DR. GYURCSEK ISTVÁN

Exercises with Two-Ports

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

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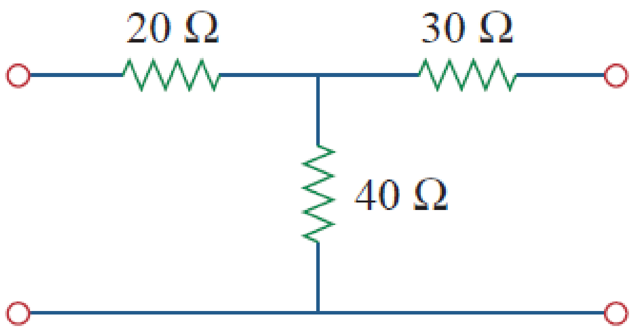
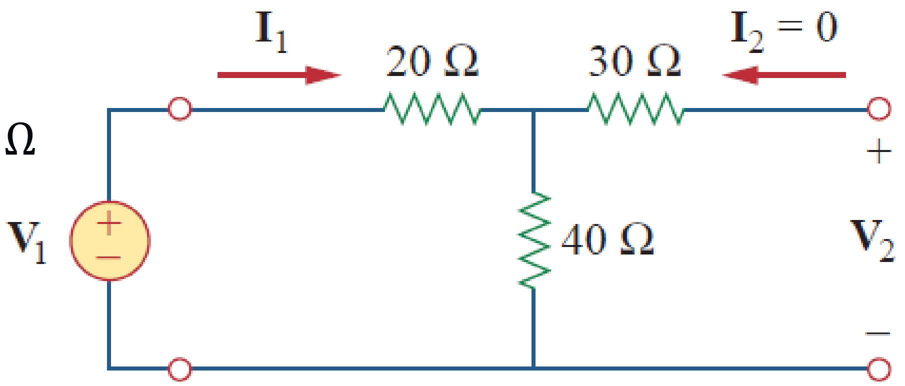
Z Parameters (OC)

Example **TWO.01** – Determine the Z parameters of the T-section circuit.



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{I_1 \cdot (20+40)}{I_1} = 60\ \Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{I_1 \cdot 40}{I_1} = 40\Omega$$



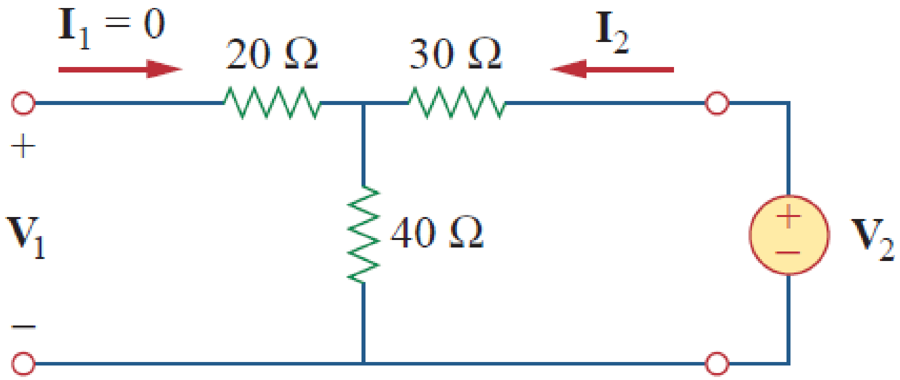
Solution

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{I_2 \cdot 40}{I_2} = 40\Omega$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{I_2 \cdot (30+40)}{I_2} = 70\Omega$$



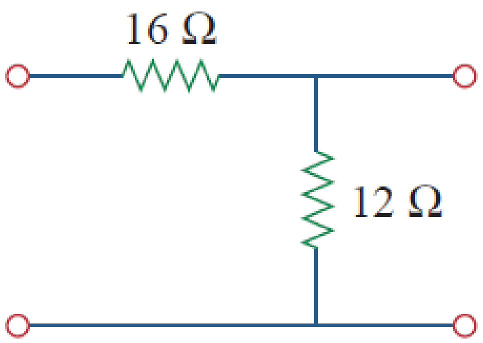
$$Z = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix} \Omega, \qquad z_{12} = z_{21} \rightarrow \mathit{reciprocal}$$

Z Parameters

Example TWO.02 – *Determine the Z parameters of the L-section circuit.*

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$



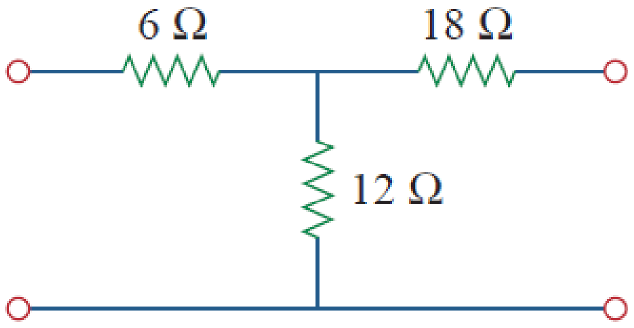
Solution $Z = \begin{bmatrix} 28 & 12 \\ 12 & 12 \end{bmatrix} \Omega$, $z_{12} = z_{21} \rightarrow reciprocal$

Y Parameters (SC)

Example TWO.03 – Calculate y parameters of the T-section circuit.

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

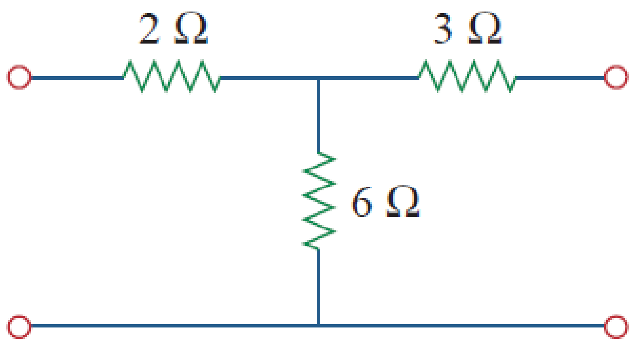


Solution

$$Y = \begin{bmatrix} 75.77 & -30.3 \\ -30.3 & 45.47 \end{bmatrix} mS\,, \qquad y_{12} = y_{21} \rightarrow \ reciprocal\ nw.$$

Hybrid Parameters

Example TWO.04 – Determine the hybrid parameters for the T-section.



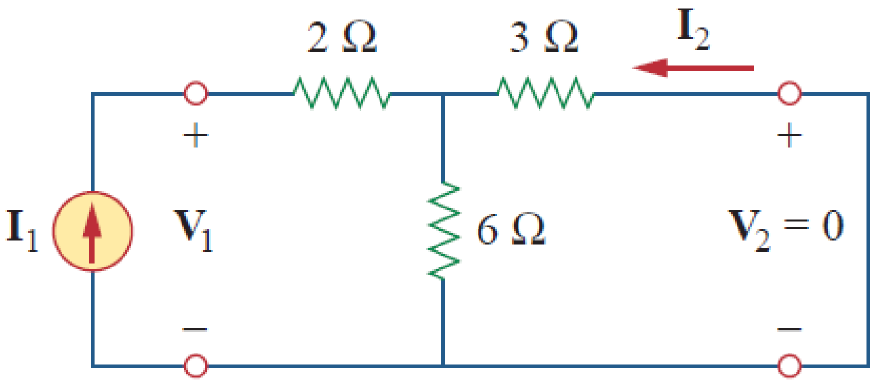
Solution $\boldsymbol{V_1 = h_{11} I_1 + h_{12} V_2}$ $\boldsymbol{I_2 = h_{21} I_1 + h_{22} V_2}$

$(\boldsymbol{h_{11}})$: $\boldsymbol{V}_1 = \boldsymbol{h}_{11}\boldsymbol{I}_1 + \boldsymbol{h}_{12}\, 0$

$$\boldsymbol{h}_{11} = \left.\frac{\boldsymbol{V}_1}{\boldsymbol{I}_1}\right|_{V_2=0} = 2 + 3 \times 6 = 4\ \Omega$$

$(\boldsymbol{h_{21}})$: $-\boldsymbol{I}_2 = \frac{6}{6+3}\boldsymbol{I}_1 = \frac{2}{3}\boldsymbol{I_1}$

$$\boldsymbol{h}_{21} = \left.\frac{\boldsymbol{I}_2}{\boldsymbol{I}_1}\right|_{V_2=0} = -\frac{2}{3}$$



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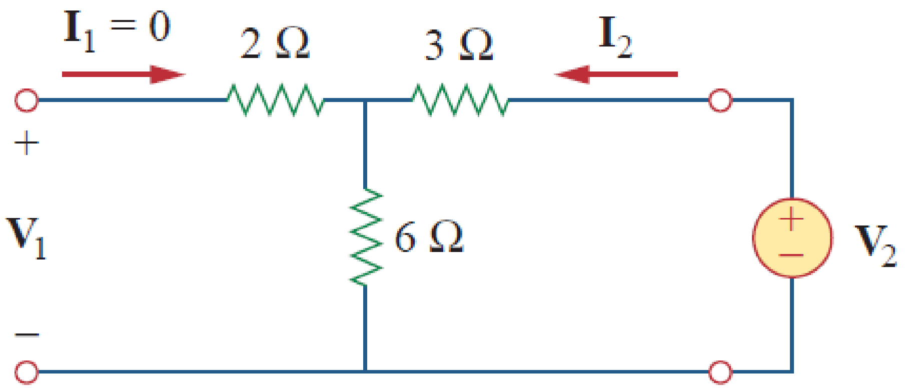
Hybrid Parameters

$V_1 = h_{11}I_1 + h_{12}V_2$ $\qquad$ $I_2 = h_{21}I_1 + h_{22}V_2$

(h_{12}): $V_1 = \dfrac{6}{6+3}V_2 = \dfrac{2}{3}V_2$ $\qquad$ $h_{12} = \left.\dfrac{V_1}{V_2}\right|_{I_1=0} = \dfrac{2}{3}$

(h_{22}): $V_2 = (3+6)\,I_2 = 9I_2$ $\qquad$ $h_{22} = \left.\dfrac{I_2}{V_2}\right|_{I_1=0} = \dfrac{1}{9}\,S$

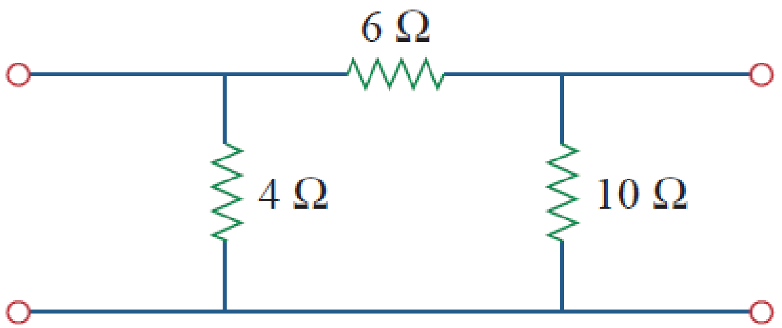


Hybrid Parameters

Example TWO.05 – Determine the hybrid parameters for the π-section.

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

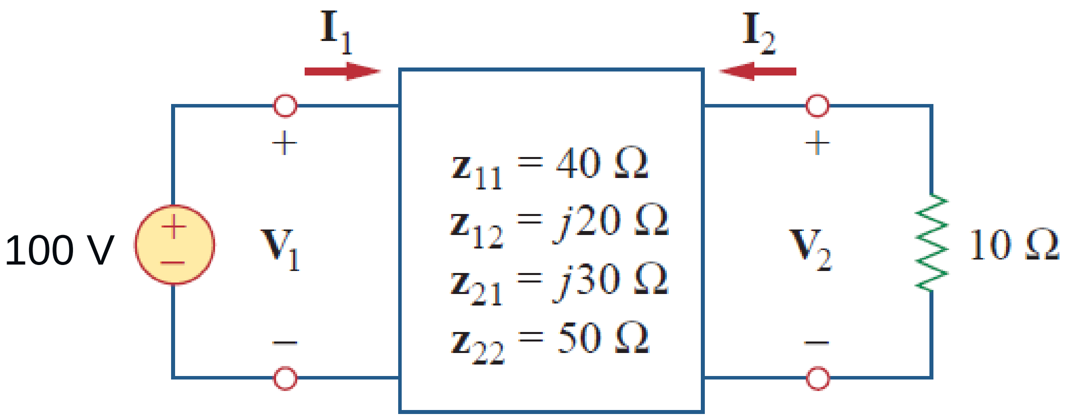


Solution

$$H = \begin{bmatrix} 2.4\ \Omega & 0.4 \\ -0.4 & 200\ mS \end{bmatrix}$$

Network Analysis

Example TWO.06 – Find I_1 and I_2 in the circuit in Figure.



Solution *(non-reciprocal network!)*

$\boldsymbol{V}_1 = 40\boldsymbol{I}_1 + j20\boldsymbol{I}_2$

$\boldsymbol{V}_2 = j30\boldsymbol{I}_1 + 50\boldsymbol{I}_2$

$\boldsymbol{V}_1 = 100\,, \boldsymbol{V}_2 = -10\boldsymbol{I}_2$

(1): $100 = 40\boldsymbol{I}_1 + j20\boldsymbol{I}_2$

(2): $-10\boldsymbol{I}_2 = j30\boldsymbol{I}_1 + 50\boldsymbol{I}_2 \rightarrow \boldsymbol{I}_1 = j2\boldsymbol{I}_2$

(2) → (1): $100 = j80\boldsymbol{I}_2 + j20\boldsymbol{I}_2 \rightarrow \boldsymbol{I}_2 = -\mathrm{j}\ \mathrm{A}$

$\boldsymbol{I}_1 = j2\boldsymbol{I}_2 = 2\ A$

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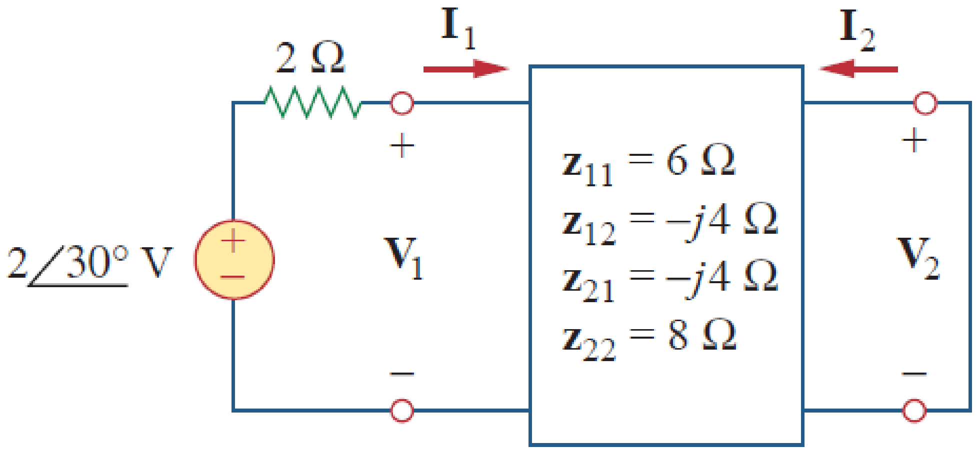
Network Analysis

Example TWO.07 – Find I_1 and I_2 in the circuit in Figure.



Solution *(reciprocal network!)*

$\boldsymbol{I_1} = 200\ e^{j30°}\ mA\ , \qquad \boldsymbol{I_2} = 100\ e^{j120°}\ mA$

Network Analysis

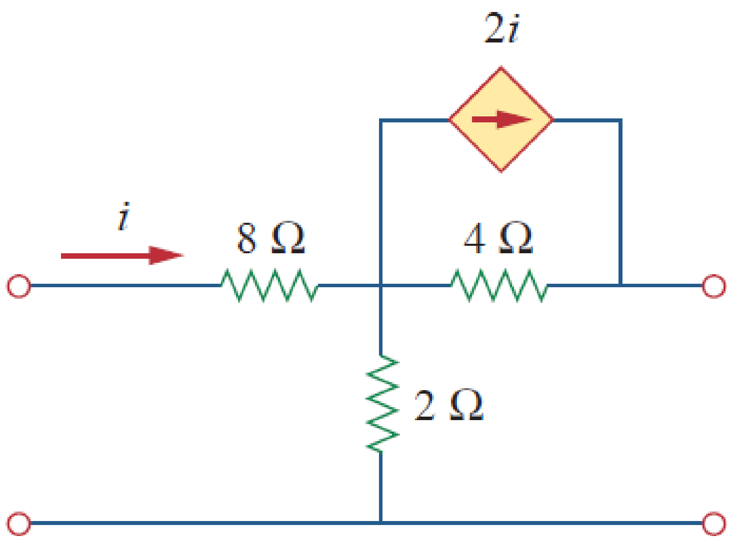
Example **TWO.08** – Determine the **Y** parameters for the two-port shown in Figure.

Solution $I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

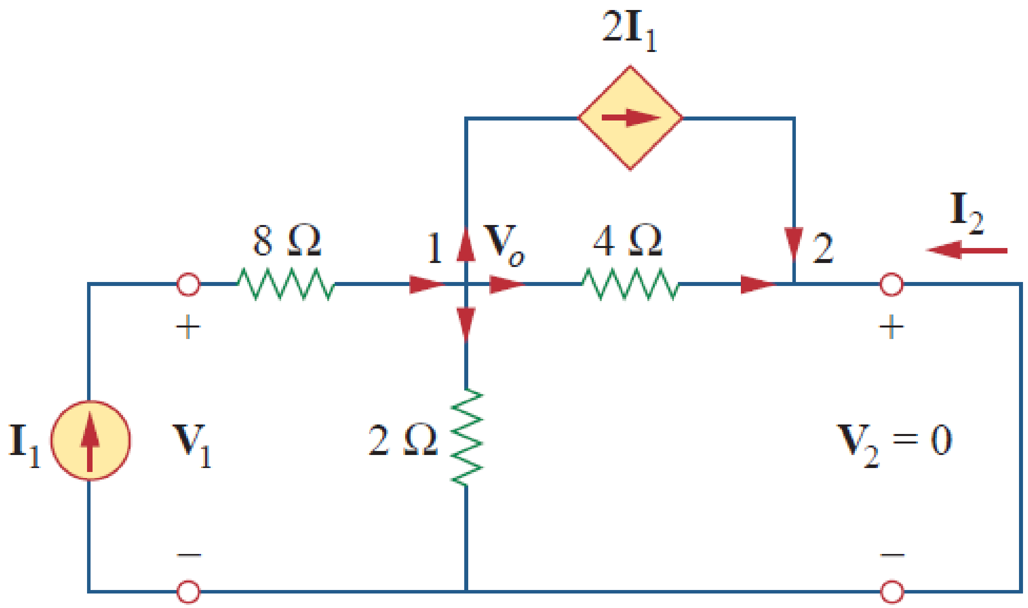
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



Network Analysis

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$



$$(\boldsymbol{node\ 1}):\ \frac{V_1 - V_0}{8} = 2I_1 + \frac{V_0}{2} + \frac{V_0 - 0}{4}$$

$$(\boldsymbol{but}):\ I_1 = \frac{V_1 - V_0}{8} \rightarrow \frac{V_1 - V_0}{8} = 2\frac{V_1 - V_0}{8} + \frac{3V_0}{4}$$

$$\rightarrow 0 = \frac{V_1 - V_0}{8} + \frac{3V_0}{4} \rightarrow$$

$$(\mathbf{1}):\ 0 = V_1 - V_0 + 6V_0 \rightarrow V_1 = -5V_0$$

$$(\mathbf{2}):\ I_1 = \frac{V_1 - V_0}{8} = \frac{-5V_0 - V_0}{8} = -0.75V_0$$

$$y_{11} = \frac{I_1}{V_1} = \frac{-0.75V_0}{-5V_0} = 0.15\ \mathrm{S}$$

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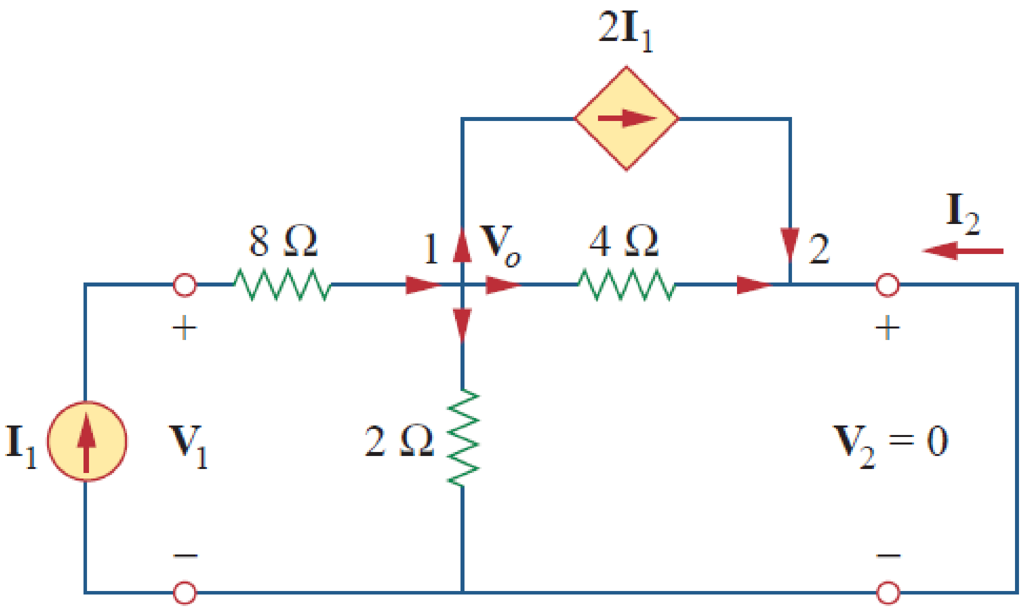
Network Analysis

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$



(1): $V_1 = -5V_0$

(2): $I_1 = -0.75V_0$

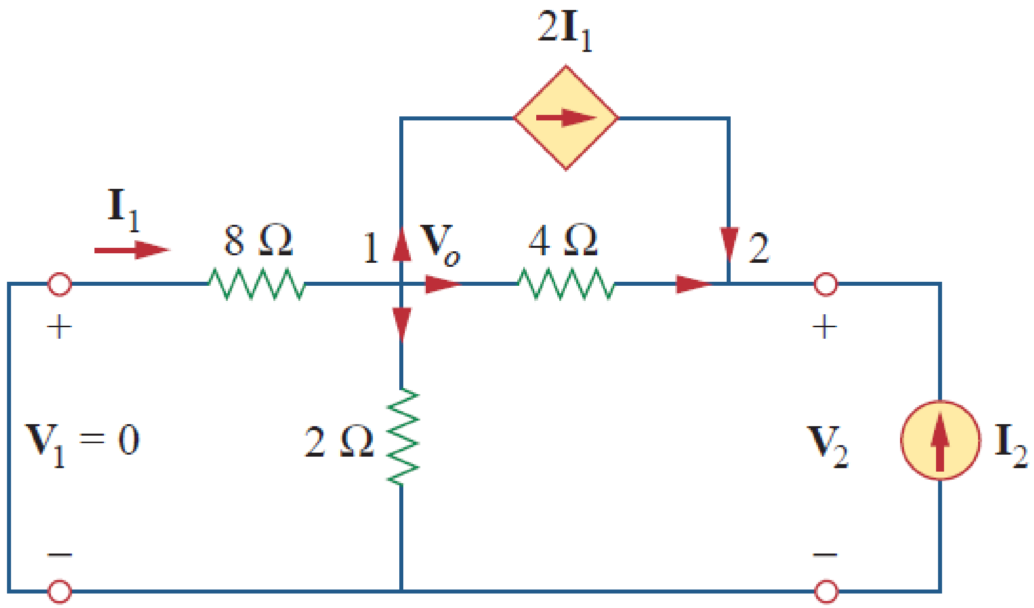
$(node\ 2)$: $\dfrac{V_0 - 0}{4} + 2I_1 + I_2 = 0$

$$\frac{V_0}{4} + 2 \cdot (-0.75V_0) = -I_2 = -1.25V_0$$

$$y_{21} = \frac{I_2}{V_1} = \frac{1.25V_0}{-5V_0} = -0.25\ S$$

Network Analysis

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$



$$(\textit{node } 1):\ \frac{0 - V_0}{8} = 2I_1 + \frac{V_0}{2} + \frac{V_0 - V_2}{4}$$

$$(\textit{but}):\ I_1 = \frac{0 - V_0}{8}$$

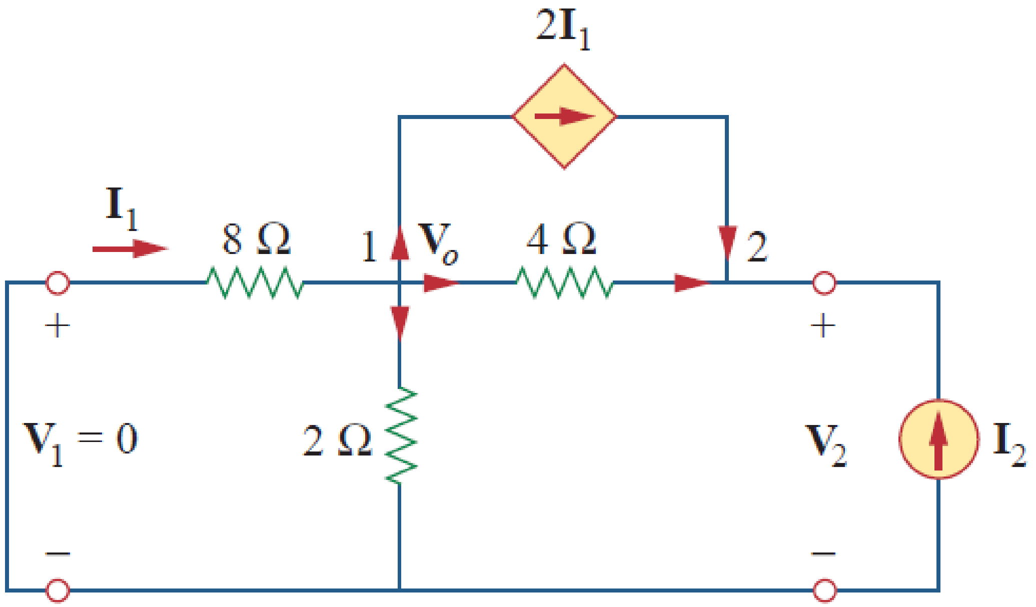
$$(\textit{but} \to \textit{node } 1):\ \frac{-V_0}{8} = \frac{-2V_0}{8} + \frac{V_0}{2} + \frac{V_0 - V_2}{4} \to$$

$$(3): \to 0 = -V_0 + 4V_0 + 2V_0 - 2V_2 \to V_2 = 2.5V_0$$

$$y_{12} = \frac{I_1}{V_2} = \frac{-V_0/8}{2.5V_0} = -0.05\ S$$

Network Analysis

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



$$(3):\ V_2 = 2.5V_0 \qquad (but):\ I_1 = \frac{0 - V_0}{8}$$

$$(node\ 2):\ \frac{V_0 - V_2}{4} + 2I_1 + I_2 = 0$$

$$-I_2 = \frac{V_0 - (2.5V_0)}{4} - 2\frac{V_0}{8} = -0.625V_0$$

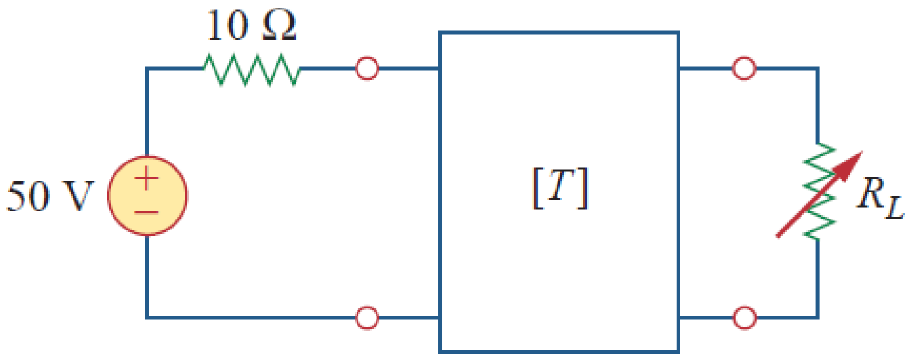
$$y_{22} = \frac{I_2}{V_2} = \frac{0.625V_0}{2.5V_0} = 0.25\ \mathrm{S}$$

$$y_{12} \neq y_{21} \rightarrow non\ reciprocal\ network$$

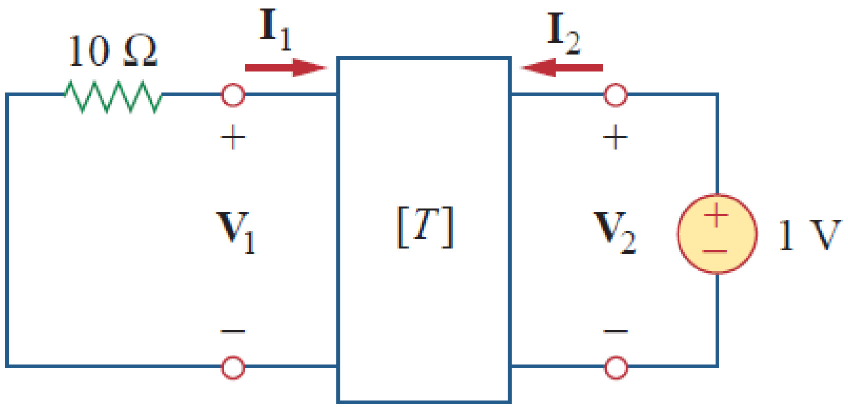
Network Analysis

Example TWO.09 – The transmission parameters of the two-port network in Figure are... $\boldsymbol{T} = \begin{bmatrix} 4 & 20\ \Omega \\ 0.1\ S & 2 \end{bmatrix}$

The output port is connected to a variable load for maximum power transfer. Find R_L and the maximum power transferred.



Solution $\boldsymbol{Z}_{Th} = ?, \boldsymbol{V}_{Th} = ?$



(1): $\boldsymbol{V}_1 = 4\boldsymbol{V}_2 - 20\boldsymbol{I}_2$

(2): $\boldsymbol{I}_1 = 0.1\boldsymbol{V}_2 - 2\boldsymbol{I}_2$

(3): $\boldsymbol{V}_1 = -10\boldsymbol{I}_1 \rightarrow (1) \rightarrow -10\boldsymbol{I}_1 = 4\boldsymbol{V}_2 - 20\boldsymbol{I}_2 \rightarrow (4)$: $\boldsymbol{I}_1 = -0.4\boldsymbol{V}_2 + 2\boldsymbol{I}_2$

$(2), (4)$: $\rightarrow 0.1\boldsymbol{V}_2 - 2\boldsymbol{I}_2 = -0.4\boldsymbol{V}_2 + 2\boldsymbol{I}_2 \rightarrow 0.5\boldsymbol{V}_2 = 4\boldsymbol{I}_2 \rightarrow \boldsymbol{Z}_{Th} = \dfrac{\boldsymbol{V}_2}{\boldsymbol{I}_2} = \dfrac{4}{0.5} = 8\ \Omega$

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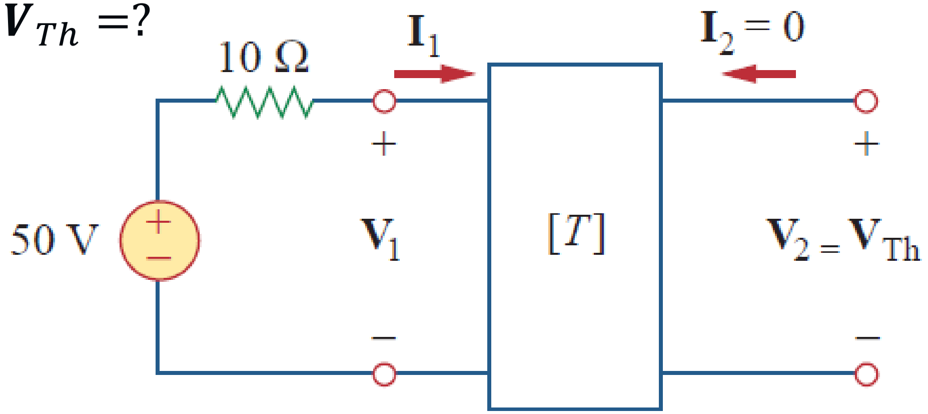
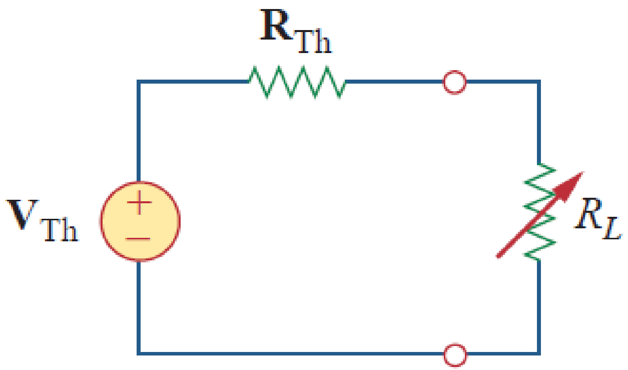
Network Analysis

$V_{Th} = ?$





(1): $V_1 = 4V_2 - 20I_2$

(2): $I_1 = 0.1V_2 - 2I_2$

$$T = \begin{bmatrix} 4 & 20\ \Omega \\ 0.1\ S & 2 \end{bmatrix}$$

(3): $I_2 = 0\,, V_1 = 50 - 10I_1$

(1), (3) → (4): $50 - 10I_1 = 4V_2$

(2) → (4): $50 - V_2 = 4V_2 \rightarrow V_2 = V_{Th} = 10\ V$

$$R_L = Z_{Th} = R_{Th} = 8\ \Omega$$

$$P = I^2 \cdot R_L = \left(\frac{V_{Th}}{2R_L}\right)^2 \cdot R_L =$$

$$= \frac{V_{Th}^2}{4R_L} = \frac{100}{4 \cdot 8} = 3.125\ W$$

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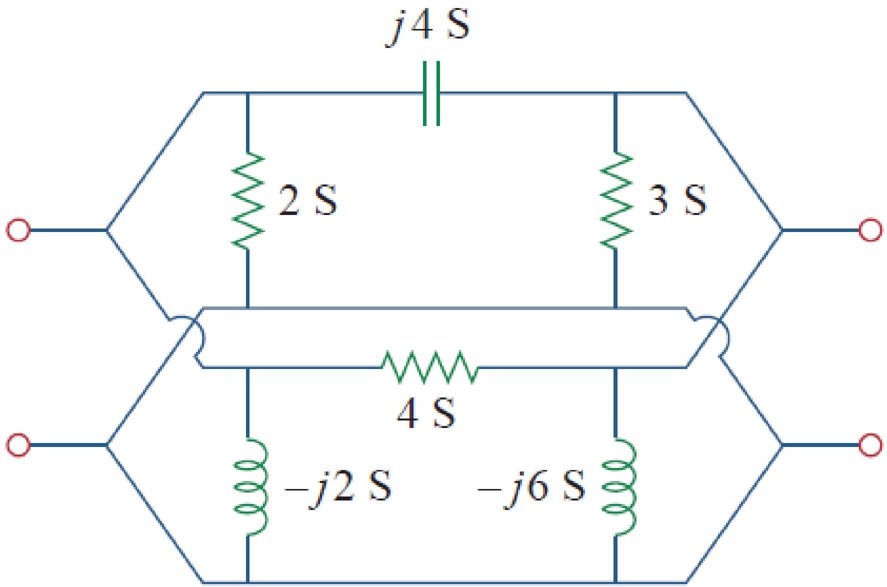
Interconnections

Example **TWO.10** – Find the *y* parameters of the two-port in Figure.

Solution

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$



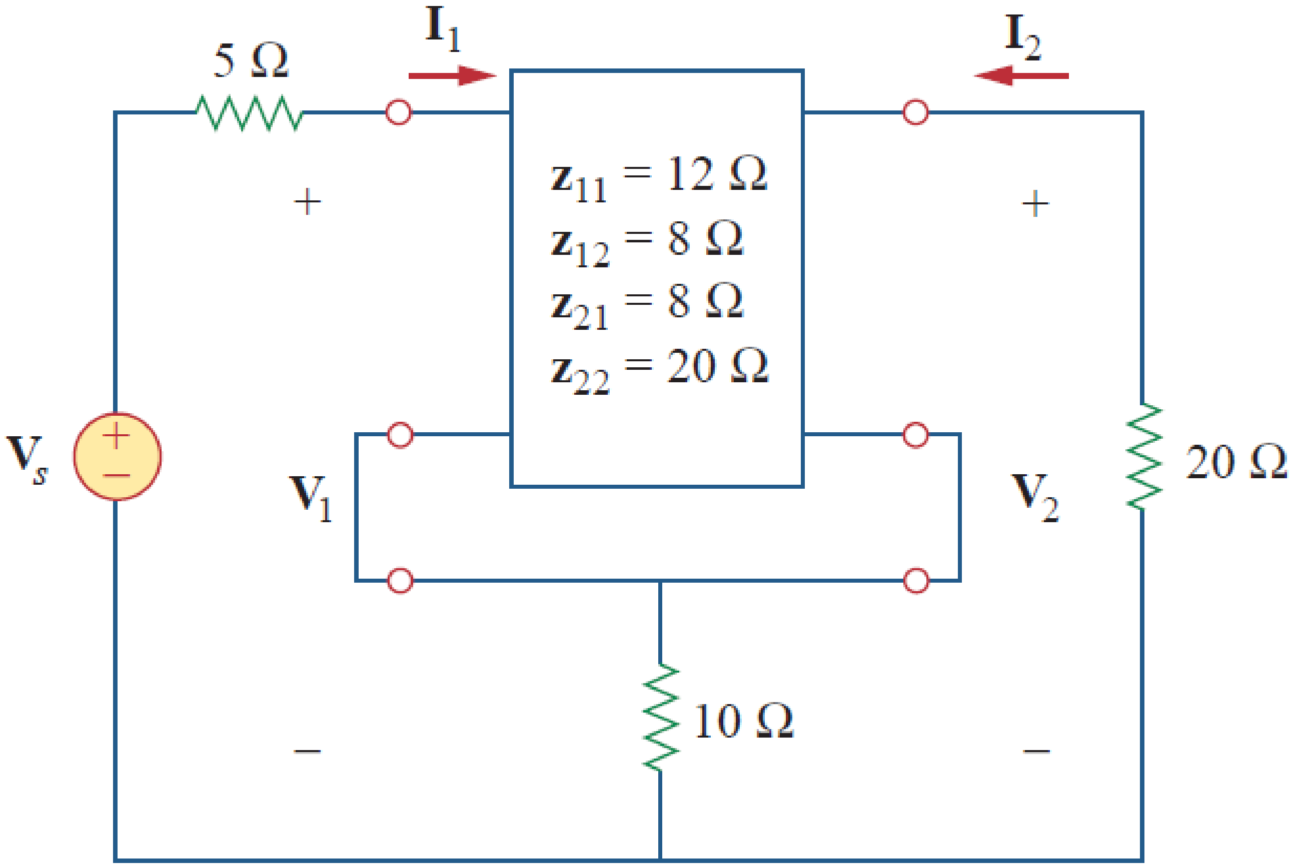
$$Y_a = \begin{bmatrix} 2+j4 & -j4 \\ -j4 & 3+j4 \end{bmatrix}$$

$$Y_b = \begin{bmatrix} 4-j2 & -4 \\ -4 & 4-j6 \end{bmatrix}$$

$$Y = Y_a + Y_b = \begin{bmatrix} 2+j4 & -j4 \\ -j4 & 3+j4 \end{bmatrix} + \begin{bmatrix} 4-j2 & -4 \\ -4 & 4-j6 \end{bmatrix} = \begin{bmatrix} 6+j2 & -4-j4 \\ -4-j4 & 7-j2 \end{bmatrix}$$

Interconnections

Example TWO.11 – Evaluate V_2/V_s in the circuit in Figure.



Solution $z_{12b} = z_{21b} = 10 = z_{11b} = z_{22b}$

$$\boldsymbol{Z} = \boldsymbol{Z}_a + \boldsymbol{Z}_b = \begin{bmatrix} 12 & 8 \\ 8 & 20 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix}$$

(1): $\boldsymbol{V}_1 = \boldsymbol{z}_{11}\boldsymbol{I}_1 + \boldsymbol{z}_{12}\boldsymbol{I}_2 = 22\boldsymbol{I}_1 + 18\boldsymbol{I}_2$

(2): $\boldsymbol{V}_2 = \boldsymbol{z}_{12}\boldsymbol{I}_1 + \boldsymbol{z}_{22}\boldsymbol{I}_2 = 18\boldsymbol{I}_1 + 30\boldsymbol{I}_2$

(3): $\boldsymbol{V}_1 = \boldsymbol{V}_S - 5\boldsymbol{I}_1$ (4): $\boldsymbol{V}_2 = -20\boldsymbol{I}_2 \rightarrow \boldsymbol{I}_2 = -\dfrac{\boldsymbol{V}_2}{20}$

(3), (4) → (1): $\boldsymbol{V}_S - 5\boldsymbol{I}_1 = 22\boldsymbol{I}_1 - \dfrac{18}{20}\boldsymbol{V}_2$

→ (5): $\boldsymbol{V}_S = 27\boldsymbol{I}_1 - 0.9\boldsymbol{V}_2$

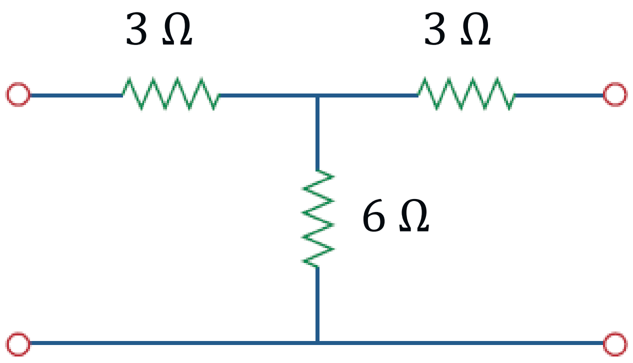
(4) → (2): $\boldsymbol{V}_2 = 18\boldsymbol{I}_1 - \dfrac{30}{20}\boldsymbol{V}_2 \rightarrow$ (6): $\boldsymbol{I}_1 = \dfrac{2.5}{18}\boldsymbol{V}_2$

(6) → (5): $\boldsymbol{V}_S = 27\dfrac{2.5}{18}\boldsymbol{V}_2 - 0.9\boldsymbol{V}_2 = 2.85\boldsymbol{V}_2 \rightarrow \dfrac{\boldsymbol{V}_2}{\boldsymbol{V}_S} = \dfrac{1}{2.85} = 0.35$

Wave Impedance

Example **TWO.12** – *Calculate the wave impedance of the symmetric T-section.*

Solution



$$\boldsymbol{Z}_{SC} = 3 + 3 \times 6 = 3 + \frac{3 \cdot 6}{3 + 6} = 5\,\Omega$$

$$\boldsymbol{Z}_{OC} = 3 + 6 = 9\,\Omega$$

$$\boldsymbol{Z}_0 = \sqrt{\boldsymbol{Z}_{SC} \cdot \boldsymbol{Z}_{OC}} = \sqrt{45} = 3\sqrt{5}\,\Omega$$

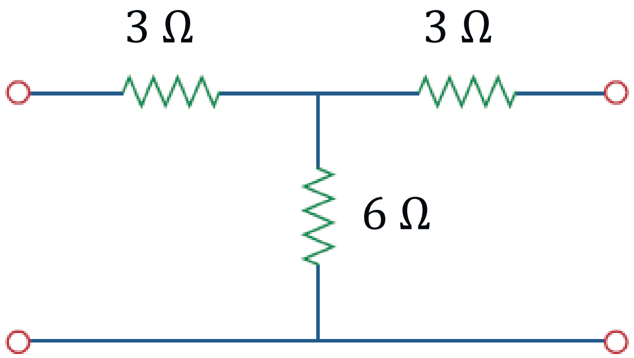
Another way… $\boldsymbol{Z}_{IN} = 3 + 6 \times (3 + \boldsymbol{Z}_0) = \boldsymbol{Z}_0 \rightarrow 3 + \dfrac{18 + 6\boldsymbol{Z}_0}{9 + \boldsymbol{Z}_0} = \boldsymbol{Z}_0$

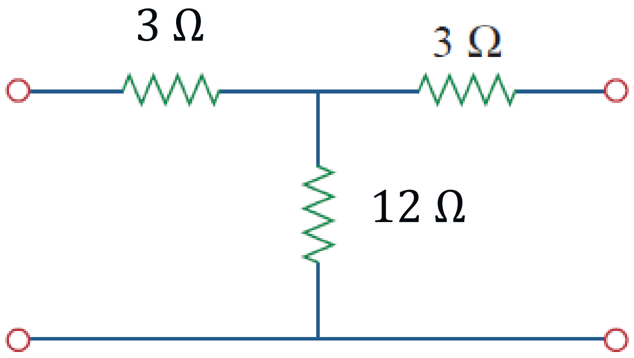
$$27 + 3\boldsymbol{Z}_0 + 18 + 6\boldsymbol{Z}_0 = 9\boldsymbol{Z}_0 + \boldsymbol{Z}_0^2$$

$$45 = \boldsymbol{Z}_0^2 \rightarrow \boldsymbol{Z}_0 = \sqrt{45} = 3\sqrt{5}\,\Omega$$

Wave Impedance

Example **TWO.13**
Calculate wave impedance and Z parameters of the symmetric T-section using Bartlett's theorem.





Solution *(with ,half-network')*

$$\boldsymbol{Z}_I = \boldsymbol{Z}_{SC-half} = (\boldsymbol{Z}_{11} - \boldsymbol{Z}_{12}) = 3\ \Omega$$

$$\boldsymbol{Z}_{II} = \boldsymbol{Z}_{OC-half} = (\boldsymbol{Z}_{11} + \boldsymbol{Z}_{12}) = 3 + 12 = 15\ \Omega$$

$$\boldsymbol{Z}_0 = \sqrt{\boldsymbol{Z}_I \cdot \boldsymbol{Z}_{II}} = \sqrt{3 \cdot 15} = 3\sqrt{5}\ \Omega \leftarrow refer\ to\ the\ prev.example$$

$$\boldsymbol{Z}_{11} = \boldsymbol{Z}_{22} = \frac{\boldsymbol{Z}_{II} + \boldsymbol{Z}_I}{2} = \frac{15+3}{2} = 9\ \Omega \qquad \boldsymbol{Z}_{12} = \boldsymbol{Z}_{21} = \frac{\boldsymbol{Z}_{II} - \boldsymbol{Z}_I}{2} = \frac{15-3}{2} = 6\ \Omega$$

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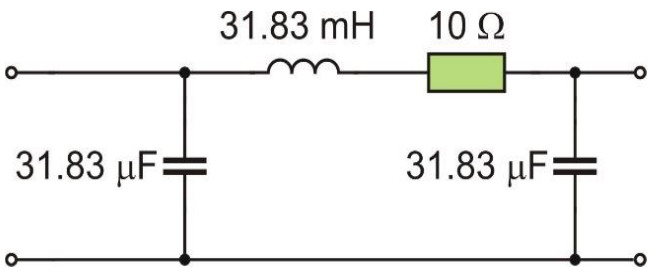
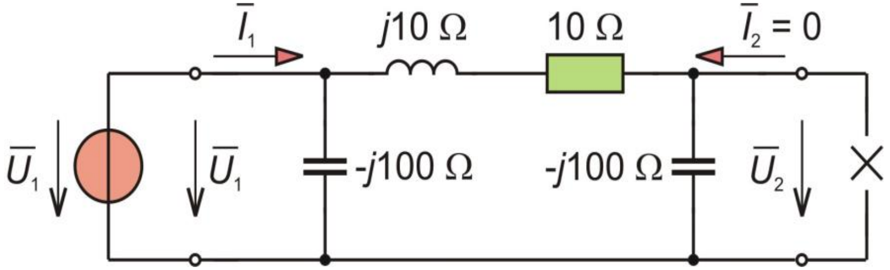
App. – Transmission Line Analysis



Example TWO.14

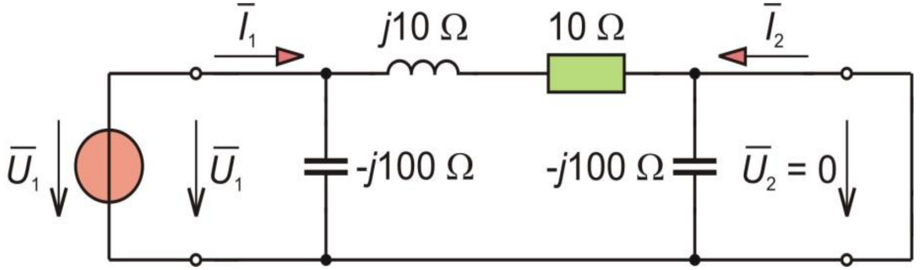
Determine the transmission parameters of the Π-equivalent circuit of a long power transmission line operating with a frequency of 50 Hz. Determine if this two-port is reciprocal or symmetrical. $(\boldsymbol{U_1} = \boldsymbol{A_{11}}\boldsymbol{U_2} + \boldsymbol{A_{12}}\boldsymbol{I_2}\,, \boldsymbol{I_1} = \boldsymbol{A_{21}}\boldsymbol{U_2} + \boldsymbol{A_{22}}\boldsymbol{I_2})$



Solution

$$X_L = \omega L = 2\pi \cdot 50 \cdot 31.83 \cdot 10^{-3} = 10\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 50 \cdot 31.83 \cdot 10^{-6}} = 100\Omega$$



$$A_{11} = \frac{U_1}{U_2} = \frac{U_1}{U_1 \dfrac{-j100}{j10 + 10 - j100}} = \frac{10 - j90}{-j100} = 0.9 + j0.1$$

$$A_{12} = \frac{U_1}{I_2} = \frac{U_1}{-I_1 \dfrac{-j100}{j10 + 10 - j100}} = \frac{U_1}{\dfrac{U_1}{-j100 \times (j10 + 10)} \cdot \dfrac{-j100}{j10 + 10 - j100}} = \cdots = (-10 - j10)\Omega$$

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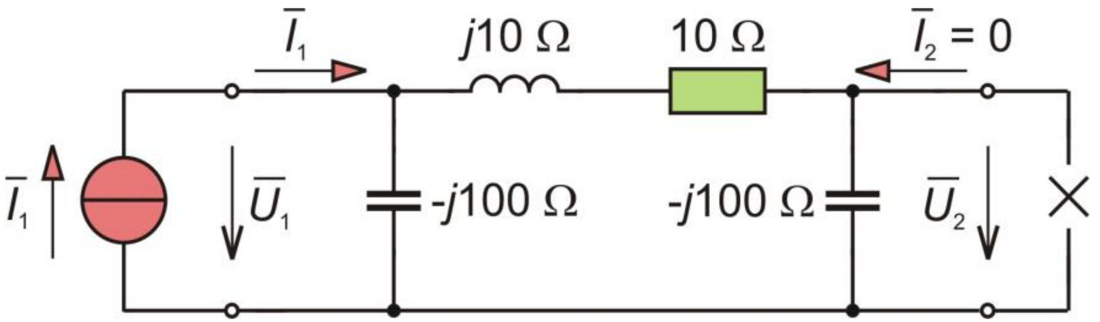
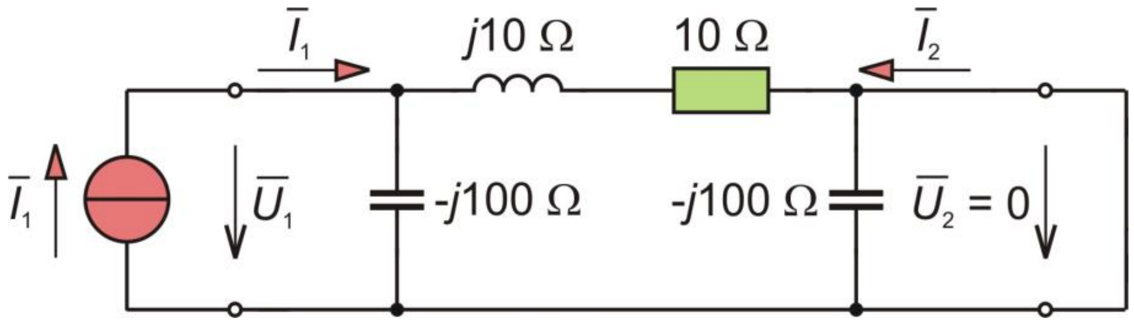
App. – Transmission Line Analysis

$$A_{21} = \frac{I_1}{U_2} = \frac{I_1}{I_1 \dfrac{-j100}{-j100 + j10 + 10 - j100}(-j100)} =$$

$$= \frac{10 - j190}{10000} = (-0.001 + j0.019)\ S$$



$$A_{22} = \frac{I_1}{I_2} = \frac{I_1}{-I_1 \dfrac{-j100}{-j100 + j10 + 10}} = \frac{-j100 + j10 + 10}{j100} = -0.9 + j0.1$$



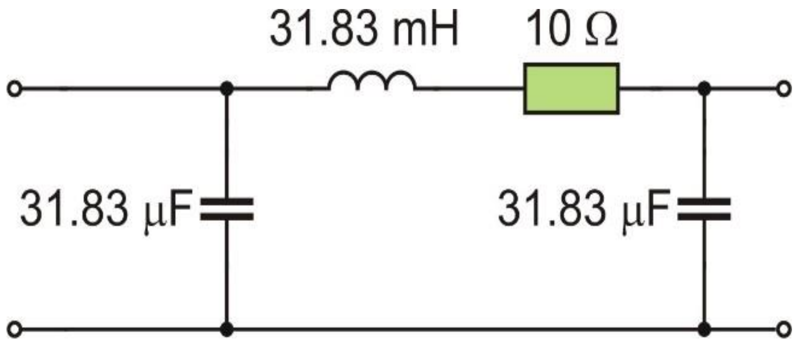
$$\boldsymbol{A} = \begin{bmatrix} 0.9 + j0.1 & (-10 - j10)\Omega \\ (-0.001 + j0.019)\ S & -0.9 - j0.1 \end{bmatrix}$$

$$\left.\begin{aligned} \Delta\boldsymbol{A} = A_{11}A_{22} - A_{12}A_{21} = \cdots = -1 \\ A_{11} = -A_{22} \end{aligned}\right\} \rightarrow \mathit{reciprocal\ \&\ symmetrical}$$

App. – Transmission Line Analysis

Example TWO.15

Calculate the wave impedance of the Π-equivalent network of a long power transmission line operating with a frequency of 50 Hz.



Solution

$$Z_I = -jX_C = -j\frac{1}{\omega C} = -j100 = 100\, e^{-j90^\circ}\Omega$$

$$Z_{II} = \left(\frac{R}{2} + j\frac{X_L}{2}\right) \times (-jX_C) = (5 + j5) \times (-j100) = \frac{\sqrt{50}\, e^{j45^\circ} 100\, e^{-j90^\circ}}{5 - j95} = \frac{707.1\, e^{-j45^\circ}}{95.13\, e^{-j86.99^\circ}} = 7.43\, e^{j41.99^\circ}\,\Omega$$

$$Z_0 = \sqrt{Z_I\, Z_{II}} = \sqrt{100\, e^{-j90^\circ} 7.43\, e^{j41.99^\circ}} = \sqrt{743\, e^{-j48.01^\circ}} = 27.26 e^{-j24^\circ} = 24.89 - j10.90\,\Omega$$

Questions