DR. GYURCSEK ISTVÁN

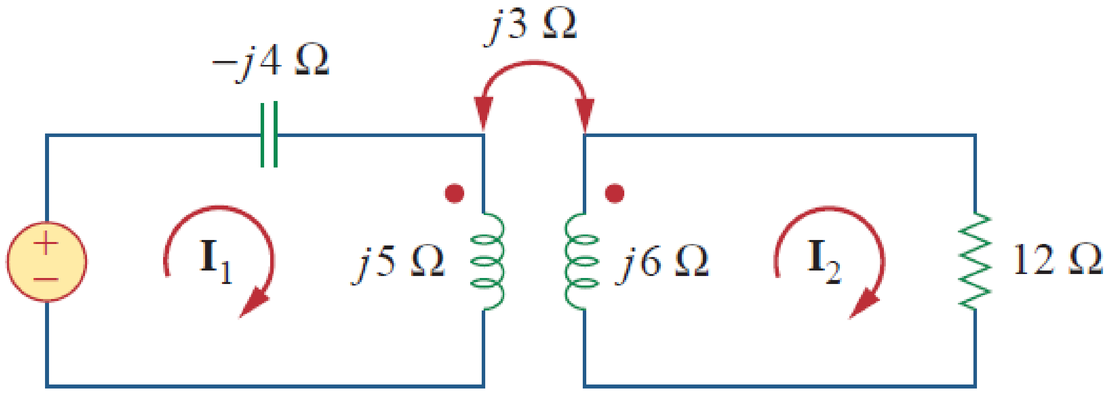
Exercises in Magnetically Coupled Circuits

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *W. M. Flanagan, Handbook of Transformer Design and Applications, 2nd ed. (New York: McGraw-Hill, 1993)*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*

TRF.01

Calculate the phasor currents in the circuit if V_S = 12 V.



Solution

$$(\mathbf{1}):\ -12+(-j4+j5)\boldsymbol{I}_1-j3\boldsymbol{I}_2\rightarrow j\boldsymbol{I}_1-j3\boldsymbol{I}_2=12$$

$$(\mathbf{2}):\ -j3\boldsymbol{I}_1+(12+j6)\boldsymbol{I}_2=0\rightarrow \boldsymbol{I}_1=\frac{(12+j6)\boldsymbol{I}_2}{j3}=(2-j4)\boldsymbol{I}_2$$

$$(\mathbf{2}\rightarrow\mathbf{1}):\ (j2+4-j3)\boldsymbol{I}_2=(4-j)=12\rightarrow \boldsymbol{I}_2=\frac{12}{(4-j)}=2.91\ e^{j14.04°}$$

$$\boldsymbol{I}_1=(2-j4)\boldsymbol{I}_2=(2-j4)2.91\ e^{j14.04°}=4.472\ e^{-j63.43°}\cdot 2.91\ e^{j14.04°}=13.01\ e^{-j49.39°}\ A$$

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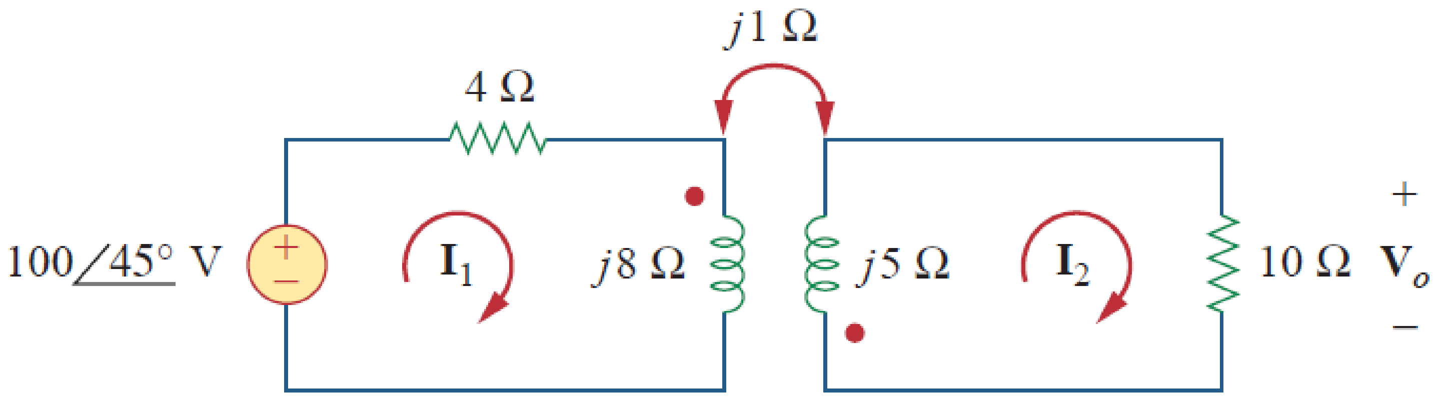
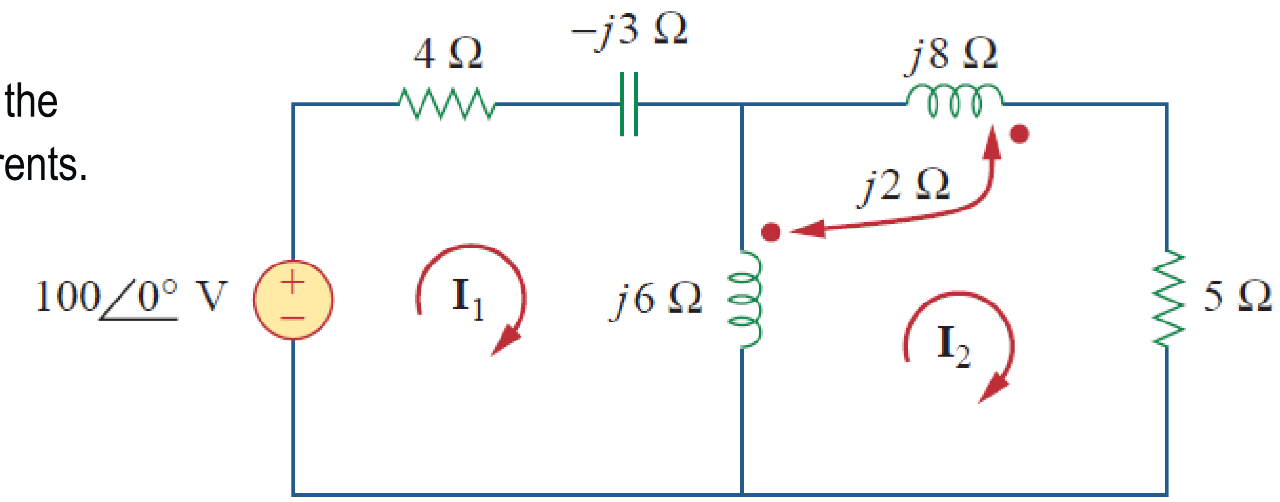
TRF.02 – V_0 = ?.

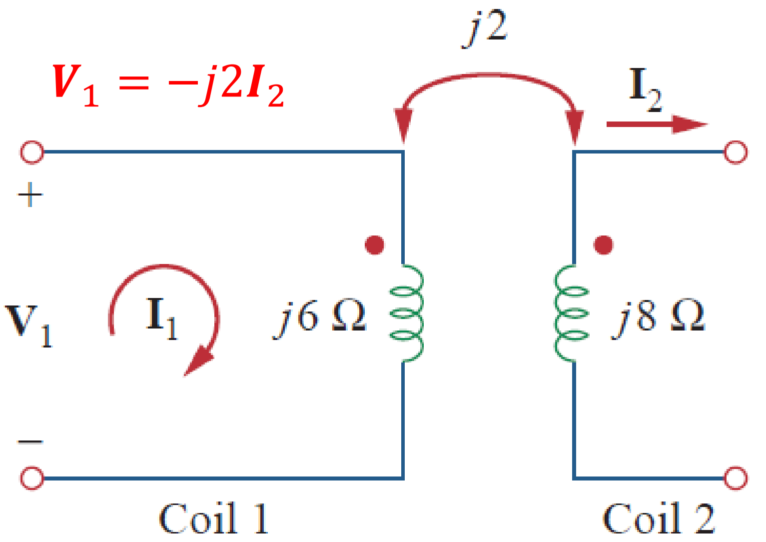


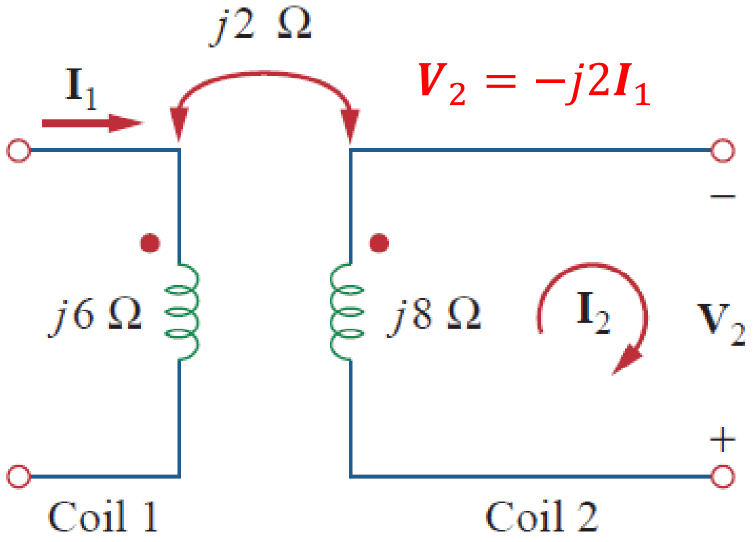
Solution $V_0 = 10\ e^{-j135°}\ V$

TRF.03

Calculate the mesh currents.







Solution

$$(\boldsymbol{mesh1}): -100 + \boldsymbol{I}_1(4 - j3 + j6) - j6\boldsymbol{I}_2 - j2\boldsymbol{I}_2 = 0$$

$$\rightarrow 100 = \boldsymbol{I}_1(4 + j3) - j8\boldsymbol{I}_2$$

$$(\boldsymbol{mesh2}):\ 0 = -j2\boldsymbol{I}_1 - j6\boldsymbol{I}_1 + (j6 + j8 + j2 \cdot 2 + 5)\boldsymbol{I}_2 = 0$$

$$\rightarrow 0 = -j8\boldsymbol{I}_1 + (5 + j18)\boldsymbol{I}_2$$

$$\begin{bmatrix} 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j3 & -j8 \\ -j8 & 5 + j18 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix}$$

Mutual Inductance

$$\begin{bmatrix}100\\0\end{bmatrix}=\begin{bmatrix}4+j3 & -j8\\ -j8 & 5+j18\end{bmatrix}\cdot\begin{bmatrix}\boldsymbol{I}_1\\ \boldsymbol{I}_2\end{bmatrix}$$

$$\Delta=\begin{vmatrix}4+j3 & -j8\\ -j8 & 5+j18\end{vmatrix}=30+j87$$

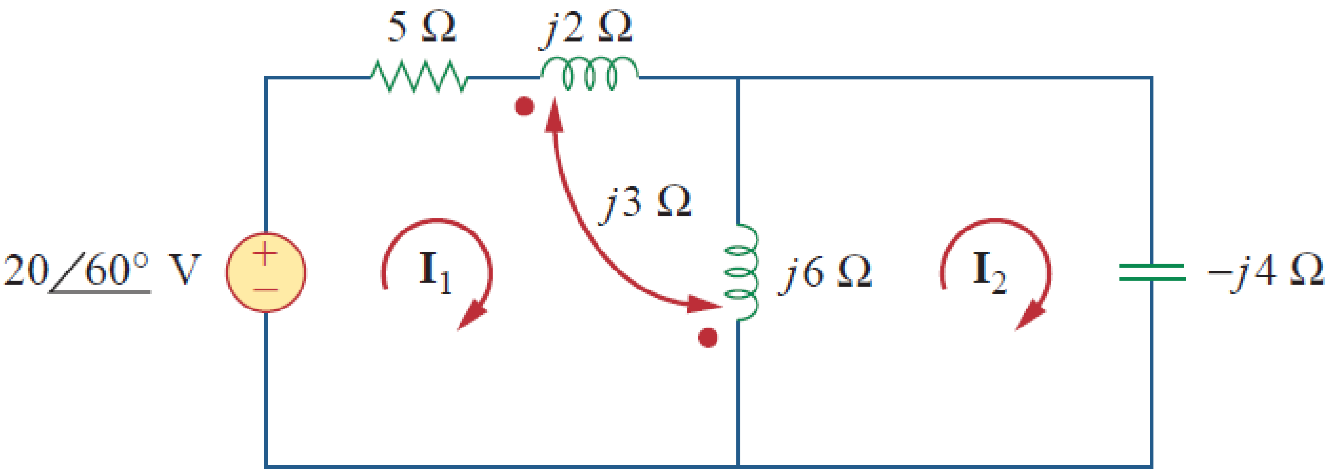
$$\Delta_1=\begin{vmatrix}100 & -j8\\ 0 & 5+j18\end{vmatrix}=100(5+j18)$$

$$\Delta_2=\begin{vmatrix}4+j3 & 100\\ -j8 & 0\end{vmatrix}=j800$$

$$\boldsymbol{I}_1=\frac{\Delta_1}{\Delta}=\frac{100(5+j18)}{30+j87}=\frac{1{,}868.2\ e^{j74.5^\circ}}{92.03\ e^{j71^\circ}}=20.3\ e^{j3.5^\circ}\ A$$

$$\boldsymbol{I}_2=\frac{\Delta_2}{\Delta}=\frac{j800}{30+j87}=\frac{800\ e^{j90^\circ}}{92.03\ e^{j71^\circ}}=8.693\ e^{j19^\circ}\ A$$

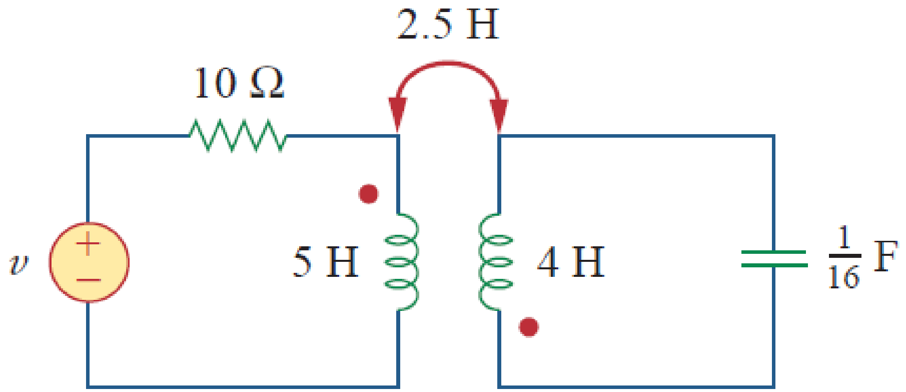
TRF.04 – Calculate the mesh currents.



Solution $\boldsymbol{I}_1 = 3.583\ e^{j86.56°}\ A$, $\boldsymbol{I}_2 = 5.383\ e^{j86.56°}\ A$

TRF.05 – Determine the coupling coefficient and calculate the energy stored in the coupled inductors at *t* = *1 s* if

$$v = 60\cos(4t + 30°)\ V$$



Solution

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{20}} = 0.56$$

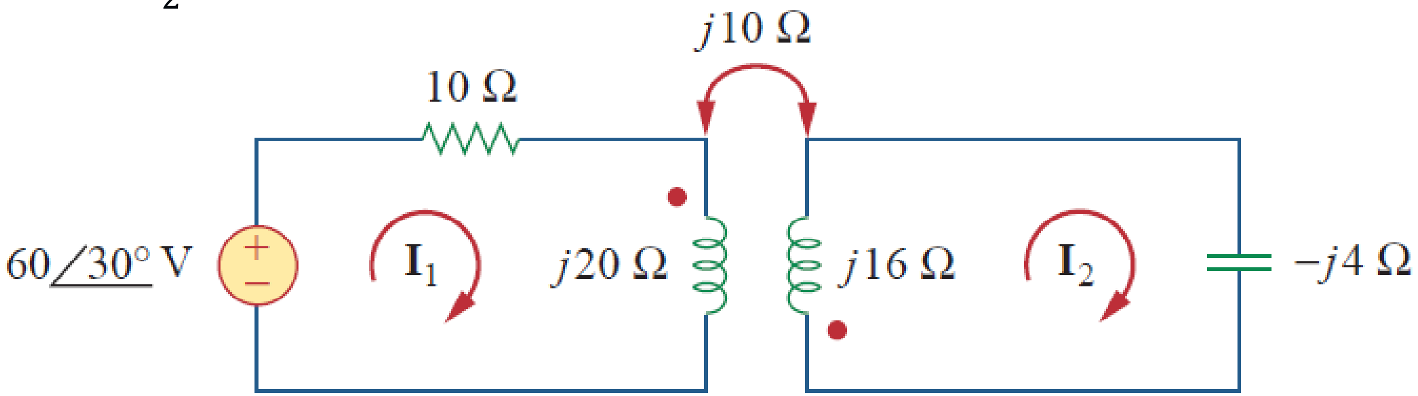
$$(\mathbf{1}):\ (10 + j20)\boldsymbol{I}_1 + j10\boldsymbol{I}_2 = 60\ e^{j30°}$$

$$(\mathbf{2}):\ j10\boldsymbol{I}_1 + (j16 - j4)\boldsymbol{I}_2 = 0 \rightarrow \boldsymbol{I}_1 = -1.2\boldsymbol{I}_2$$

$$(\mathbf{2} \rightarrow \mathbf{1}):\ (-12 - j14)\boldsymbol{I}_2 = 60\ e^{j30°}$$

$$\rightarrow \boldsymbol{I}_2 = 3.254\ e^{j160.6°}\ A$$

$$\boldsymbol{I}_1 = -1.2\boldsymbol{I}_2 = 3.905\ e^{-j19.4°}\ A$$



Energy in Coupled Circuit

$$\boldsymbol{I}_1 = 3.905\, e^{-j19.4°} \rightarrow i_1(t) = 3.905 \cos(4t - 19.4°)$$

$$\boldsymbol{I}_2 = 3.254\, e^{j160.6°} \rightarrow i_2(t) = 3.254 \cos(4t + 160.6°)$$

$$t = 1\, s \rightarrow 4t = 4\, rad = 229.2°$$

$$I_1 = 3.905 \cos(229.2° - 19.4°) = -\,3.389\, A$$

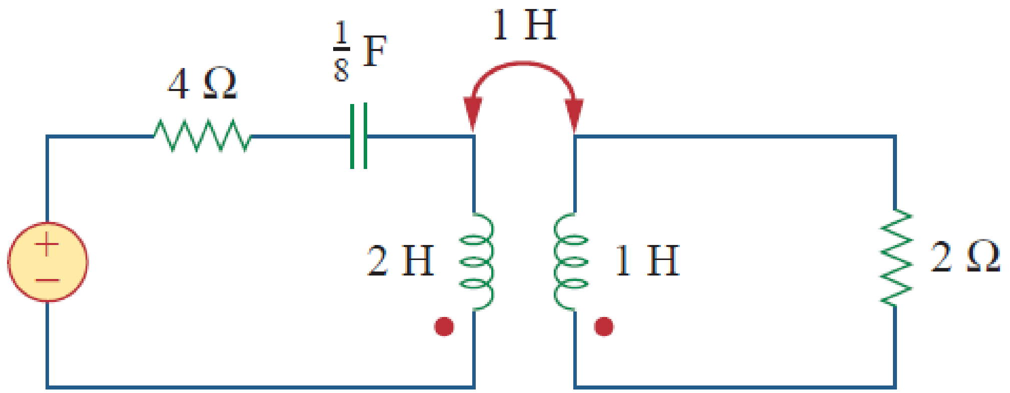
$$I_2 = 3.254 \cos(229.2° + 160.6°) = 2.824\, A$$

$$w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M I_1 I_2 = \frac{1}{2} \cdot 5 \cdot (-3.389)^2 + \frac{1}{2} \cdot 4 \cdot (2.824)^2 + 2.5 \cdot (-3.389) \cdot 2.824 = 20.73\, J$$

Energy in Coupled Circuit

TRF.06

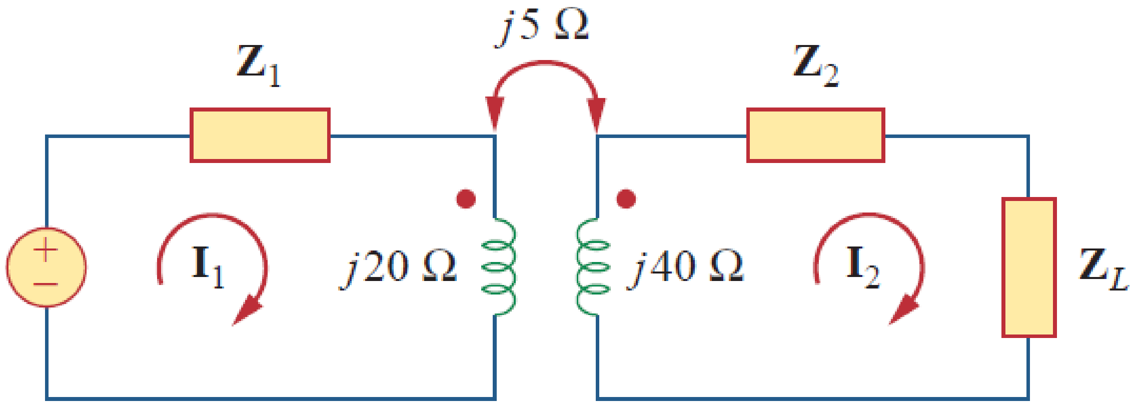
Determine the coupling coefficient and calculate the energy stored in the coupled inductors at *t* = *1.5 s* if $v_S = 40\cos 4t$



Solution *k = 0.7071, w = 39.4 J*

TRF.07 – Find the input impedance and current I_1.

$$\boldsymbol{V_S} = 50\,e^{j60^\circ}\,V, \boldsymbol{Z}_1 = 60 - j100\,\Omega\,, \boldsymbol{Z}_2 = 30 + j40\Omega\,, \boldsymbol{Z}_L = 80 + j60\,\Omega$$



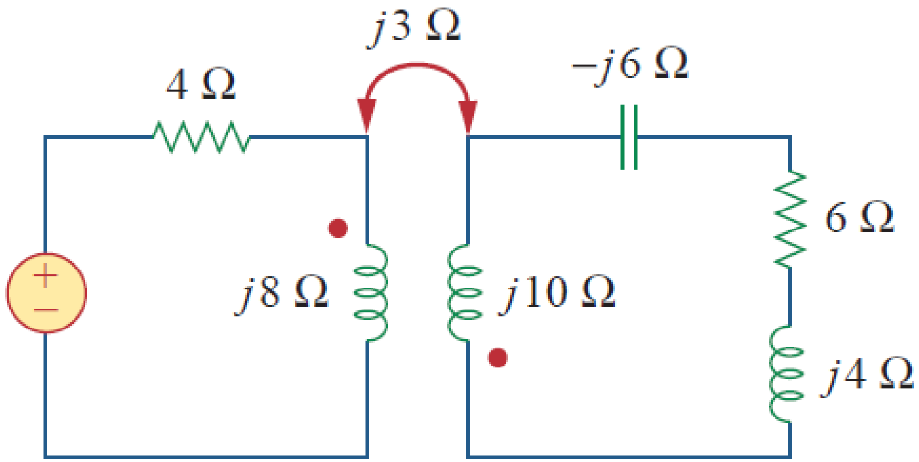
Solution

$$\boldsymbol{Z}_{in} = \boldsymbol{Z}_1 + j20 + \frac{5^2}{j40 + \boldsymbol{Z}_2 + \boldsymbol{Z}_L} = 60 - j100 + j20 + \frac{25}{110 + j140} = 60 - j80 + 1.14\,e^{-j51.84^\circ}$$

$$= 60.09 - j80.11 = 100.14\,e^{-j53.1^\circ}\,\Omega$$

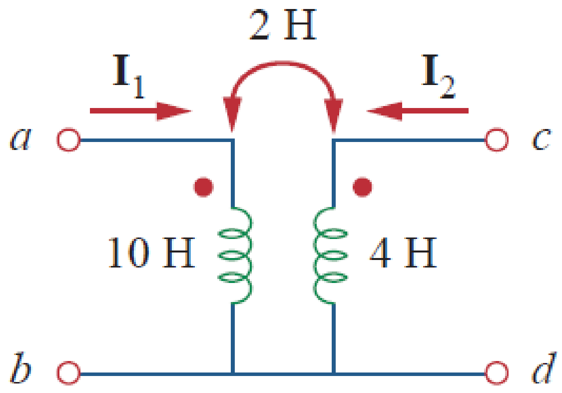
$$\boldsymbol{I}_1 = \frac{\boldsymbol{V}}{\boldsymbol{Z}_{in}} = \frac{50\,e^{j60^\circ}}{100.14\,e^{-j53.1^\circ}} = 0.5\,e^{j113.1^\circ}\,A$$

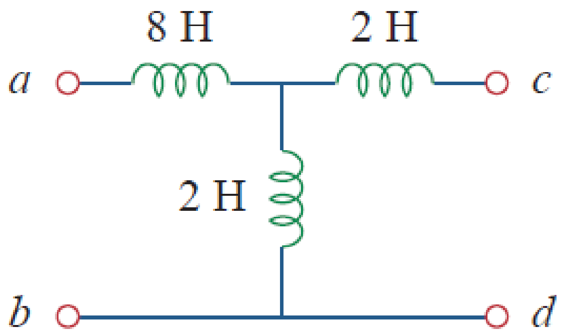
TRF.08 – Find the input impedance and current from V_S if $\boldsymbol{V_S} = 20\ V$



Solution $\boldsymbol{Z_{in}} = 8.58\ e^{j58.05°}\ \Omega\,,\qquad \boldsymbol{I_S} = 2.331\ e^{-j58.05°}\ A$

TRF.09 – Determine the T-equivalent circuit of the linear transformer

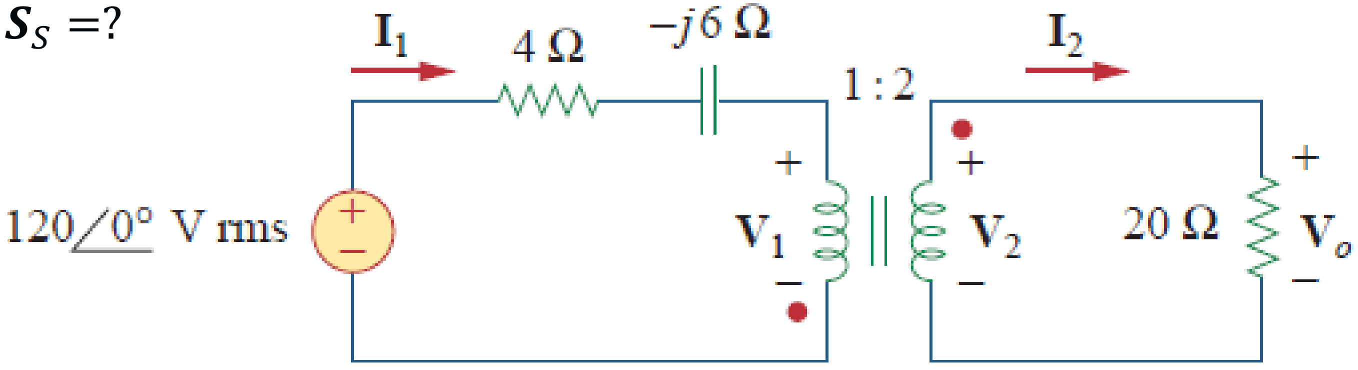




Solution $L_1 = 10\ H\,,\qquad L_2 = 4\ H\,,\qquad M = 2\ H\,,$

$$L_a = L_1 - M = 8\ H\,,\qquad L_b = L_2 - M = 2\ H\,,\qquad L_C = M = 2\ H$$

TRF.10 – $\boldsymbol{I}_1 = ?$, $\boldsymbol{V}_O = ?$, $\boldsymbol{S}_S = ?$



Solution

$$\boldsymbol{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5\ \Omega$$

$$\boldsymbol{Z}_{in} = 4 - j6 + \boldsymbol{Z}_R = 9 - j6 = 10.82\ e^{-j33.69°}\ \Omega$$

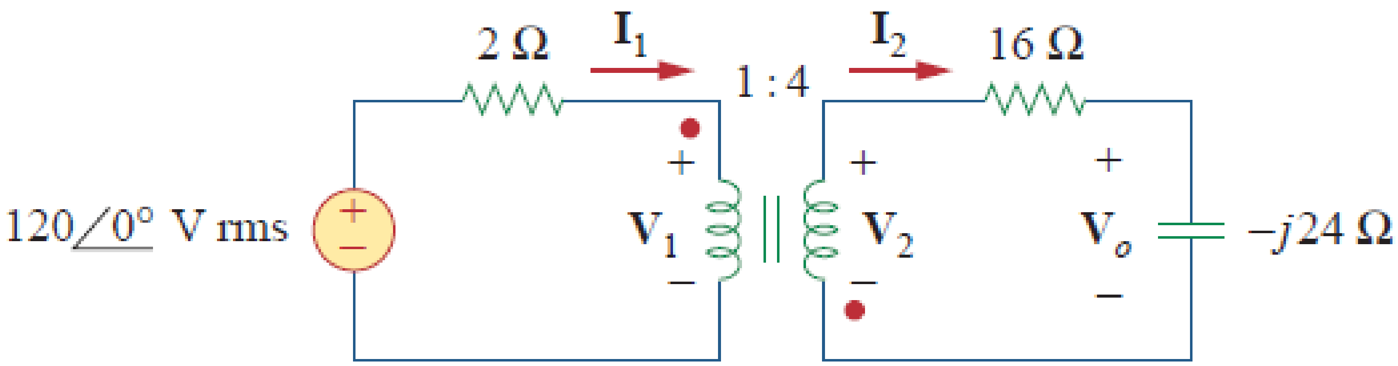
$$\boldsymbol{I}_1 = \frac{120}{\boldsymbol{Z}_{in}} = 11.09\ e^{j33.69°}\ A$$

$$\boldsymbol{I}_2 = -\frac{1}{n}\boldsymbol{I}_1 = -5.545\ e^{j33.69°} = 5.545\ e^{j213.69°} A$$

$$\boldsymbol{V}_O = 20\boldsymbol{I}_2 = 110.9\ e^{j213.69°}\ V$$

$$\boldsymbol{S}_S = \boldsymbol{V}_S \boldsymbol{I}_1^* = 120 \cdot 11.09\ e^{-j33.69°} = 1{,}330.8\ e^{-j33.69°}\ VA$$

TRF.11 – $\boldsymbol{V_O} = ?$, $\boldsymbol{S_S} = ?$



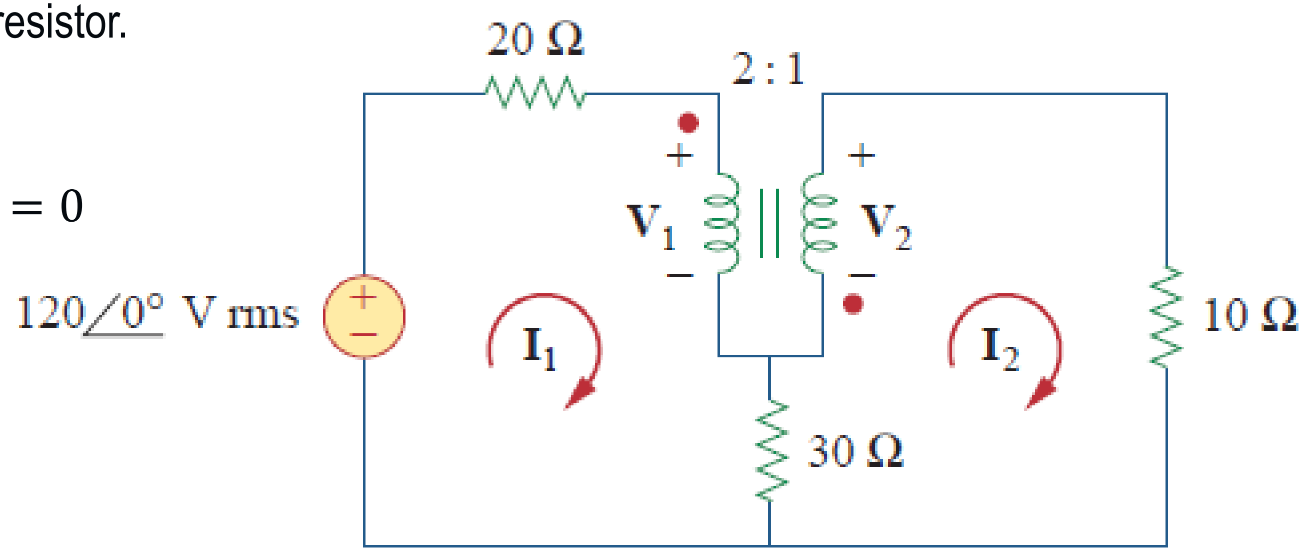
Solution $\boldsymbol{V_O} = 214.7\, e^{j116.56°}\, A$ $\boldsymbol{S_S} = 4.293\, e^{-j26.56°}\, kVA$

TRF.12 – Calculate the power supplied to the load resistor.

Solution

$$(\mathbf{mesh1}):\ -120 + (20+30)\boldsymbol{I}_1 - 30\boldsymbol{I}_2 + \boldsymbol{V}_1 = 0$$

$$\rightarrow 50\boldsymbol{I}_1 - 30\boldsymbol{I}_2 + \boldsymbol{V}_1 = 120$$



$$(\mathbf{mesh2}):\ \boldsymbol{V}_2 + (10+30)\boldsymbol{I}_2 - 30\boldsymbol{I}_1 = 0$$

$$\rightarrow -30\boldsymbol{I}_1 + 40\boldsymbol{I}_2 - \boldsymbol{V}_2 = 0$$

$$\boldsymbol{V}_2 = -\frac{1}{2}\boldsymbol{V}_1\,, \qquad \boldsymbol{I}_2 = -2\boldsymbol{I}_1$$

$$(\mathbf{mesh1} \leftarrow \boldsymbol{V}_1, \boldsymbol{I}_1):\ -55\boldsymbol{I}_2 - 2\boldsymbol{V}_2 = 120$$

$$(\mathbf{mesh2} \leftarrow \boldsymbol{V}_1, \boldsymbol{I}_1):\ 15\boldsymbol{I}_2 + 40\boldsymbol{I}_2 - \boldsymbol{V}_2 = 0 \rightarrow \boldsymbol{V}_2 = 55\boldsymbol{I}_2$$

$$-165\boldsymbol{I}_2 = 120 \rightarrow \boldsymbol{I}_2 = -0.7272\ A$$

$$P = (-0.7272)^2 \cdot 10 = 5.3\ W$$

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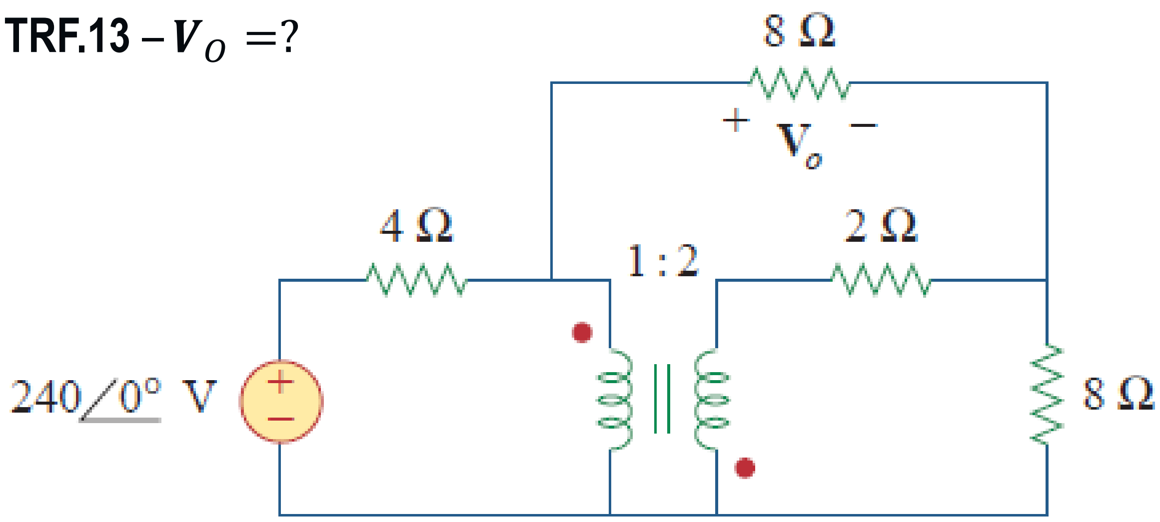
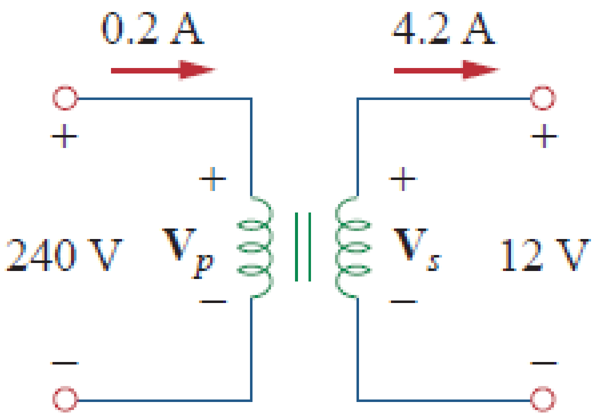
Ideal Transformers

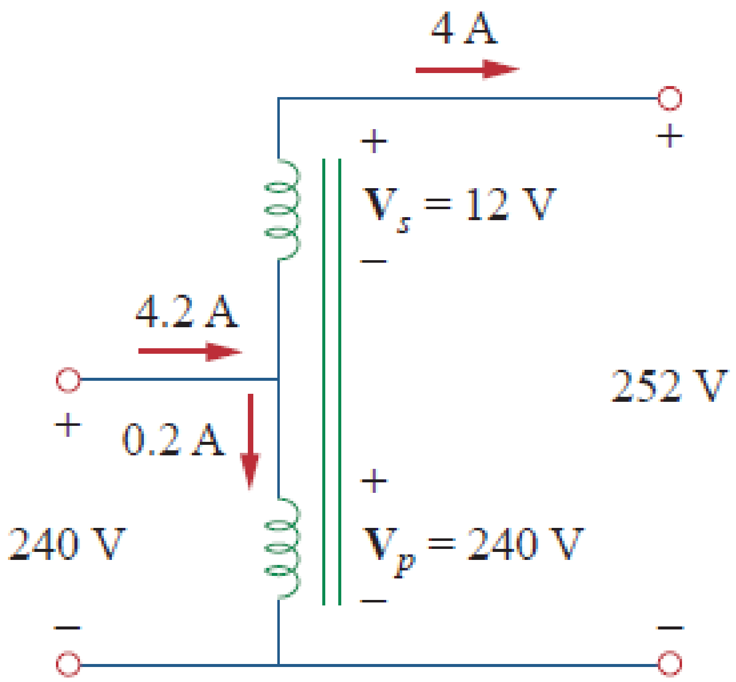
TRF.13 – $V_O = ?$



Solution … 96 V

TRF.14 – Compare the power ratings of the transformers.





Solution

$(AUTOTR.\ 21\ TIMES\ HIGHER!!!)$

$$AUTOTR. \rightarrow S_1 = 4.2 \cdot 240 = 1{,}008\ VA\,, \qquad S_2 = 4 \cdot 252 = 1{,}008\ VA$$

$$2\ WINDING\ TR. \rightarrow S_1 = 0.2 \cdot 240 = 48\ VA\,, \qquad S_2 = 4 \cdot 12 = 48\ VA$$

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TRF.15 – $\boldsymbol{Z}_L = (8 + j6)\ \Omega\,, \boldsymbol{I}_1 = ?\,, \boldsymbol{I}_2 = ?\,, \boldsymbol{I}_O = ?\,, \boldsymbol{S}_L = ?$

Solution

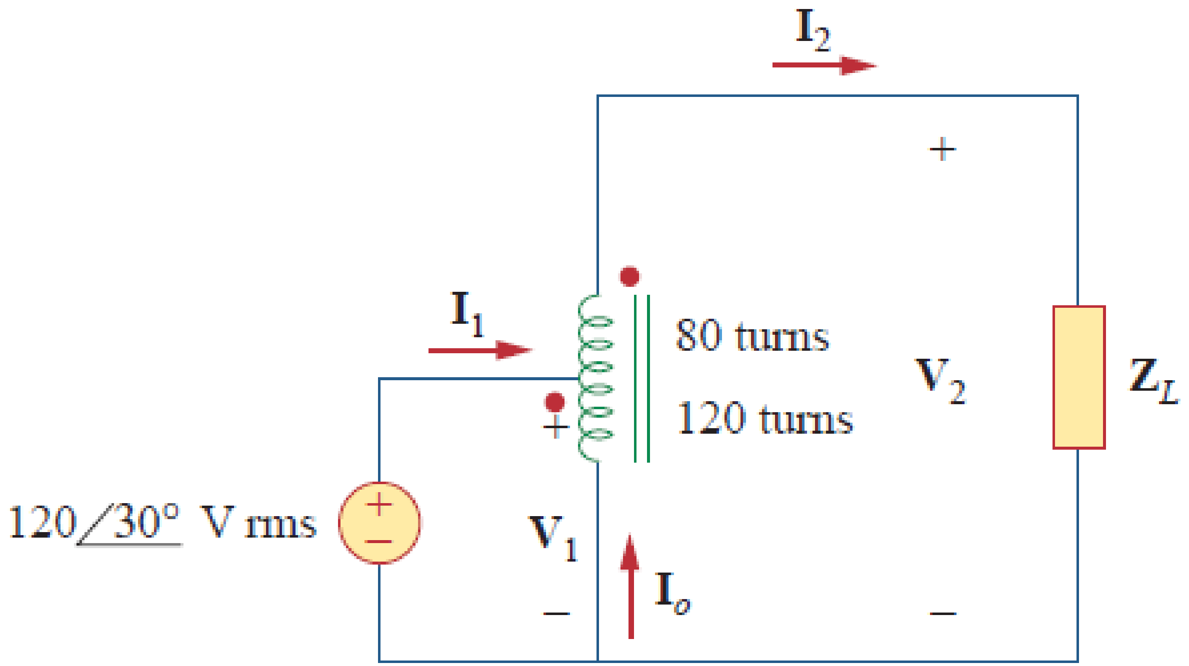
$$\frac{\boldsymbol{V}_1}{\boldsymbol{V}_2} = \frac{N_1}{N_1 + N_2} = \frac{80}{200}$$

$$\boldsymbol{V}_2 = \frac{200}{80}\boldsymbol{V}_1 = \frac{200}{80} 120\ e^{j30°} = 300\ e^{j30°}\ V$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{V}_2}{\boldsymbol{Z}_L} = \frac{300\ e^{j30°}}{8 + j6} = \frac{300\ e^{j30°}}{10\ e^{j36.87°}} = 30\ e^{-j6.87°}\ A$$

$$\frac{\boldsymbol{I}_1}{\boldsymbol{I}_2} = \frac{N_1 + N_2}{N_1} = \frac{200}{80}$$

$$\boldsymbol{I}_1 = \frac{200}{80}\boldsymbol{I}_2 = \frac{200}{80} 30\ e^{-j6.87°} = 75\ e^{-j6.87°}$$

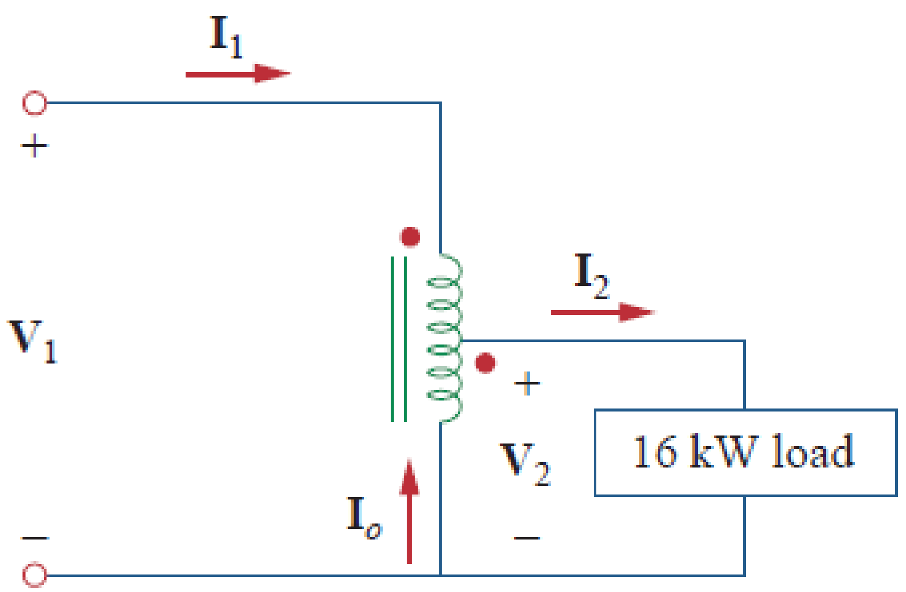


$$\boldsymbol{I}_1 + \boldsymbol{I}_0 = \boldsymbol{I}_2 \to \boldsymbol{I}_0 = \boldsymbol{I}_2 - \boldsymbol{I}_1 = 30\ e^{-j6.87°} - 75\ e^{-j6.87°} = 45\ e^{j173.13°}$$

$$\boldsymbol{S}_2 = \boldsymbol{V}_2 \boldsymbol{I}_2^* = I_2^2 \boldsymbol{Z}_L = 30^2 \cdot 10\ e^{j36.87°} = 9\ e^{j36.87°}\ kVA$$

Ideal Autotransformers

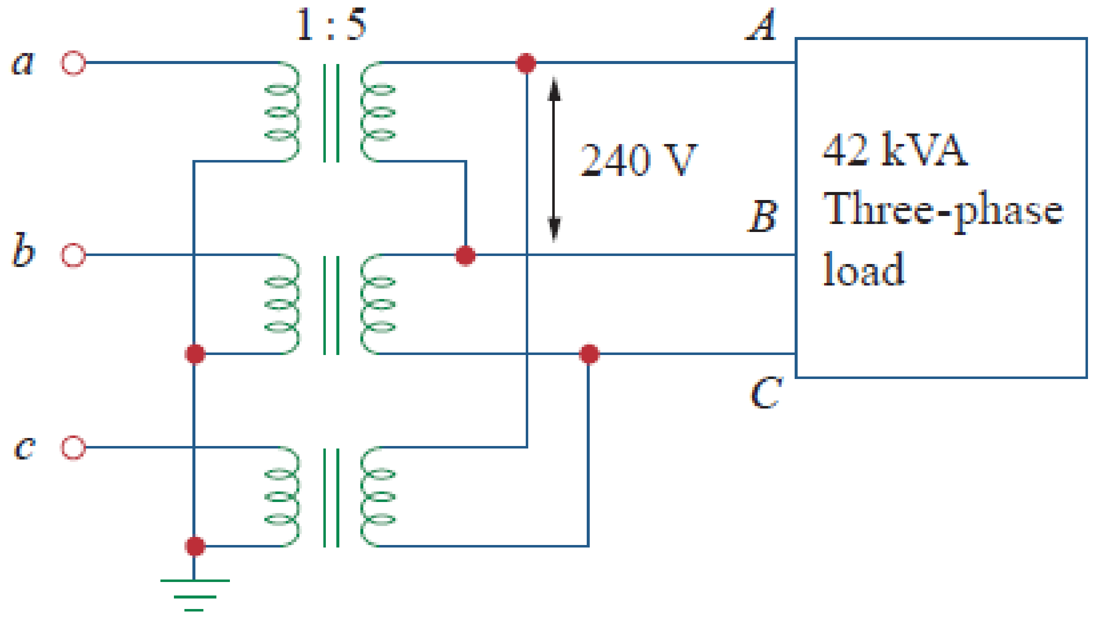
TRF.16 – $\boldsymbol{V}_1 = 1{,}250\ V\ ,\boldsymbol{V}_2 = 500\ V\ ,\boldsymbol{I}_1 = ?\ ,\boldsymbol{I}_2 = ?\ ,\boldsymbol{I}_O = ?$



Solution 12.8 A, 32 A, 19.2 A

TRF.17 – Determine the type of transformer connections. Find the line voltage and current on the primary side.
Determine the kVA rating of each transformer used in the transformer bank. (The transformers are ideal.)

Solution *... (YD)*



$$I_{Ls} = \frac{S_{tot}}{\sqrt{3}V_{Ls}} = \frac{42{,}000}{\sqrt{3}\cdot 240} = 101\ A$$

$$I_{Lp} = \frac{n}{\sqrt{3}} I_{Ls} = \frac{5\cdot 101}{\sqrt{3}} = 292\ A$$

$$V_{Lp} = \frac{\sqrt{3}}{n} V_{Ls} = \frac{\sqrt{3}\cdot 240}{5} = 83.14\ V$$

$$S = \frac{S_{tot}}{3} = 14\ kVA\ \ ...or\ S_s = V_{Ps}I_{Ps} = V_{Ls}\frac{I_{Ls}}{\sqrt{3}} = 240\cdot\frac{101}{\sqrt{3}} = 14\ kVA\ \ ...or\ S_P = V_{Pp}I_{Pp} = \frac{V_{Lp}}{\sqrt{3}} I_{Lp} = \frac{83.14}{\sqrt{3}}\cdot 292 = 14\ kVA$$

TRF.18 – A three-phase DD transformer is used to step down a line voltage of 625 kV, to supply a plant operating at a line voltage of 12.5 kV. The plant draws 40 MW with a lagging power factor of 85 percent. Find:

a) the current drawn by the plant
b) the turns ratio
c) the current on the primary side of the transformer
d) the load carried by each transformer

Solution (a) 2.1736 kA, (b) 0.02, (c) 43.47 A, (d) 15.69 MVA.

Questions