DR. GYURCSEK ISTVÁN

First-Order Circuits - Examples

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

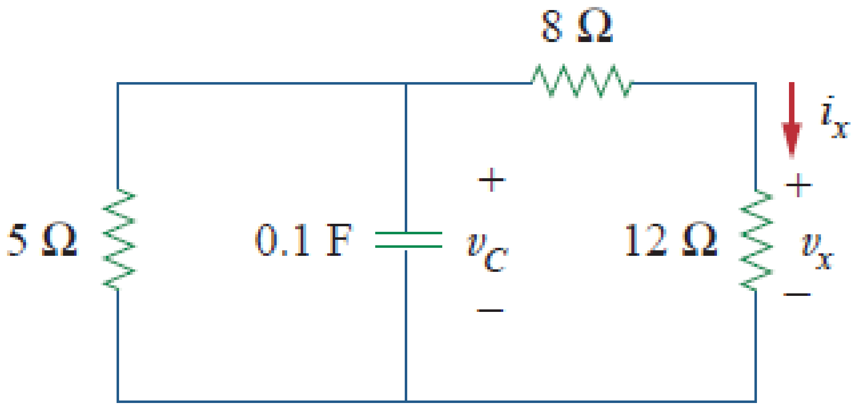
Examples

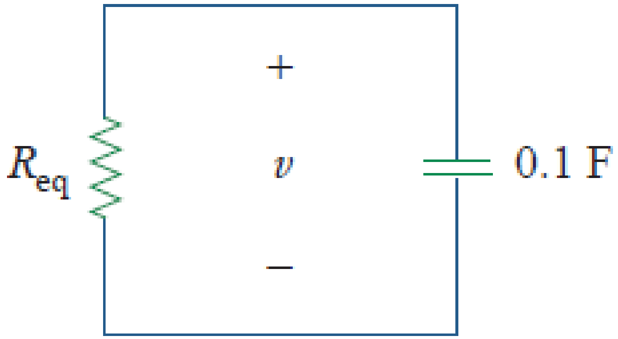
FOC.01 – Calculate the insulation resistance of a 220 nF capacitor if the voltage of the charged capacitor decays into its third part in 10 mins.

Solution $v(t) = V\,e^{-\frac{t}{\tau}} \rightarrow \frac{V}{3} = V\,e^{-\frac{600}{\tau}} \rightarrow \tau = RC = \frac{600}{ln\,3}$

$$R = \frac{600}{C \cdot ln\,3} = \frac{600}{220 \cdot 10^{-9} \cdot ln\,3} = 2.48 \cdot 10^{9} = 2.48\ G\Omega$$

FOC.02 – Find $v_C(t), v_X(t), i_X(t)$ for $t \geq 0$ if $v_C(0) = 15\ V$





Solution

$$R_{eq} = 20 \times 5 = 4\ \Omega$$

$$\tau = R_{eq}C = 4 \cdot 0.1 = 0.4\ s$$

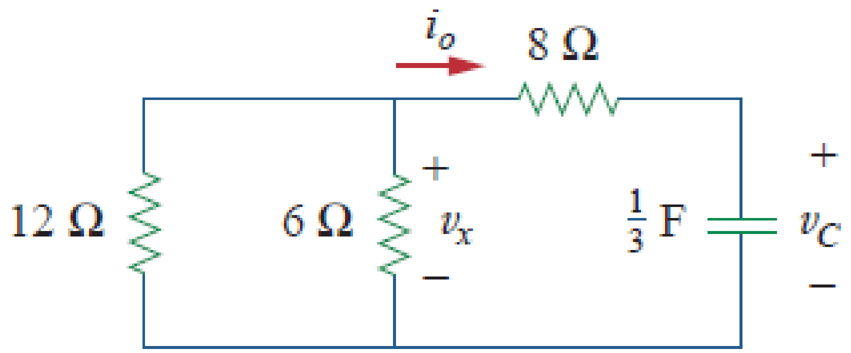
$$v = v(0)\ e^{-\frac{t}{\tau}} = 15\ e^{-\frac{t}{0.4}}$$

$$v_C = 15\ e^{-2.5t}\ V$$

$$v_X = \frac{12}{12+8} v = 0.6 \cdot 15\ e^{-2.5t} = 9\ e^{-2.5t}\ V$$

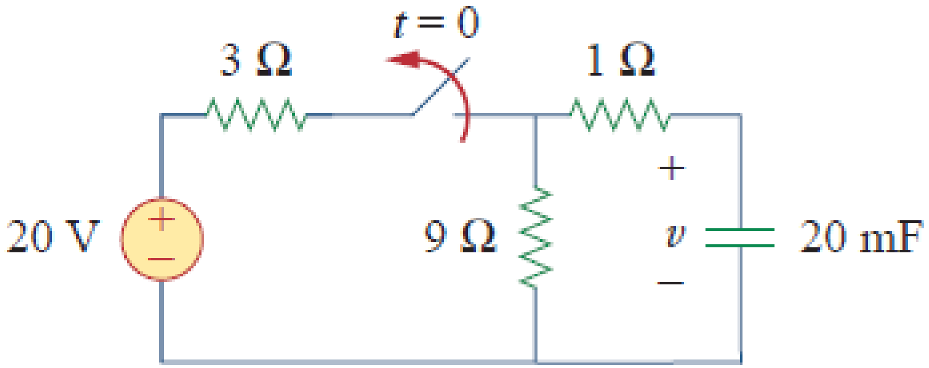
$$i_X = \frac{v_X}{12} = 0.75\ e^{-2.5t}\ A$$

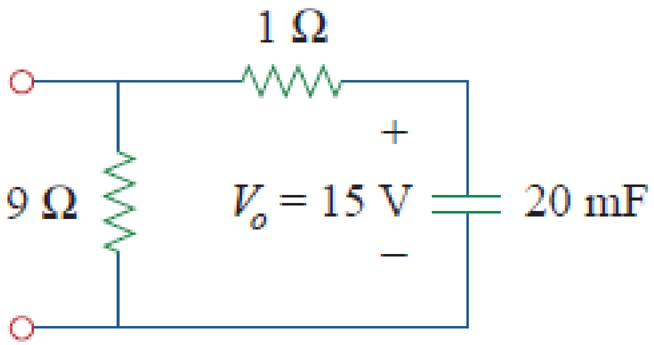
FOC.03 – Find $v_C(t), v_X(t), i_0(t)$ for $t \geq 0$ if $v_C(0) = 45\ V$



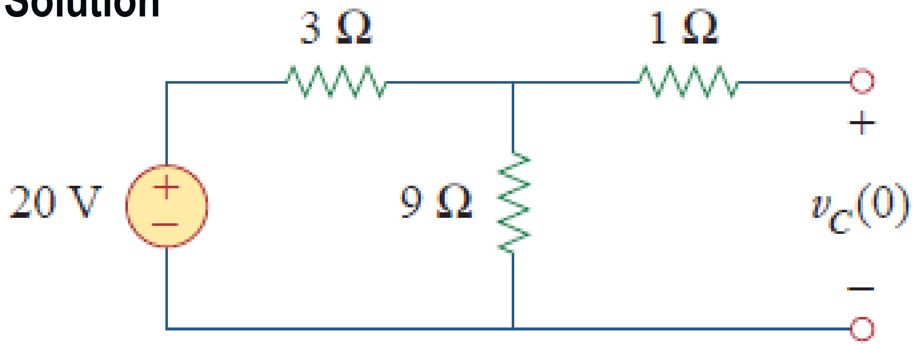
Solution $v_C = 45\ e^{-0.25t}\ V, v_X = 15\ e^{-0.25t}\ V\ i_0 = -3.75\ e^{-0.25t}\ A$

FOC.04 – The switch has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t \geq 0$. Calculate the initial energy stored in the capacitor.





Solution



$$v_C(-0) = 20 \cdot \frac{9}{9+3} = 15\,V = v_C(+0) = V_0$$

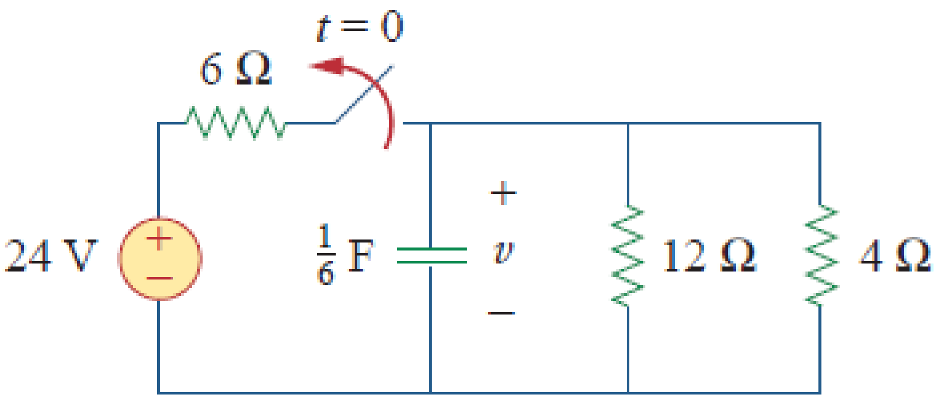
$$R_{eq} = 1 + 9 = 10\,\Omega$$

$$\tau = R_{eq}C = 10 \cdot 20 \cdot 10^{-3} = 0.2\,s$$

$$v(t) = v_C(0)\,e^{-\frac{t}{\tau}} = 15e^{-\frac{t}{0.2}} = 15e^{-5t}\,V$$

$$w_C(0) = \frac{1}{2}Cv_C^2(0) = \frac{1}{2} \cdot 20 \cdot 10^{-3} \cdot 15^2 = 2.25\,J$$

FOC.05 – If the switch opens at $t = 0$ find $v(t)$ for $t \geq 0$ and $w_C(0)$.



Solution $8\ e^{-2t}\ V, 5.33\ J$

Source-Free RL Circuit

FOC.06 – Assuming that *i(0)* = 10 A calculate *i(t)* and $i_x(t)$.

Method 1 *(find Thevenin equivalant resistance)*

$$2(i_1 - i_2) + 1 = 0 \rightarrow i_1 - i_2 = -\frac{1}{2}$$

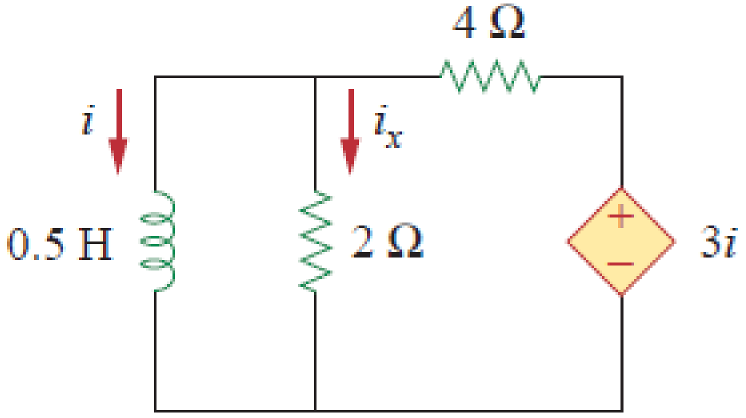
$$6i_2 - 2i_1 - 3i_1 = 0 \rightarrow i_2 = \frac{5}{6} i_1$$

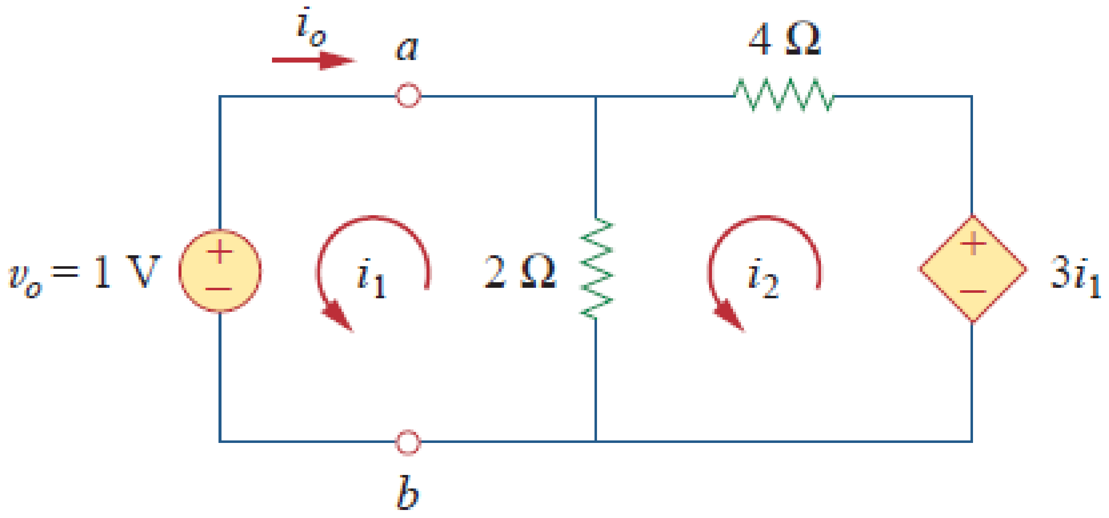
$$i_1 = -3\ A\,, \qquad i_0 = -i_1 = 3\ A$$

$$R_{eq} = R_{Th} = \frac{v_0}{i_0} = \frac{1}{3}\ \Omega \rightarrow \tau = \frac{L}{R_{eq}} = \frac{3}{2}\ s$$

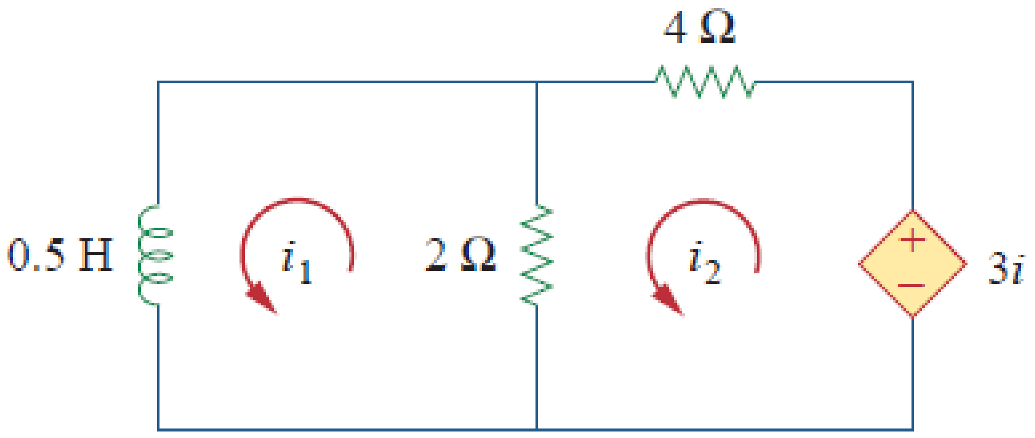
$$i(t) = i(0)\ e^{-\frac{t}{\tau}} = 10\ e^{-\frac{2}{3}t}\ A\,, t \geq 0$$

$$i_x(t) = \frac{v_L(t)}{R} = \frac{L\, {}^{di}/_{dt}}{R} = -\frac{5}{3}\ e^{-\frac{2}{3}t}\ A\,, t \geq 0$$





Method 2 *(Korchhoff Voltage Law)*



$$(\boldsymbol{loop1}): 0.5\frac{di_1}{dt} + 2(i_1 - i_2) = 0 \rightarrow \frac{di_1}{dt} + 4i_1 - 4i_2 = 0$$

$$(\boldsymbol{loop2}): 6i_2 - 2i_1 - 3i_1 = 0 \rightarrow i_2 = \frac{5}{6}i_1$$

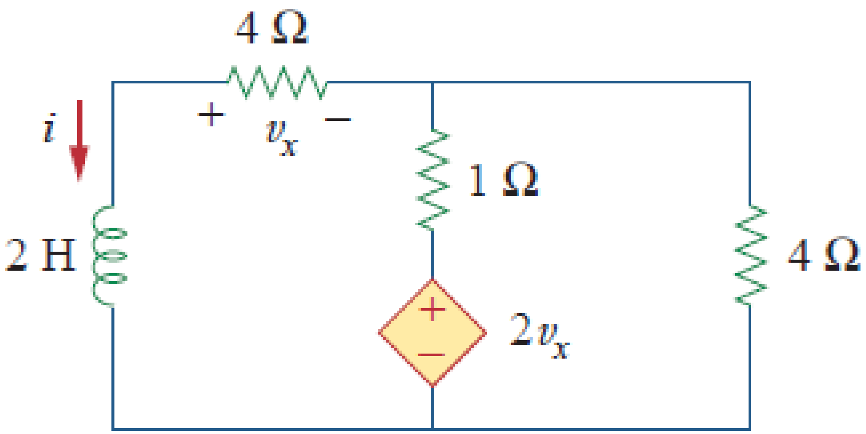
$$(\boldsymbol{loop2} \rightarrow \boldsymbol{loop1}): \frac{di_1}{dt} + \frac{2}{3}i_1 = 0$$

$$\frac{di_1}{i_1} = -\frac{2}{3}dt = 0 \rightarrow \ln i\Big|_{i(0)}^{i(t)} = -\frac{2}{3}t\Big|_0^t \qquad \rightarrow \ln\frac{i(t)}{i(0)} = -\frac{2}{3}t \qquad \rightarrow i(t) = i(0)\, e^{-\frac{t}{\tau}} = 10\, e^{-\frac{2}{3}t}\, A\,, t \geq 0$$

$$i_x(t) = \frac{v_L(t)}{R} = \frac{L\, {di}/{dt}}{R} = -\frac{5}{3}\, e^{-\frac{2}{3}t}\, A\,, t \geq 0$$

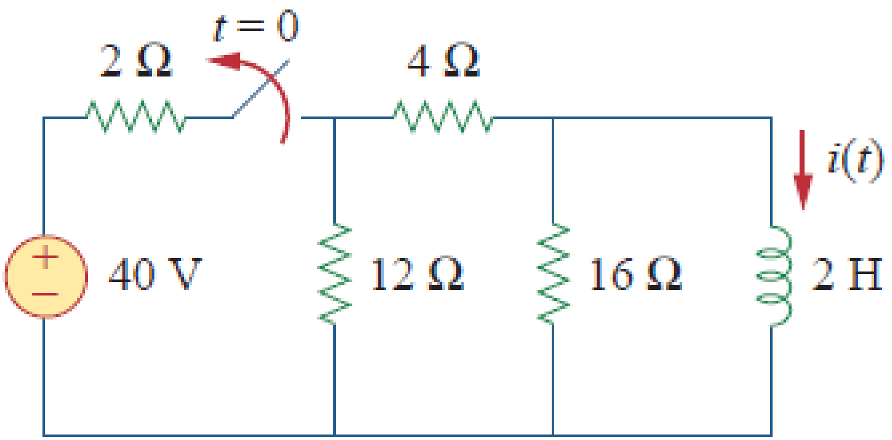
gyurcsek.istvan@mik.pte.hu

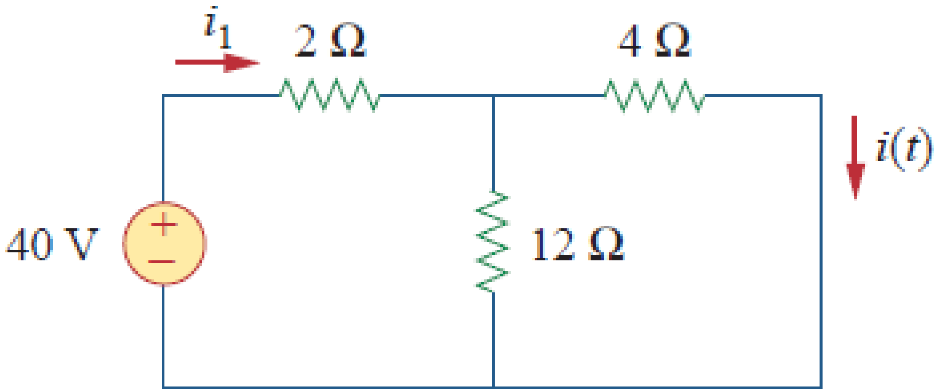
FOC.07 – Find $i(t)$ and $v_x(t)$ if $i(0) = 5$ A.



Solution $5\, e^{-4\,t}\, A\,,\;\; 20\, e^{-4\,t}\, V$

FOC.08 – *i(t)* = ? for *t* > 0

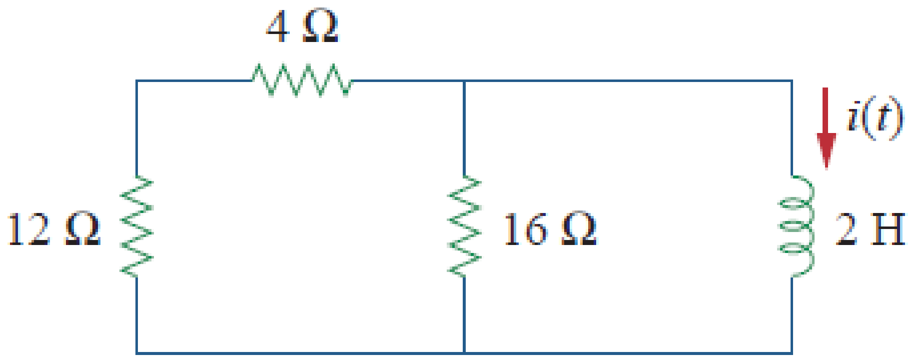




Solution

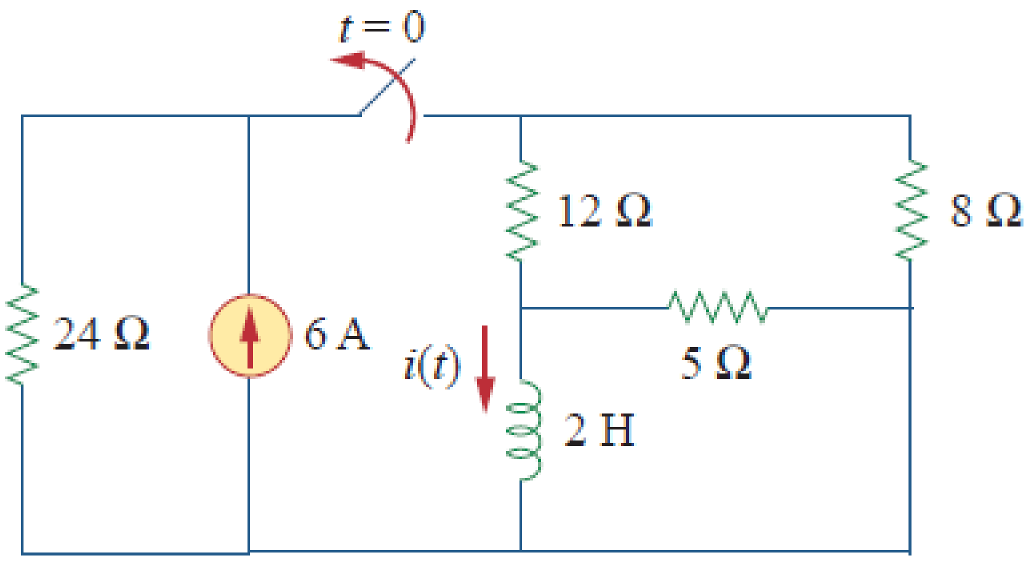
$$4 \times 12 = \frac{4 \cdot 12}{4+12} = 3\ \Omega \rightarrow i_1 = \frac{40}{2+3} = 8\ A$$

$$i(0^-) = \frac{12}{12+4} i_1 = 6\ A\ (t<0) \qquad i(0) = i(0^-) = 6\ A$$



$$R_{eq} = (12+4) \times 16 = 8\ \Omega \rightarrow \tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4}\ s \qquad i(t) = i(0)\, e^{-\frac{2}{3}t} = 6\, e^{-4\,t}\ A\,, \qquad t > 0$$

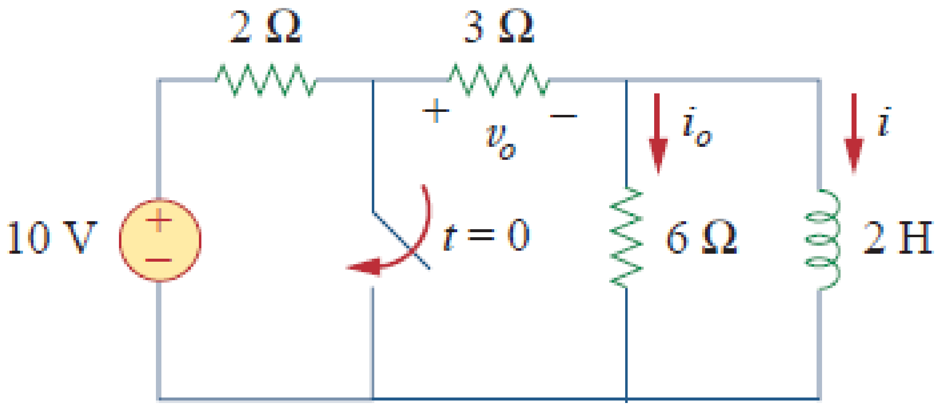
FOC.09 – *i(t)* = ? for *t* > 0



Solution $2\ e^{-2\ t}\ A\ ,\ \ t > 0$

Source-Free RL Circuit

FOC.10 – Find i_0, v_0 and i for all time, assuming that the switch was open for a long time.

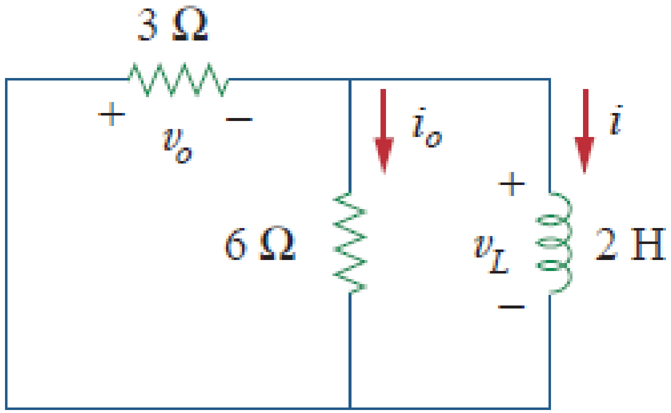


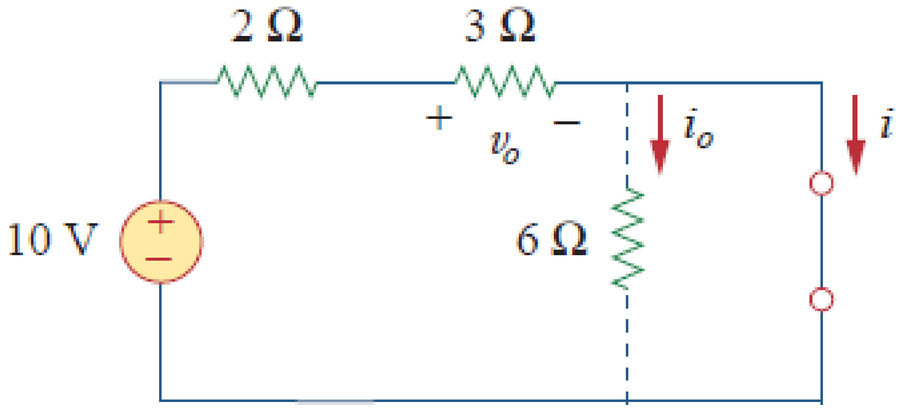
Solution

$$i(0^-) = i(0) = \frac{10}{2+3} = 2\ A$$

$$v_0(t) = 3\ i(t) = 6\ V\ (t < 0)$$

$$R_{Th} = 3 \times 6 = 2\ \Omega \rightarrow \tau = \frac{L}{R_{Th}} = 1\ s$$





$$i(t) = i(0)\, e^{-\frac{t}{\tau}} = 2\, e^{-t}\, A\,, \qquad t > 0$$

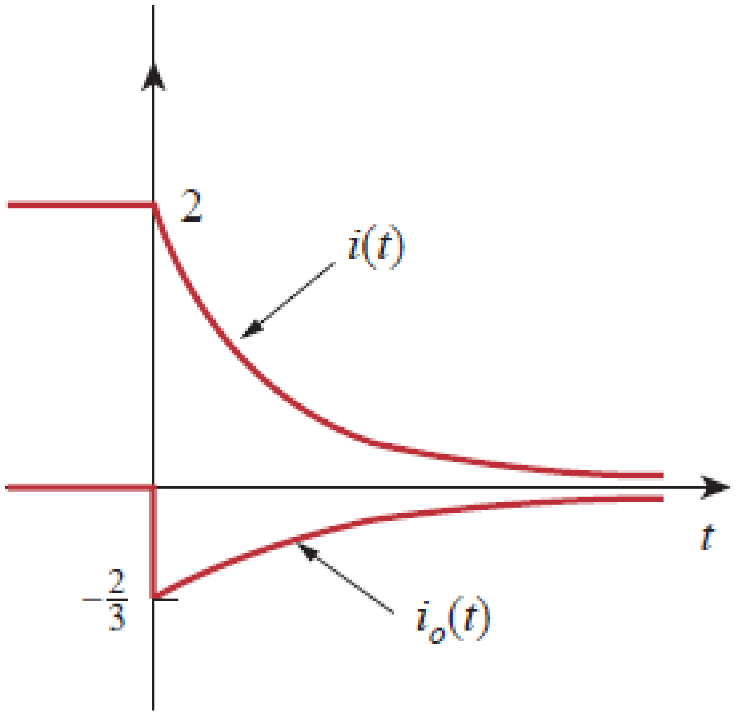
$$v_0(t) = -v_L = -L\frac{di}{dt} = -2 \cdot (-2\, e^{-t}) = 4\, e^{-t}\, V\,, \qquad t > 0$$

$$i_0(t) = \frac{v_L(t)}{6} = -\frac{2}{3}\, e^{-t}\, A\,, \qquad t > 0$$

$$v_0(t) = \begin{cases} 6\, V & t < 0 \\ 4\, e^{-t}\, V & t > 0 \end{cases}$$

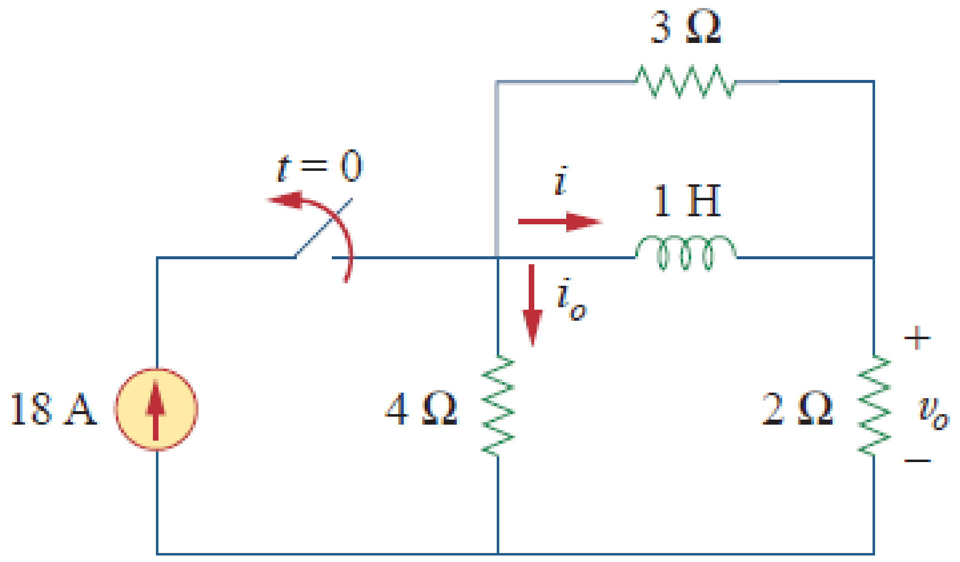
$$i_0(t) = \begin{cases} 0\, A & t < 0 \\ -\dfrac{2}{3}\, e^{-t}\, A & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2\, A & t < 0 \\ 2\, e^{-t}\, A & t \geq 0 \end{cases}$$



FOC.11 – Determine i, i_0, and v_0 for all t. Assume that the switch was closed for a long time.

It should be noted that opening a switch in series with an ideal current source creates an infinite voltage at the current source terminals. Clearly this is impossible. For the purposes of problem solving, we can place a shunt resistor in parallel with the source (which now makes it a voltage source in series with a resistor). In more practical circuits, devices that act like current sources are, for the most part, electronic circuits. These circuits will allow the source to act like an ideal current source over its operating range but voltage-limit it when the load resistor becomes too large (as in an open circuit).



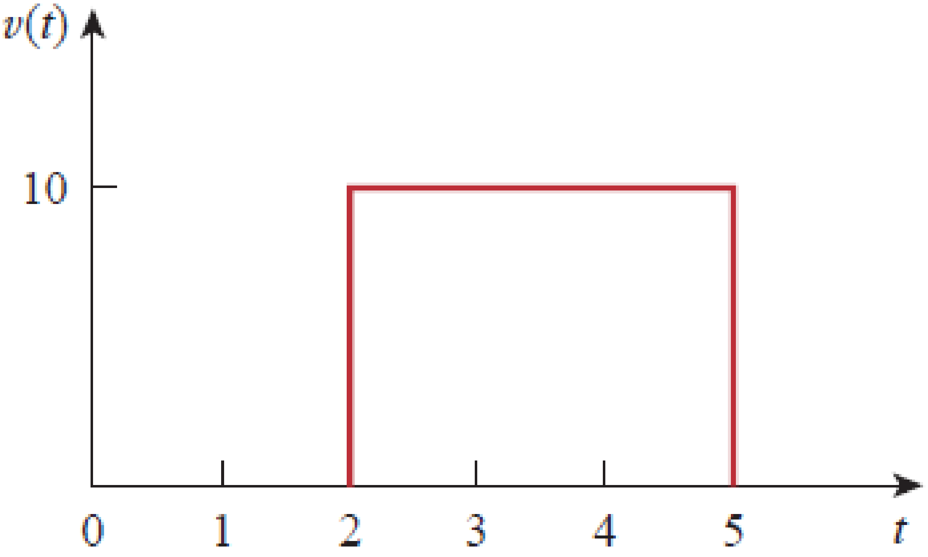
Solution

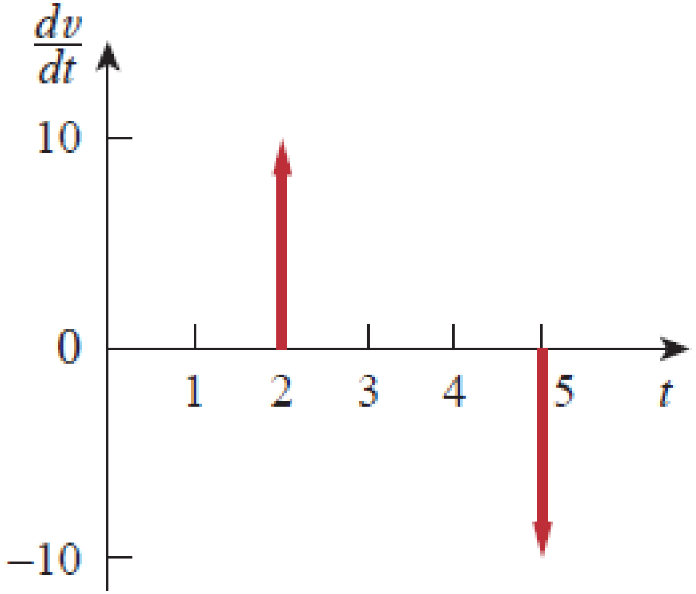
$$i(t) = \begin{cases} 12\,A & t < 0 \\ 12\,e^{-2t}\,A & t \geq 0 \end{cases}$$

$$i_0(t) = \begin{cases} 6\,A & t < 0 \\ -4\,e^{-2t}\,A & t > 0 \end{cases}$$

$$v_0(t) = \begin{cases} 24\,V & t < 0 \\ 8\,e^{-2t}\,V & t > 0 \end{cases}$$

FOC.12 – Express the voltage pulse in terms of the unit step. Calculate its derivative and sketch it.



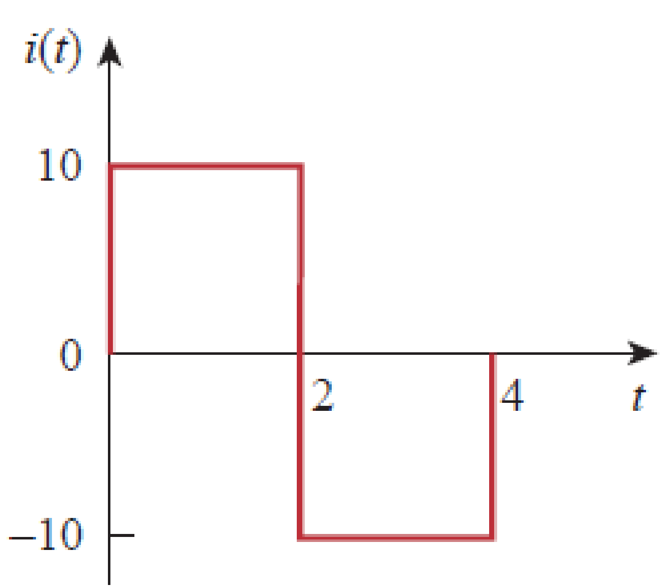


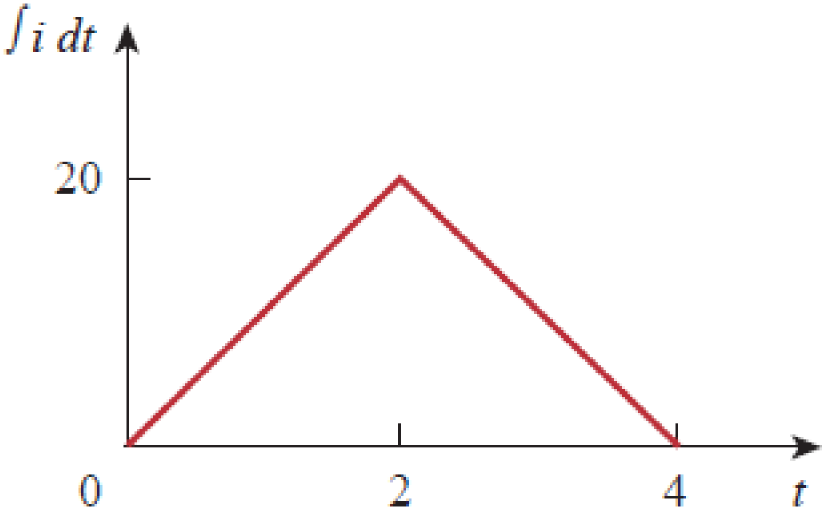
Solution

$$v(t) = 10\,u(t-2) - 10\,u(t-5) = 10\,[u(t-2) - (t-5)]$$

$$\frac{dv}{dt} = 10\,[\delta(t-2) - \delta(t-5)]$$

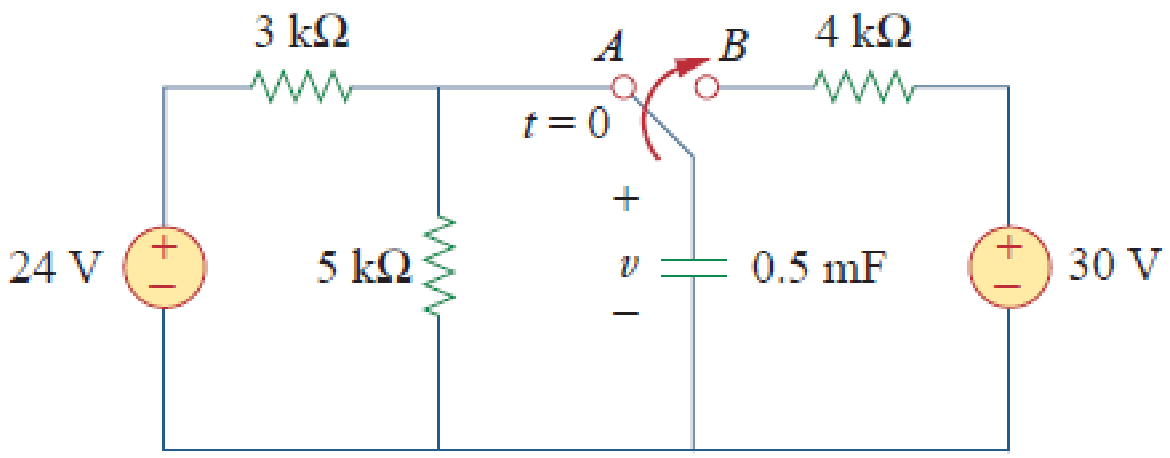
FOC.13 – Express the current pulse in terms of the unit step. Find its integral and sketch it.





Solution $10[u(t)-2u(t-2)+u(t-4)]$, $10[r(t)-2r(t-2)+r(t-4)]$

FOC.14 – The switch has been in position A for a long time. At $\mathrm{t} = 0$ the switch moves to B. Determine $v(t)$ for $\mathrm{t} > 0$ and calculate its value at $t = 1\ s$ and $t = 4\ s$.



Solution

$$v(0^-) = 24 \cdot \frac{5}{5+3} = 15\ V = v(0^+)$$

$$v(\infty) = 30\ V$$

$$\tau = R_{Th}C = 4 \cdot 10^3 \cdot 0.5 \cdot 10^{-3} = 2\ s$$

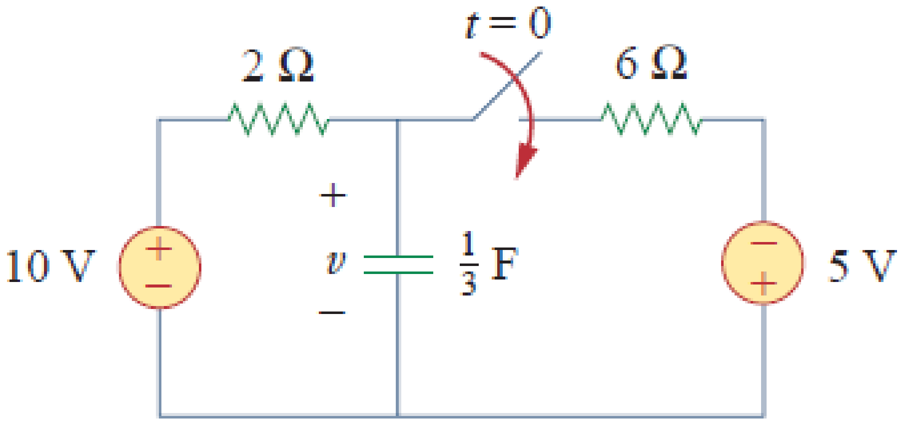
$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} = 30 + (15 - 30)e^{-\frac{t}{2}} = (30 - 15e^{-0.5t})\ V$$

$$v(1) = 30 - 15\ e^{-0.5} = 20.9\ V$$

$$v(4) = 30 - 15\ e^{-2} = 27.97\ V$$

FOC.15

The switch has been open for a long time and is closed at $\mathrm{t} = 0$. Determine $v(t)$ for $t > 0$ and $v(t)$ at $t = 0.5\ s$.

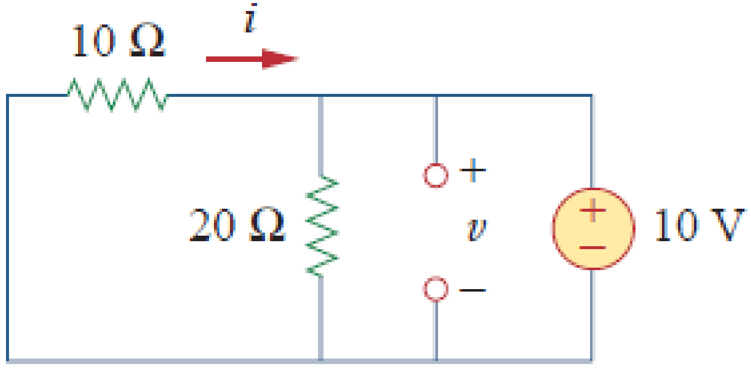


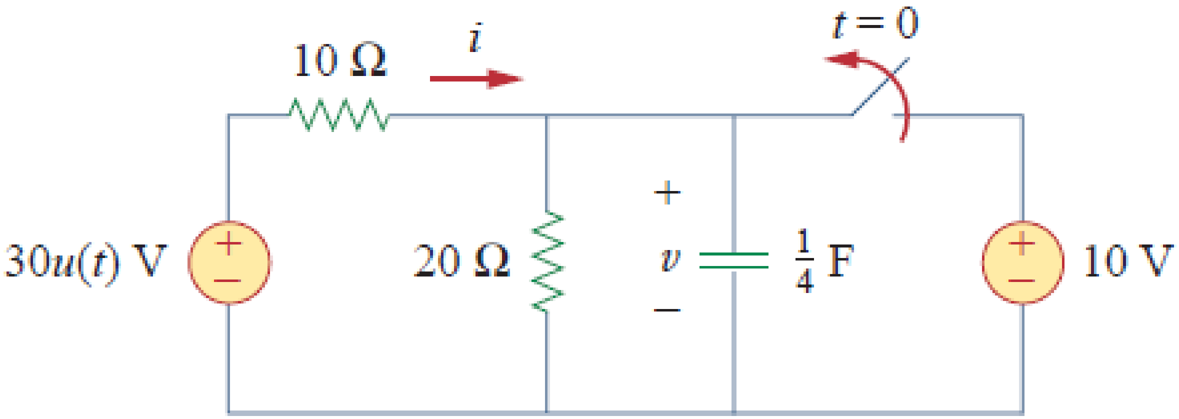
Solution $(6.25 + 3.75\ e^{-2t})\ V,\ \ 7.63\ V$

FOC.16 – The switch has been closed for a long time and is opened at $t = 0$. Find $\mathrm{i}(t)$ and $v(t)$ for all time.

Solution

$$t < 0 \rightarrow 30u(t) = 0 \leftarrow short\ circuit$$





$$t < 0 \rightarrow v = 10\ V, i = -\frac{v}{10} = -1\ A$$

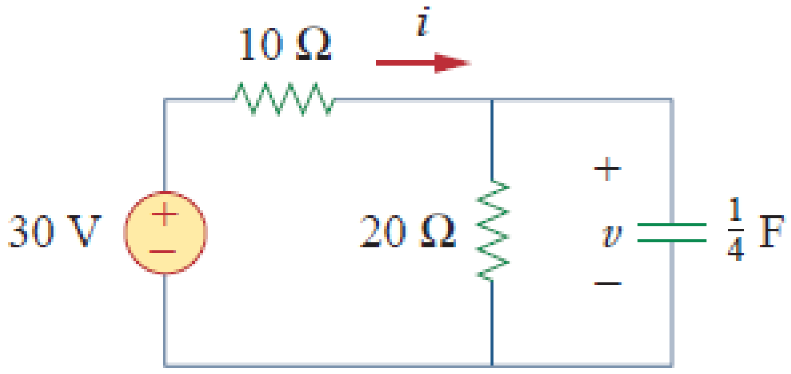
$$v(0) = v(0^-) = 10\ V, v(\infty) = 30 \cdot \frac{20}{20+10} = 20\ V$$

$$R_{Th} = 10 \times 20 = \frac{10 \cdot 20}{10+20} = \frac{20}{3}\ \Omega \rightarrow \tau = R_{Th}C = \frac{20}{3} \cdot \frac{1}{4} = \frac{5}{3}\ s$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} = 20 + (10-20)e^{-\left(\frac{3}{5}\right)t}$$

$$= (20 - 10e^{-0.6t})\ V$$

Step Response of an RC Circuit

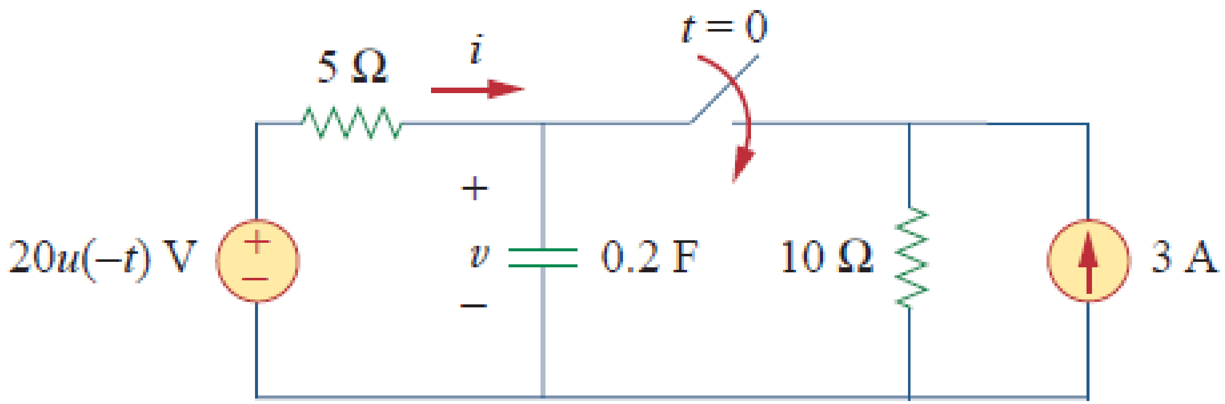


$$i = \frac{v}{20} + C\frac{dv}{dt} = (1 - 0.5e^{-0.6t}) + 0.25(-0.6)(-10)e^{-0.6t} = (1 + e^{-0.6t})\ A$$

$$check: v + 10i = 30$$

$$v = \begin{cases} 10\ V & t < 0 \\ (20 - 10e^{-0.6t})\ V & t \geq 0 \end{cases} \qquad i = \begin{cases} -1\ A & t < 0 \\ (1 + e^{-0.6t})\ A & t > 0 \end{cases}$$

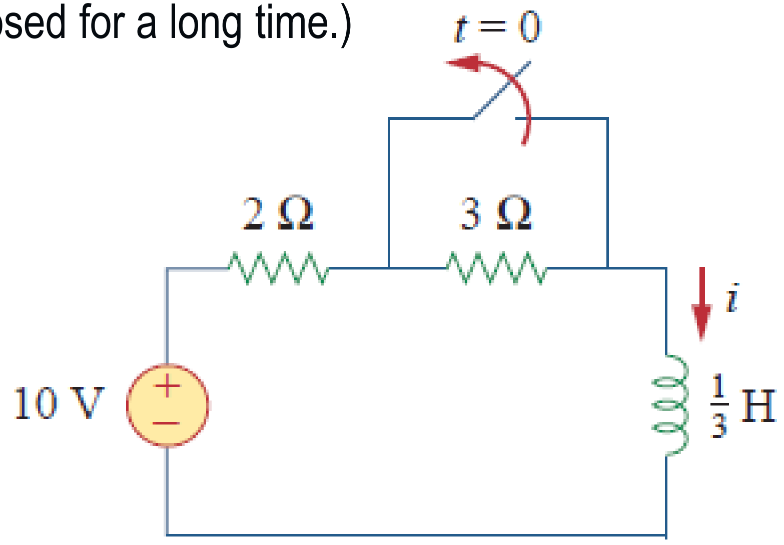
FOC.17 – The switch is closed at $t = 0$. Find $\mathrm{i}(t)$ and $v(t)$ for all time.



Solution $u(-t) = 1 - u(t)$

$$i = \begin{cases} 0 & t < 0 \\ -2(1 + e^{-1.5t})\ A & t > 0 \end{cases} \qquad v = \begin{cases} 20\ V & t < 0 \\ 10(1 + e^{-1.5t})\ V & t > 0 \end{cases}$$

FOC.18 – Find $i(t)$ for $t > 0$. (Switch has been colosed for a long time.)



Solution

$$i(0^-) = \frac{10}{2} = 5\,A = i(0^+)$$

$$i(\infty) = \frac{10}{2+3} = 2\,A$$

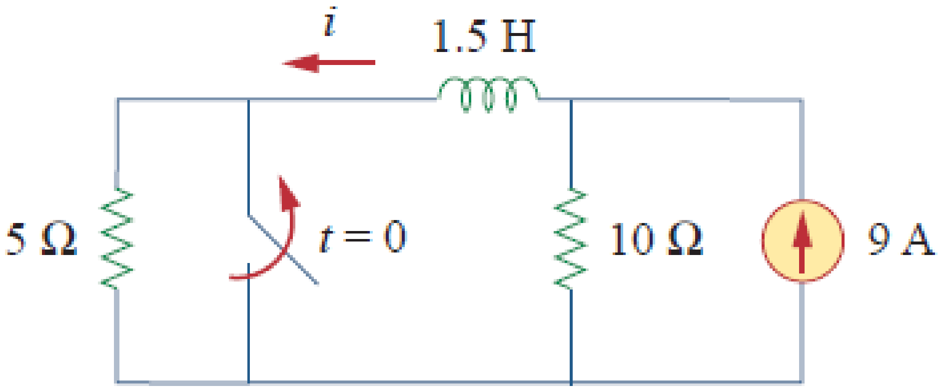
$$R_{Th} = 2 + 3 = 5\,\Omega$$

$$\tau = \frac{L}{R_{Th}} = \frac{1/3}{5} = \frac{1}{15}\,s$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} = 2 + (5-2)e^{-15t} = 2 + 3e^{-15t}\,A, \qquad t > 0$$

$$ckeck\ (KVL): 10 = 5i + L\frac{di}{dt} \to 5i + L\frac{di}{dt} = [10 + 15e^{-15t}] + \left[\frac{1}{3}\cdot 3\cdot(-15)e^{-15t}\right] = 10$$

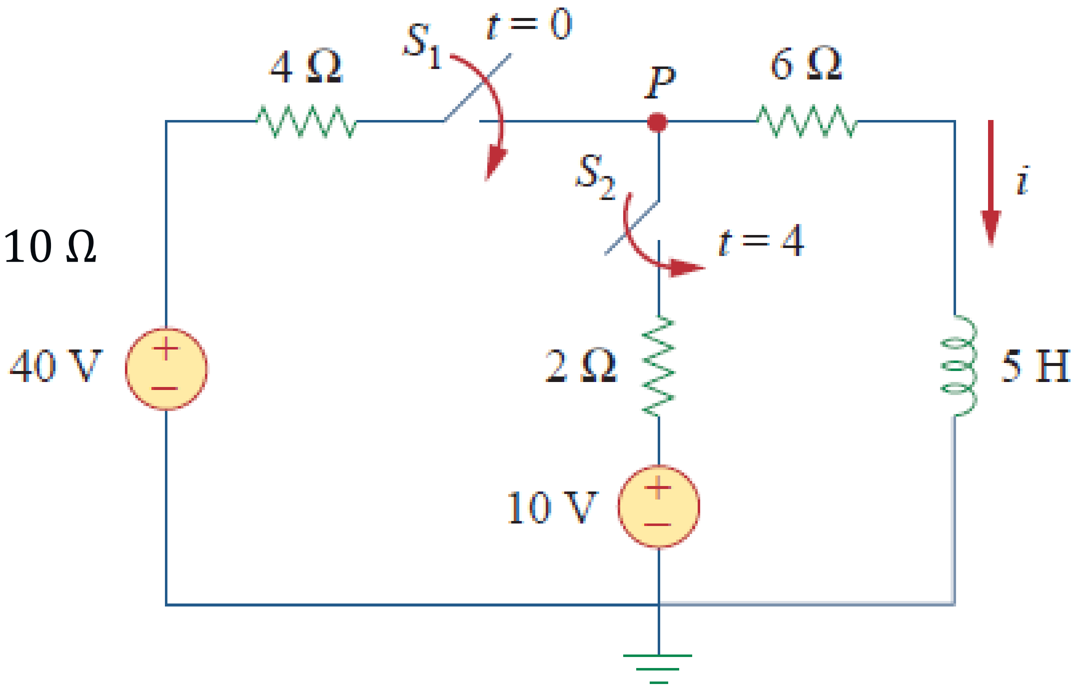
FOC.19 – Find $i(t)$ for $t > 0$.
(Switch has been colosed for a long time.)



Solution $(6 + 3e^{-10t})\ A,\ \ t > 0$

Step Response of an RL Circuit

FOC.20 – Find $i(t)$ for $t > 0$. Calculate $i(2)$ and $i(5)$.



Solution $\quad i(0^-) = i(0^+) = 0$

$$0 \le t \le 4 \to i(\infty) = \frac{40}{4+6} = 10\ A\,, \qquad R_{Th} = 4 + 6 = 10\ \Omega$$

$$0 \le t \le 4 \to \tau = \frac{L}{R_{Th}} = \frac{5}{10} = \frac{1}{2}\ s$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} = 4 + (0-4)e^{-2t}$$

$$= 4(1 - e^{-2t})\ A, \qquad 0 \le t \le 4$$

$$t \ge 4 \to i(4^-) = i(4^+) = 4(1 - e^{-8}) \cong 4\ A$$

$$i(\infty) =?\,(KCL): \frac{40 - v_P}{4} + \frac{10 - v_P}{2} = \frac{v_P}{6} \to v_P = \frac{180}{11}\ V$$

$$i(\infty) = \frac{v_P}{6} = \frac{30}{11} = 2.727\ A$$

$$R_{Th} = 4 \times 2 + 6 = \frac{8}{6} + 6 = \frac{22}{3}\ \Omega \to \tau = \frac{L}{R_{Th}} = \frac{5}{22/3} = \frac{15}{22}\ s$$

Step Response of an RL Circuit

$$i(t) = i(\infty) + [i(4) - i(\infty)]e^{-(t-4)/\tau}, \qquad t \geq 4$$

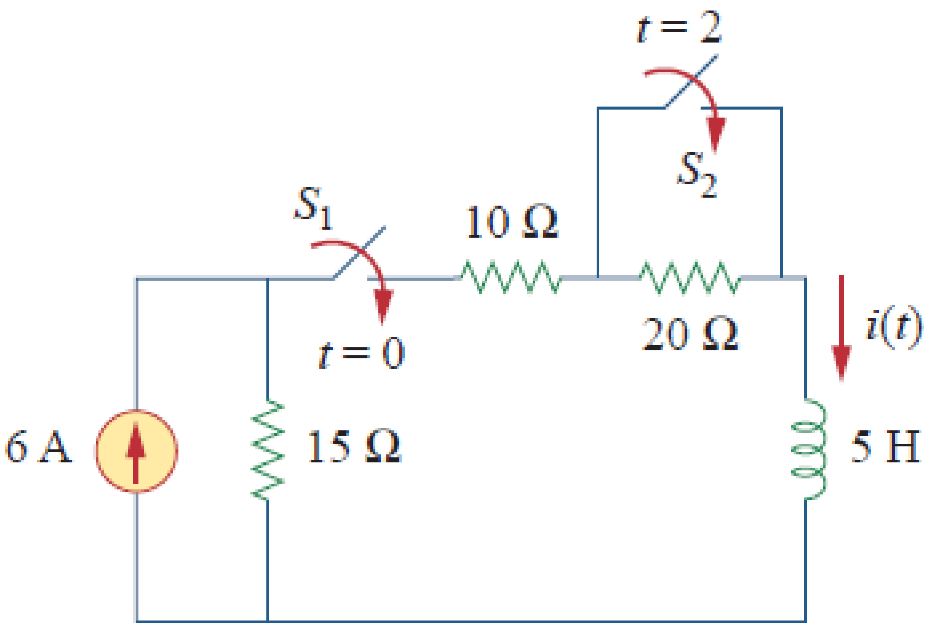
$$i(t) = 2.727 + [4 - 2.727]e^{-(t-4)/\tau} = 2.727 + 1.273e^{-1.4667(t-4)}, \qquad t \geq 4$$

$$i(t) = \begin{cases} 0 & t \leq 0 \\ 4(1 - e^{-2t})\ A & 0 \leq t \leq 4 \\ \left[2.727 + 1.273e^{-1.4667(t-4)}\right] A & t \geq 4 \end{cases}$$

$$i(2) = 4(1 - e^{-4}) = 3.93\ A$$

$$i(5) = 2.727 + 1.273e^{-1.4667} = 3.02\ A$$

FOC.21 – Find $i(t)$ for all t. Calculate $i(1)$ and $i(3)$.



Solution

$$i(t) = \begin{cases} 0 & t < 0 \\ 2(1 - e^{-9t})\ A & 0 < t < 2 \\ \left[3.6 - 1.6\, e^{-5(t-2)}\right] A & t > 2 \end{cases}$$

$$i(1) = 1.9997\ A\,, \qquad i(3) = 3.589\ A$$

gyurcsek.istvan@mik.pte.hu

Questions

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