DR. GYURCSEK ISTVÁN

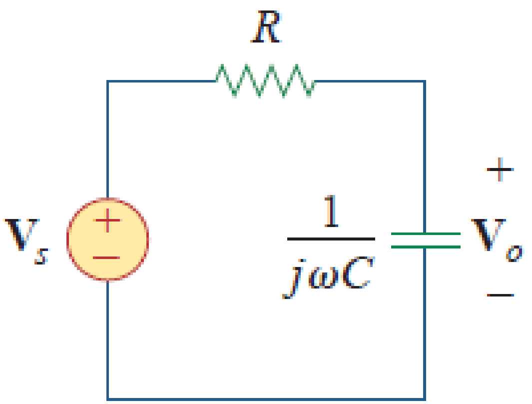
Exercises in Frequency Response

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*
- *Source: http://www.swarthmore.edu/*
- *https://en.wikipedia.org/wiki/Bode_plot*

Transfer Function (Low Pass Filter)

FRQ.01 – For the *RC* circuit obtain the transfer function and its frequency response.

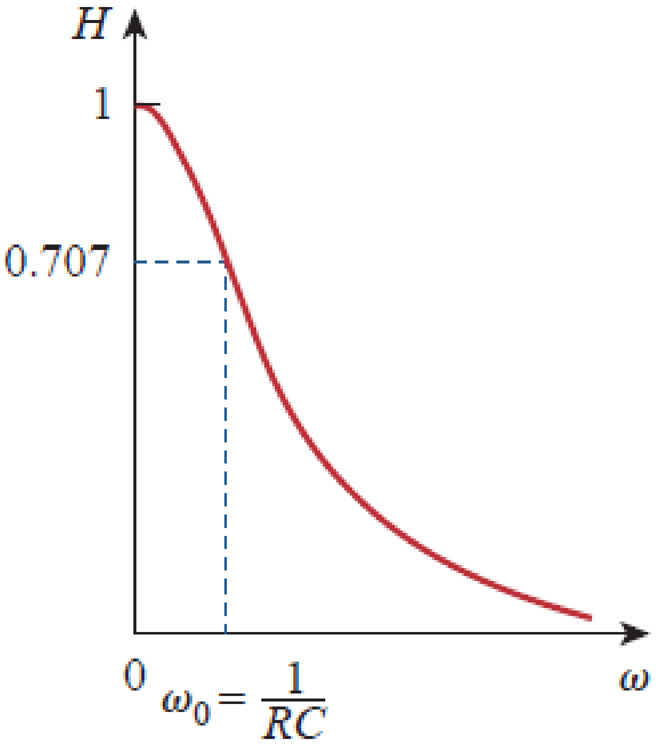


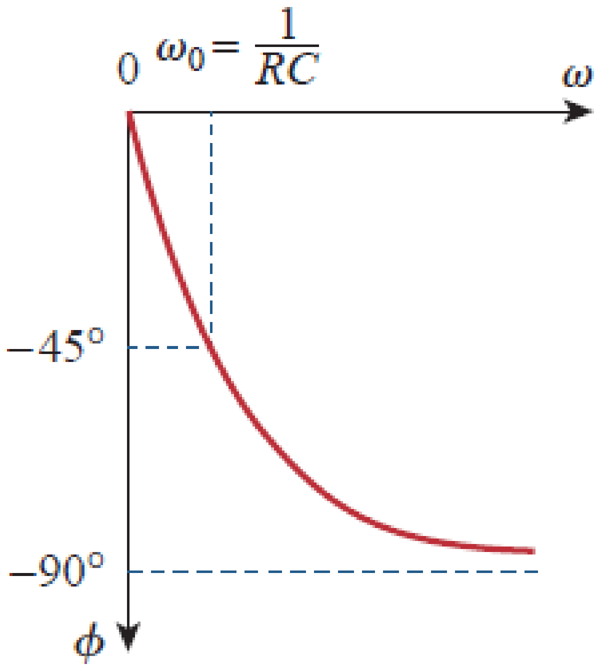
$$\boldsymbol{H}(\omega) = \frac{\boldsymbol{V}_0(\omega)}{\boldsymbol{V}_S(\omega)}$$

$$\boldsymbol{H}(\omega) = \frac{{}^{1}/_{j\omega C}}{R + {}^{1}/_{j\omega C}}$$

$$\boldsymbol{H}(\omega) = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\,{}^{\omega}/_{\omega_0}} \leftarrow \omega_0 = \frac{1}{RC}$$

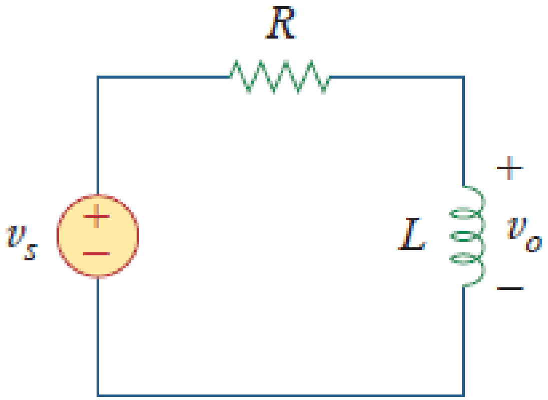
$$H(\omega) = \frac{1}{\sqrt{1 + \left({}^{\omega}/_{\omega_0}\right)^2}}, \phi = -tan^{-1}\frac{\omega}{\omega_0}$$





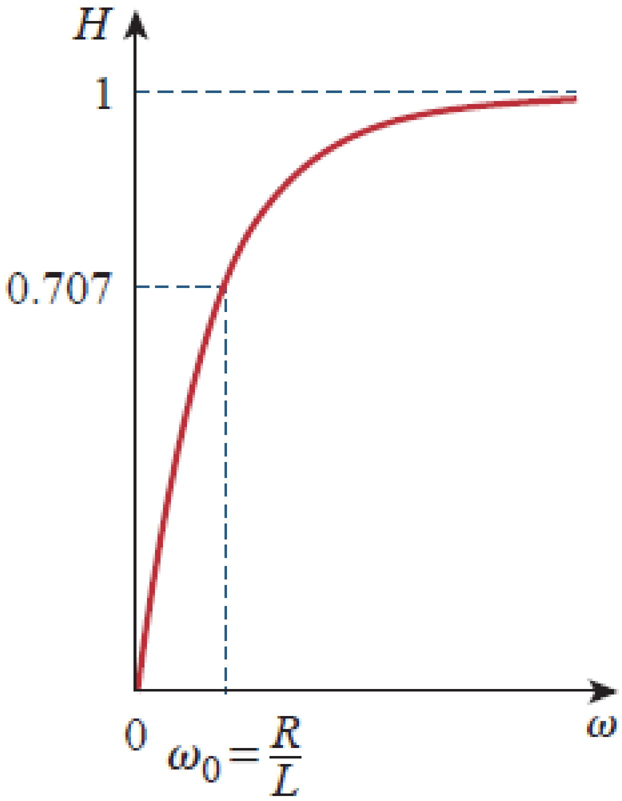
Transfer Function (High Pass Filter)

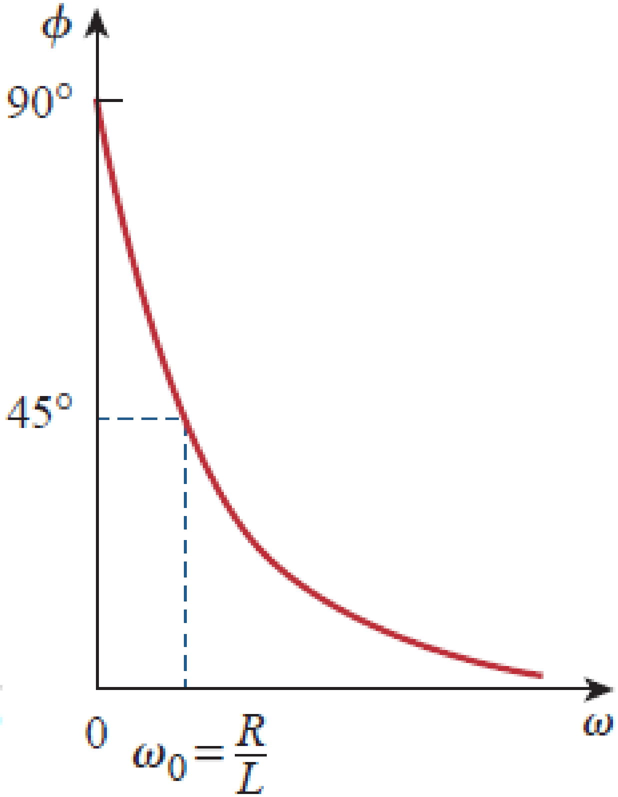
FRQ.02 – For the *RL* circuit obtain the transfer function and its frequency response.



$$\boldsymbol{H}(\omega) = \frac{j\omega L}{R + j\omega L}$$

$$\boldsymbol{H}(\omega) = \frac{j\,{}^{\omega}/_{\omega_0}}{1 + j\,{}^{\omega}/_{\omega_0}} \leftarrow \omega_0 = \frac{R}{L}$$

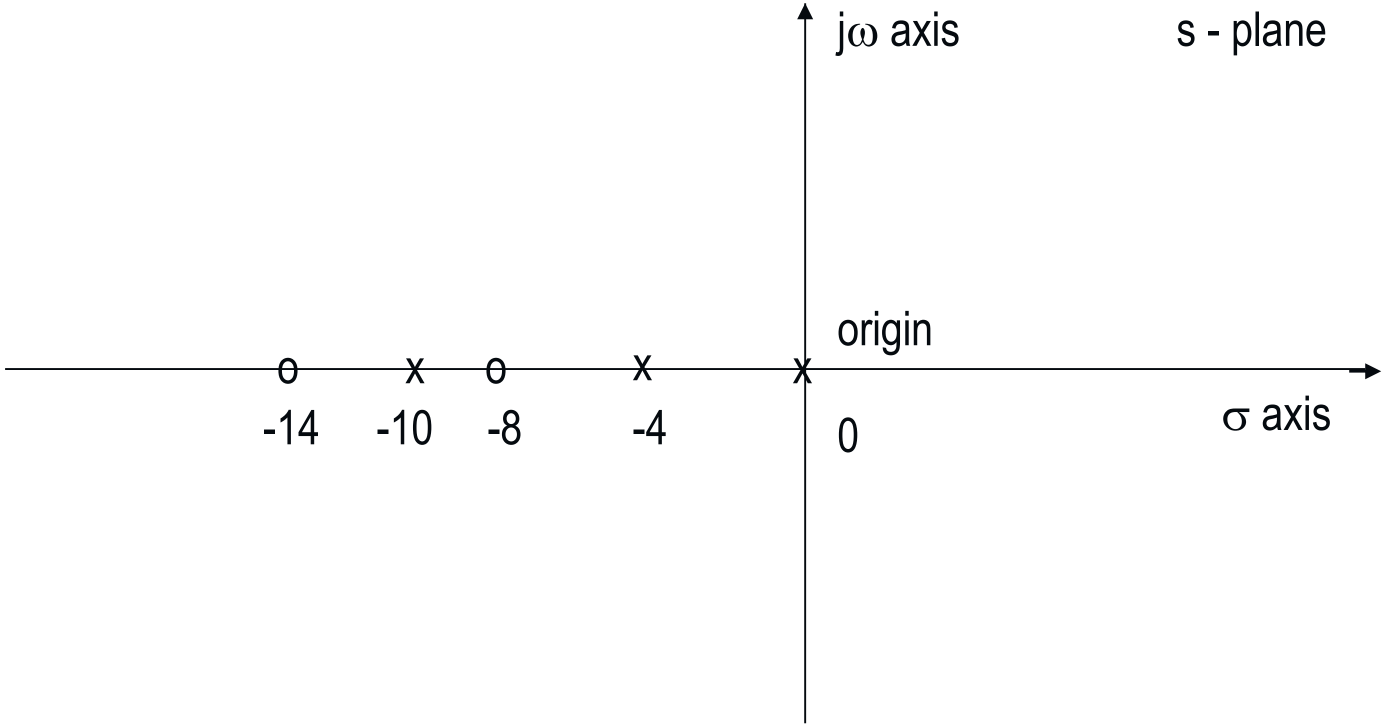




Bode Plots

FRQ.03 – You are given the following transfer function.
Show the roots *(determine poles and zeros)* in the s-plane.

$$G(s) = \frac{(s+8)(s+14)}{s(s+4)(s+10)}$$



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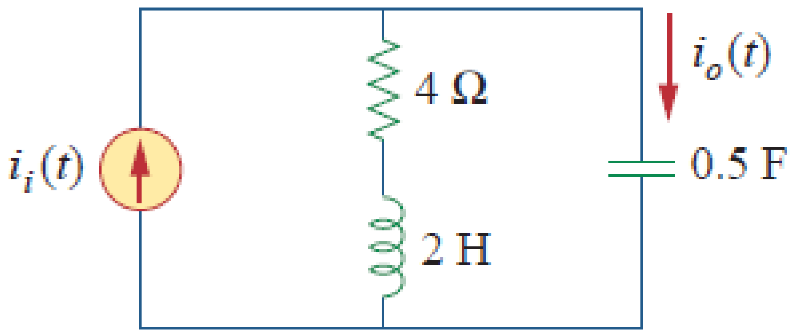
Transfer Function

FRQ.04 – Calculate the current gain I_0/I_i and its poles and zeros.

$$\boldsymbol{I}_0(\omega) = \frac{4 + j2\omega}{4 + j2\omega + {}^{1}/_{j0.5\omega}} \boldsymbol{I}_i(\omega)$$

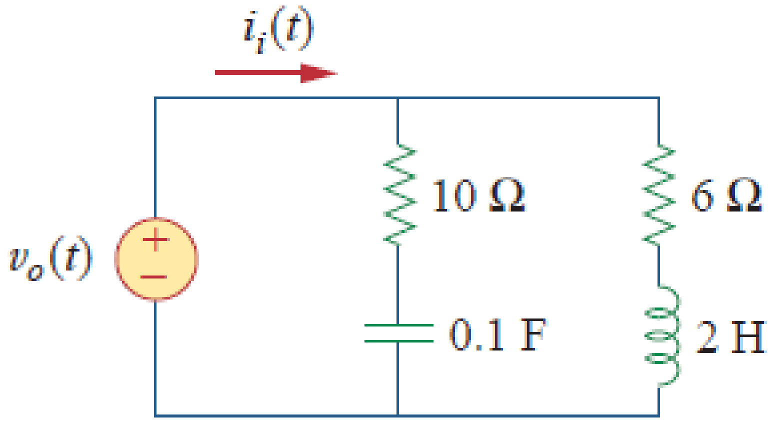


$$\boldsymbol{H}(\omega) = \frac{\boldsymbol{I}_0(\omega)}{\boldsymbol{I}_i(\omega)} = \frac{j0.5\omega(4 + j2\omega)}{1 + j2\omega + (j\omega)^2} = \frac{s(s+2)}{s^2 + 2s + 1} = \frac{s(s+2)}{(s+1)^2} \leftarrow s = j\omega$$

$$z_1 = 0\,, z_2 = -2\,, p_{12} = -1 \leftarrow repeated\ (double)\ poles$$

Transfer Function

FRQ.05 – Calculate the transfer impedance of V_0/I_i and its poles and zeros.



$$\boldsymbol{Z}(\omega)=\frac{\boldsymbol{V}_0(\omega)}{\boldsymbol{I}_i(\omega)}=\left(10+\frac{10}{s}\right)\times(6+2s)=(10+10s^{-1})\times(6+2s)$$

$$=\frac{60+\ 60s^{-1}+20s+20}{16+10s^{-1}+2s}=\frac{10(8s+6+2s^2)}{2(8s+5+s^2)}=10\frac{(s^2+4s+3)}{s^2+8s+5}$$

$$zeros\rightarrow s_{1,2}=\frac{-4\pm\sqrt{16-12}}{2}=\begin{cases}-1\rightarrow z_1=1\\-3\rightarrow z_2=3\end{cases}$$

$$poles\rightarrow s_{3,4}=\frac{-8\pm\sqrt{64-20}}{2}=\begin{cases}-0.683\rightarrow p_1=0.683\\-7.317\rightarrow p_2=7.317\end{cases}$$

$$\boldsymbol{Z}(\omega)=\frac{\boldsymbol{V}_0(\omega)}{\boldsymbol{I}_i(\omega)}=\frac{10(s+1)(s+3)}{(s+0.68)(s+7.32)}$$

Nyquist and Bode Plots

FRQ.06 – Find the Bode and Nyquist plots for the first order system described with the transfer function of…

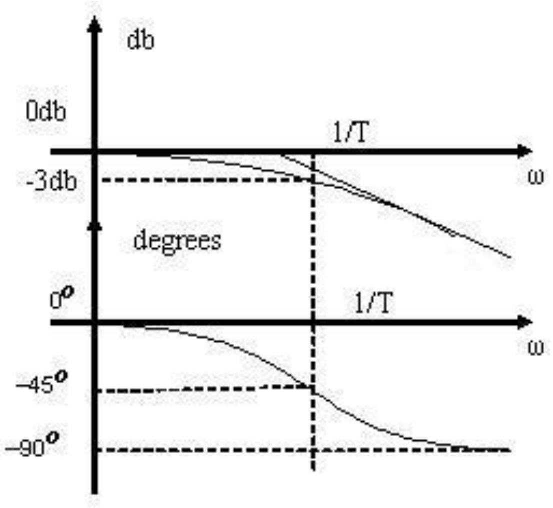
$$\boldsymbol{H}(\omega) = \frac{1}{1+j\omega T} = \frac{1}{1+j\omega T}\cdot\frac{1-j\omega T}{1-j\omega T} = \frac{1}{1+\omega^2T^2} - j\cdot\frac{\omega T}{1+\omega^2T^2}$$

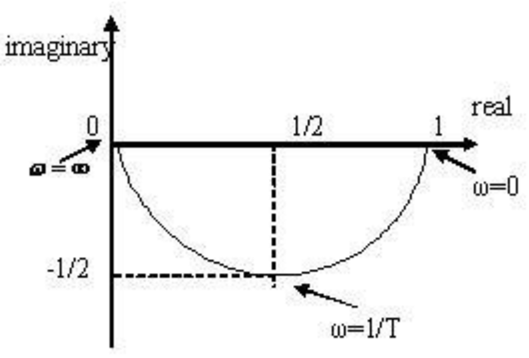
$$Gain = 20\log|\boldsymbol{H}| = 20\log\left|\frac{1}{1+j\omega T}\right| = -20\log|1+j\omega T|$$

$$Phase = tan^{-1}\frac{Im\{\boldsymbol{H}\}}{Re\{\boldsymbol{H}\}} = tan^{-1}(-\omega T) = -tan^{-1}(\omega T)$$

ω	Gain
0	1
1/T	0.5-j0.5
infinity	0

ω	Gain (dB)	Phase (°)
0	0	0
1/T	-3	-45
infinity	- infinity	-90





[low-pass filter]

Bode Plots

FRQ.07
Construct the Bode plots for the transfer function

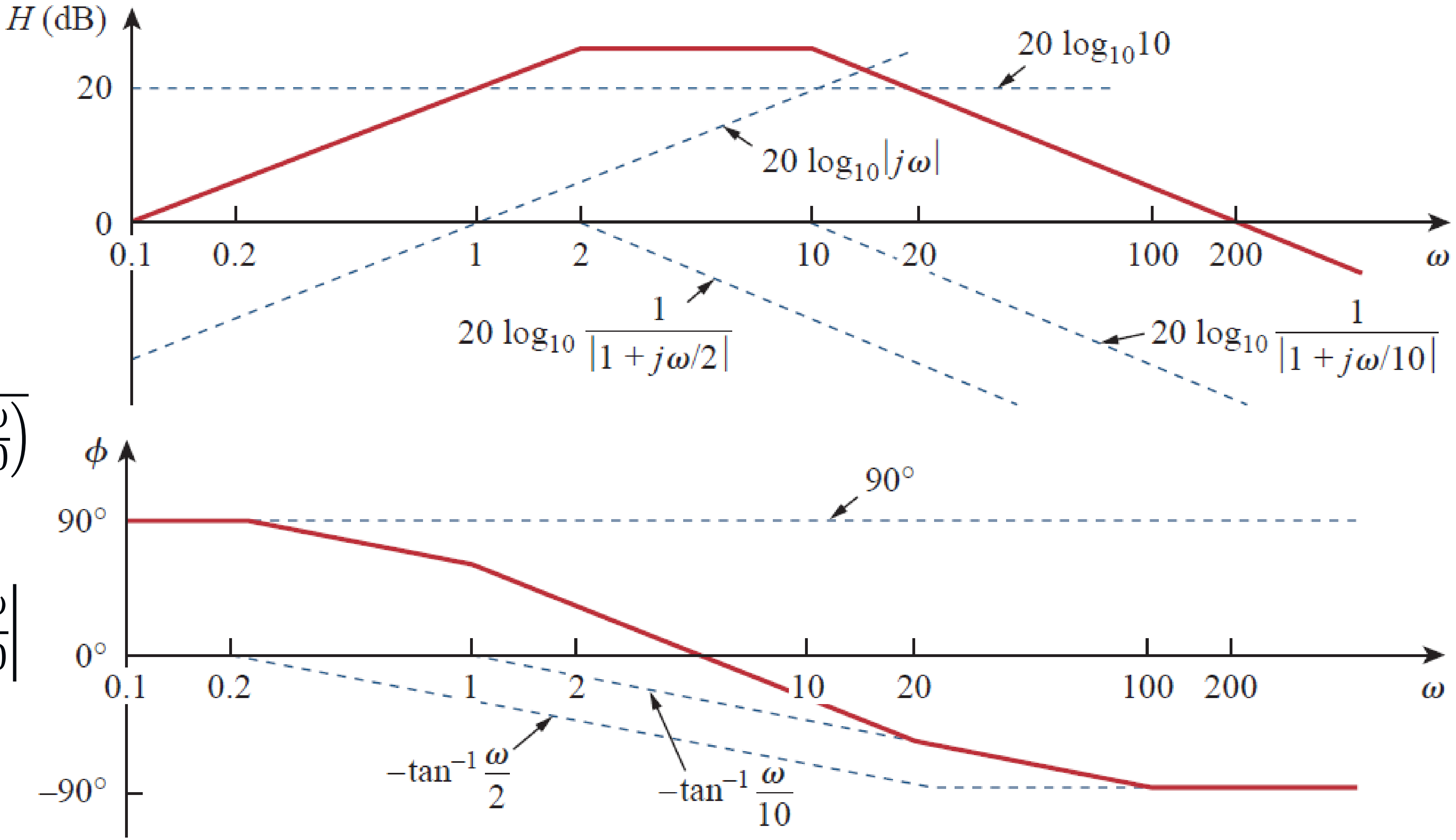
$$\boldsymbol{H}(\omega) = \frac{j200\omega}{(j\omega + 2)(j\omega + 10)}$$

standard form →

$$\boldsymbol{H}(\omega) = \frac{200}{2 \cdot 10} \cdot \frac{j\omega}{\left(1 + \frac{j\omega}{2}\right)\left(1 + \frac{j\omega}{10}\right)}$$

$$H_{dB} = 20 \log 10 + 20 \log|j\omega| - -20 \log\left|1 + \frac{j\omega}{2}\right| - 20 \log\left|1 + \frac{j\omega}{10}\right|$$

$$\phi = 90° - tan^{-1}\frac{\omega}{2} - tan^{-1}\frac{\omega}{10}$$



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Bode Plots

FRQ.08 – Draw the Bode plots for the transfer function

$$\boldsymbol{H}(\omega) = \frac{5(j\omega + 2)}{j\omega(j\omega + 10)}$$

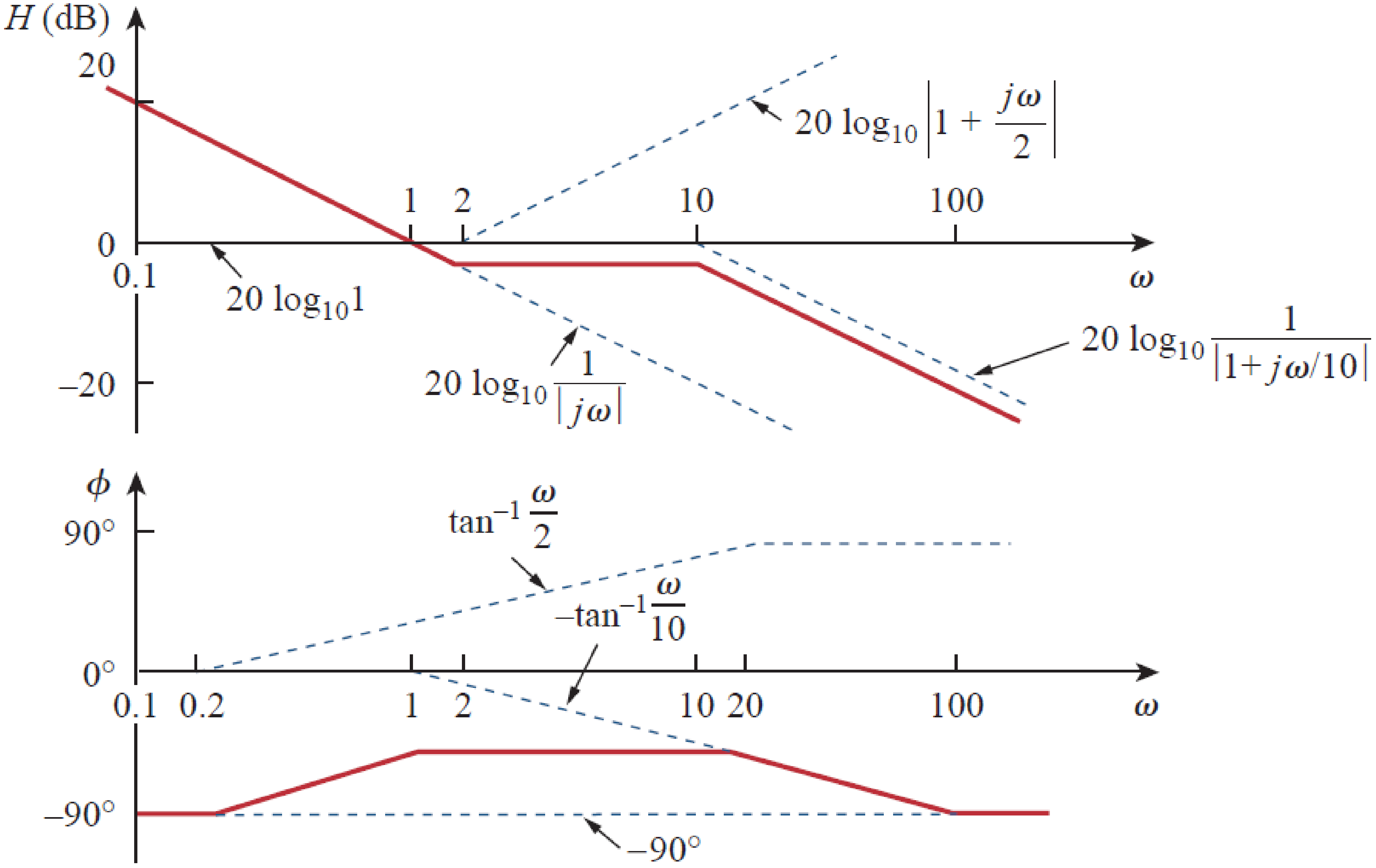
standard form →

$$\boldsymbol{H}(\omega) = \frac{10}{10} \cdot \frac{1 + j\omega/2}{j\omega(1 + j\omega/10)}$$

$$H_{dB} = -20\log|j\omega| + 20\log\left|1 + \frac{j\omega}{2}\right|$$

$$-20\log\left|1 + \frac{j\omega}{10}\right|$$

$$\phi = -90° + tan^{-1}\frac{\omega}{2} - tan^{-1}\frac{\omega}{10}$$



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Bode Plots

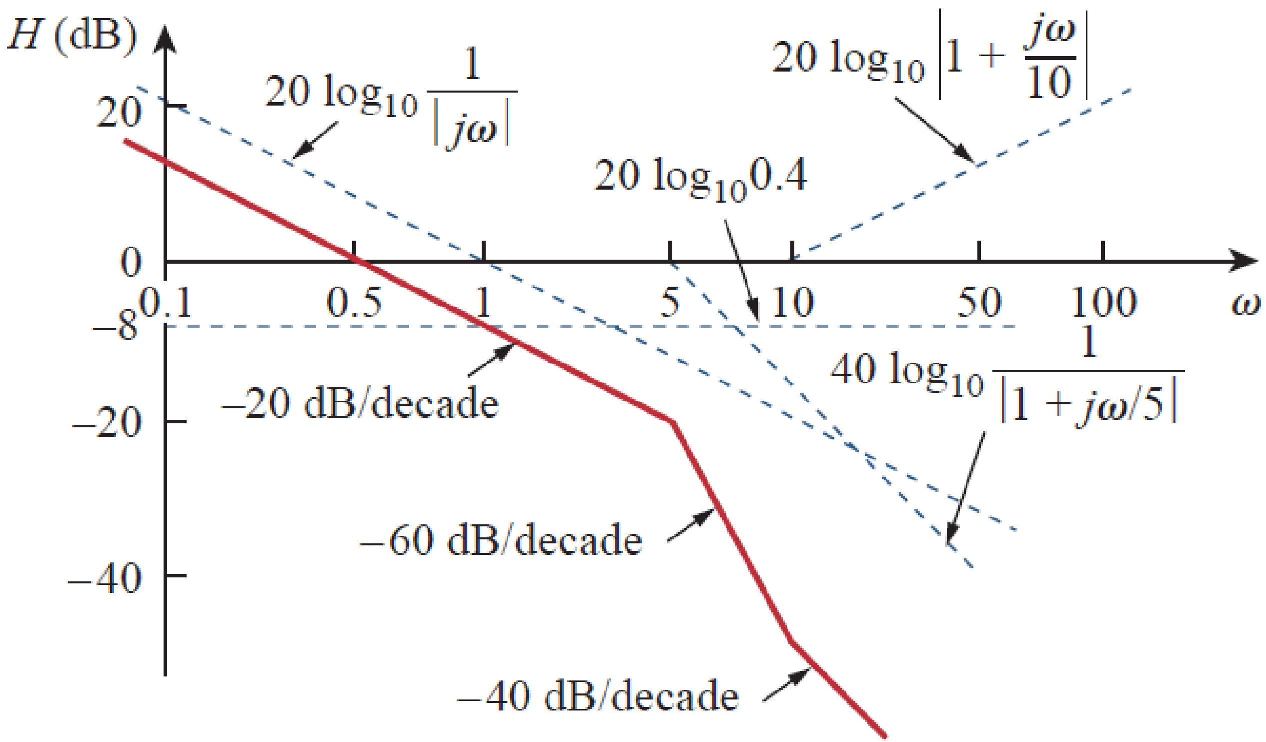
FRQ.09 – Obtain the Bode plots for the transfer function $\boldsymbol{H}(\omega) = \dfrac{j\omega + 10}{j\omega(j\omega + 5)^2}$

standard form → $\boldsymbol{H}(\omega) = \dfrac{0.4(1 + j\omega/10)}{j\omega(1 + j\omega/5)^2}$

$$H_{dB} = 20\log 0.4 + 20\log\left|1 + \frac{j\omega}{10}\right| - 20\log|j\omega|$$

$$-40\log\left|1 + \frac{j\omega}{5}\right|$$

$$\phi = tan^{-1}\frac{\omega}{10} - 90° - 2tan^{-1}\frac{\omega}{5}$$



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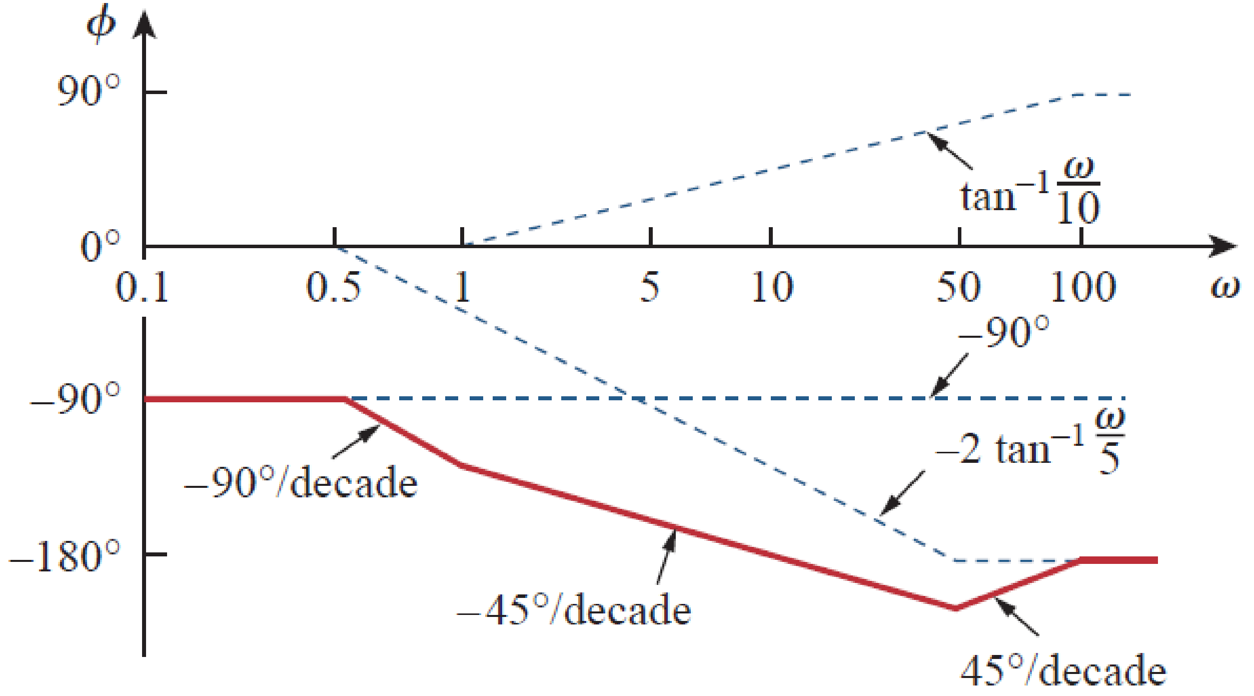
Bode Plots

$$\phi = tan^{-1}\frac{\omega}{10} - 90° - 2tan^{-1}\frac{\omega}{5}$$



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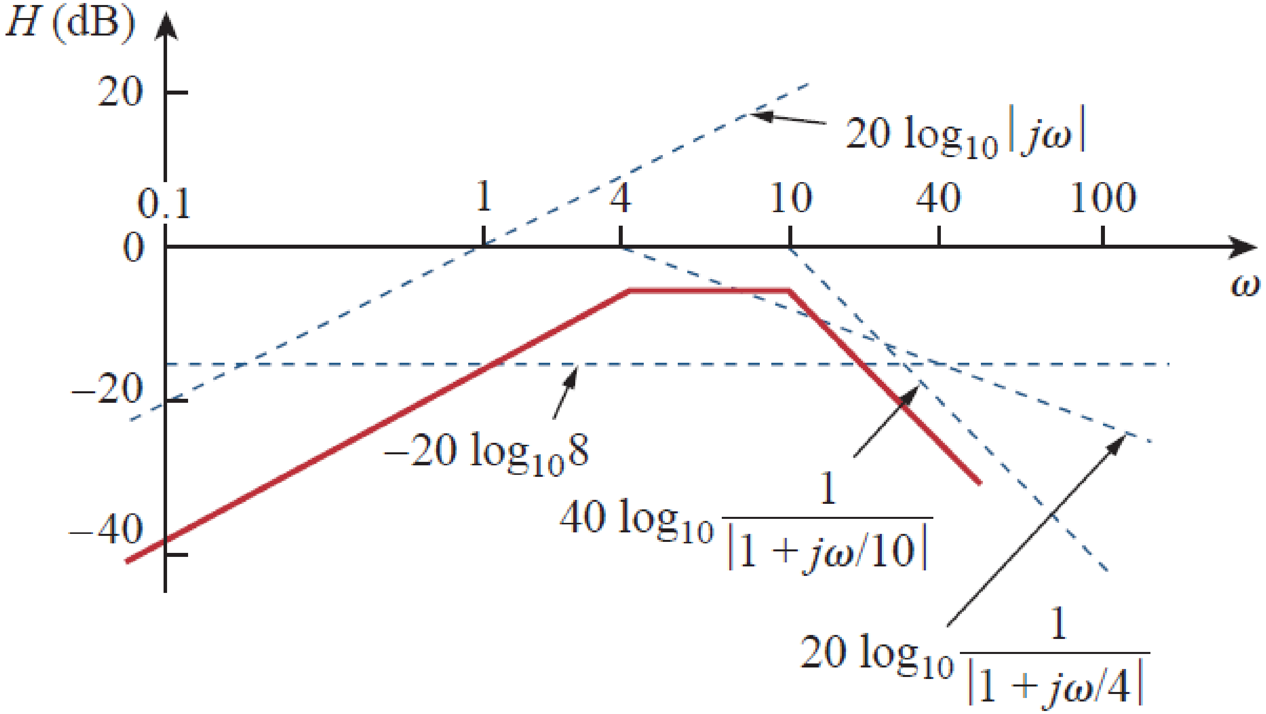
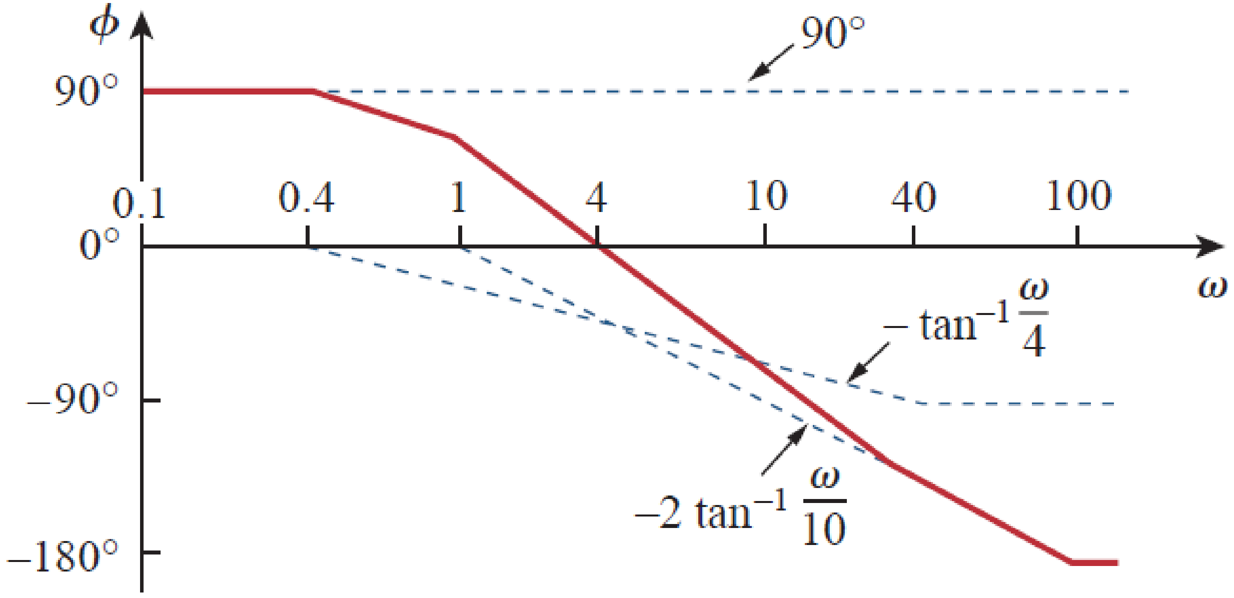
Bode Plots

FRQ.10 – Sketch the Bode plots for $\boldsymbol{H}(\omega) = \dfrac{50j\omega}{(j\omega + 4)(j\omega + 10)^2}$ $\rightarrow \boldsymbol{H}(\omega) = \dfrac{50}{4 \cdot 100} \cdot \dfrac{j\omega}{(1 + j\omega/4)(1 + j\omega/10)^2}$





Bode Plots

FRQ.11 – Draw the Bode plots for

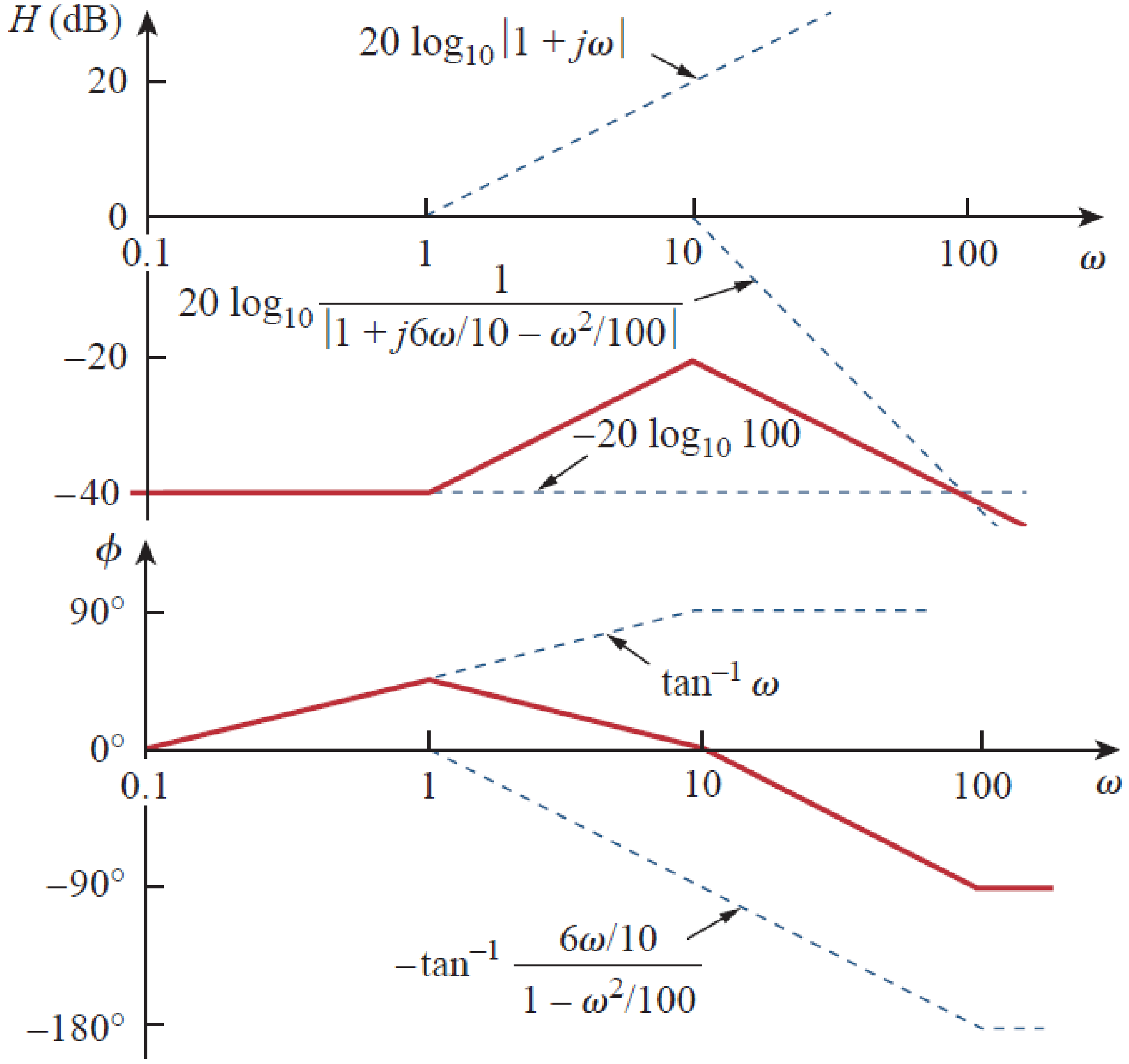
$$\boldsymbol{H}(\omega) = \frac{s+1}{s^2+12s+100}$$

$$\rightarrow \boldsymbol{H}(\omega) = \frac{1}{100}\frac{1+j\omega}{1+j\omega 1.2/10+(j\omega/10)^2}$$

$$H_{dB} = -20\log 100 + 20\log|1+j\omega|$$

$$-20\log|1+j\omega 1.2/10+(j\omega/10)^2|$$

$$\phi = tan^{-1}\omega - 2tan^{-1}\frac{\omega 1.2/10}{1-\omega^2/100}$$



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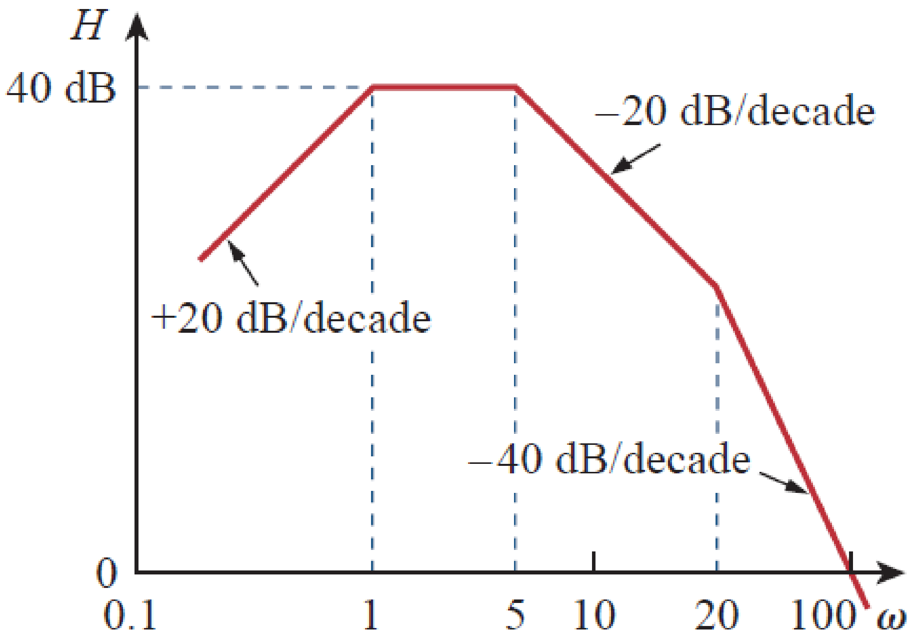
Bode Plots

FRQ.12 – Find the transfer function for the given Bode plot.



$$zero\ at\ origin \rightarrow \boldsymbol{H}(\omega) = \frac{j\omega(\dots)}{(\dots)}$$

$$40\ dB = 20\log K \rightarrow K = 100 \rightarrow \boldsymbol{H}(\omega) = 100\frac{j\omega}{(\dots)}$$

$$pole\ at\ \omega = 1 \rightarrow \boldsymbol{H}(\omega) = 100\frac{j\omega}{(1+j\omega)(\dots)}$$

$$pole\ at\ \omega = 5 \rightarrow \boldsymbol{H}(\omega) = 100\frac{j\omega}{(1+j\omega)(1+j\omega/5)(\dots)}$$

$$pole\ at\ \omega = 20 \rightarrow \boldsymbol{H}(\omega) = 100\frac{j\omega}{(1+j\omega)(1+j\omega/5)(1+j\omega/20)}$$

$$\boldsymbol{H}(\omega) = \frac{10^4 j\omega}{(j\omega+1)(j\omega+5)(j\omega+20)} \rightarrow \boldsymbol{H}(s) = \frac{10^4 s}{(s+1)(s+5)(s+20)}$$

Bode Plot

FRQ.13 – Obtain the transfer function corresponding to the Bode plot.

$$\boldsymbol{H}(\omega) = K \cdot \frac{(1 + j\omega/5)}{(1 + j\omega/10)(1 + j\omega/100)^2}$$

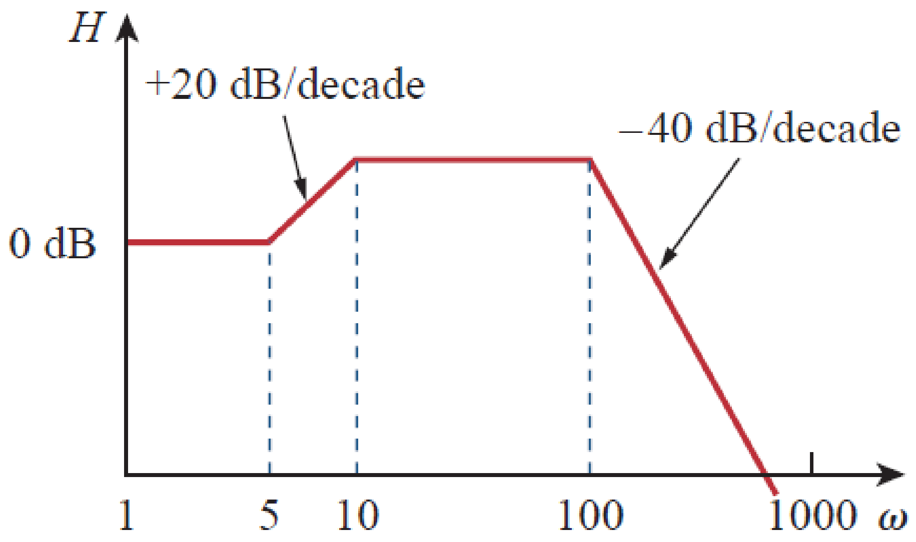
$$\#dec = log\, {}^{\omega_2}/_{\omega_1} = log\, {}^{10}/_{5} = 0.3$$

$$\#dec \cdot 20\, {}^{dB}/_{dec} = 6\, dB$$

$$(6 - 20)\, dB\ at\ \omega = 1 \rightarrow -14\, dB = 20\, log\, K \rightarrow K = 0.2$$

$$\boldsymbol{H}(\omega) = 0.2 \cdot \frac{1}{5} \cdot \frac{10}{1} \cdot \frac{10^4}{1} \cdot \frac{(j\omega + 5)}{(j\omega + 10)(j\omega + 100)^2}$$

$$\boldsymbol{H}(\omega) = \frac{4 \cdot 10^3 (j\omega + 5)}{(j\omega + 10)(j\omega + 100)^2}$$



$$\rightarrow \boldsymbol{H}(s) = \frac{4 \cdot 10^3 (s + 5)}{(s + 10)(s + 100)^2}$$

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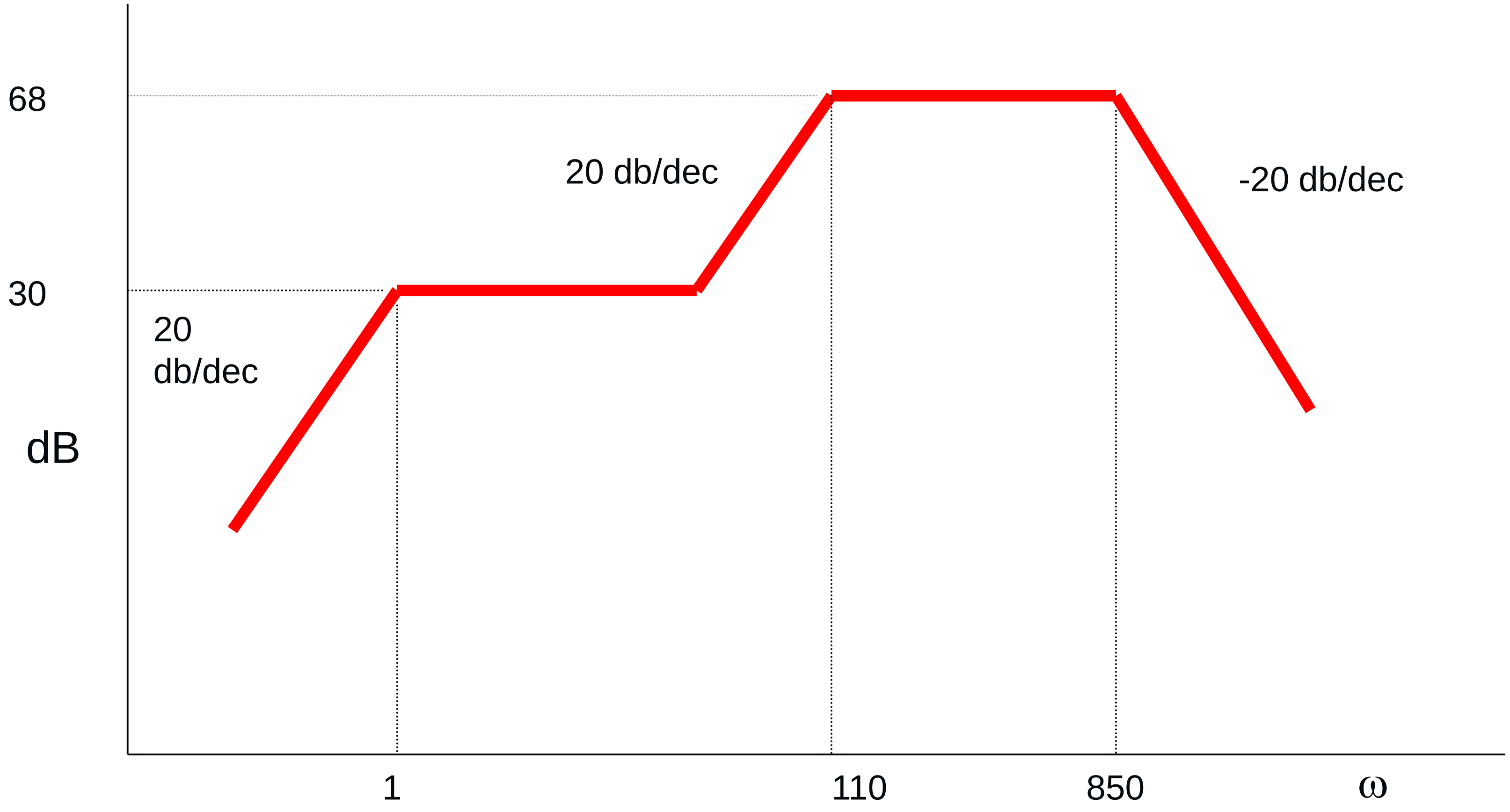
Bode Plots

Check your skill – From the partial Bode diagram, determine the transfer function



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Questions