



DR. GYURCSEK ISTVÁN

Exercises with First-Order Circuits

Sources and additional materials (recommended)

- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

Examples

FOC.01 – Calculate the insulation resistance of a $10 \mu F$ oil-filled capacitor if the voltage of the charged capacitor decays into its third part in 30 minutes. Find its stored energy 10 minutes after the 300 V dc source disconnected.

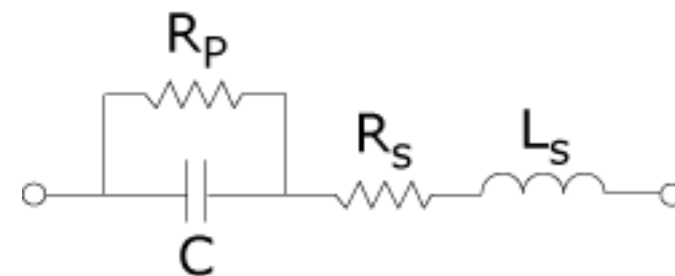
Solution

$$v(t) = V e^{-\frac{t}{\tau}}$$

$$\frac{V}{3} = V e^{-\frac{1800}{\tau}}$$

$$\rightarrow \tau = R_p C = \frac{1800}{\ln 3}$$

$$R_p = \frac{1800}{C \cdot \ln 3} = \frac{1800}{10 \cdot 10^{-6} \cdot \ln 3} = 163.84 \cdot 10^6 = 163.64 M\Omega$$



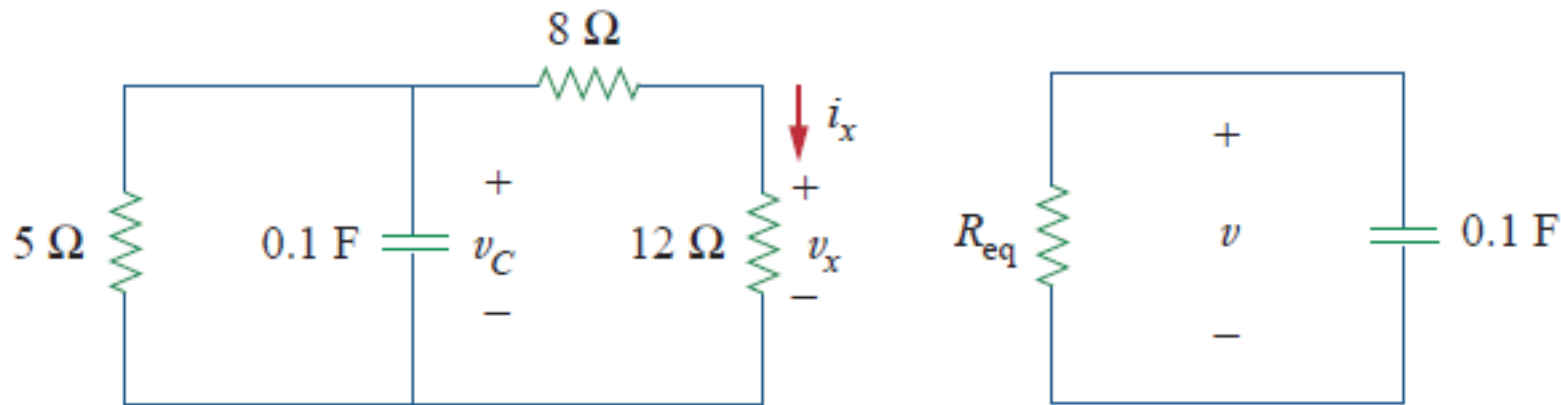
$$\tau = R_p C = 163.84 \cdot 10^6 \cdot 10 \cdot 10^{-6} = 1638.43 \text{ s}$$

$$v(600) = V(0) e^{-\frac{600}{\tau}} = 208 \text{ V}$$

$$W(600) = \frac{1}{2} C V^2 = 0.216 \text{ J}$$

Source-Free RC Circuit

FOC.02 – Find $v_C(t)$, $v_X(t)$, $i_X(t)$ for $t \geq 0$ if $v_C(0) = 15\text{ V}$



Solution $R_{eq} = 20 \times 5 = 4\ \Omega$

$$\tau = R_{eq}C = 4 \cdot 0.1 = 0.4\text{ s}$$

$$v = v(0) e^{-\frac{t}{\tau}} = 15 e^{-\frac{t}{0.4}}$$

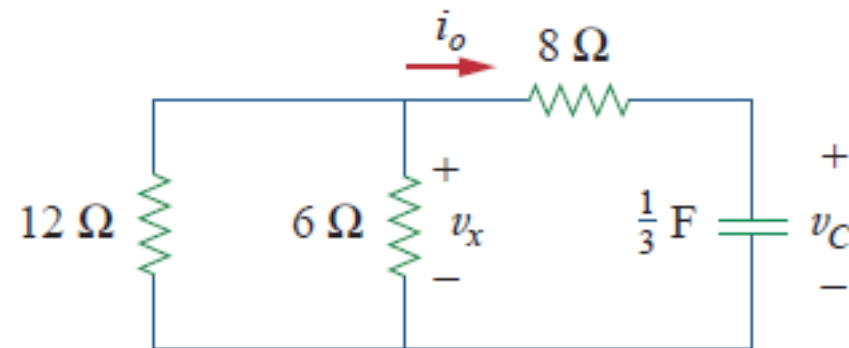
$$v_C = 15 e^{-2.5t}\text{ V}$$

$$v_X = \frac{12}{12 + 8} v_C = 0.6 \cdot 15 e^{-2.5t} = 9 e^{-2.5t}\text{ V}$$

$$i_X = \frac{v_X}{12} = 0.75 e^{-2.5t}\text{ A}$$

Source-Free RC Circuit

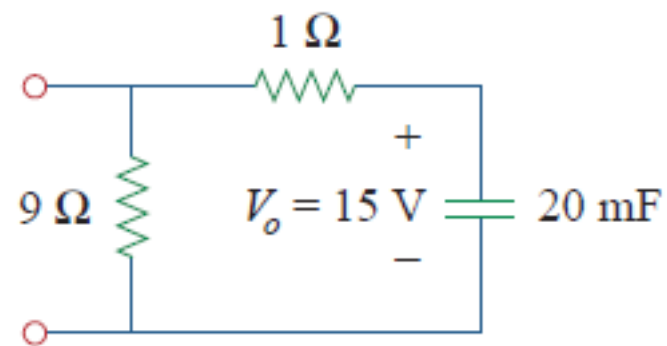
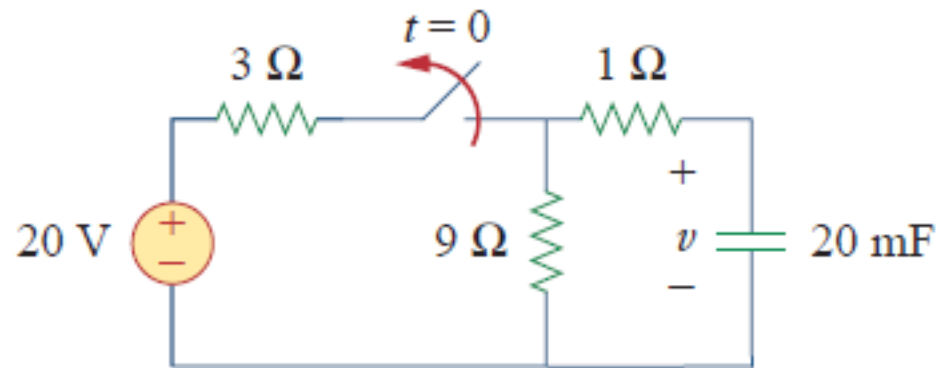
FOC.03 – Find $v_C(t)$, $v_X(t)$, $i_0(t)$ for $t \geq 0$ if $v_C(0) = 45 \text{ V}$



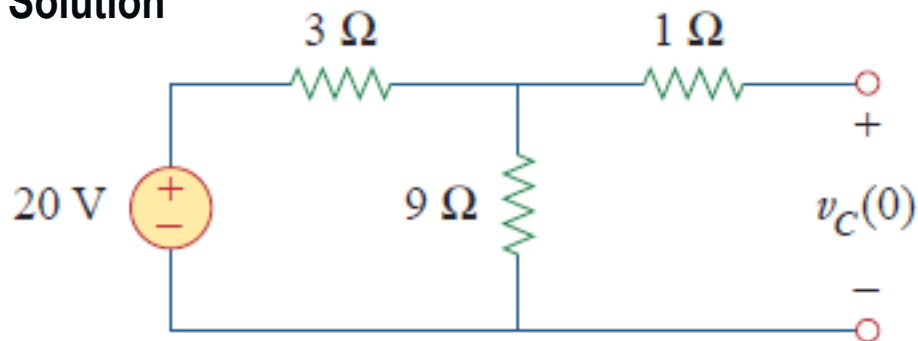
Solution $v_C = 45 e^{-0.25t} \text{ V}$, $v_X = 15 e^{-0.25t} \text{ V}$, $i_0 = -3.75 e^{-0.25t} \text{ A}$

Source-Free RC Circuit with Switch

FOC.04 – The switch has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t \geq 0$. Calculate the initial energy stored in the capacitor.



Solution



$$v_C(-0) = 20 \cdot \frac{9}{9 + 3} = 15 \text{ V} = v_C(+0) = V_0$$

$$R_{eq} = 1 + 9 = 10 \text{ } \Omega$$

$$\tau = R_{eq}C = 10 \cdot 20 \cdot 10^{-3} = 0.2 \text{ s}$$

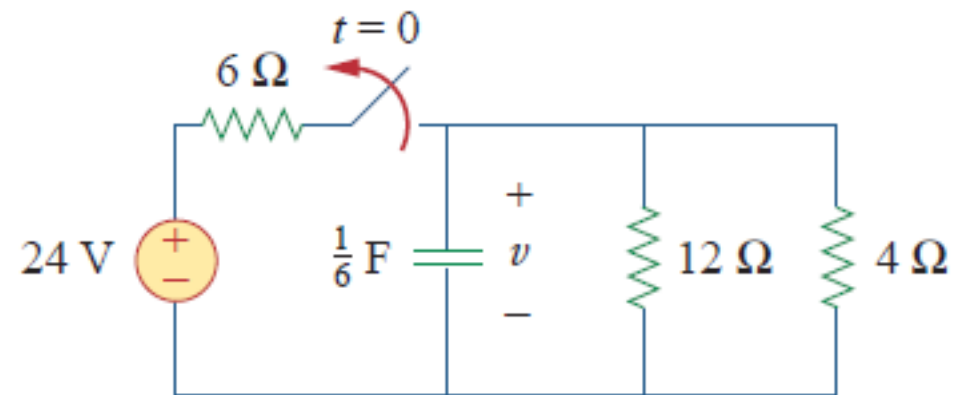
$$v(t) = v_C(0) e^{-\frac{t}{\tau}} = 15 e^{-\frac{t}{0.2}} = 15 e^{-5t} \text{ V}$$

$$w_C(0) = \frac{1}{2} C v_C^2(0) = \frac{1}{2} \cdot 20 \cdot 10^{-3} \cdot 15^2 = 2.25 \text{ J}$$

Source-Free RC Circuit



FOC.05 – If the switch opens at $t = 0$ find $v(t)$ for $t \geq 0$ and $w_C(0)$.



Solution $8 e^{-2t}$ V, 5.33 J

Source-Free RL Circuit

FOC.06 – Assuming that $i(0) = 10$ A calculate $i(t)$ and $i_x(t)$.

Method 1 (find Thevenin equivalent resistance)

$$2(i_1 - i_2) + 1 = 0 \rightarrow i_1 - i_2 = -\frac{1}{2}$$

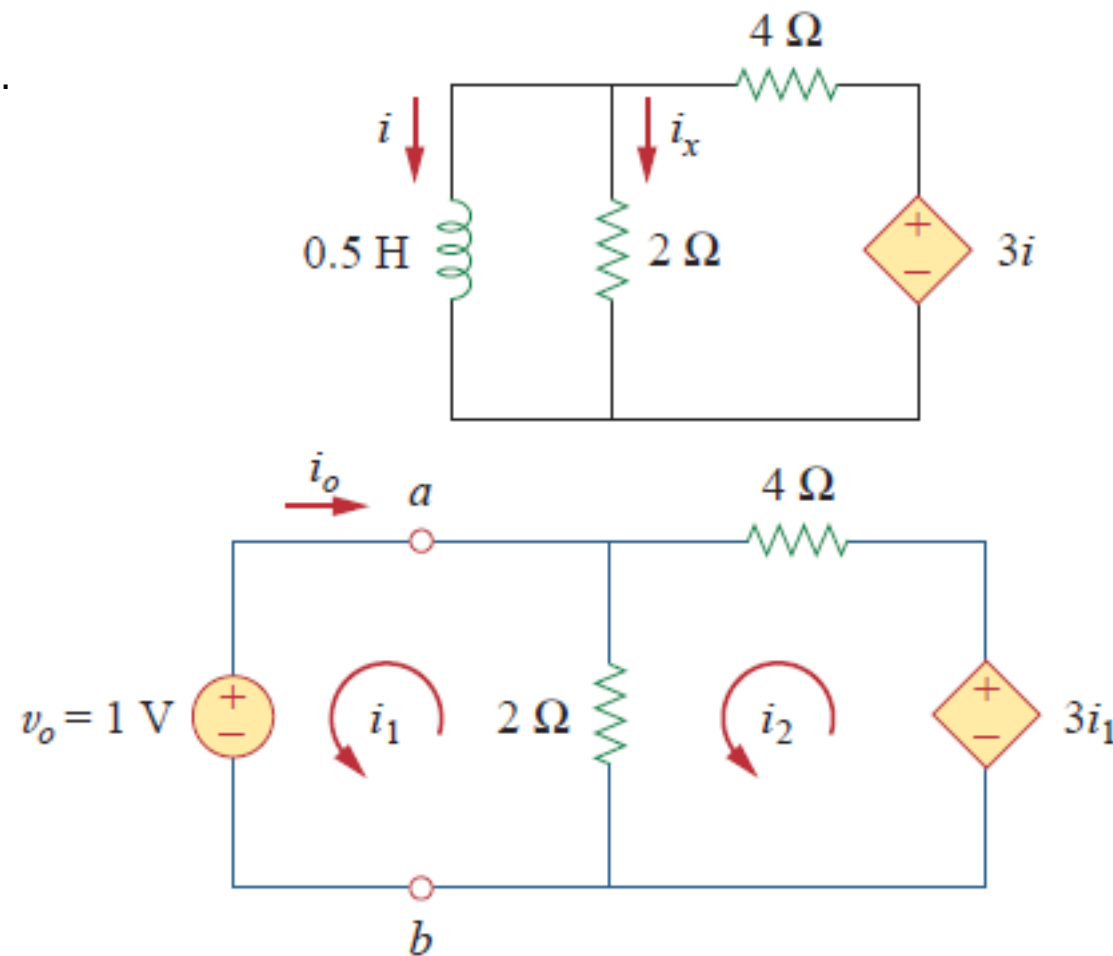
$$6i_2 - 2i_1 - 3i_1 = 0 \rightarrow i_2 = \frac{5}{6}i_1$$

$$i_1 = -3 \text{ A}, \quad i_0 = -i_1 = 3 \text{ A}$$

$$R_{eq} = R_{Th} = \frac{v_0}{i_0} = \frac{1}{3} \Omega \rightarrow \tau = \frac{L}{R_{eq}} = \frac{3}{2} \text{ s}$$

$$i(t) = i(0) e^{-\frac{t}{\tau}} = 10 e^{-\frac{2}{3}t} \text{ A}, t \geq 0$$

$$i_x(t) = \frac{v_L(t)}{R} = \frac{L \frac{di}{dt}}{R} = -\frac{5}{3} e^{-\frac{2}{3}t} \text{ A}, t \geq 0$$



Source-Free RL Circuit

Method 2 (Kirchhoff Voltage Law)

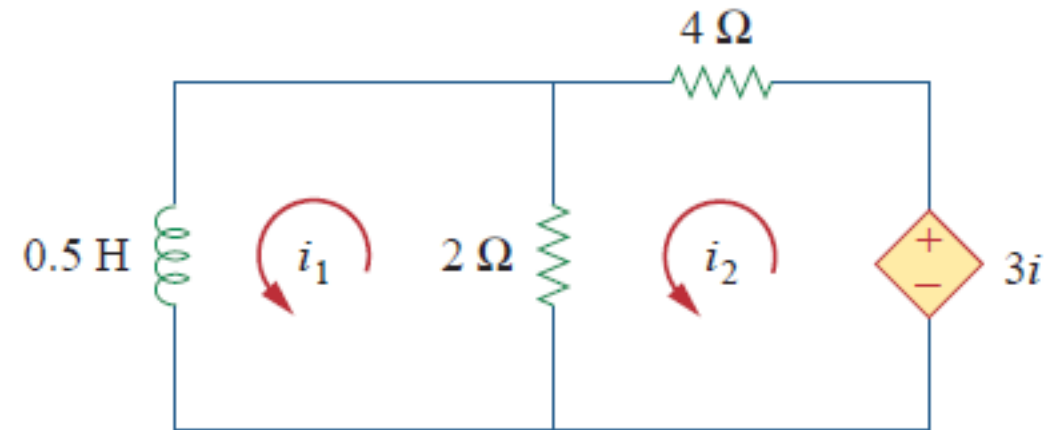
$$(\text{loop1}): 0.5 \frac{di_1}{dt} + 2(i_1 - i_2) = 0 \rightarrow \frac{di_1}{dt} + 4i_1 - 4i_2 = 0$$

$$(\text{loop2}): 6i_2 - 2i_1 - 3i_1 = 0 \rightarrow i_2 = \frac{5}{6}i_1$$

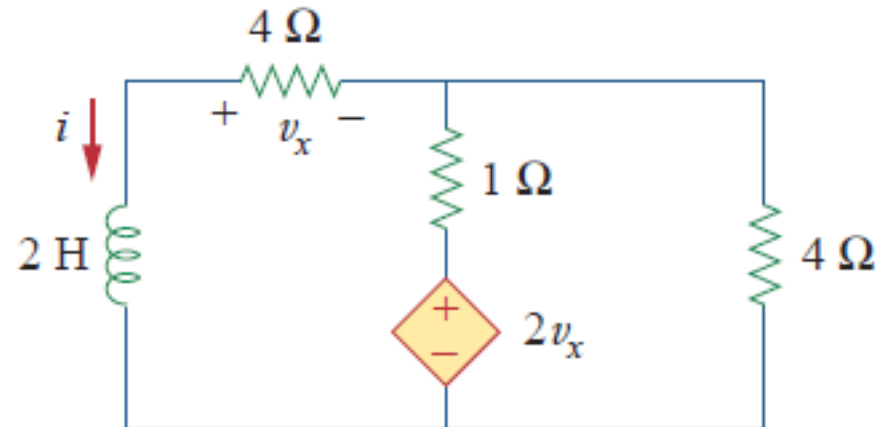
$$(\text{loop2} \rightarrow \text{loop1}): \frac{di_1}{dt} + \frac{2}{3}i_1 = 0$$

$$\frac{di_1}{i_1} = -\frac{2}{3} dt = 0 \rightarrow \ln i \Big|_{i(0)}^{i(t)} = -\frac{2}{3} t \Big|_0^t \quad \rightarrow \ln \frac{i(t)}{i(0)} = -\frac{2}{3} t \quad \rightarrow i(t) = i(0) e^{-\frac{2}{3} t} = 10 e^{-\frac{2}{3} t} \text{ A}, t \geq 0$$

$$i_x(t) = \frac{v_L(t)}{R} = \frac{L \frac{di}{dt}}{R} = -\frac{5}{3} e^{-\frac{2}{3} t} \text{ A}, t \geq 0$$



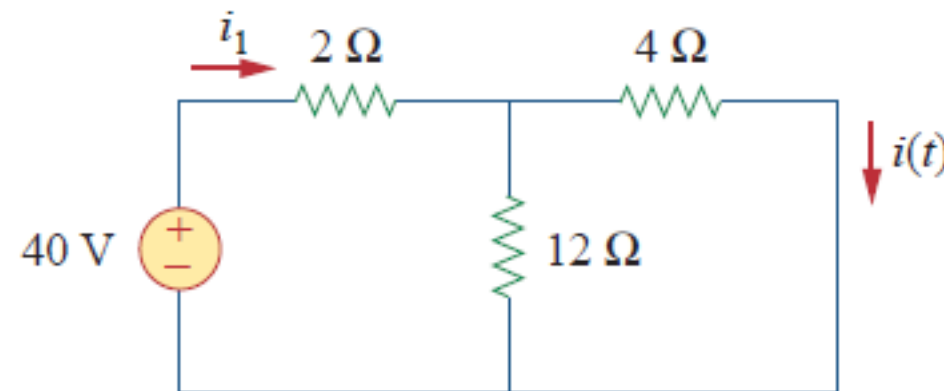
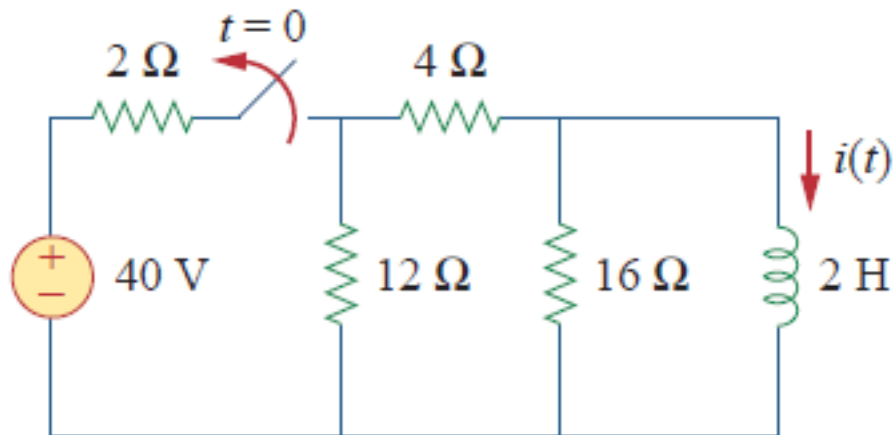
FOC.07 – Find $i(t)$ and $v_x(t)$ if $i(0) = 5$ A.



Solution $5 e^{-4t}$ A, $20 e^{-4t}$ V

Source-Free RL Circuit with Switch

FOC.08 – $i(t) = ?$ for $t > 0$

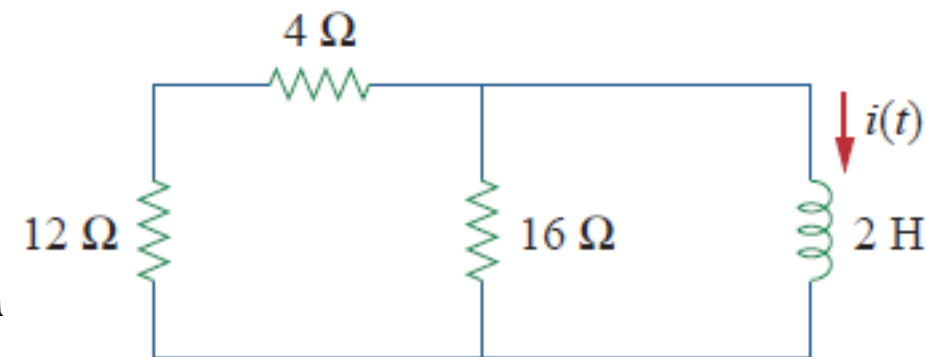


Solution

$$4 \times 12 = \frac{4 \cdot 12}{4 + 12} = 3 \Omega \rightarrow i_1 = \frac{40}{2 + 3} = 8 A$$

$$i(0^-) = \frac{12}{12 + 4} i_1 = 6 A \quad (t < 0) \quad i(0) = i(0^-) = 6 A$$

$$R_{eq} = (12 + 4) \times 16 = 8 \Omega \rightarrow \tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4} s$$

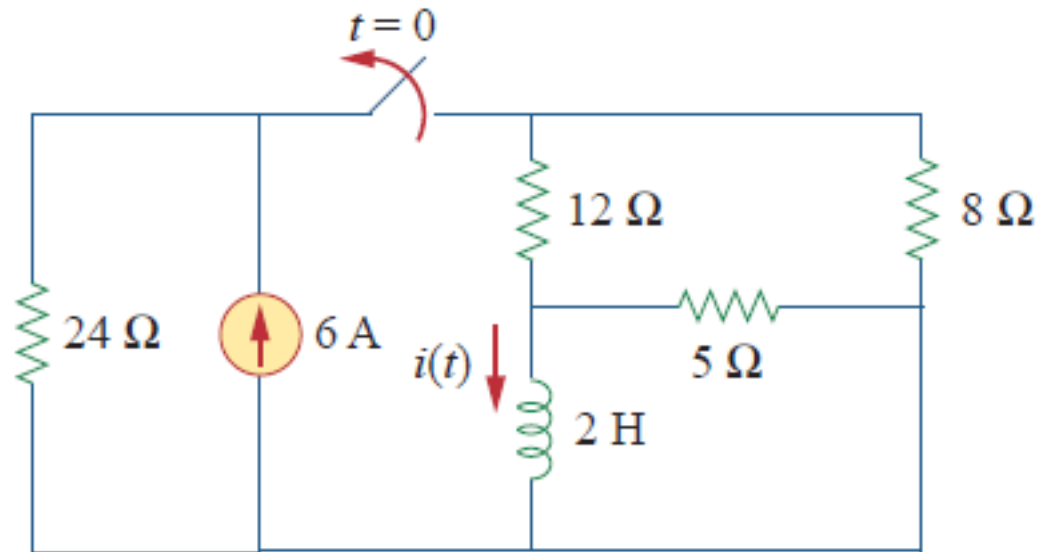


$$i(t) = i(0) e^{-\frac{2}{3}t} = 6 e^{-4t} A, \quad t > 0$$

Source-Free RL Circuit



FOC.09 – $i(t) = ?$ for $t > 0$



Solution $2 e^{-2t} A, t > 0$

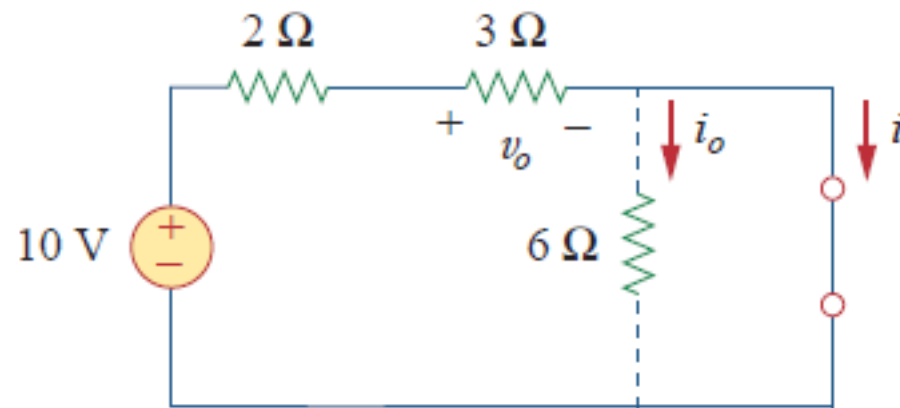
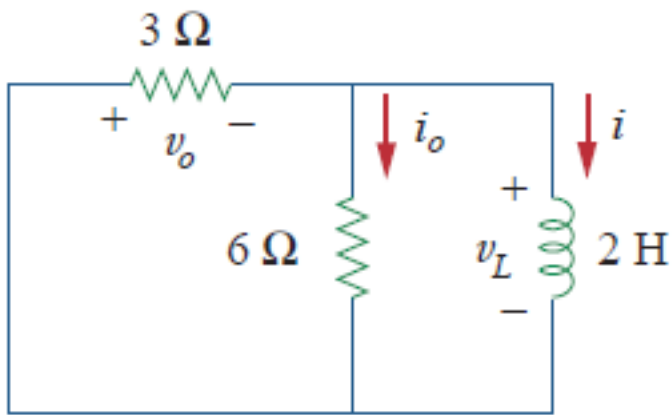
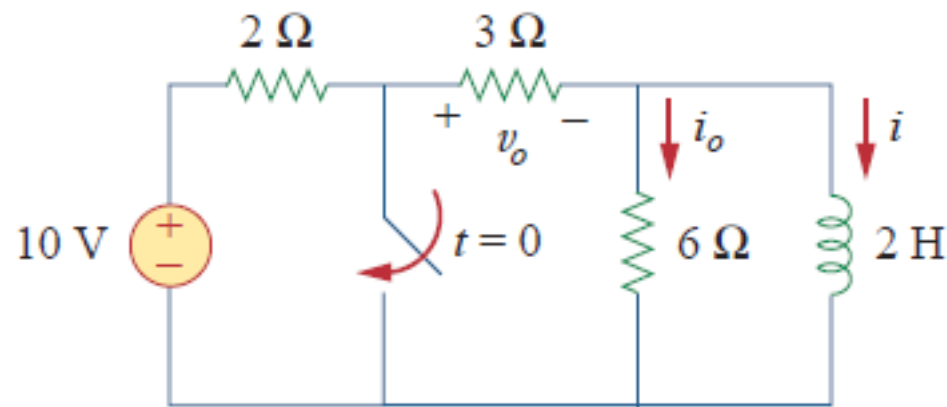
Source-Free RL Circuit

FOC.10 – Find i_0 , v_0 and i for all time, assuming that the switch was open for a long time.

Solution $i(0^-) = i(0) = \frac{10}{2 + 3} = 2 \text{ A}$

$$v_0(t) = 3 i(t) = 6 \text{ V } (t < 0)$$

$$R_{Th} = 3 \times 6 = 2 \Omega \rightarrow \tau = \frac{L}{R_{Th}} = 1 \text{ s}$$



Source-Free RL Circuit

$$i(t) = i(0) e^{-\frac{t}{\tau}} = 2 e^{-t} A, \quad t > 0$$

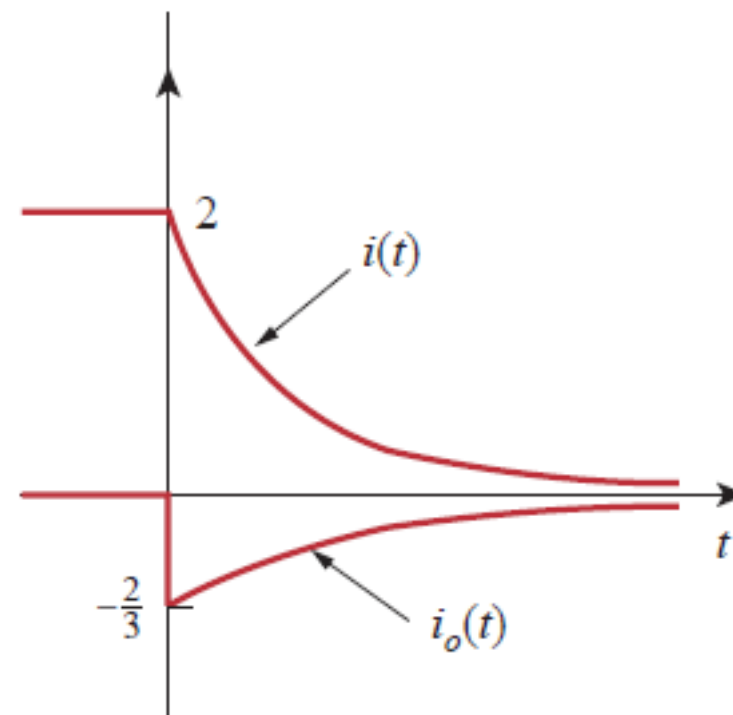
$$v_0(t) = -v_L = -L \frac{di}{dt} = -2 \cdot (-2 e^{-t}) = 4 e^{-t} V, \quad t > 0$$

$$i_o(t) = \frac{v_L(t)}{6} = -\frac{2}{3} e^{-t} A, \quad t > 0$$

$$i_o(t) = \begin{cases} 0 A & t < 0 \\ -\frac{2}{3} e^{-t} A & t > 0 \end{cases}$$

$$v_0(t) = \begin{cases} 6 V & t < 0 \\ 4 e^{-t} V & t > 0 \end{cases}$$

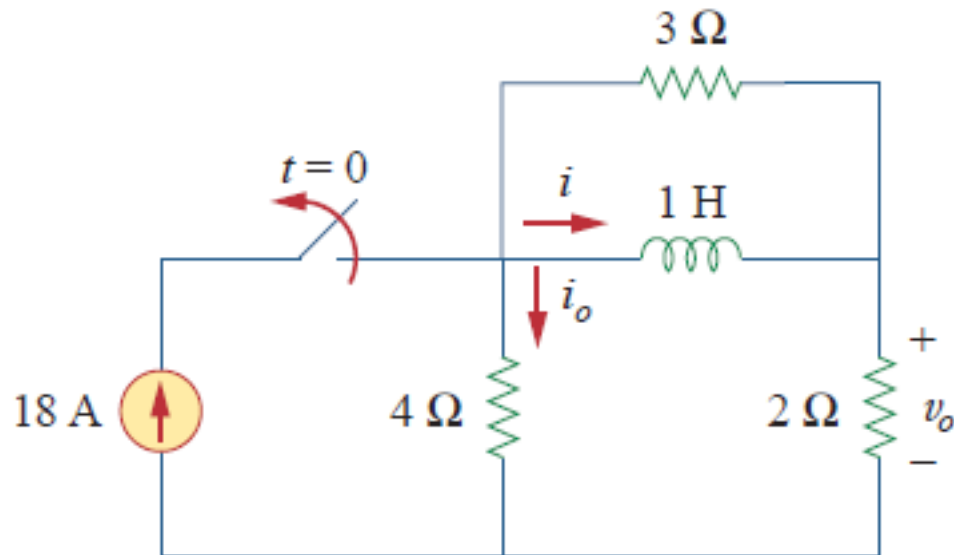
$$i(t) = \begin{cases} 2 A & t < 0 \\ 2 e^{-t} A & t \geq 0 \end{cases}$$



Source-Free RL Circuit

FOC.11 – Determine i , i_o , and v_o for all t . Assume that the switch was closed for a long time.

It should be noted that opening a switch in series with an ideal current source creates an infinite voltage at the current source terminals. Clearly this is impossible. For the purposes of problem solving, we can place a shunt resistor in parallel with the source (which now makes it a voltage source in series with a resistor). In more practical circuits, devices that act like current sources are, for the most part, electronic circuits. These circuits will allow the source to act like an ideal current source over its operating range but voltage-limit it when the load resistor becomes too large (as in an open circuit).



Solution

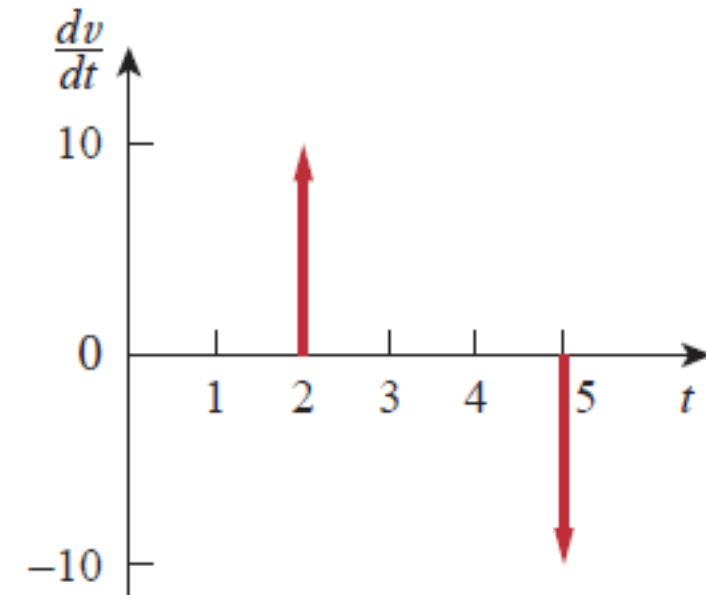
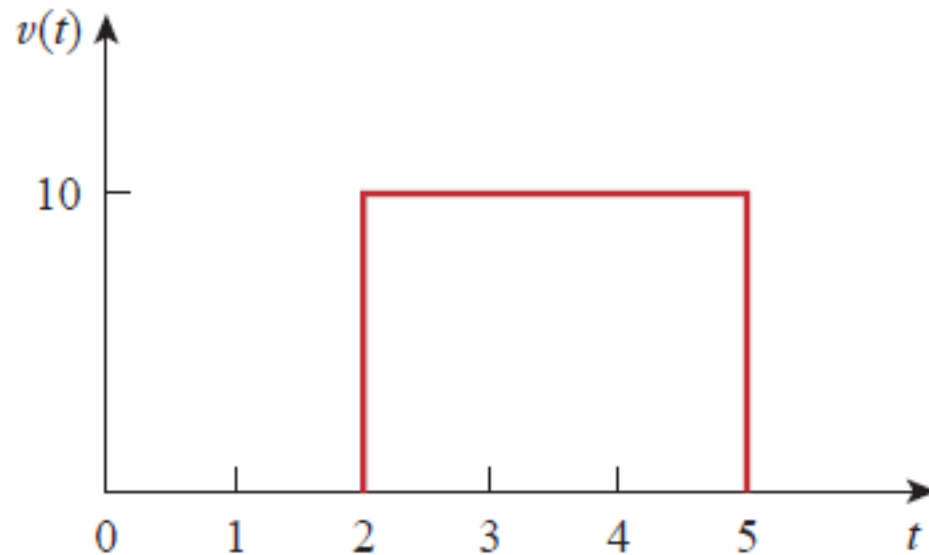
$$i(t) = \begin{cases} 12 \text{ A} & t < 0 \\ 12 e^{-2t} \text{ A} & t \geq 0 \end{cases}$$

$$i_o(t) = \begin{cases} 6 \text{ A} & t < 0 \\ -4 e^{-2t} \text{ A} & t > 0 \end{cases}$$

$$v_o(t) = \begin{cases} 24 \text{ V} & t < 0 \\ 8 e^{-2t} \text{ V} & t > 0 \end{cases}$$

Singularity Functions

FOC.12 – Express the voltage pulse in terms of the unit step. Calculate its derivative and sketch it.



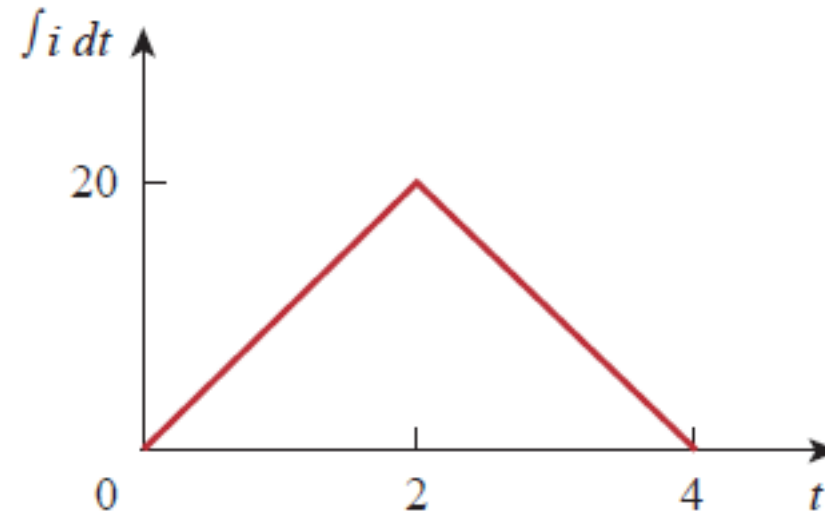
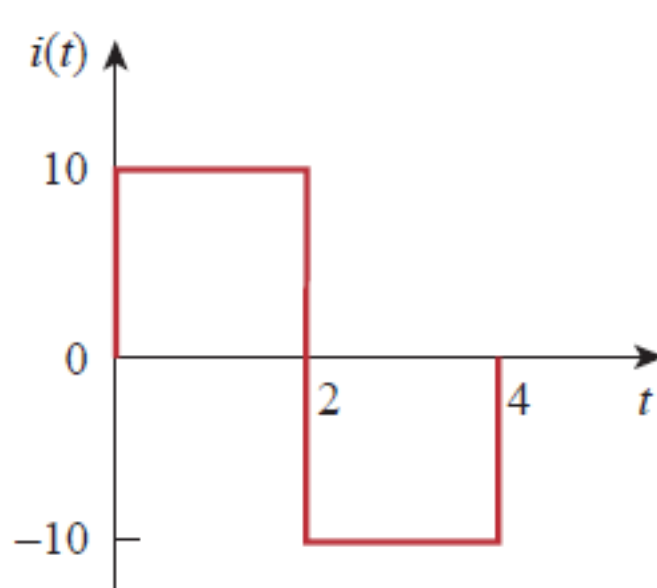
Solution

$$v(t) = 10 u(t - 2) - 10 u(t - 5) = 10 [u(t - 2) - (t - 5)]$$

$$\frac{dv}{dt} = 10 [\delta(t - 2) - \delta(t - 5)]$$

Singularity Functions

FOC.13 – Express the current pulse in terms of the unit step. Find its integral and sketch it.



Solution $10[u(t) - 2u(t - 2) + u(t - 4)]$, $10[r(t) - 2r(t - 2) + r(t - 4)]$

Step Response of an RC Circuit

FOC.14 – The switch has been in position ‘A’ for a long time. At $t = 0$ the switch moves to position ‘B’. Determine $v(t)$ for $t > 0$ and calculate its value at $t = 1$ s and $t = 4$ s.

Solution

$$v(0^-) = 24 \cdot \frac{5}{5 + 3} = 15 \text{ V} = v(0^+)$$

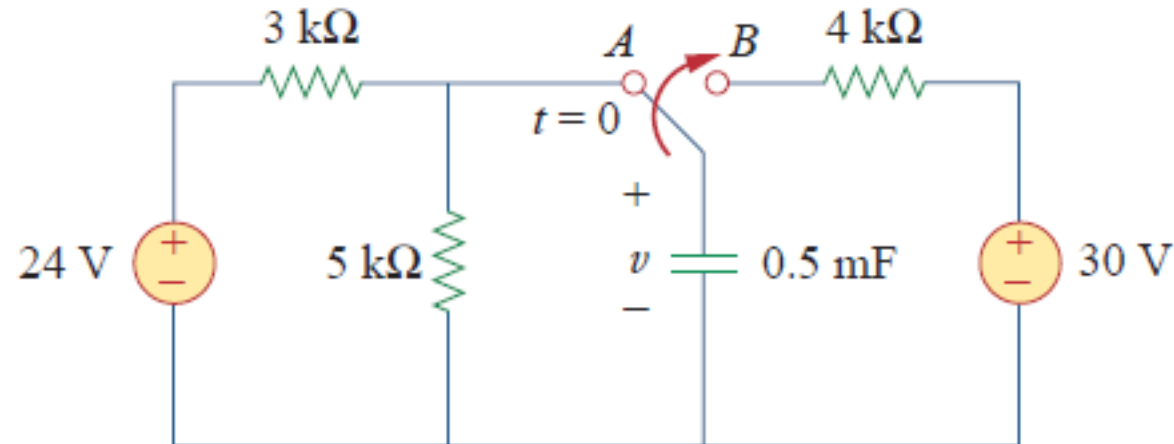
$$v(\infty) = 30 \text{ V}$$

$$\tau = R_{Th}C = 4 \cdot 10^3 \cdot 0.5 \cdot 10^{-3} = 2 \text{ s}$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} = 30 + (15 - 30)e^{-\frac{t}{2}} = (30 - 15e^{-0.5t}) \text{ V}$$

$$v(1) = 30 - 15e^{-0.5} = 20.9 \text{ V}$$

$$v(4) = 30 - 15e^{-2} = 27.97 \text{ V}$$

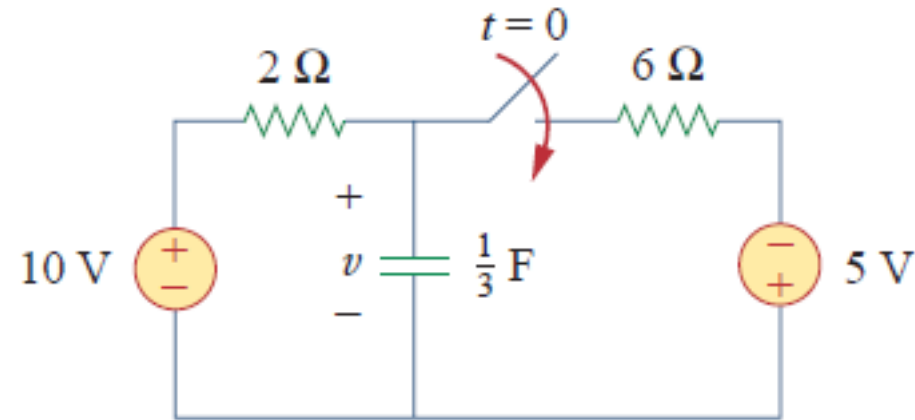


Step Response of an RC Circuit

FOC.15

The switch has been open for a long time and is closed at $t = 0$. Determine $v(t)$ for $t > 0$ and $v(t)$ at $t = 0.5$ s.

Solution $(6.25 + 3.75 e^{-2t})$ V, 7.63 V

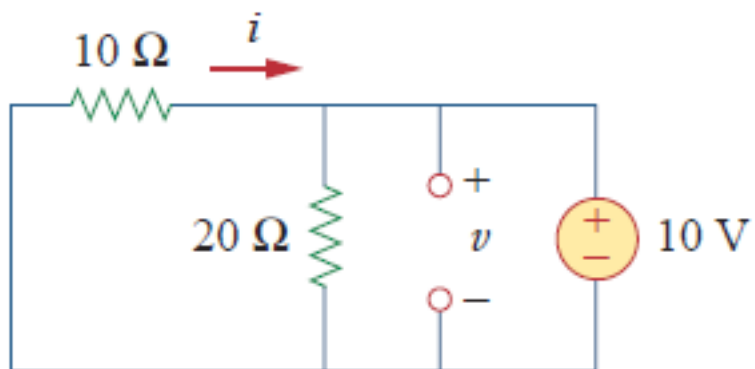


Step Response of an RC Circuit

FOC.16 – The switch has been closed for a long time and is opened at $t = 0$. Find $i(t)$ and $v(t)$ for all time.

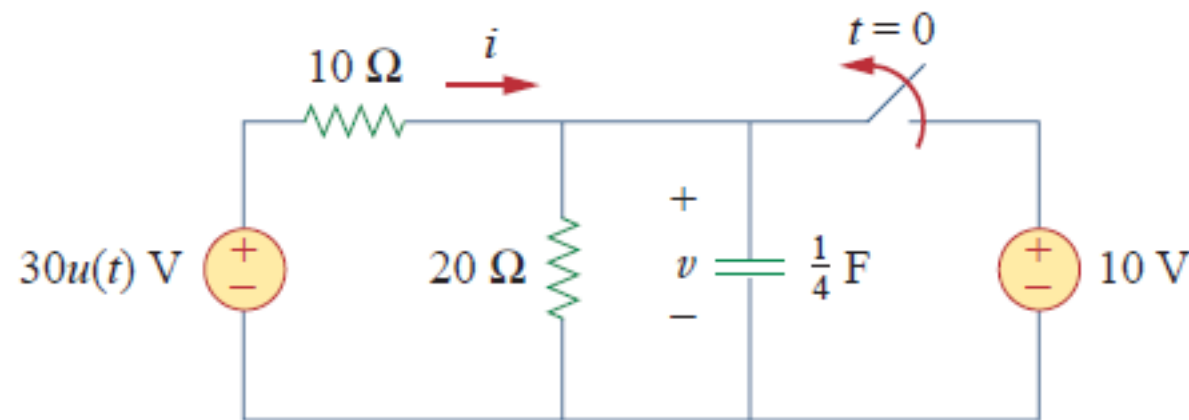
Solution

$t < 0 \rightarrow 30u(t) = 0 \leftarrow$ short circuit



$$t < 0 \rightarrow v = 10 \text{ V}, i = -\frac{v}{10} = -1 \text{ A}$$

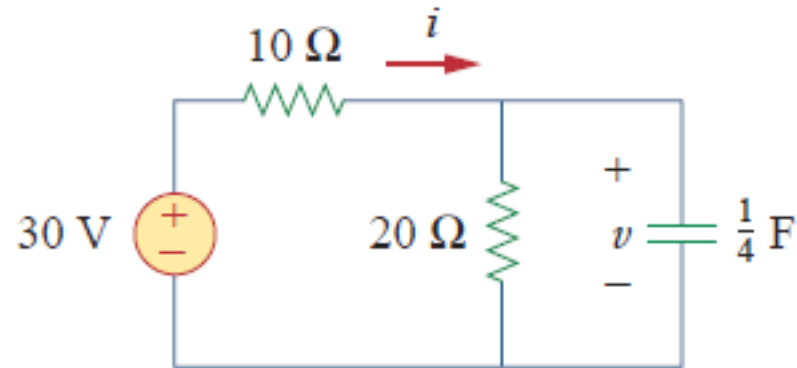
$$v(0) = v(0^-) = 10 \text{ V}, v(\infty) = 30 \cdot \frac{20}{20 + 10} = 20 \text{ V}$$



$$R_{Th} = 10 \times 20 = \frac{10 \cdot 20}{10 + 20} = \frac{20}{3} \Omega \rightarrow \tau = R_{Th}C = \frac{20}{3} \cdot \frac{1}{4} = \frac{5}{3} \text{ s}$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} = 20 + (10 - 20)e^{-\left(\frac{3}{5}\right)t} = (20 - 10e^{-0.6t}) \text{ V}$$

Step Response of an RC Circuit



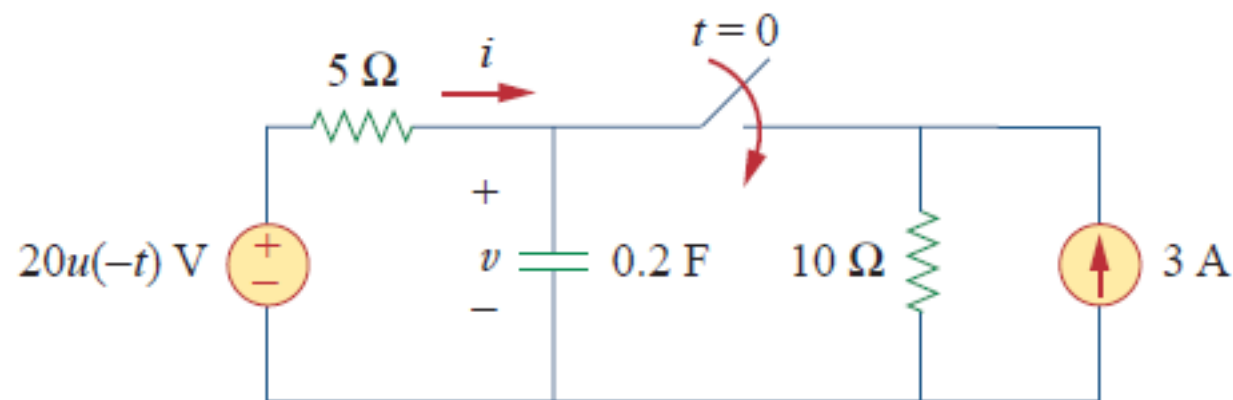
$$i = \frac{v}{20} + C \frac{dv}{dt} = (1 - 0.5e^{-0.6t}) + 0.25(-0.6)(-10)e^{-0.6t} = (1 + e^{-0.6t}) \text{ A}$$

check: $v + 10i = 30$

$$v = \begin{cases} 10 \text{ V} & t < 0 \\ (20 - 10e^{-0.6t}) \text{ V} & t \geq 0 \end{cases} \quad i = \begin{cases} -1 \text{ A} & t < 0 \\ (1 + e^{-0.6t}) \text{ A} & t > 0 \end{cases}$$

Step Response of an RC Circuit

FOC.17 – The switch is closed at $t = 0$. Find $i(t)$ and $v(t)$ for all time.



Solution $u(-t) = 1 - u(t)$

$$i = \begin{cases} 0 & t < 0 \\ -2(1 + e^{-1.5t}) \text{ A} & t > 0 \end{cases} \quad v = \begin{cases} 20 \text{ V} & t < 0 \\ 10(1 + e^{-1.5t}) \text{ V} & t > 0 \end{cases}$$

Step Response of an RL Circuit

FOC.18 – Find $i(t)$ for $t > 0$. (Switch has been closed for long time.)

Solution

$$i(0^-) = \frac{10}{2} = 5 \text{ A} = i(0^+)$$

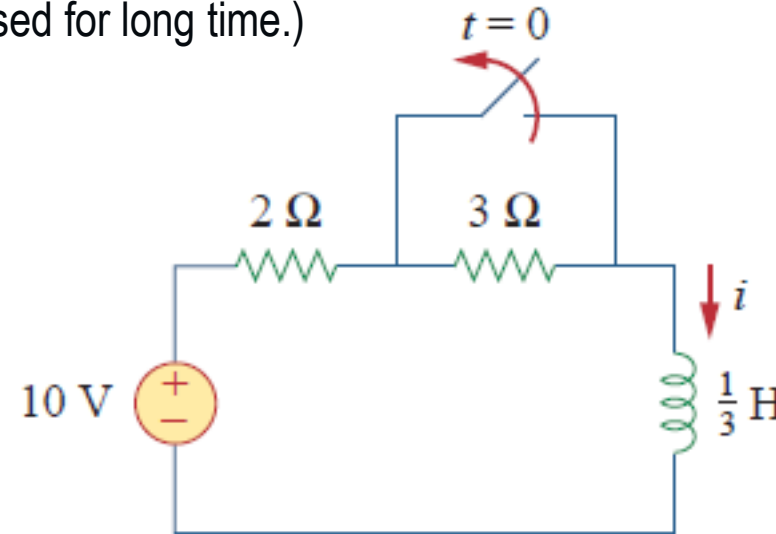
$$i(\infty) = \frac{10}{2 + 3} = 2 \text{ A}$$

$$R_{Th} = 2 + 3 = 5 \Omega$$

$$\tau = \frac{L}{R_{Th}} = \frac{1/3}{5} = \frac{1}{15} \text{ s}$$

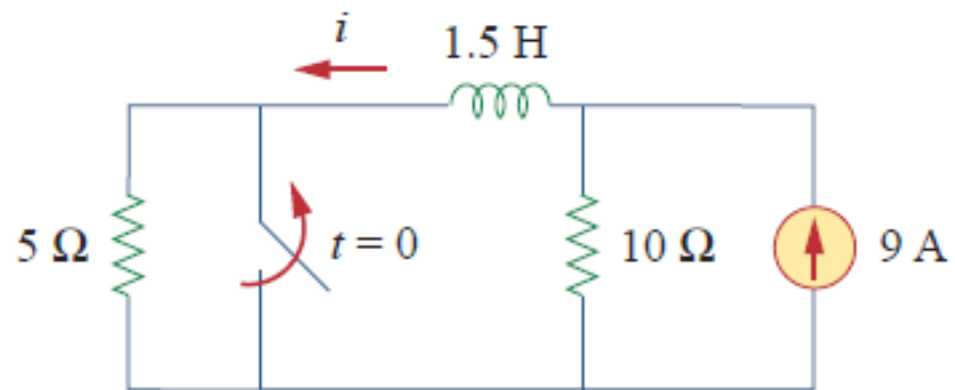
$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} = 2 + (5 - 2)e^{-15t} = 2 + 3e^{-15t} \text{ A}, \quad t > 0$$

$$\text{check (KVL): } 10 = 5i + L \frac{di}{dt} \rightarrow 5i + L \frac{di}{dt} = \left[10 + 15e^{-15t} \right] + \left[\frac{1}{3} \cdot 3 \cdot (-15)e^{-15t} \right] = 10$$



Step Response of an RL Circuit

FOC.19 – Find $i(t)$ for $t > 0$.
(Switch has been closed for a long time.)



Solution $(6 + 3e^{-10t})\ \text{A}, t > 0$

Step Response of an RL Circuit

FOC.20 – Find $i(t)$ for $t > 0$. Calculate $i(2)$ and $i(5)$.

Solution $i(0^-) = i(0^+) = 0$

$$0 \leq t \leq 4 \rightarrow i(\infty) = \frac{40}{4 + 6} = 10 \text{ A}, \quad R_{Th} = 4 + 6 = 10 \Omega$$

$$0 \leq t \leq 4 \rightarrow \tau = \frac{L}{R_{Th}} = \frac{5}{10} = \frac{1}{2} \text{ s}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} = 4 + (0 - 4)e^{-2t}$$

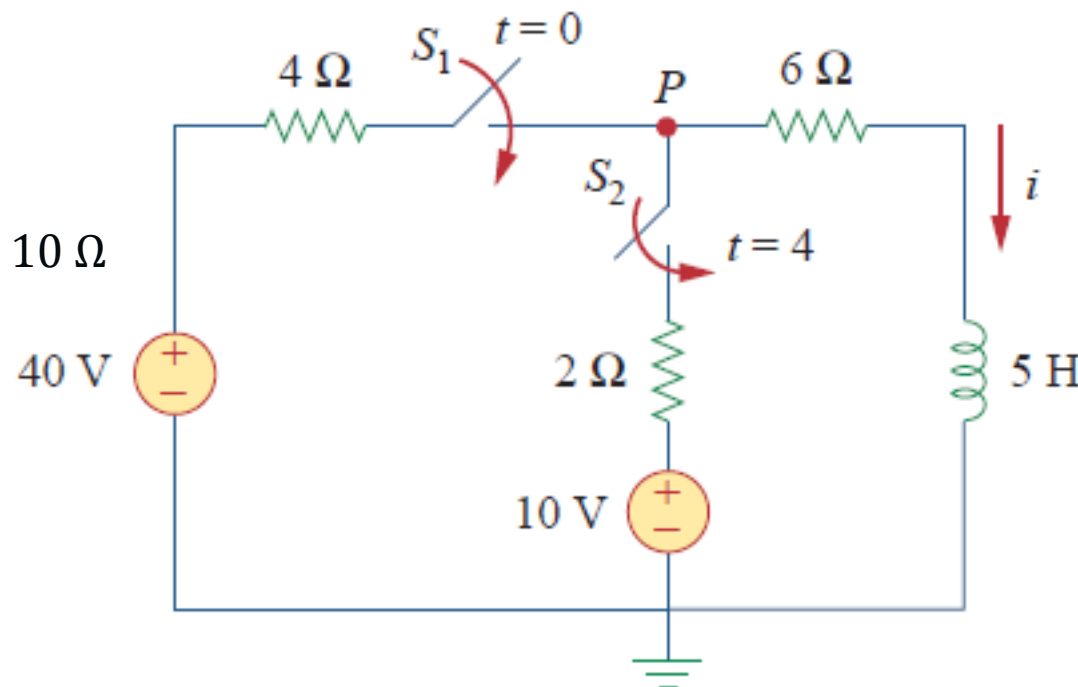
$$= 4(1 - e^{-2t}) \text{ A}, \quad 0 \leq t \leq 4$$

$$t \geq 4 \rightarrow i(4^-) = i(4^+) = 4(1 - e^{-8}) \cong 4 \text{ A}$$

$$i(\infty) = ? \text{ (KCL): } \frac{40 - v_P}{4} + \frac{10 - v_P}{2} = \frac{v_P}{6} \rightarrow v_P = \frac{180}{11} \text{ V}$$

$$i(\infty) = \frac{v_P}{6} = \frac{30}{11} = 2.727 \text{ A}$$

$$R_{Th} = 4 \times 2 + 6 = \frac{8}{6} + 6 = \frac{22}{3} \Omega \rightarrow \tau = \frac{L}{R_{Th}} = \frac{5}{22/3} = \frac{15}{22} \text{ s}$$



Step Response of an RL Circuit



$$i(t) = i(\infty) + [i(4) - i(\infty)]e^{-(t-4)/\tau}, \quad t \geq 4$$

$$i(t) = 2.727 + [4 - 2.727]e^{-(t-4)/\tau} = 2.727 + 1.273e^{-1.4667(t-4)}, \quad t \geq 4$$

$$i(t) = \begin{cases} 0 & t \leq 0 \\ 4(1 - e^{-2t}) A & 0 \leq t \leq 4 \\ [2.727 + 1.273e^{-1.4667(t-4)}] A & t \geq 4 \end{cases}$$

$$i(2) = 4(1 - e^{-4}) = 3.93 A$$

$$i(5) = 2.727 + 1.273e^{-1.4667} = 3.02 A$$

Step Response of an RL Circuit

FOC.21 – Find $i(t)$ for all t . Calculate $i(1)$ and $i(3)$.

Solution

$$i(t) = \begin{cases} 0 & t < 0 \\ 2(1 - e^{-9t}) \text{ A} & 0 < t < 2 \\ [3.6 - 1.6 e^{-5(t-2)}] \text{ A} & t > 2 \end{cases}$$

$$i(1) = 1.9997 \text{ A}, \quad i(3) = 3.589 \text{ A}$$

