DR. GYURCSEK ISTVÁN

# Exercises with Magnetically Coupled Circuits

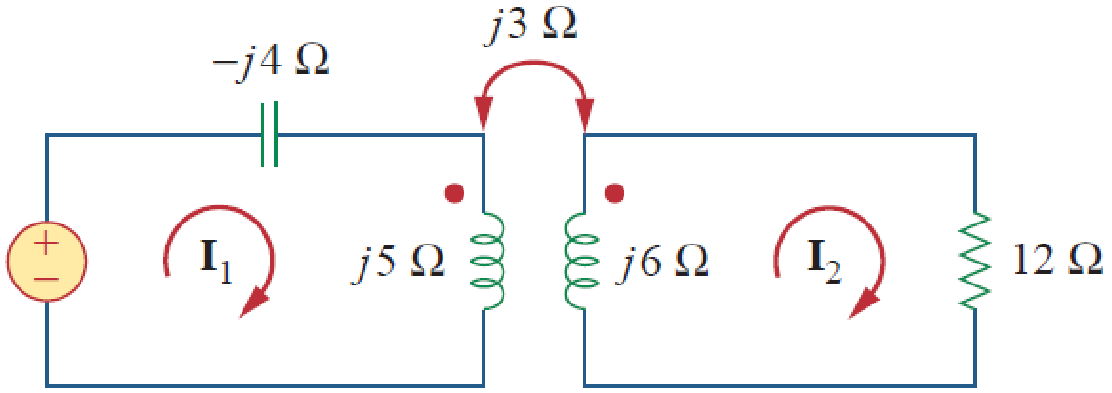
***Sources and additional materials (recommended)***


- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *W. M. Flanagan, Handbook of Transformer Design and Applications, 2nd ed. (New York: McGraw-Hill, 1993)*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Zombory L.: Elektromágneses terek. MK Budapest 2006, (www.electro.uni-miskolc.hu)*

# Mutual Inductance

TRF.01

Calculate the phasor currents in the circuit if $\boldsymbol{V}_S$ = 12 V.




**Solution**

$$(\mathbf{1}):\ -12 + (-j4 + j5)\boldsymbol{I}_1 - j3\boldsymbol{I}_2 \rightarrow j\boldsymbol{I}_1 - j3\boldsymbol{I}_2 = 12$$

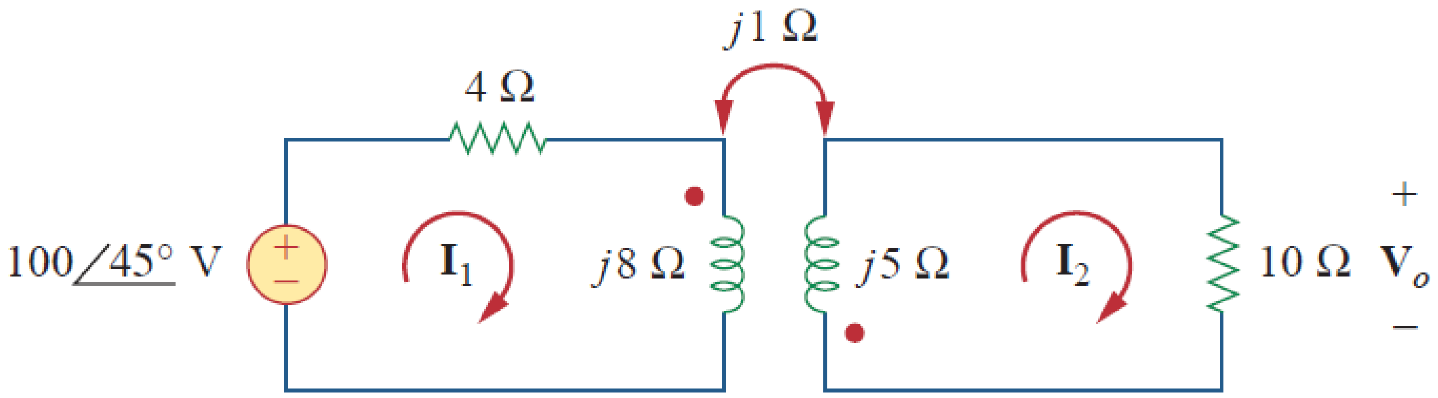
$$(\mathbf{2}):\ -j3\boldsymbol{I}_1 + (12 + j6)\boldsymbol{I}_2 = 0 \rightarrow \boldsymbol{I}_1 = \frac{(12 + j6)\boldsymbol{I}_2}{j3} = (2 - j4)\boldsymbol{I}_2$$

$$(\mathbf{2} \rightarrow \mathbf{1}):\ (j2 + 4 - j3)\boldsymbol{I}_2 = (4 - j) = 12 \rightarrow \boldsymbol{I}_2 = \frac{12}{(4 - j)} = 2.91\ e^{j14.04°}$$

$$\boldsymbol{I}_1 = (2 - j4)\boldsymbol{I}_2 = (2 - j4)2.91\ e^{j14.04°} = 4.472\ e^{-j63.43°} \cdot 2.91\ e^{j14.04°} = 13.01\ e^{-j49.39°}\ A$$

# Mutual Inductance

**TRF.02** – $V_0$ = ?.




**Solution** $V_0 = 10\ e^{-j135°}\ V$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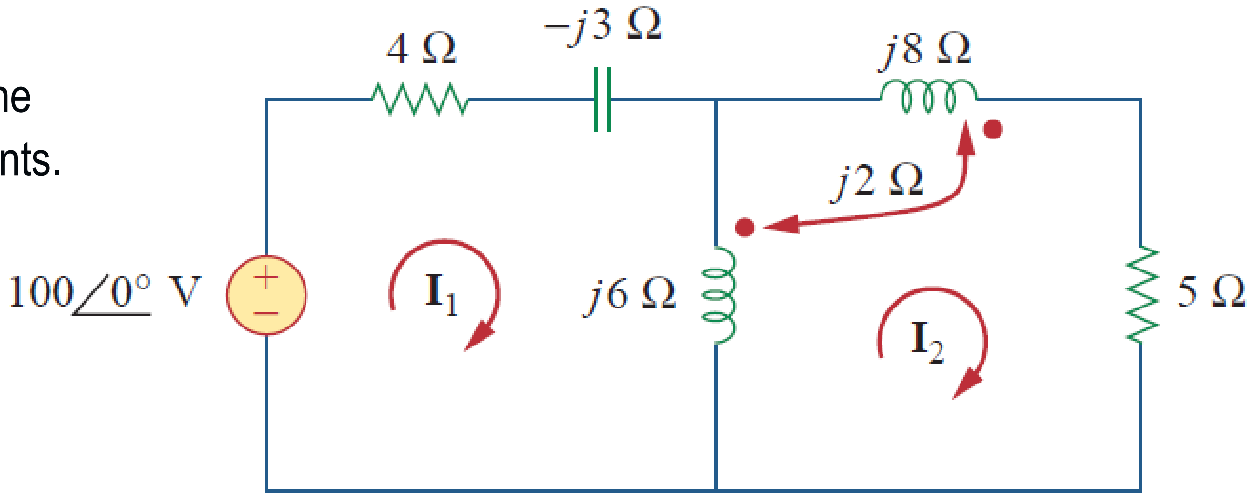
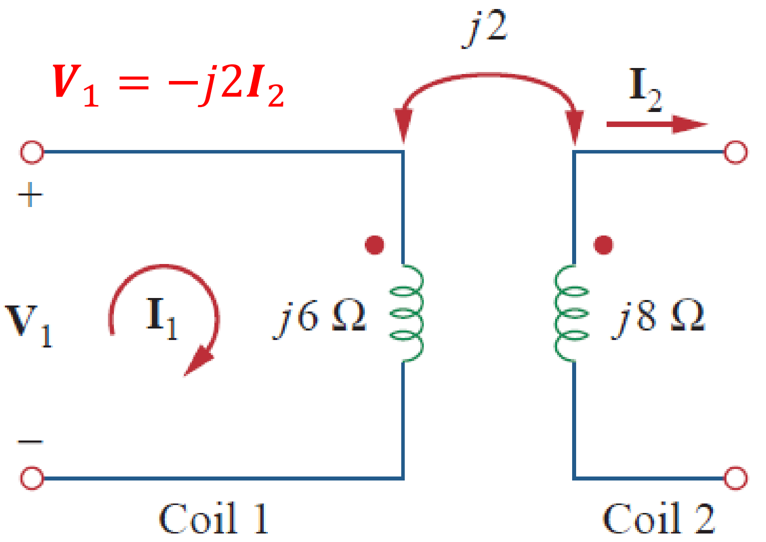
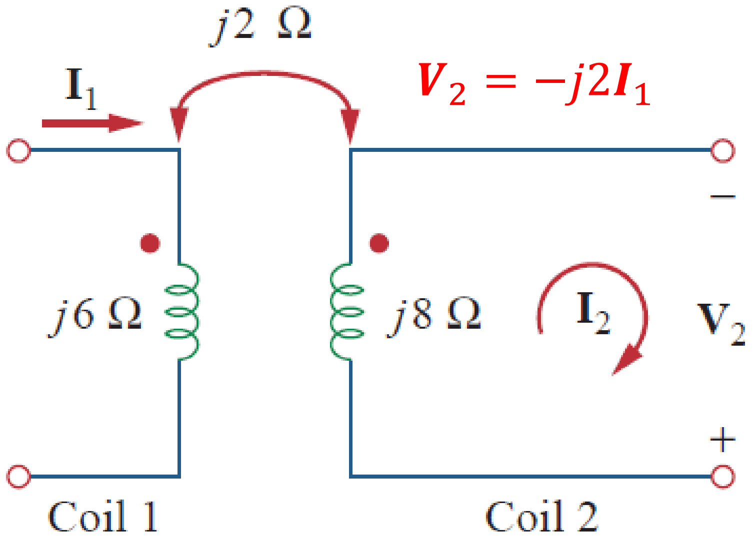
# Mutual Inductance

**TRF.03**

Calculate the mesh currents.




**Solution**

$$(\boldsymbol{mesh1}): -100 + \boldsymbol{I}_1(4 - j3 + j6) - j6\boldsymbol{I}_2 - j2\boldsymbol{I}_2 = 0$$

$$\rightarrow 100 = \boldsymbol{I}_1(4 + j3) - j8\boldsymbol{I}_2$$

$$(\boldsymbol{mesh2}):\ 0 = -j2\boldsymbol{I}_1 - j6\boldsymbol{I}_1 + (j6 + j8 + j2 \cdot 2 + 5)\boldsymbol{I}_2 = 0$$

$$\rightarrow 0 = -j8\boldsymbol{I}_1 + (5 + j18)\boldsymbol{I}_2$$

$$\begin{bmatrix} 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j3 & -j8 \\ -j8 & 5 + j18 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix}$$

# Mutual Inductance

$$\begin{bmatrix}100\\0\end{bmatrix}=\begin{bmatrix}4+j3 & -j8\\-j8 & 5+j18\end{bmatrix}\cdot\begin{bmatrix}\boldsymbol{I}_1\\\boldsymbol{I}_2\end{bmatrix}\qquad \Delta=\begin{vmatrix}4+j3 & -j8\\-j8 & 5+j18\end{vmatrix}=30+j87$$

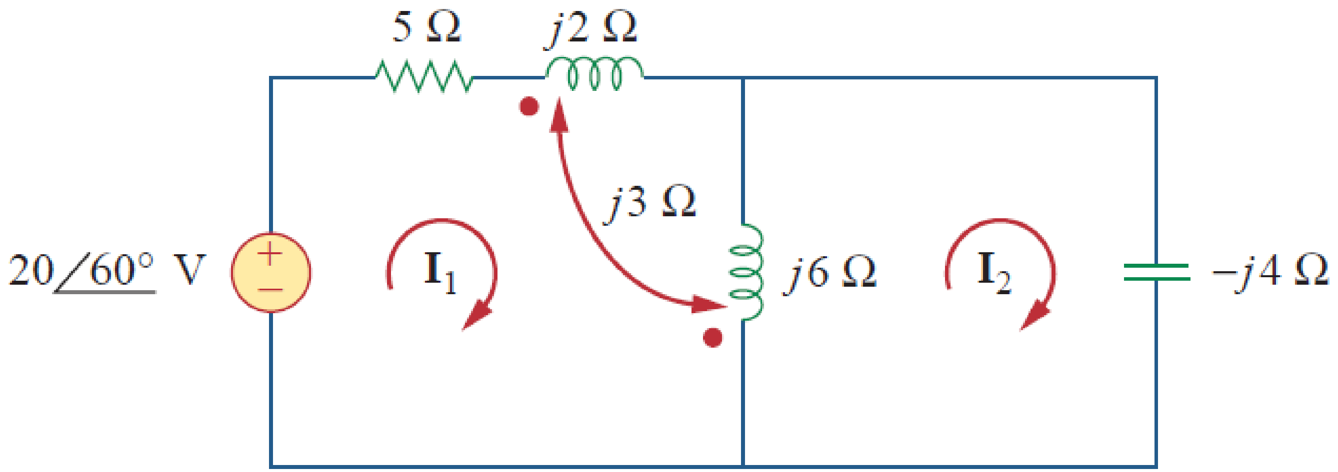
$$\Delta_1=\begin{vmatrix}100 & -j8\\0 & 5+j18\end{vmatrix}=100(5+j18)\qquad \Delta_2=\begin{vmatrix}4+j3 & 100\\-j8 & 0\end{vmatrix}=j800$$

$$\boldsymbol{I}_1=\frac{\Delta_1}{\Delta}=\frac{100(5+j18)}{30+j87}=\frac{1{,}868.2\ e^{j74.5^\circ}}{92.03\ e^{j71^\circ}}=20.3\ e^{j3.5^\circ}\ A$$

$$\boldsymbol{I}_2=\frac{\Delta_2}{\Delta}=\frac{j800}{30+j87}=\frac{800\ e^{j90^\circ}}{92.03\ e^{j71^\circ}}=8.693\ e^{j19^\circ}\ A$$

# Mutual Inductance

**TRF.04** – Calculate the mesh currents.

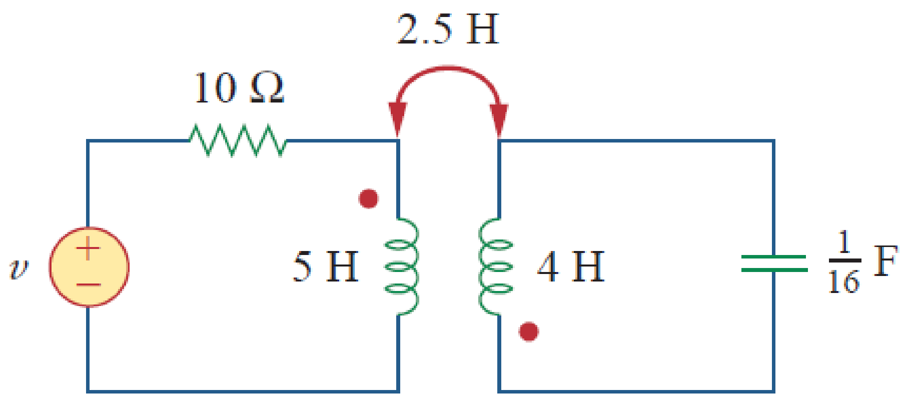



**Solution** $\boldsymbol{I}_1 = 3.583\ e^{j86.56°}\ A$, $\boldsymbol{I}_2 = 5.383\ e^{j86.56°}\ A$

# Energy in Coupled Circuit

**TRF.05** – Determine the coupling coefficient and calculate the energy stored in the coupled inductors at *t* = *1 s* if

$$v = 60\cos(4t + 30°)\ V$$




**Solution** $$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{20}} = 0.56$$

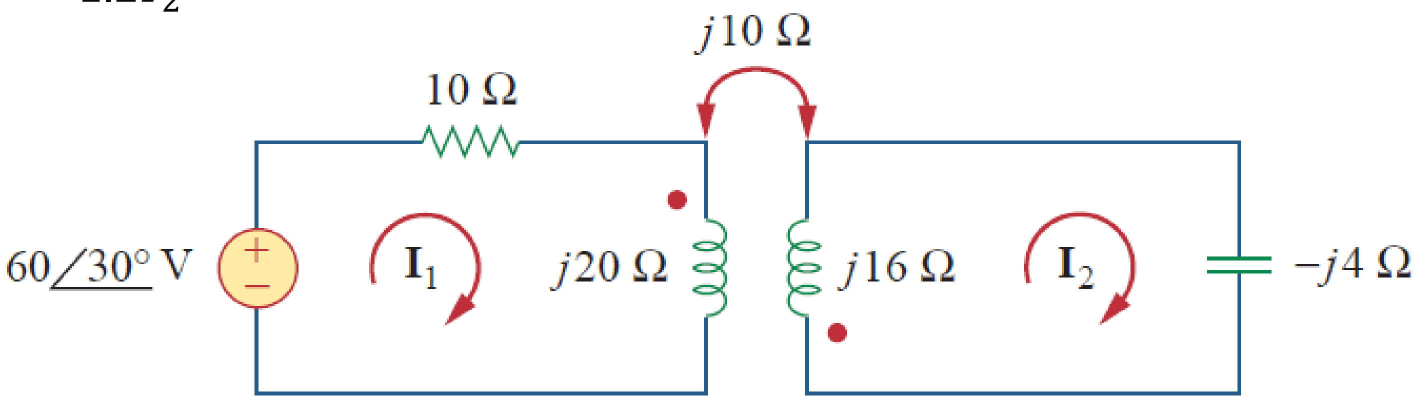
$$(\mathbf{1}):\ (10 + j20)\boldsymbol{I}_1 + j10\boldsymbol{I}_2 = 60\ e^{j30°}$$

$$(\mathbf{2}):\ j10\boldsymbol{I}_1 + (j16 - j4)\boldsymbol{I}_2 = 0 \rightarrow \boldsymbol{I}_1 = -1.2\boldsymbol{I}_2$$

$$(\mathbf{2} \rightarrow \mathbf{1}):\ (-12 - j14)\boldsymbol{I}_2 = 60\ e^{j30°}$$

$$\rightarrow \boldsymbol{I}_2 = 3.254\ e^{j160.6°}\ A$$

$$\boldsymbol{I}_1 = -1.2\boldsymbol{I}_2 = 3.905\ e^{-j19.4°}\ A$$



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# Energy in Coupled Circuit

$$\boldsymbol{I}_1 = 3.905\, e^{-j19.4°} \rightarrow i_1(t) = 3.905 \cos(4t - 19.4°)$$

$$\boldsymbol{I}_2 = 3.254\, e^{j160.6°} \rightarrow i_2(t) = 3.254 \cos(4t + 160.6°)$$

$$t = 1\, s \rightarrow 4t = 4\, rad = 229.2°$$

$$I_1 = 3.905 \cos(229.2° - 19.4°) = -\,3.389\, A$$

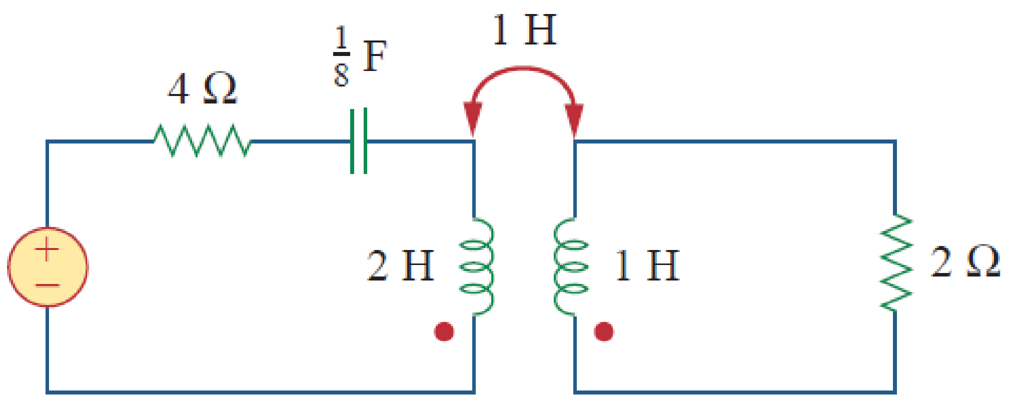
$$I_2 = 3.254 \cos(229.2° + 160.6°) = 2.824\, A$$

$$w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M I_1 I_2 = \frac{1}{2} \cdot 5 \cdot (-3.389)^2 + \frac{1}{2} \cdot 4 \cdot (2.824)^2 + 2.5 \cdot (-3.389) \cdot 2.824 = 20.73\, J$$

# Energy in Coupled Circuit

**TRF.06**

Determine the coupling coefficient and calculate the energy stored in the coupled inductors at *t* = *1.5 s* if $v_S = 40\cos 4t$

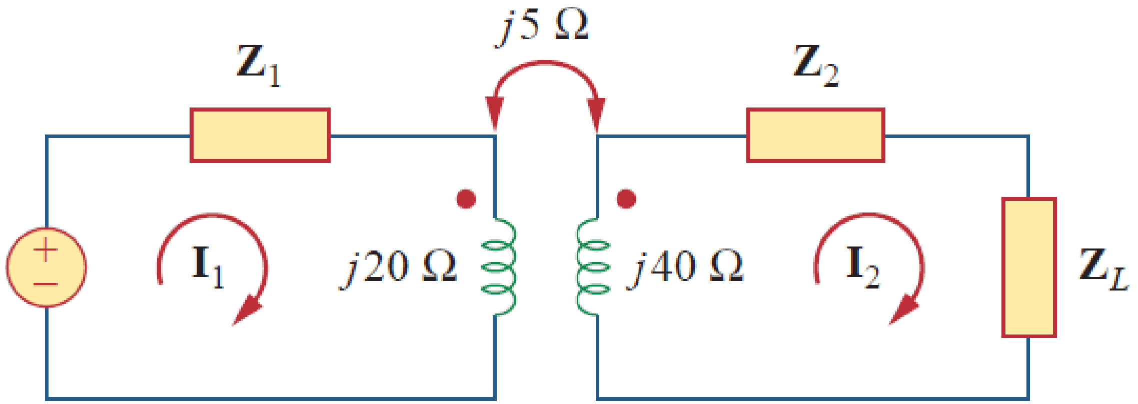



**Solution** *k = 0.7071, w = 39.4 J*

# Linear Transformers

**TRF.07** – Find the input impedance and current $I_1$.

$$\boldsymbol{V_S} = 50\, e^{j60°}\, V, \boldsymbol{Z_1} = 60 - j100\, \Omega\, , \boldsymbol{Z_2} = 30 + j40\Omega\, , \boldsymbol{Z_L} = 80 + j60\, \Omega$$



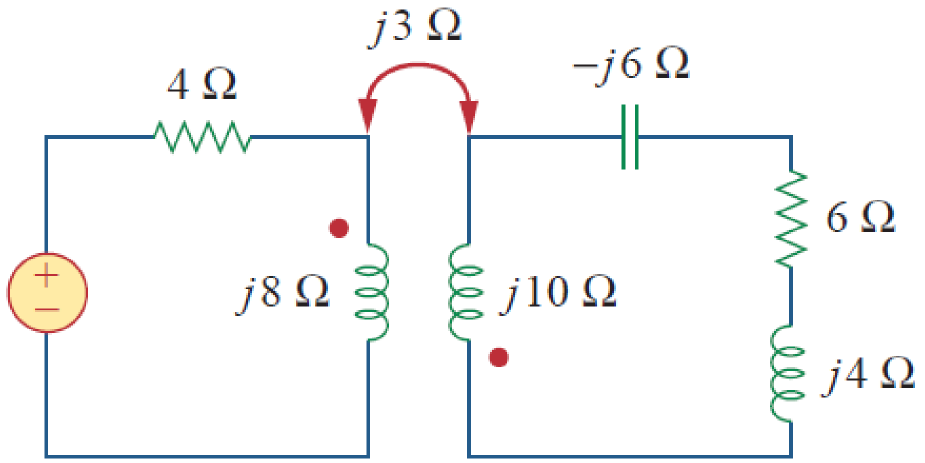

**Solution**

$$\boldsymbol{Z_{in}} = \boldsymbol{Z_1} + j20 + \frac{5^2}{j40 + \boldsymbol{Z_2} + \boldsymbol{Z_L}} = 60 - j100 + j20 + \frac{25}{110 + j140} = 60 - j80 + 1.14\, e^{-j51.84°}$$

$$= 60.09 - j80.11 = 100.14\, e^{-j53.1°}\, \Omega \qquad \boldsymbol{I_1} = \frac{\boldsymbol{V}}{\boldsymbol{Z_{in}}} = \frac{50\, e^{j60°}}{100.14\, e^{-j53.1°}} = 0.5\, e^{j113.1°}\ A$$

# Linear Transformers

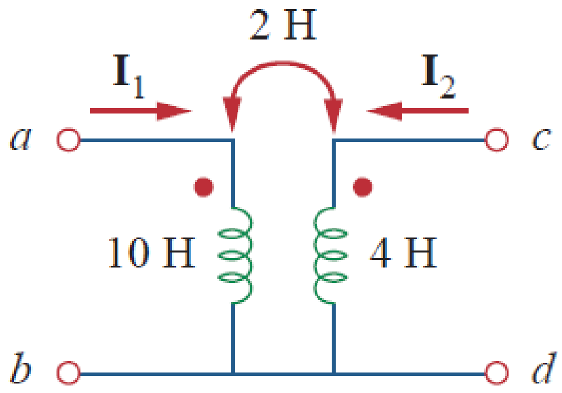
**TRF.08** – Find the input impedance and current from $V_S$ if $\boldsymbol{V_S} = 20\ V$

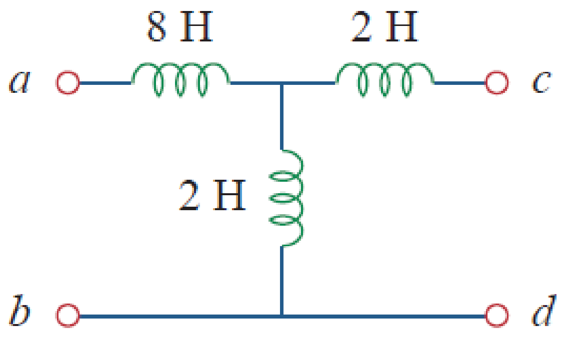



**Solution** $\boldsymbol{Z_{in}} = 8.58\ e^{j58.05°}\ \Omega\ ,\qquad \boldsymbol{I_S} = 2.331\ e^{-j58.05°}\ A$

# Linear Transformers

**TRF.09** – Determine the T-equivalent circuit of the linear transformer



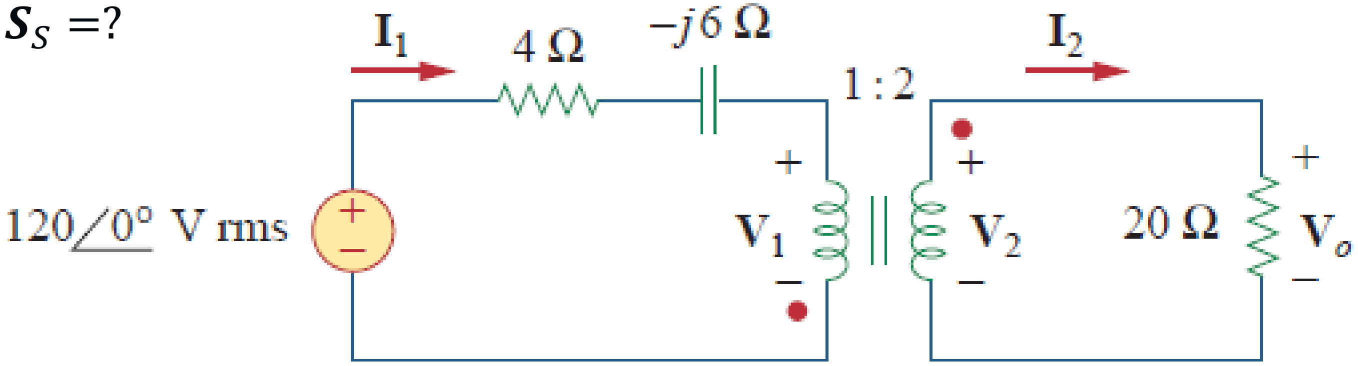

**Solution** $L_1 = 10\ H$ , $L_2 = 4\ H$ , $M = 2\ H$ ,

$L_a = L_1 - M = 8\ H$ , $L_b = L_2 - M = 2\ H$ , $L_C = M = 2\ H$

# Ideal Transformers

TRF.10 – $\boldsymbol{I}_1$ =?, $\boldsymbol{V}_O$ =?, $\boldsymbol{S}_S$ =?

**Solution**




$$\boldsymbol{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5\ \Omega$$

$$\boldsymbol{Z}_{in} = 4 - j6 + \boldsymbol{Z}_R = 9 - j6 = 10.82\ e^{-j33.69°}\ \Omega$$

$$\boldsymbol{I}_1 = \frac{120}{\boldsymbol{Z}_{in}} = 11.09\ e^{j33.69°}\ A$$

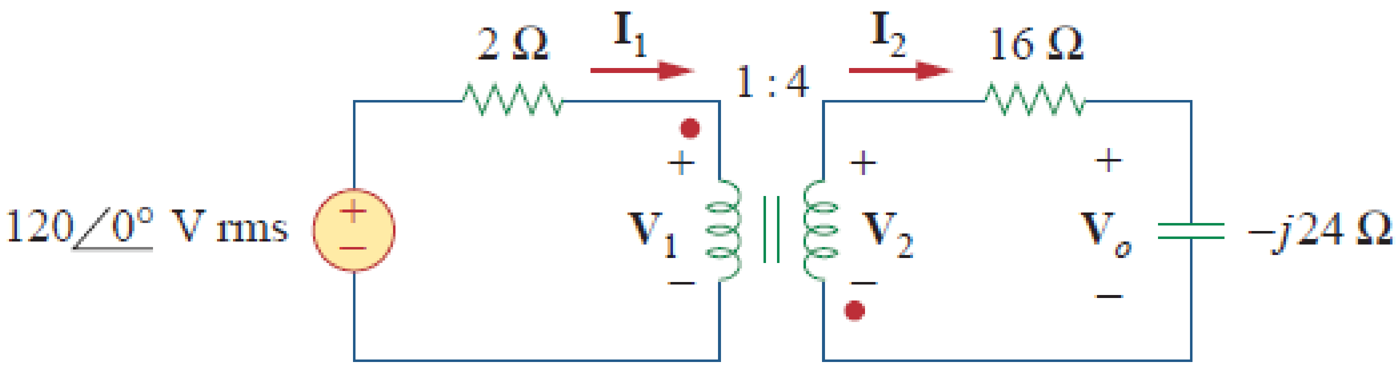
$$\boldsymbol{I}_2 = -\frac{1}{n}\boldsymbol{I}_1 = -5.545\ e^{j33.69°} = 5.545\ e^{j213.69°} A$$

$$\boldsymbol{V}_O = 20\boldsymbol{I}_2 = 110.9\ e^{j213.69°}\ V$$

$$\boldsymbol{S}_S = \boldsymbol{V}_S \boldsymbol{I}_1^* = 120 \cdot 11.09\ e^{-j33.69°} = 1{,}330.8\ e^{-j33.69°}\ VA$$

# Ideal Transformers

**TRF.11 –** $\boldsymbol{V_O} = ?, \qquad \boldsymbol{S_S} = ?$

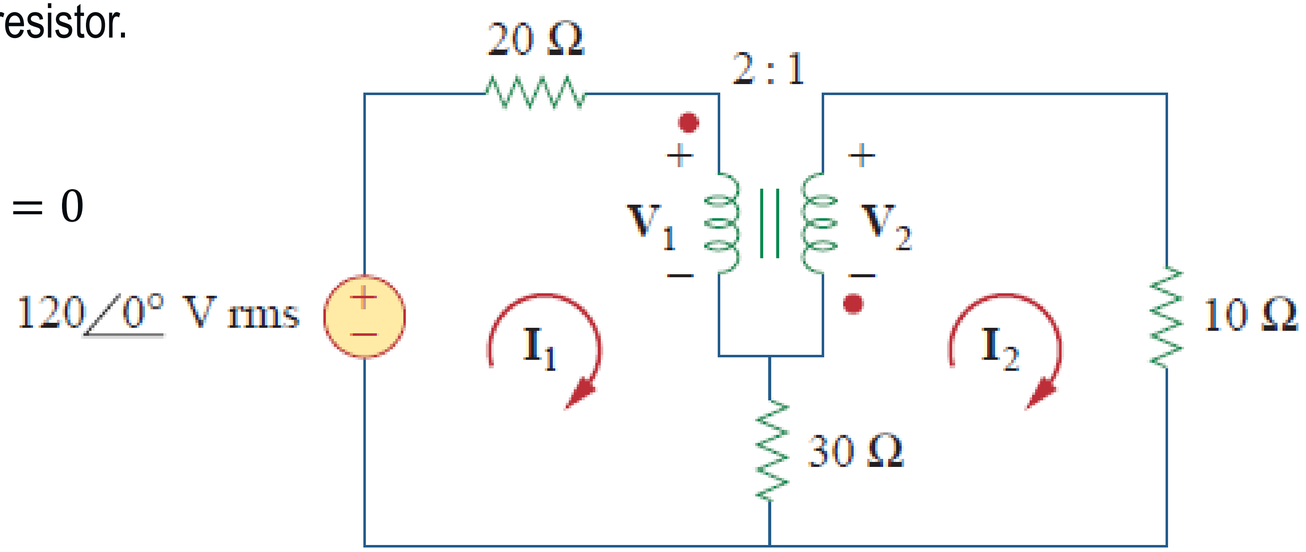



**Solution** $\boldsymbol{V_O} = 214.7\, e^{j116.56°}\, A \qquad \boldsymbol{S_S} = 4.293\, e^{-j26.56°}\, kVA$

# Ideal Transformers

**TRF.12** – Calculate the power supplied to the load resistor.

**Solution**




$$(\mathbf{mesh1}):\ -120+(20+30)\boldsymbol{I}_1-30\boldsymbol{I}_2+\boldsymbol{V}_1=0$$

$$\rightarrow 50\boldsymbol{I}_1-30\boldsymbol{I}_2+\boldsymbol{V}_1=120$$

$$(\mathbf{mesh2}):\ \boldsymbol{V}_2+(10+30)\boldsymbol{I}_2-30\boldsymbol{I}_1=0$$

$$\rightarrow -30\boldsymbol{I}_1+40\boldsymbol{I}_2-\boldsymbol{V}_2=0$$

$$\boldsymbol{V}_2=-\frac{1}{2}\boldsymbol{V}_1\,,\qquad \boldsymbol{I}_2=-2\boldsymbol{I}_1$$

$$(\mathbf{mesh1}\leftarrow \boldsymbol{V}_1\,,\boldsymbol{I}_1):\ -55\boldsymbol{I}_2-2\boldsymbol{V}_2=120$$

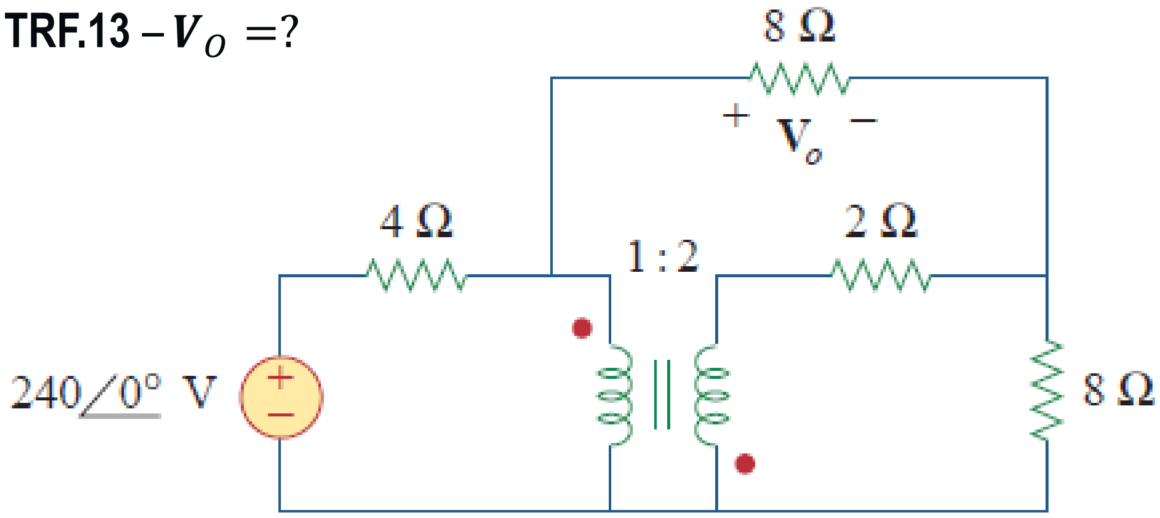
$$(\mathbf{mesh2}\leftarrow \boldsymbol{V}_1\,,\boldsymbol{I}_1): 15\boldsymbol{I}_2+40\boldsymbol{I}_2-\boldsymbol{V}_2=0\rightarrow \boldsymbol{V}_2=55\boldsymbol{I}_2$$

$$-165\boldsymbol{I}_2=120\rightarrow \boldsymbol{I}_2=-0.7272\ A$$

$$P=(-0.7272)^2\cdot 10=5.3\ W$$

# Ideal Transformers

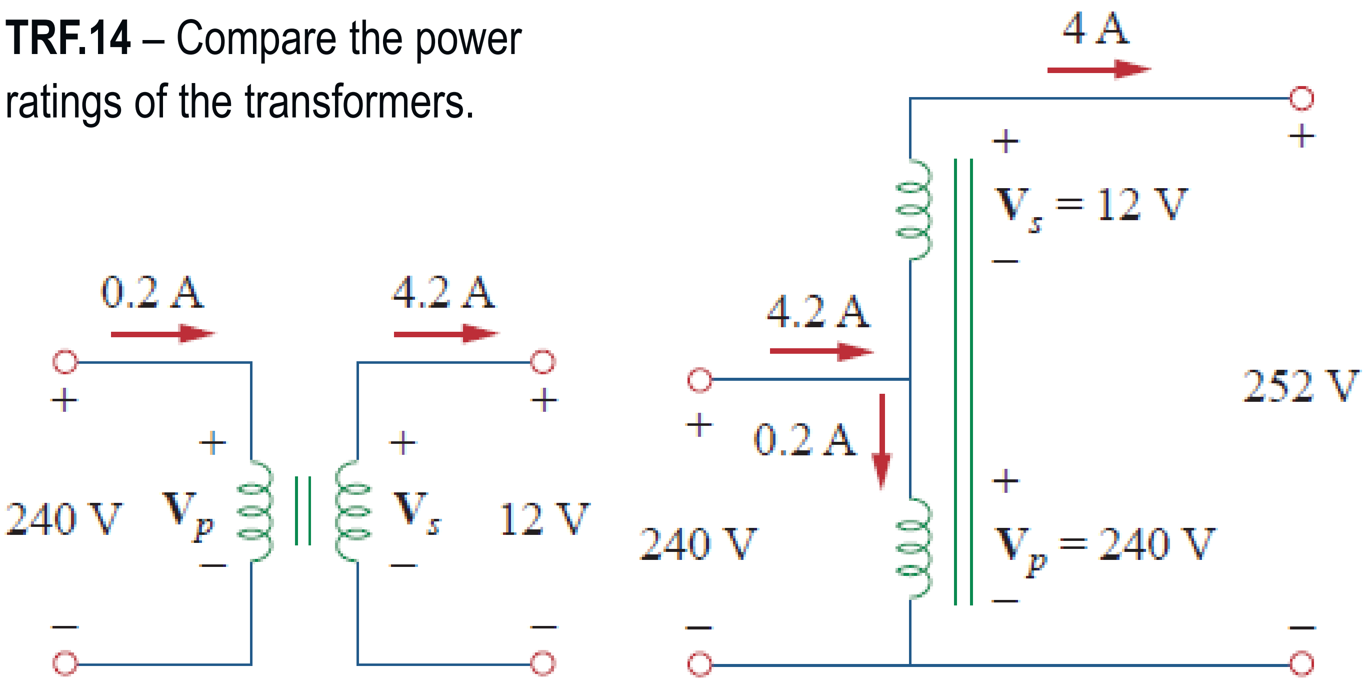
TRF.13 – $V_O = ?$




**Solution …** 96 V

# Ideal Autotransformers

**TRF.14** – Compare the power ratings of the transformers.




**Solution**

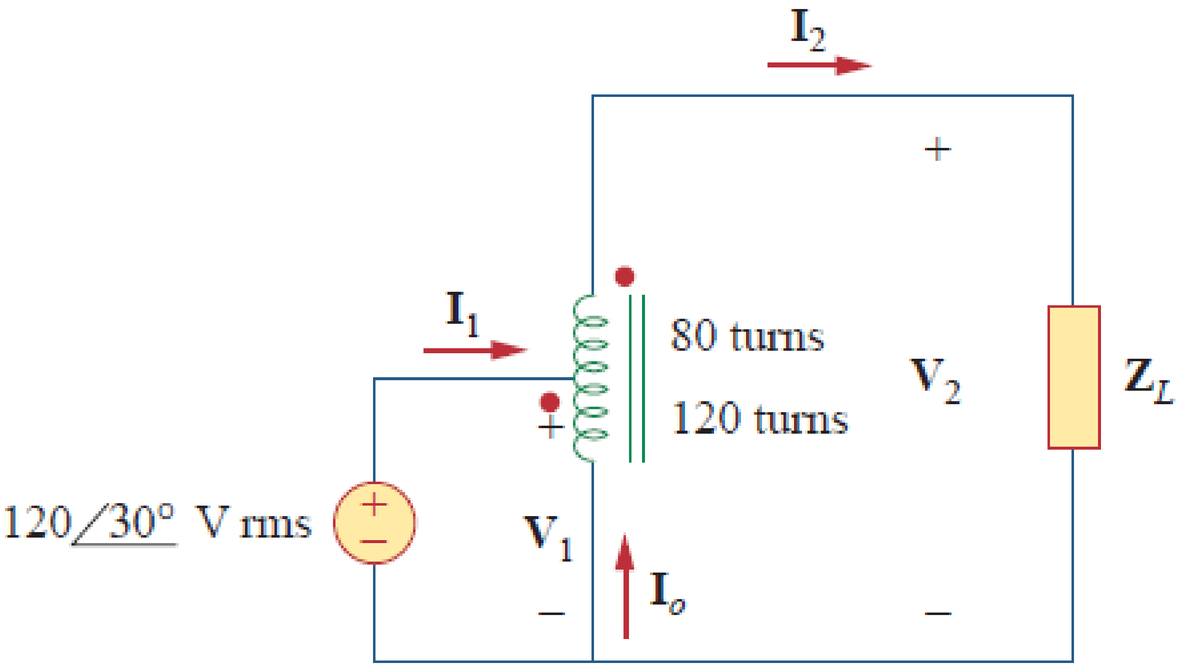
$(AUTOTR.\ 21\ TIMES\ HIGHER!!!)$

$$AUTOTR. \rightarrow S_1 = 4.2 \cdot 240 = 1{,}008\ VA\,, \qquad S_2 = 4 \cdot 252 = 1{,}008\ VA$$

$$2\ WINDING\ TR. \rightarrow S_1 = 0.2 \cdot 240 = 48\ VA\,, \qquad S_2 = 4 \cdot 12 = 48\ VA$$

# Ideal Autotransformers

**TRF.15** – $\boldsymbol{Z}_L = (8 + j6)\ \Omega\,, \boldsymbol{I}_1 = ?\,, \boldsymbol{I}_2 = ?\,, \boldsymbol{I}_O = ?\,, \boldsymbol{S}_L = ?$




**Solution**

$$\frac{\boldsymbol{V}_1}{\boldsymbol{V}_2} = \frac{N_1}{N_1 + N_2} = \frac{80}{200}$$

$$\boldsymbol{V}_2 = \frac{200}{80}\boldsymbol{V}_1 = \frac{200}{80} 120\ e^{j30°} = 300\ e^{j30°}\ V$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{V}_2}{\boldsymbol{Z}_L} = \frac{300\ e^{j30°}}{8 + j6} = \frac{300\ e^{j30°}}{10\ e^{j36.87°}} = 30\ e^{-j6.87°}\ A$$

$$\frac{\boldsymbol{I}_1}{\boldsymbol{I}_2} = \frac{N_1 + N_2}{N_1} = \frac{200}{80}$$

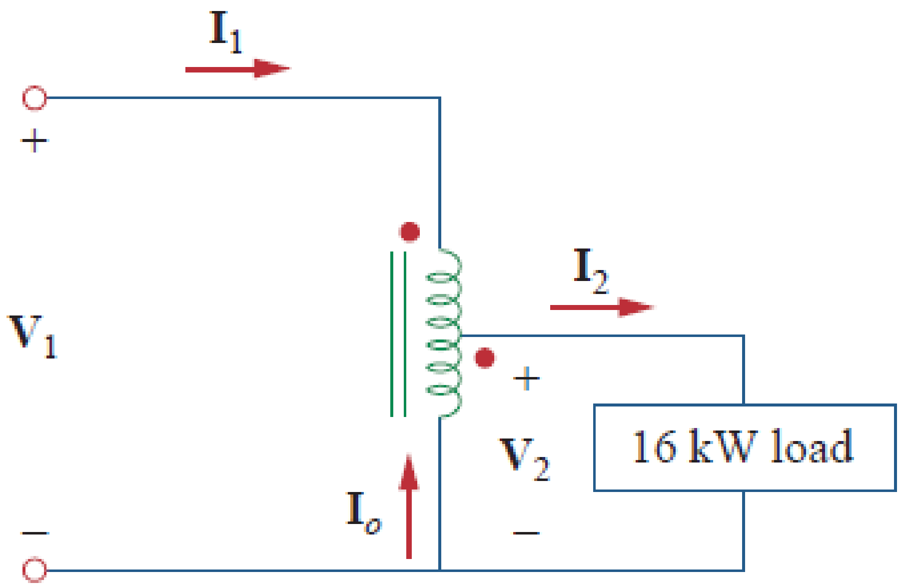
$$\boldsymbol{I}_1 + \boldsymbol{I}_0 = \boldsymbol{I}_2 \rightarrow \boldsymbol{I}_0 = \boldsymbol{I}_2 - \boldsymbol{I}_1 = 30\ e^{-j6.87°} - 75\ e^{-j6.87°} = 45\ e^{j173.13°}$$

$$\boldsymbol{I}_1 = \frac{200}{80}\boldsymbol{I}_2 = \frac{200}{80} 30\ e^{-j6.87°} = 75\ e^{-j6.87°}$$

$$\boldsymbol{S}_2 = \boldsymbol{V}_2 \boldsymbol{I}_2^* = I_2^2 \boldsymbol{Z}_L = 30^2 \cdot 10\ e^{j36.87°} = 9\ e^{j36.87°}\ kVA$$

# Ideal Autotransformers

**TRF.16** – $V_1 = 1{,}250\ V\ , V_2 = 500\ V\ , I_1 = ?\ , I_2 = ?\ , I_O = ?$

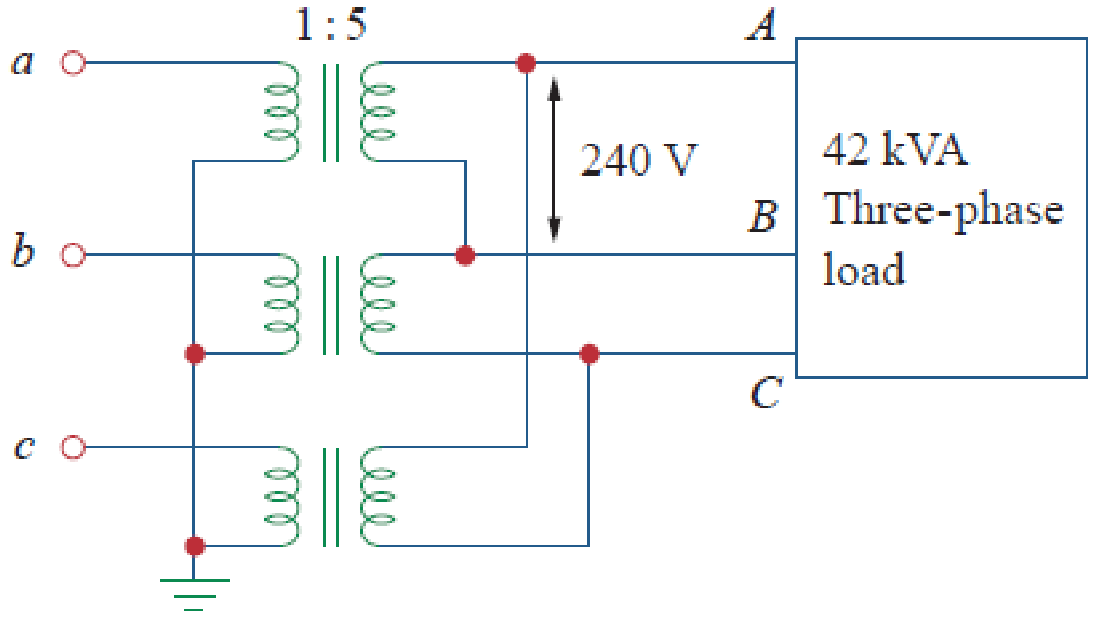



**Solution** 12.8 A, 32 A, 19.2 A

# Three-Phase Transformers

**TRF.17** – Determine the type of transformer connections. Find the line voltage and current on the primary side.
Determine the kVA rating of each transformer used in the transformer bank. (The transformers are ideal.)

**Solution** … *(YD)*




$$I_{Ls} = \frac{S_{tot}}{\sqrt{3}V_{Ls}} = \frac{42{,}000}{\sqrt{3}\cdot 240} = 101\ A$$

$$I_{Lp} = \frac{n}{\sqrt{3}} I_{Ls} = \frac{5\cdot 101}{\sqrt{3}} = 292\ A$$

$$V_{Lp} = \frac{\sqrt{3}}{n} V_{Ls} = \frac{\sqrt{3}\cdot 240}{5} = 83.14\ V$$

$$S = \frac{S_{tot}}{3} = 14\ kVA\ \ \ldots or\ S_s = V_{Ps}I_{Ps} = V_{Ls}\frac{I_{Ls}}{\sqrt{3}} = 240\cdot\frac{101}{\sqrt{3}} = 14\ kVA\ \ \ldots or\ S_P = V_{Pp}I_{Pp} = \frac{V_{Lp}}{\sqrt{3}} I_{Lp} = \frac{83.14}{\sqrt{3}}\cdot 292 = 14\ kVA$$

# Three-Phase Transformers

**TRF.18** – A three-phase DD transformer is used to step down a line voltage of 625 kV, to supply a plant operating at a line voltage of 12.5 kV. The plant draws 40 MW with a lagging power factor of 85 percent. Find:

a) the current drawn by the plant
b) the turns ratio
c) the current on the primary side of the transformer
d) the load carried by each transformer

**Solution** (a) 2.1736 kA, (b) 0.02, (c) 43.47 A, (d) 15.69 MVA.

# Questions