

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2 \cdot 120 \cdot 10^{-6}}} = 32.5 \text{ Hz}$$

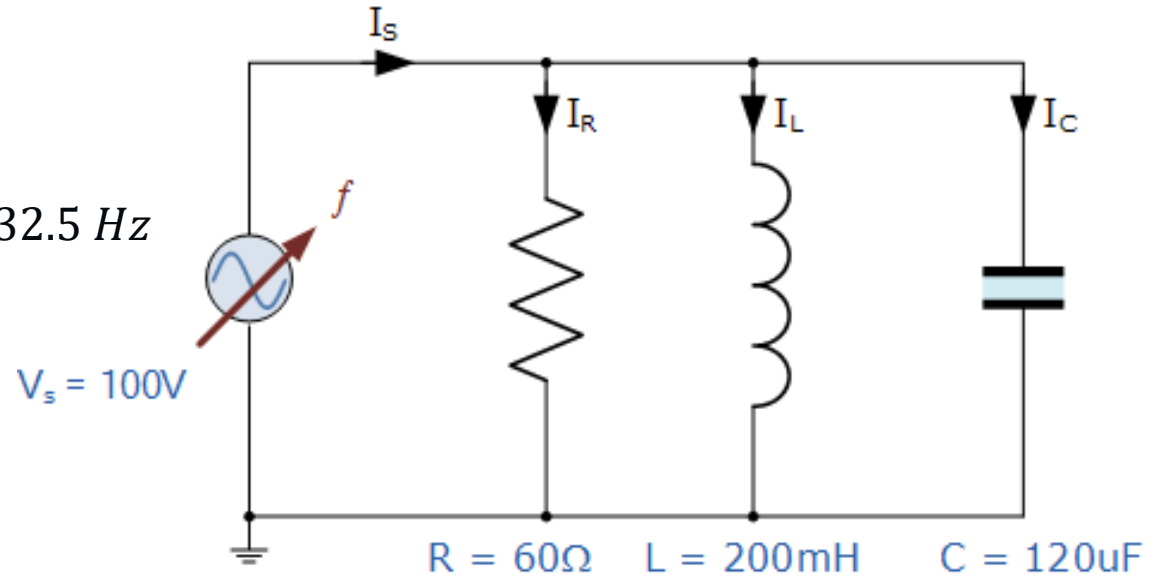
$$X_L = \omega \cdot L = 2\pi \cdot 32.5 \cdot 0.2 = 40.8 \ \Omega$$

$$Q = \frac{R}{X_L} = \frac{60}{40.8} = 1.47$$

$$BW = \frac{f_r}{Q} = \frac{32.5}{1.47} = 22 \text{ Hz}$$

$$f_L = f_r - \frac{1}{2}BW = 32.5 - 11 = 21.5 \text{ Hz}$$

$$f_H = f_r + \frac{1}{2}BW = 32.5 + 11 = 43.5 \text{ Hz}$$



$$I_S = \frac{V_S}{R} = \frac{100}{60} = 1.67 \text{ A}$$

$$I_{MAG} = Q \cdot I_S = 1.47 \cdot 1.67 = 2.45 \text{ A}$$

$$I_L = \frac{V_S}{X_L} = \frac{100}{40.8} = 2.45 \text{ A}$$

# Resonance Circuits



**RES.07** – A series-connected circuit has  $R = 2 \Omega$  and  $L = 1 \text{ mH}$ ,  $C = 400 \text{ nF}$ ,  $v_S(t) = 20 \sin \omega t \text{ V}$ .

- (a) Find  $\omega_0, \omega_1, \omega_2$
- (b) Calculate Q and B.
- (c) Determine amplitude of the current at  $\omega_0, \omega_1, \omega_2$

**Solution (a):**  $50 \text{ krad/s}, 49 \text{ krad/s}, 51 \text{ krad/s}$  (b):  $2 \text{ krad/s}, 25$  (c):  $10 \text{ A}, 7.071 \text{ A}$

**RES.08** – A series-connected circuit has  $R = 4 \Omega$  and  $L = 25 \text{ mH}$ .  $v_S(t) = 100 \sin \omega t \text{ V}$ .

- (a) Calculate the value of C that will produce a quality factor of 50.
- (b) Find  $\omega_1, \omega_2$  and B.
- (c) Determine the average power dissipated at  $\omega_0, \omega_1, \omega_2$

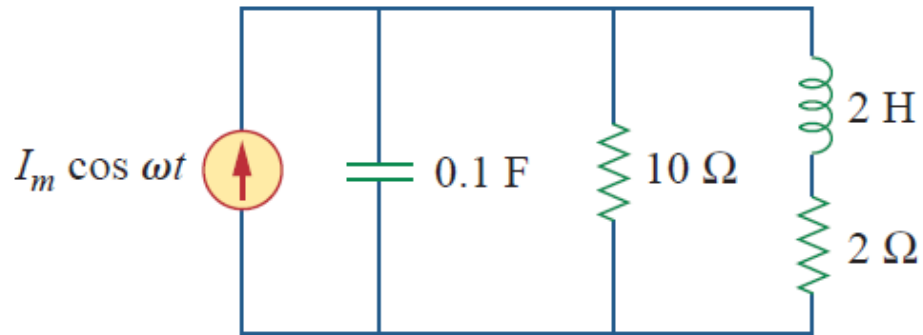
**Solution (a):**  $625 \text{ nF}$ , (b):  $7.92 \text{ krad/s}, 8.08 \text{ krad/s}, 160 \text{ rad/s}$  (c):  $1.25 \text{ kW}, 0.625 \text{ kW}, 0.625 \text{ kW}$

# Resonance Circuits



## RES.09

Determine the resonant frequency of the circuit.



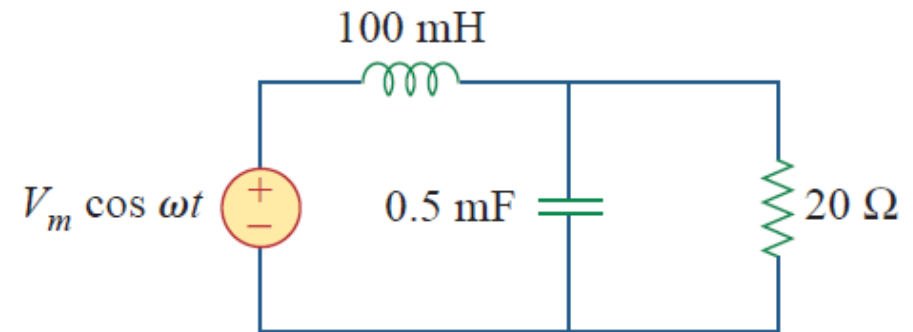
### Solution

$$Y_{eq} = j0.1\omega + \frac{1}{10} + \frac{1}{2 + j2\omega} = 0.1 + j0.1\omega + \frac{2 - j2\omega}{4 + 4\omega^2}$$

$$\text{Im}[Y_{eq}] = 0 \rightarrow 0.1\omega_0 - \frac{2\omega_0}{4 + 4\omega_0^2} = 0 \quad \omega_0 = 2 \text{ rad/s}$$

## RES.10

Determine the resonant frequency of the circuit.



**Solution** 100 rad/s

# Questions

