



DR. GYURCSEK ISTVÁN

# Exercises with Second-Order Circuits

*Sources and additional materials (recommended)*

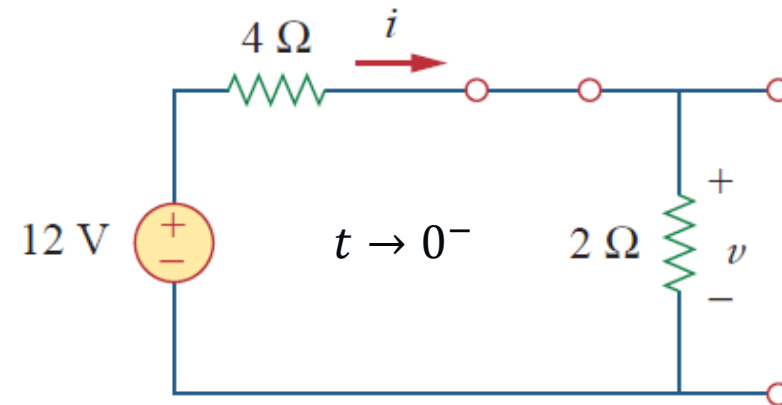
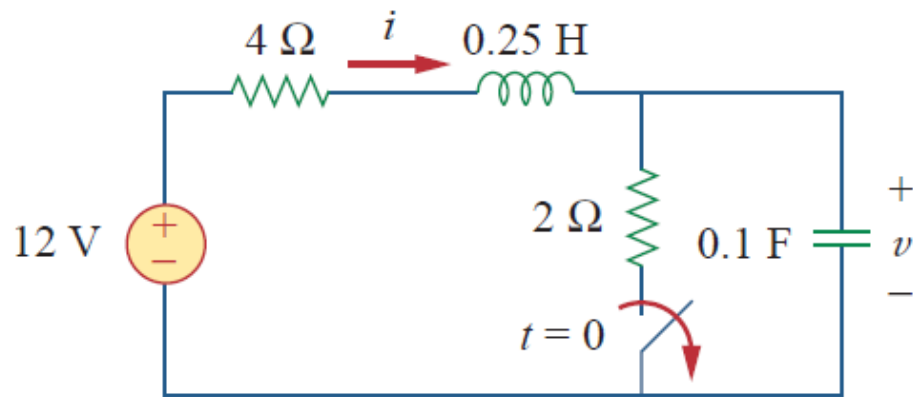
- ❑ *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- ❑ *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- ❑ *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- ❑ *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

# Initial and Final Conditions



**SOC.01** – Find initial and steady-state values. (*key to work*)

(a):  $i(0^+) = ?$ ,  $v(0^+) = ?$ , (b):  $\frac{di(0^+)}{dt} = ?$ ,  $\frac{dv(0^+)}{dt} = ?$ , (c):  $i(\infty) = ?$ ,  $v(\infty) = ?$

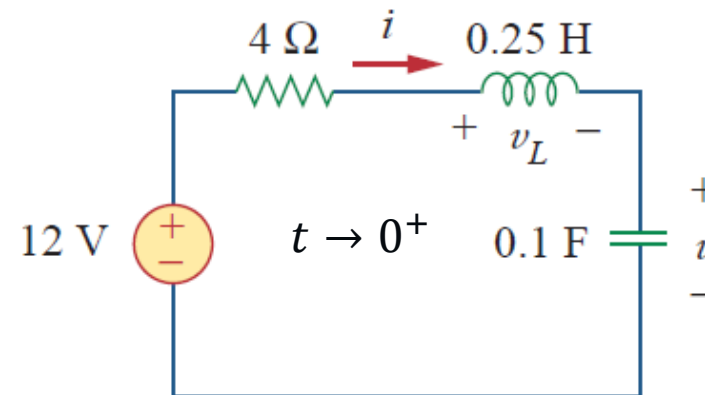
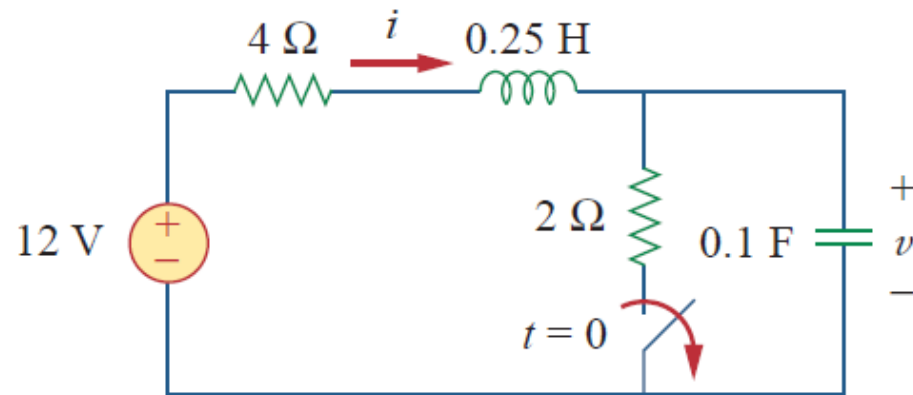


**Solution** (a):  $i(0^+) = ?$ ,  $v(0^+) = ?$

$$i(0^-) = \frac{12}{4 + 2} = 2 \text{ A}, \quad v(0^-) = 2i = 4 \text{ V}$$

$$i(0^+) = i(0^-) = 2 \text{ A}, \quad v(0^+) = v(0^-) = 4 \text{ V}$$

# Initial and Final Conditions



$$(b): \frac{dv(0^+)}{dt} = ?, \quad \frac{di(0^+)}{dt} = ?$$

$$i(0^+) = i_C(0^+) = 2 \text{ A}$$

$$i_C = C \frac{dv}{dt} \rightarrow \frac{dv}{dt} = \frac{i_C}{C}$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{2}{0.1} = 20 \frac{\text{V}}{\text{s}}$$

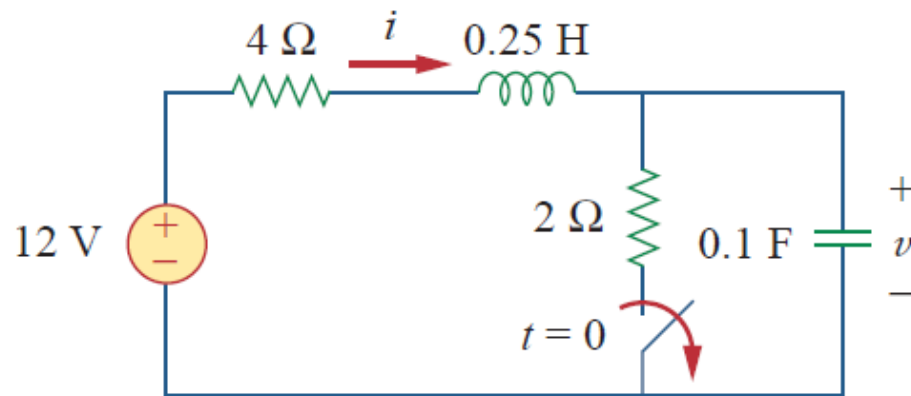
$$v_L = L \frac{di}{dt} \rightarrow \frac{di}{dt} = \frac{v_L}{L}$$

$$(KVL): -12 + 4i(0^+) + v_L(0^+) + v(0^+) = 0$$

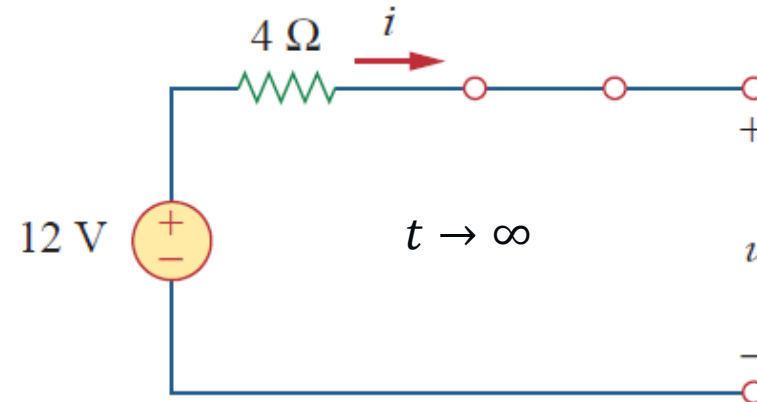
$$v_L(0^+) = 12 - 8 - 4 = 0$$

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{0}{0.25} = 0 \frac{\text{A}}{\text{s}}$$

# Initial and Final Conditions

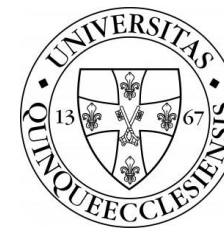


(c):  $i(\infty) = ?$ ,  $v(\infty) = ?$



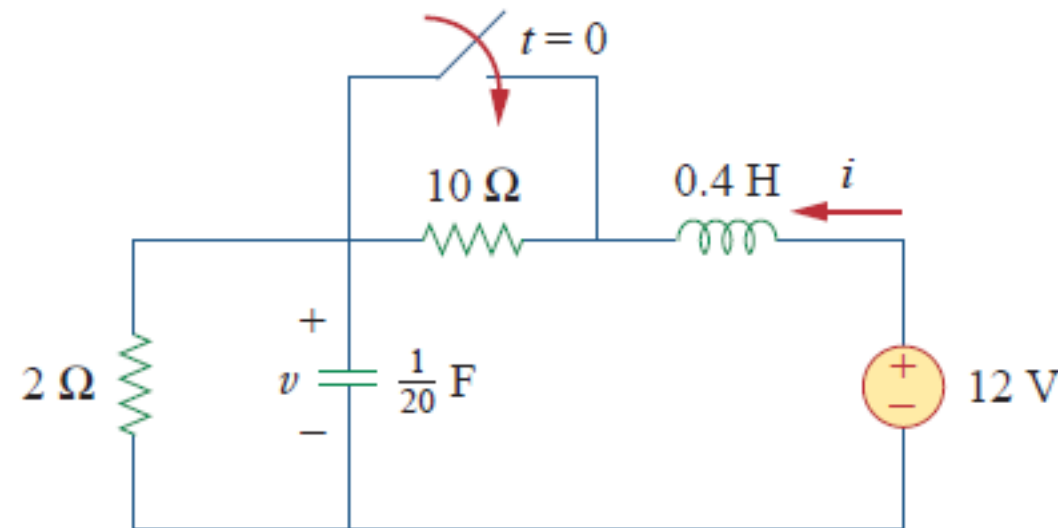
$i(\infty) = 0$ ,  $v(\infty) = 12 V$

# Initial and Final Conditions



**SOC.02** – The switch was open for a long time but closed at  $t = 0$ .

(a):  $i(0^+) = ?$ ,  $v(0^+) = ?$ , (b):  $\frac{di(0^+)}{dt} = ?$ ,  $\frac{dv(0^+)}{dt} = ?$ , (c):  $i(\infty) = ?$ ,  $v(\infty) = ?$



**Solution** (a):  $i(0^+) = 1 \text{ A}$ ,  $v(0^+) = 2 \text{ V}$ , (b):  $\frac{di(0^+)}{dt} = 25 \frac{\text{A}}{\text{s}}$ ,  $\frac{dv(0^+)}{dt} = 0 \frac{\text{V}}{\text{s}}$ , (c):  $i(\infty) = 6 \text{ A}$ ,  $v(\infty) = 12 \text{ V}$

# Initial and Final Conditions

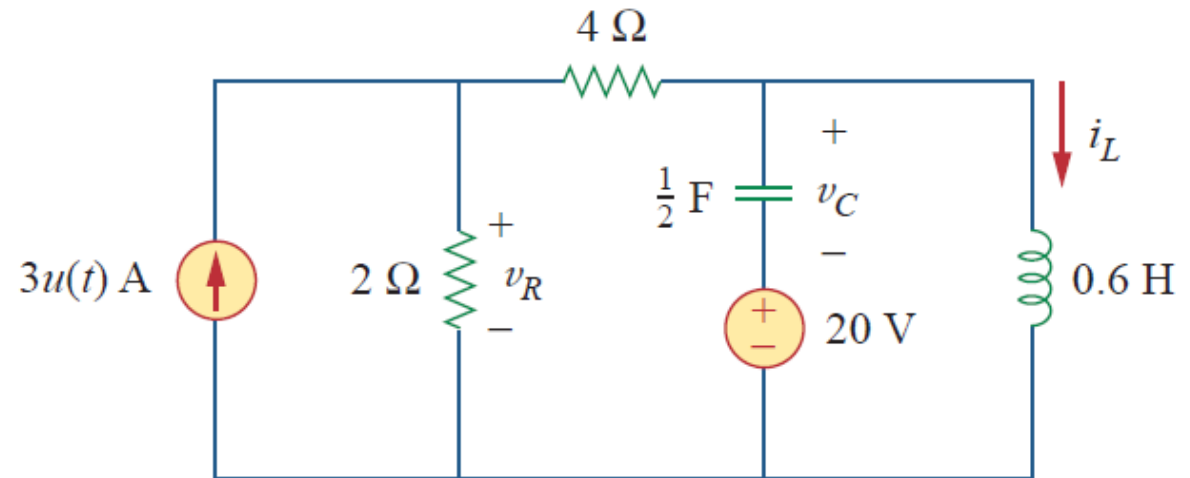


## SOC.03

(a):  $i_L(0^+) = ?$ ,  $v_C(0^+) = ?$ ,  $v_R(0^+) = ?$

(b):  $\frac{di_L(0^+)}{dt} = ?$ ,  $\frac{dv_C(0^+)}{dt} = ?$ ,  $\frac{dv_R(0^+)}{dt} = ?$

(c):  $i_L(\infty) = ?$ ,  $v_C(\infty) = ?$ ,  $v_R(\infty) = ?$



## Solution

$$i_L(0^-) = 0, v_C(0^-) = -20 \text{ V}, v_R(0^-) = 0$$

$$i_L(0^+) = i_L(0^-) = 0, v_C(0^+) = v_C(0^-) = -20 \text{ V}$$

# Initial and Final Conditions



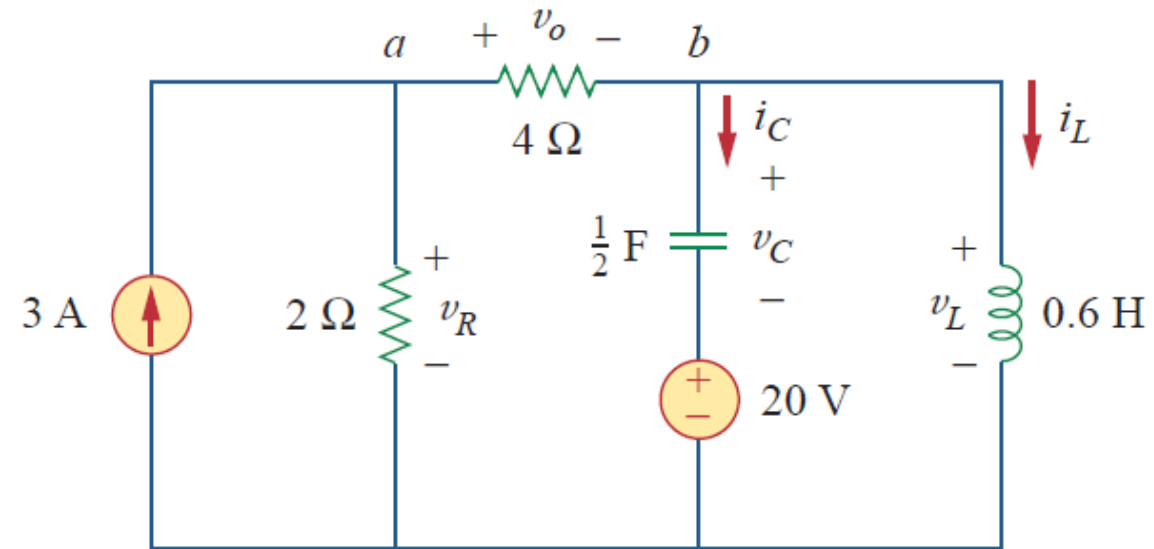
$$v_R(0^+) = ?$$

$$\text{(Node a): } 3 = \frac{v_R(0^+)}{2} + \frac{v_0(0^+)}{4}$$

$$\text{(KVL2): } -v_R(0^+) + v_0(0^+) + v_C(0^+) + 20 = 0$$

$$\text{(but): } v_C(0^+) = -20 \text{ V} \rightarrow v_R(0^+) = v_0(0^+)$$

$$\text{(but} \rightarrow \text{Node a): } v_R(0^+) = v_0(0^+) = 4 \text{ V}$$



$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} \quad \text{(KVL3): } v_L(0^+) = v_C(0^+) + 20 = 0 \rightarrow \frac{di_L(0^+)}{dt} = 0$$

$$\frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C} \quad \text{(Node b): } \frac{v_0(0^+)}{4} = i_C(0^+) + i_L(0^+) \rightarrow \frac{4}{4} = i_C(0^+) + 0 \rightarrow i_C(0^+) = 1 \text{ A} \rightarrow \frac{dv_C(0^+)}{dt} = \frac{1}{0.5} = 2 \frac{\text{V}}{\text{s}}$$

# Initial and Final Conditions



$$\frac{dv_R(0^+)}{dt} = ? \quad (\text{Node } a): 3 = \frac{v_R(0^+)}{2} + \frac{v_0(0^+)}{4} \quad (\dots \leftarrow \frac{d}{dt})$$

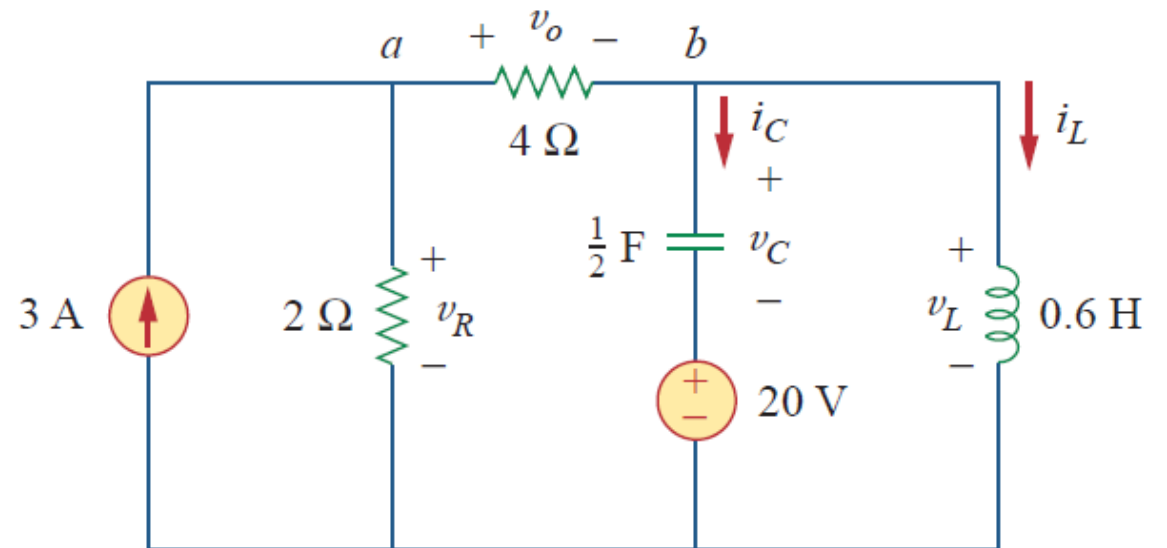
$$(1): 0 = 2 \frac{dv_R(0^+)}{dt} + \frac{dv_0(0^+)}{dt}$$

$$(KVL2): -v_R(0^+) + v_0(0^+) + v_C(0^+) + 20 = 0 \quad (\dots \leftarrow \frac{d}{dt})$$

$$-\frac{dv_R(0^+)}{dt} + \frac{dv_0(0^+)}{dt} + \frac{dv_C(0^+)}{dt} = 0$$

$$\frac{dv_C(0^+)}{dt} = 2 \rightarrow (2): \frac{dv_R(0^+)}{dt} = 2 + \frac{dv_0(0^+)}{dt}$$

$$(1 \rightarrow 2): \frac{dv_R(0^+)}{dt} = \frac{2V}{3s}$$

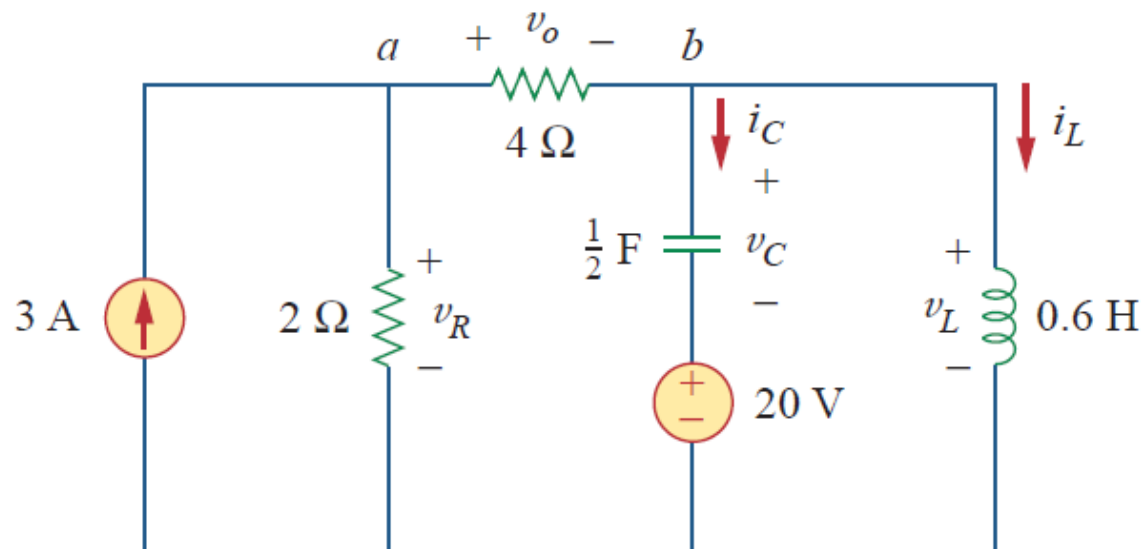


$$\frac{di_R(0^+)}{dt} = ?$$

$$i_R(0^+) = \frac{v_R(0^+)}{2} \rightarrow \frac{di_R(0^+)}{dt} = \frac{1}{2} \frac{dv_R(0^+)}{dt} = \frac{1A}{3s}$$



# Initial and Final Conditions



$$i_L(\infty) = 3 \cdot \frac{2}{2 + 4} = 1 \text{ A}$$

$$v_C(\infty) = -20 \text{ V}$$

$$v_R(\infty) = 3 \cdot \frac{4}{2 + 4} \cdot 2 = 4 \text{ V}$$

# Initial and Final Conditions

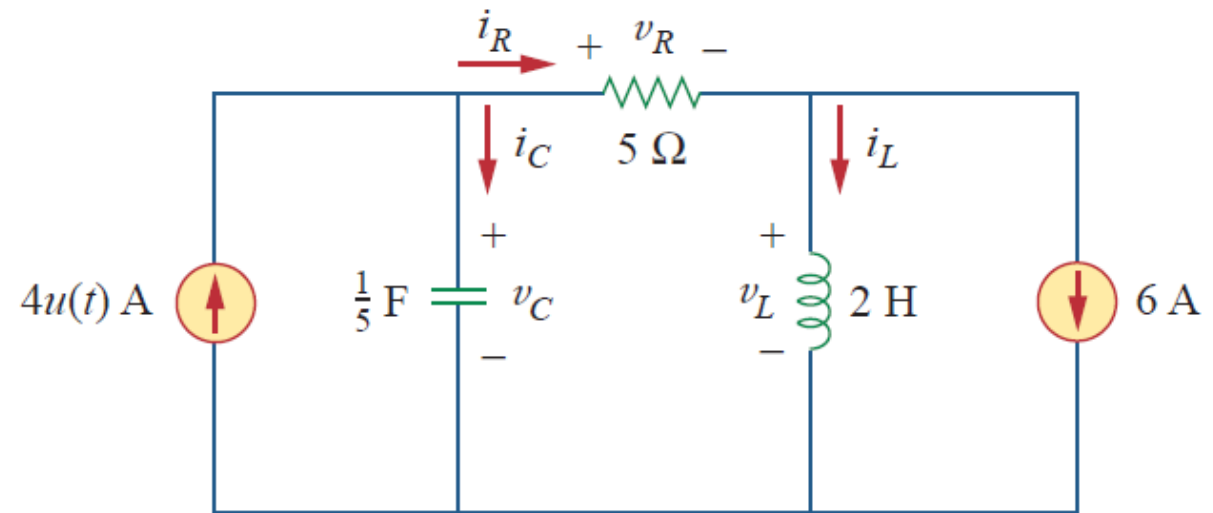


SOC.04

(a):  $i_L(0^+) = ?$ ,  $v_C(0^+) = ?$ ,  $v_R(0^+) = ?$

(b):  $\frac{di_L(0^+)}{dt} = ?$ ,  $\frac{dv_C(0^+)}{dt} = ?$ ,  $\frac{dv_R(0^+)}{dt} = ?$

(c):  $i_L(\infty) = ?$ ,  $v_C(\infty) = ?$ ,  $v_R(\infty) = ?$



**Solution** (a) -6 A, 0, 0, (b) 0, 20 V/s, 0, (c) -2 A, 20 V, 20 V

# Source-Free Series RLC Circuit



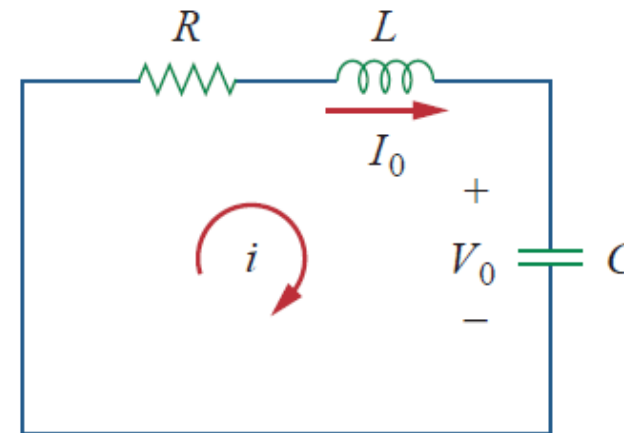
**SOC.05** – Calculate the characteristic roots of the circuit.  $R = 40 \Omega$ ,  $L = 4 H$ ,  $C = 1/4 F$ . Is the natural response overdamped, underdamped, or critically damped?

**Solution**

$$\delta = \frac{R}{2L} = 5, \omega_0 = \frac{1}{\sqrt{LC}} = 1$$

$$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -5 \pm \sqrt{25 - 1}$$

$$s_1 = -0.101, \quad s_2 = -9.899$$



$\delta > \omega \rightarrow$  *overdamped (real negative roots)*

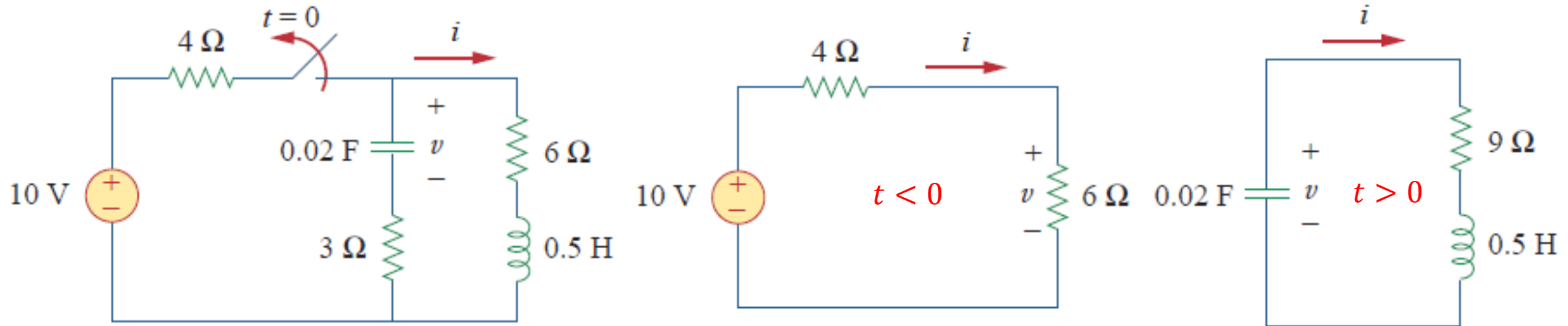
**SOC.06** – Calculate the characteristic roots of the series RLC circuit if the circuit elements are the followings.  $R = 10 \Omega$ ,  $L = 5 H$ ,  $C = 2 mF$ . Is the natural response overdamped, underdamped, or critically damped?

**Solution**  $\delta = 1, \omega_0 = 10, s_{1,2} = -1 \pm j9.95, \text{underdamped}$

# Source-Free Series RLC Circuit



**SOC.07** – Find  $i(t)$  in the circuit for  $t > 0$ . (Steady-state has been reached at  $t = 0^-$ .)



**Solution**  $i(0) = \frac{10}{4 + 6} = 1 \text{ A}, \quad v(0) = 6i = 6 \text{ V}$

$$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -9 \pm \sqrt{81 - 100}$$

$$\delta = \frac{R}{2L} = \frac{9}{2 \cdot 0.5} = 9, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5 \cdot 0.02}} = 10$$

$$s_{1,2} = -9 \pm j4.359, \quad \delta < \omega \rightarrow \textit{underdamped}$$

# Source-Free Series RLC Circuit



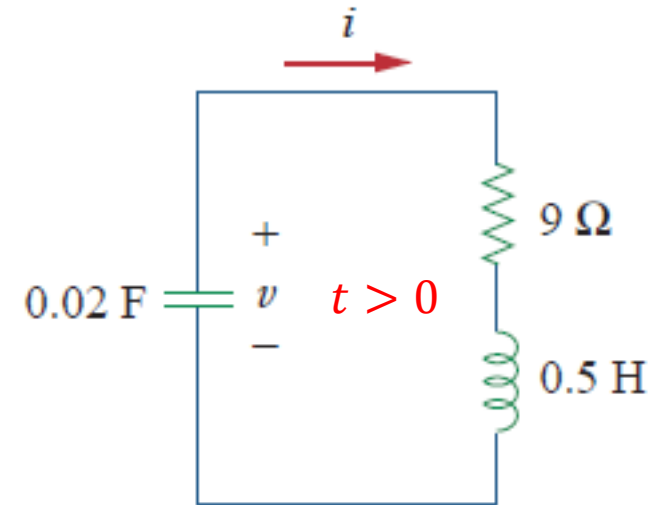
$$s_{1,2} = -9 \pm j4.359, \delta < \omega \rightarrow \text{underdamped}$$

$$i(t) = e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t)$$

( $A_1 = ?$ ,  $A_2 = ?$ )  $\leftarrow$  from initial conditions

$$i(0) = 1 = e^0(A_1 \cos 0 + A_2 \sin 0) = A_1$$

$$(KVL): \frac{di(0)}{dt} = -\frac{1}{L}[Ri(0) + v(0)] = -2[9 \cdot 1 - 6] = -6 \frac{A}{s}$$



$$\frac{di(t)}{dt} = -9e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t) + e^{-9t}4.359(-A_1 \sin 4.359t + A_2 \cos 4.359t)$$

$$t = 0 \rightarrow -6 = -9(A_1 \cos 0 + A_2 \sin 0) + 4.359(-A_1 \sin 0 + A_2 \cos 0) = -9A_1 + 4.359A_2$$

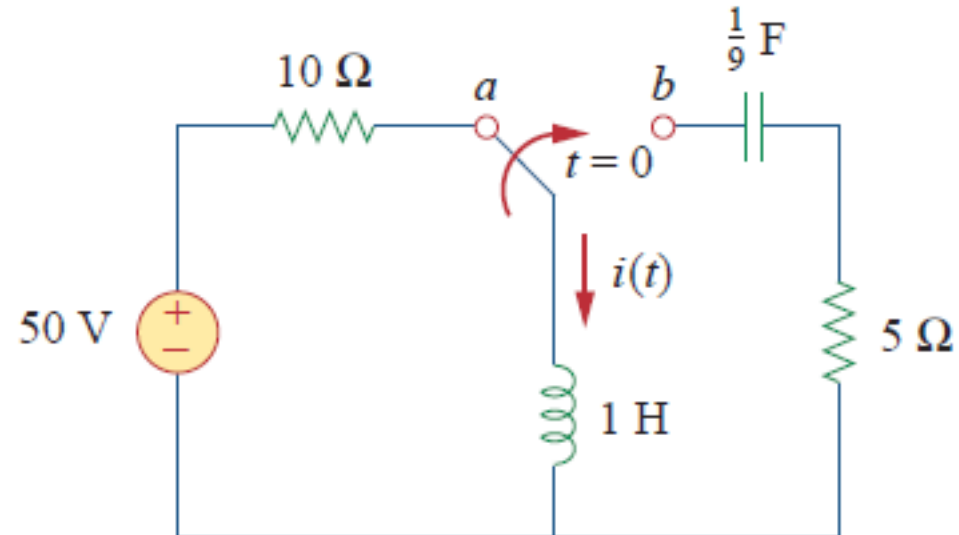
$$A_1 = 1 \rightarrow -6 = -9 + 4.359A_2 \rightarrow A_2 = 0.6882$$

$$i(t) = e^{-9t}(\cos 4.359t + 0.6882 \sin 4.359t) A$$

# Source-Free Series RLC Circuit



**SOC.08** – Find  $i(t)$  in the circuit for  $t > 0$ . (Steady-state has been reached at  $t = 0^-$ .)



**Solution**  $i(t) = e^{-2.5t}(5 \cos 1.6583t - 7.5378 \sin 1.6583t) \text{ A}$

# Source-Free Parallel RLC Circuit



## SOC.09

In a parallel RLC circuit  $R = 500 \Omega$ ,  $C = 1 \mu F$ ,  $L = 200 mH$ . The initial conditions are  $I_0 = 50 mA$ ,  $V_0 = 0$ . Find the zero-input response of inductor current, resistor current, and capacitor voltage.

## Solution

$$LCs^2 + GLs + 1 = 2 \cdot 10^{-7}s^2 + 4 \cdot 10^{-4}s + 1 = 0 \quad \delta = \frac{G}{2C}, \quad \omega_0 = \frac{1}{\sqrt{LC}}, \quad s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$$

$$s_{1,2} = \frac{4 \cdot 10^{-4} \pm \sqrt{16 \cdot 10^{-8} - 8 \cdot 10^{-7}}}{4 \cdot 10^{-7}} = -1000 \pm j2000 \leftarrow \text{underdamped}$$

$$i_L(t) = K_1 e^{-1000t} \cos 2000t + K_2 e^{-1000t} \sin 2000t, t \geq 0$$

$$i_L(0) = I_0 = K_1 e^0 \cos 0 + K_2 e^0 \sin 0, t \geq 0 \quad v_C(t) = L \frac{di_L}{dt}$$

$$\frac{di_L}{dt} = -2000K_1 e^{-1000t} \sin 2000t - 1000K_1 e^{-1000t} \cos 2000t - 1000K_2 e^{-1000t} \sin 2000t + 2000K_2 e^{-1000t} \cos 2000t$$

# Source-Free Parallel RLC Circuit



$$\left. \frac{di_L}{dt} \right|_{t=0} = -2000K_1 e^0 \sin 0 - 1000K_1 e^0 \cos 0 - 1000K_2 e^0 \sin 0 + 2000K_2 e^0 \cos 0 = -1000K_1 + 2000K_2 = 0$$

$$i_L(t) = 50e^{-1000t} \cos 2000t + 25e^{-1000t} \sin 2000t \text{ mA}, t \geq 0$$

$$v_C(t) = v_L(t) = L \frac{di_L}{dt} = -25e^{-1000t} \sin 2000t \text{ V}, t \geq 0$$

$$i_R(t) = \frac{v_L(t)}{R} = -50e^{-1000t} \sin 2000t \text{ mA}, t \geq 0$$



# Source-Free Parallel RLC Circuit

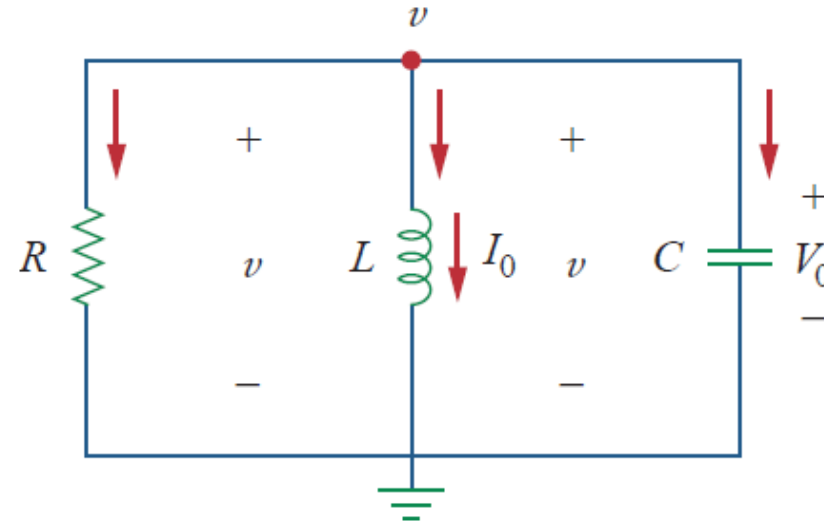


## SOC.10

In a parallel RLC circuit  $R = 2 \Omega$ ,  $C = 25 \text{ mF}$ ,  $L = 400 \text{ mH}$ . The initial conditions are  $I_0 = 10 \text{ mA}$ ,  $V_0 = 0$ . Find the zero-input response of capacitor voltage.

### Solution

$$v(t) = -400 t e^{-105t} u(t) \text{ mV}$$



# Step Response of a Series RLC Circuit



**SOC.11** – The circuit is in the zero state at  $t = 0$ . Find the capacitor voltage for  $t \geq 0$ .

$$V_S = 10 \text{ V}, \quad C = 500 \text{ nF}, \quad L = 2 \text{ H}, \quad R = 1 \text{ k}\Omega$$

**Solution**  $LC \frac{d^2v}{dt^2} + RC \frac{dv}{dt} + v = V_S$

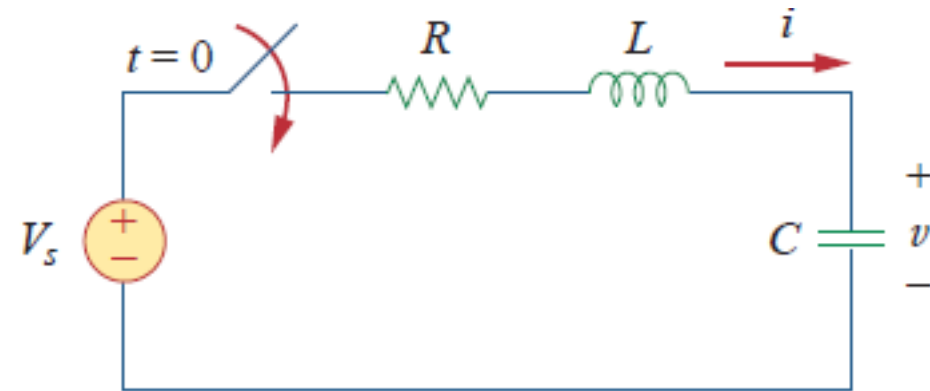
$$10^{-6} \frac{d^2v}{dt^2} + 0.5 \cdot 10^{-3} \frac{dv}{dt} + v = 10, t \geq 0$$

$$\frac{d^2v}{dt^2} + 500 \frac{dv}{dt} + 10^6 \cdot v = 0 \rightarrow$$

$$\delta = \frac{R}{2L} = 250 \frac{\text{rad}}{\text{s}}, \omega_0 = \frac{1000 \text{rad}}{\text{s}} \quad (\rightarrow \text{underdamped!}), \omega_d = \sqrt{\omega_0^2 - \delta^2} = 968 \text{ rad/s}$$

$$v_{tr}(t) = K_1 e^{-250t} \cos 968t + K_2 e^{-250t} \sin 968t, t \geq 0 \quad v(t) = v_{ss}(t) + v_{tr}(t)$$

$$v(\infty) = V_S = 10 \text{ V} \rightarrow v(t) = 10 + K_1 e^{-250t} \cos 968t + K_2 e^{-250t} \sin 968t, t \geq 0$$



# Step Response of a Series RLC Circuit



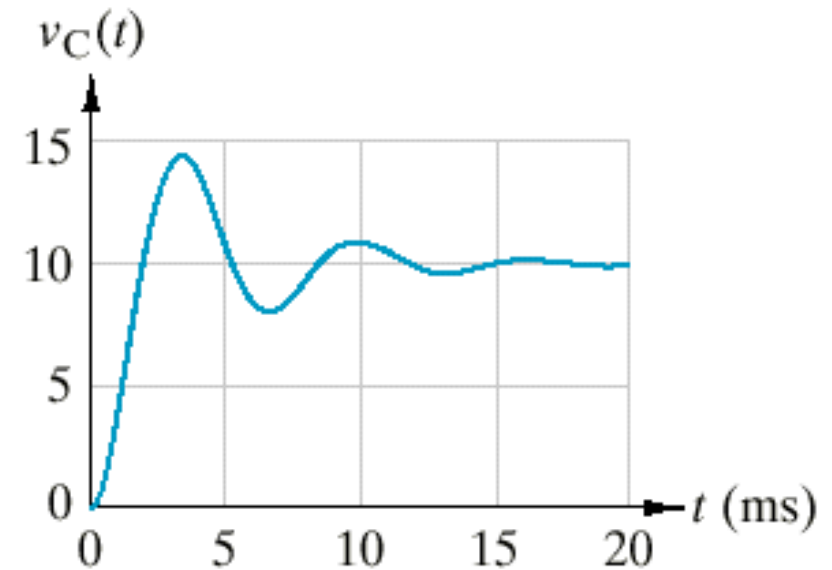
The constants ( $K_1$  and  $K_2$ ) are determined by the initial conditions.

$$v_C(0) = 10 + K_1 = 0$$

$$\left. \frac{dv_C}{dt} \right|_{t=0} = -250K_1 + 968K_2 = 0$$

$$K_1 = -10, \quad K_2 = -2.58$$

$$v(t) = 10 - 10e^{-250t} \cos 968t - 2.58e^{-250t} \sin 968t, t \geq 0$$



# Step Response of a Series RLC Circuit



**SOC.12** – Find  $v(t)$  and  $i(t)$  for  $t > 0$ . Consider the cases  $R = 5 \Omega$ ,  $R = 4 \Omega$ ,  $R = 1 \Omega$ .

**Case**  $R = 5 \Omega$

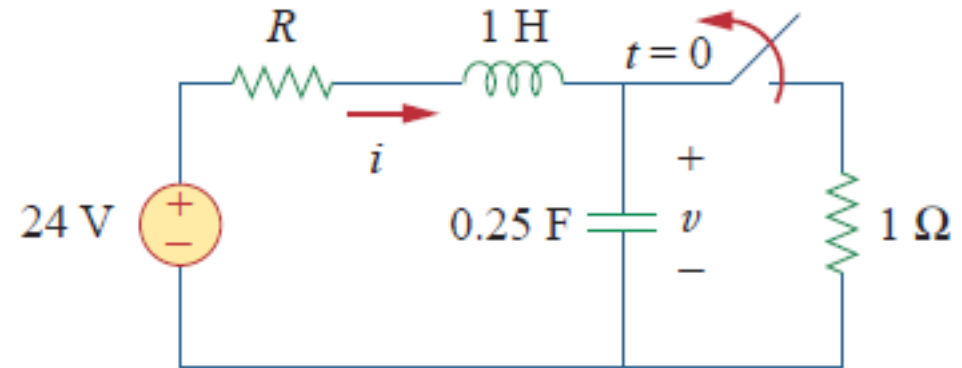
$$i(0) = \frac{24}{5 + 1} = 4 \text{ A}, v(0) = 1 \cdot i = 4 \text{ V}$$

$$\delta = \frac{R}{2L} = \frac{5}{2 \cdot 1} = 2.5, \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \cdot 0.25}} = 2$$

$$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -1, -4 \quad (\leftarrow \text{overdamped natural response})$$

$$v(t) = v_{SS} + (A_1 e^{-t} + A_2 e^{-4t}) \quad (\leftarrow \text{total response}) \quad v_{SS} = 24 \text{ V} \quad (\leftarrow \text{steady - state response})$$

$$t = 0 \rightarrow v(0) = 4 = 24 + (A_1 e^0 + A_2 e^0) \rightarrow A_1 + A_2 = -20$$



# Step Response of a Series RLC Circuit



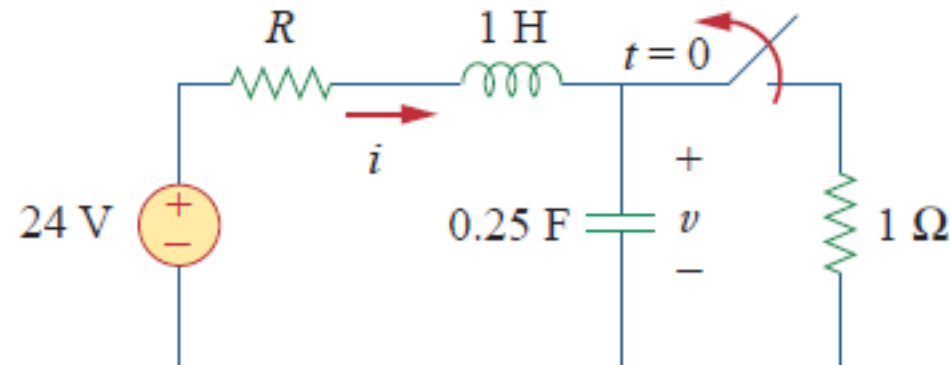
$$i(0) = C \frac{dv(0)}{dt} = 4 \rightarrow \frac{dv(0)}{dt} = \frac{4}{C} = \frac{4}{0.25} = 16$$

$$\frac{dv}{dt} = -A_1 e^{-t} - 4A_2 e^{-4t} \rightarrow \frac{dv(0)}{dt} = 16 = -A_1 - 4A_2$$

$$\left. \begin{array}{l} A_1 + A_2 = -20 \\ 16 = -A_1 - 4A_2 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} A_1 = -64/3 \\ A_2 = 4/3 \end{array} \right.$$

$$v(t) = 24 + \frac{4}{3}(-16e^{-t} + e^{-4t}) \text{ V}$$

$$i(t) = C \frac{dv(t)}{dt} = \frac{4}{3}(4e^{-t} - e^{-4t}) \text{ A}$$



Note:  $i(0) = 4 \text{ A}$  as expected.

# Step Response of a Series RLC Circuit



Case  $R = 4 \Omega$

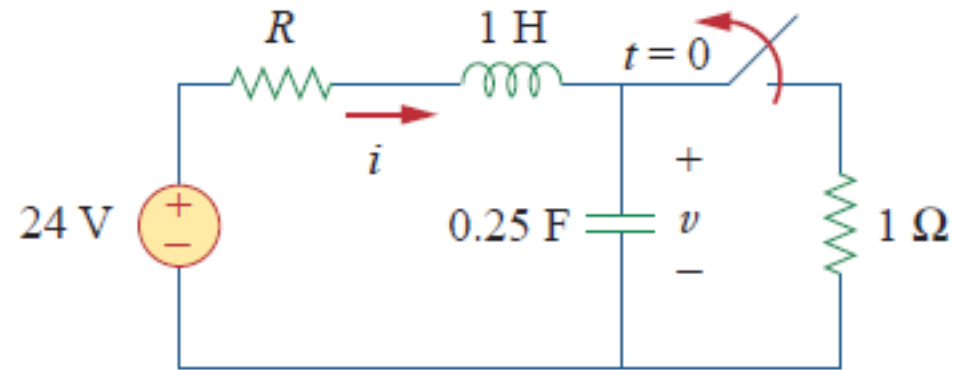
$$i(0) = \frac{24}{4 + 1} = 4.8 \text{ A}, v(0) = 1 \cdot i = 4.8 \text{ V}$$

$$\delta = \frac{R}{2L} = \frac{4}{2 \cdot 1} = 2, \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \cdot 0.25}} = 2$$

$$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -2 \quad (\leftarrow \text{critically damped natural response})$$

$$v(t) = v_{SS} + (A_1 + A_2 t)e^{-2t} \quad (\leftarrow \text{total response}) \quad v_{SS} = 24 \text{ V} \quad (\leftarrow \text{steady - state response})$$

$$t = 0 \rightarrow v(0) = 4.8 = 24 + A_1 \rightarrow A_1 = -19.2$$



# Step Response of a Series RLC Circuit



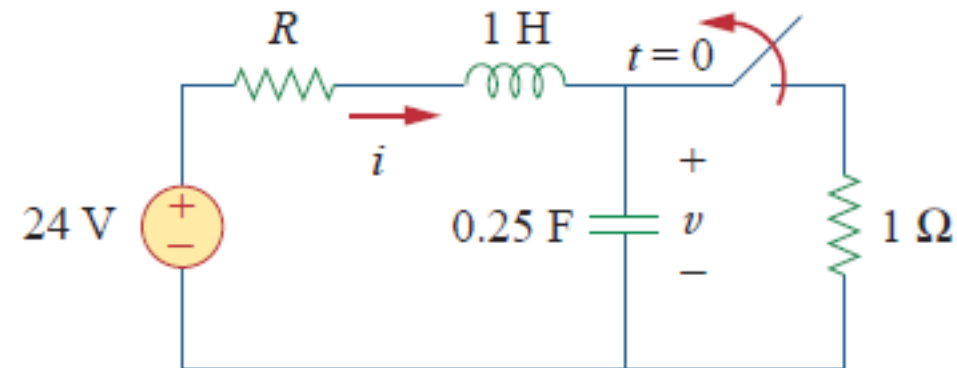
$$i(0) = C \frac{dv(0)}{dt} = 4.8 \rightarrow \frac{dv(0)}{dt} = \frac{4.8}{C} = \frac{4.8}{0.25} = 19.2$$

$$\frac{dv}{dt} = (-2A_1 - 2tA_2 + A_2)e^{-2t} \rightarrow \frac{dv(0)}{dt} = 19.2 = -2A_1 + A_2$$

$$\left. \begin{array}{l} A_1 = -19.2 \\ 19.2 = -2A_1 + A_2 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} A_1 = -19.2 \\ A_2 = -19.2 \end{array} \right.$$

$$v(t) = 24 - 19.2(1 + t)e^{-2t} \text{ V}$$

$$i(t) = C \frac{dv(t)}{dt} = (4.8 + 9.6t)e^{-2t} \text{ A}$$



Note:  $i(0) = 4.8 \text{ A}$  as expected.

# Step Response of a Series RLC Circuit



Case  $R = 1 \Omega$

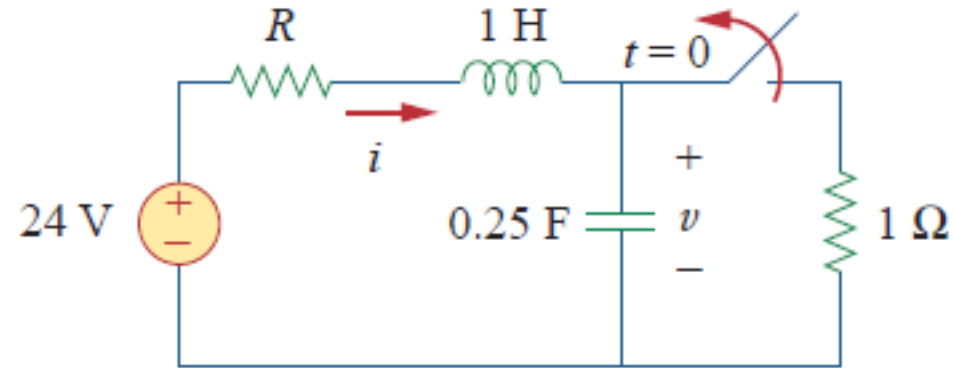
$$i(0) = \frac{24}{1+1} = 12 \text{ A}, v(0) = 1 \cdot i = 12 \text{ V}$$

$$\delta = \frac{R}{2L} = \frac{1}{2 \cdot 1} = 0.5, \quad \omega_0 = 2$$

$$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -0.5 \pm j1.936 \quad (\leftarrow \textit{underdamped natural response})$$

$$v(t) = 24 + (A_1 \cos 1.936t + A_2 \sin 1.936t)e^{-0.5t} \quad (\leftarrow \textit{total response})$$

$$t = 0 \rightarrow v(0) = 12 = 24 + A_1 \rightarrow A_1 = -12$$





# Step Response of a Series RLC Circuit



$$i(0) = C \frac{dv(0)}{dt} = 12 \rightarrow \frac{dv(0)}{dt} = \frac{12}{C} = \frac{12}{0.25} = 48$$

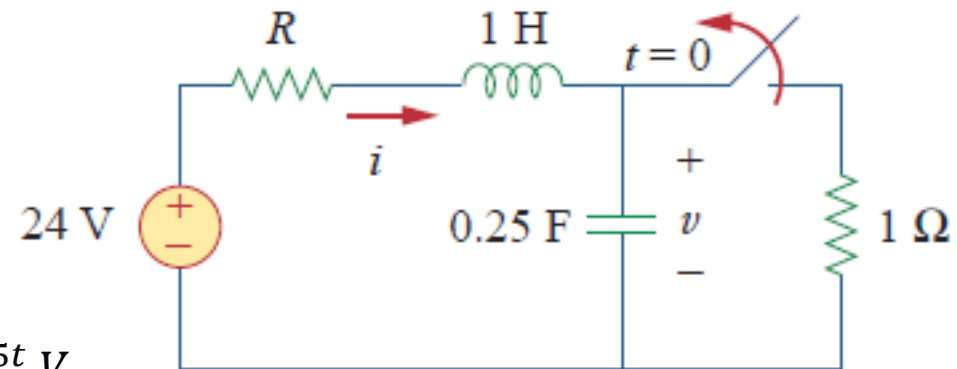
$$\frac{dv}{dt} = e^{-0.5t}(-1.936A_1 \sin 1.936t + 1.936A_2 \cos 1.936t) - 0.5e^{-0.5t}(A_1 \cos 1.936t + A_2 \sin 1.936t)$$

$$\rightarrow \frac{dv(0)}{dt} = 48 = (-0 + 1.936A_2) - 0.5(A_1 + 0)$$

$$\left. \begin{array}{l} A_1 = -12 \\ 48 = 1.936A_2 - 0.5A_1 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} A_1 = -12 \\ A_2 = +21.694 \end{array} \right.$$

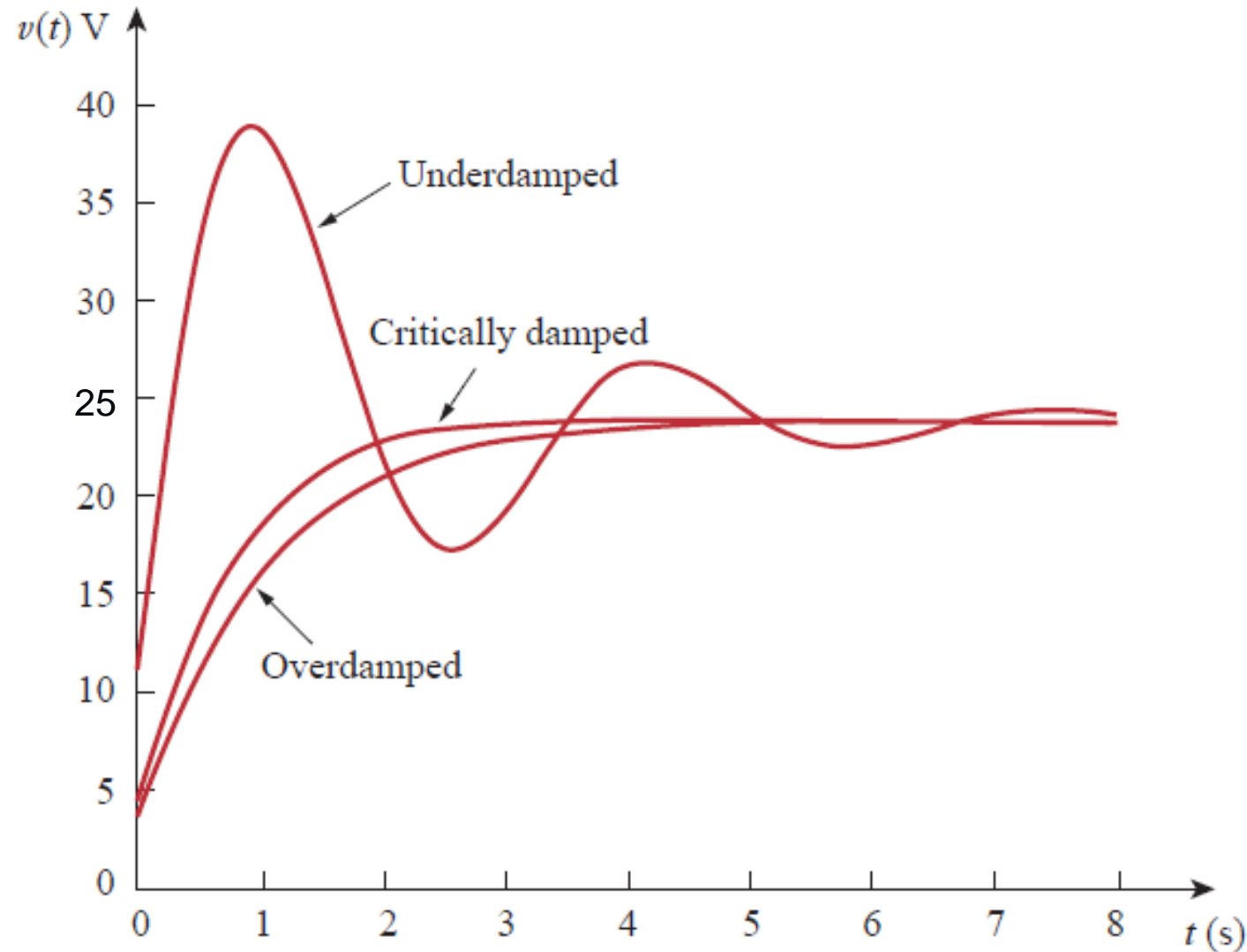
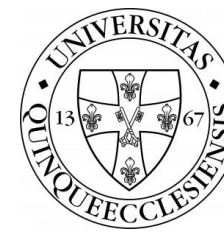
$$v(t) = 24 + (21.694 \sin 1.936t - 12 \cos 1.936t)e^{-0.5t} \text{ V}$$

$$i(t) = C \frac{dv(t)}{dt} = (3.1 \sin 1.936t + 12 \cos 1.936t)e^{-0.5t} \text{ A}$$



Note:  $i(0) = 12 \text{ A}$  as expected.

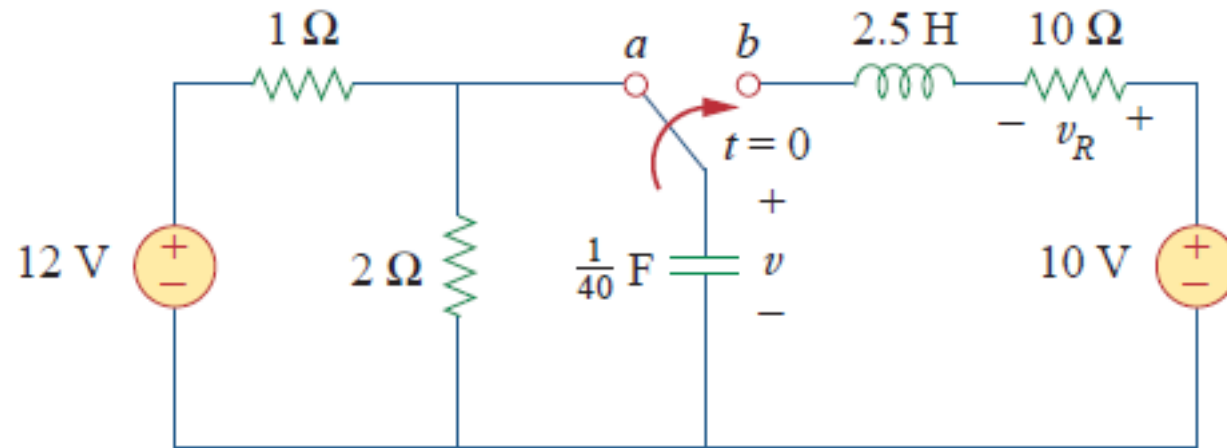
# Step Response of a Series RLC Circuit



# Step Response of a Series RLC Circuit

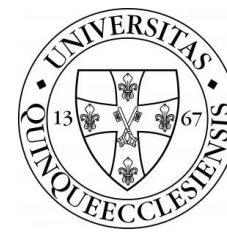


SOC.13 – Find  $v(t)$  and  $v_R(t)$  for  $t > 0$ .



**Solution**  $v(t) = 10 - (1.1547 \sin 3.464t + 2 \cos 3.464t)e^{-2t} \text{ V}$   
 $v_R(t) = 2.31e^{-2t} \sin 3.464t \text{ V}$

# Step Response of a Parallel RLC Circuit



**SOC.14** – Find  $i(t)$  and  $i_R(t)$  for  $t > 0$ .

**Solution**  $t < 0 \rightarrow i(0) = 4 \text{ A}$

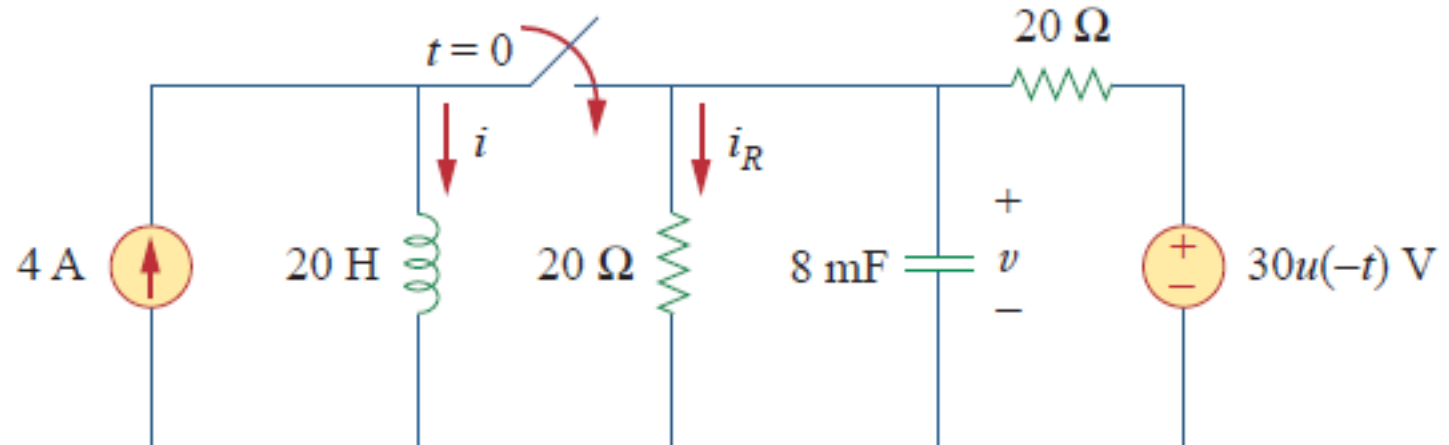
$$t < 0 \rightarrow v(0) = 30 \cdot \frac{20}{20 + 20} = 15 \text{ V}$$

$$t > 0 \rightarrow R = 20 \times 20 = 10 \Omega$$

$$\delta = \frac{1}{2RC} = \dots = 6.25$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \dots = 2.5$$

$$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = \begin{cases} -11.978 \\ -0.5218 \end{cases}$$

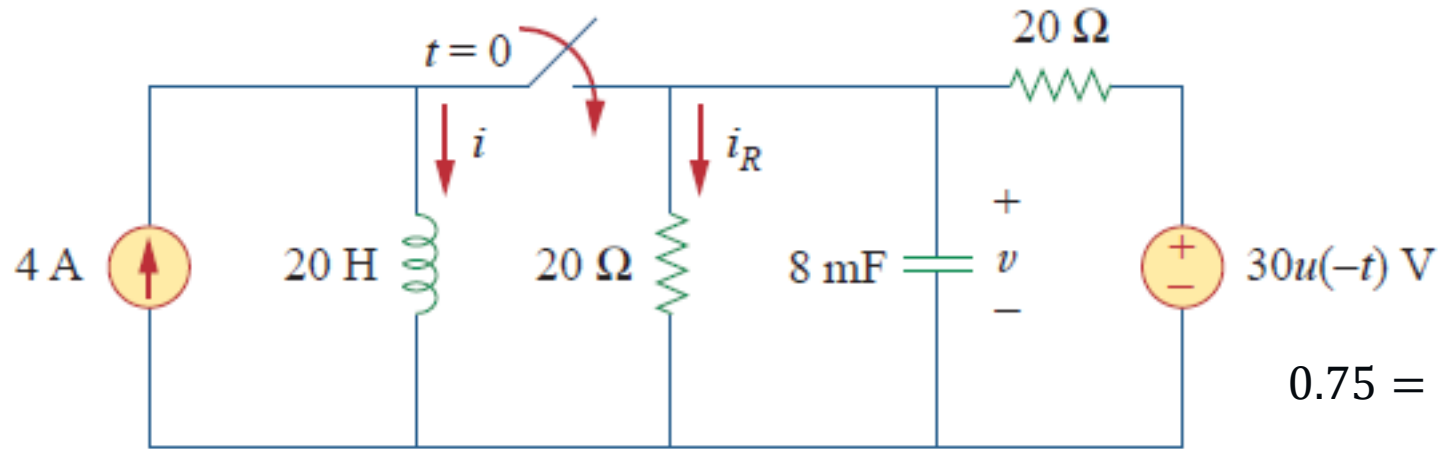


$\delta > \omega \rightarrow$  overdamped (real negative roots)

$$i(t) = I_S + A_1 e^{-11.978t} + A_2 e^{-0.5218t}$$

$$i(0) = 4 = 4 + A_1 e^0 + A_2 e^0 \rightarrow A_2 = -A_1$$

# Step Response of a Parallel RLC Circuit



$$(but): \frac{di(0)}{dt} = \frac{V(0)}{L} = \frac{15}{20} = 0.75$$

$$0.75 = (11.978 - 0.5218)A_2 \rightarrow A_2 = 0.0655$$

$$i(0) = 4 + A_1e^0 + A_2e^0 \rightarrow A_2 = -A_1$$

$$A_1 = -0.0655$$

$$\frac{di}{dt} = -11.978A_1e^{-11.978t} - 0.5218A_2e^{-0.5218t}$$

$$i(t) = 4 + 0.0655(e^{-0.5218t} - e^{-11.978t}) A$$

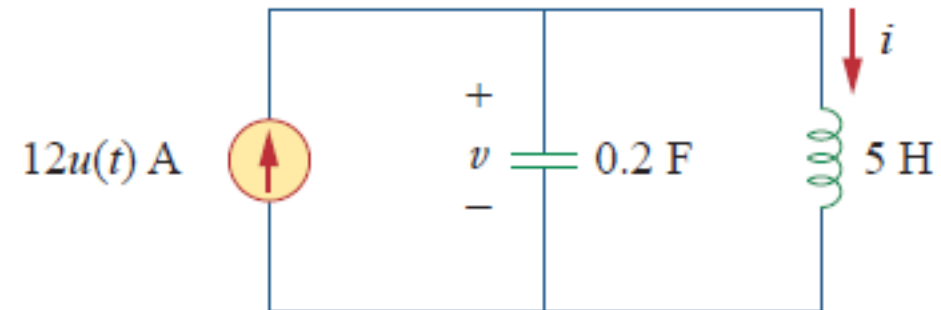
$$\frac{di(0)}{dt} = -11.978A_1e^0 - 0.5218A_2e^0$$

$$i_R(t) = \frac{v(t)}{20} = L \frac{di}{dt} = 0.785e^{-11.978t} - 0.0342e^{-0.5218t} A$$

# Step Response of a Parallel RLC Circuit



SOC.15 – Find  $i(t)$  and  $v(t)$  for  $t > 0$ .



**Solution**  $12(1 - \cos t)$  A,  $60 \sin t$  V

# General Second-Order Circuits



**SOC.16** – Find the complete response  $v(t)$  and then  $i(t)$  for  $t > 0$ .

**(1) Initial and final conditions**

$$v(0^+) = v(0^-) = 12 \text{ V}, \quad i(0^+) = i(0^-) = 0$$

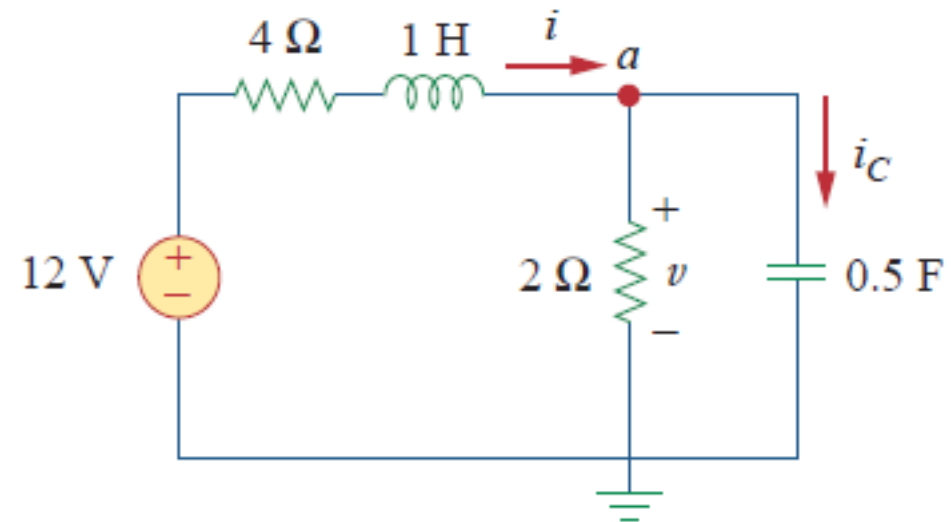
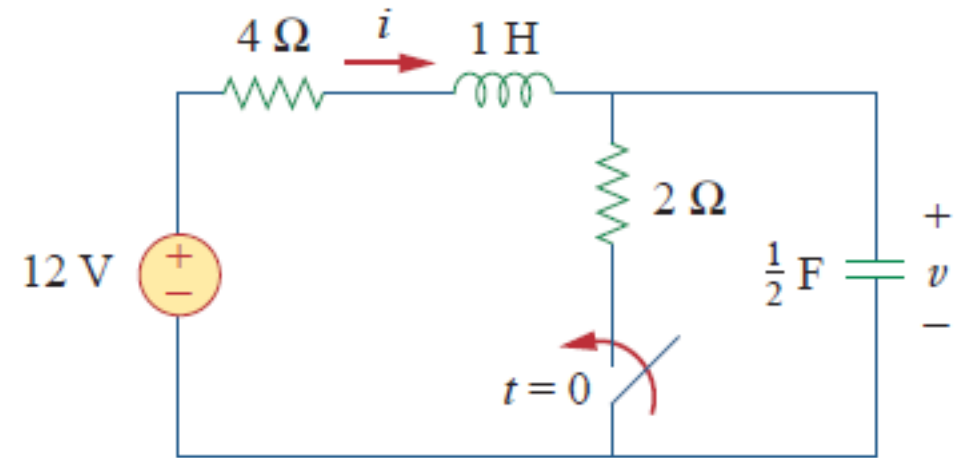
$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C}$$

**(node a):**  $i(0^+) = i_C(0^+) + \frac{v(0^+)}{2}$

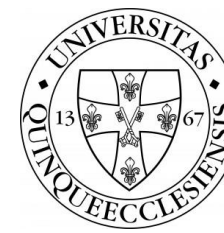
$$0 = i_C(0^+) + \frac{12}{2} \rightarrow i_C(0^+) = -6 \text{ A}$$

$$\frac{dv(0^+)}{dt} = \frac{-6}{0.5} = -12 \frac{\text{V}}{\text{s}}$$

$$i(\infty) = \frac{12}{4 + 2} = 2 \text{ A}, \quad v(\infty) = 2i(\infty) = 4 \text{ V}$$



# General Second-Order Circuits



(2) Transient response (source-free)

$$(KCL): i = \frac{v}{2} + \frac{1}{2} \frac{dv}{dt}$$

$$(KVL): 4i + 1 \frac{di}{dt} + v = 0$$

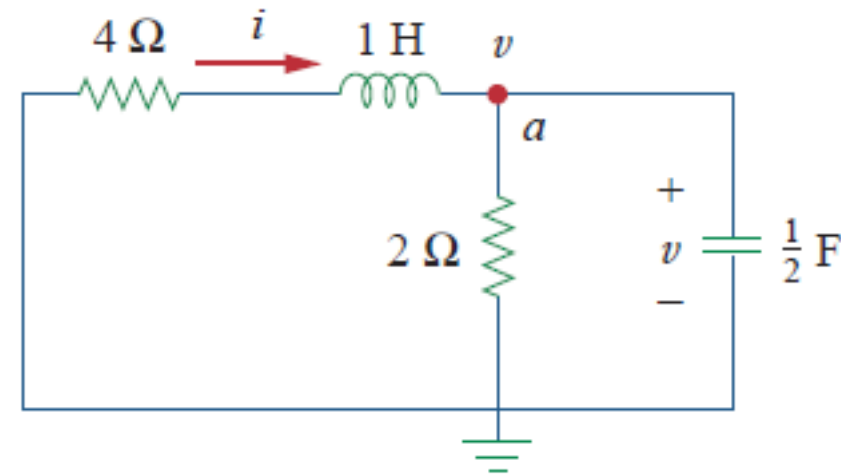
$$(KCL \rightarrow KVL): 2v + 2 \frac{dv}{dt} + \frac{1}{2} \frac{dv}{dt} + \frac{1}{2} \frac{d^2v}{dt^2} + v = 0$$

$$\frac{d^2v}{dt^2} + 5 \frac{dv}{dt} + 6v = 0 \rightarrow \text{characteristic eq.}$$

$$s^2 + 5s + 6 = 0 \rightarrow s_{1,2} = -2, \quad -3 \text{ (overdamped)}$$

$$v_{tr}(t) = Ae^{-2t} + Be^{-3t}$$

(3) Steady-state response  $v_{ss}(t) = v(\infty) = 4$



(4) Complete response

$$v(t) = v_{ss}(t) + v_{tr}(t) = 4 + Ae^{-2t} + Be^{-3t}$$



# General Second-Order Circuits

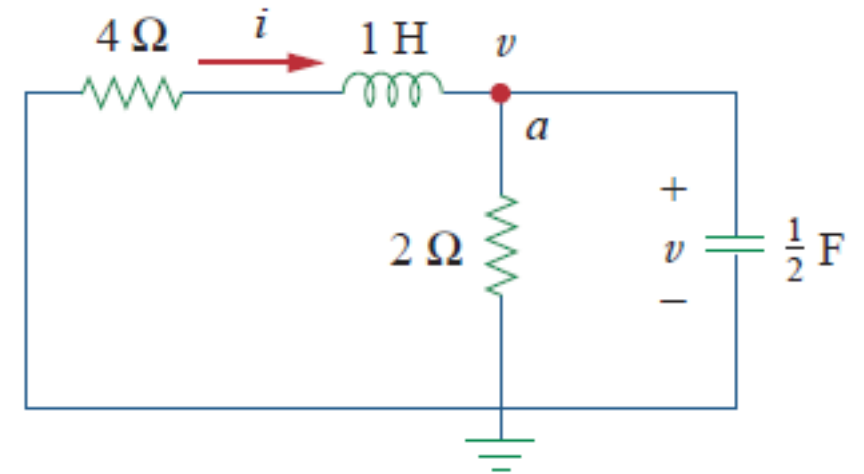


(5) Find A, B using initial values  $v(t) = 4 + Ae^{-2t} + Be^{-3t}$

$$v(0) = 12 = 4 + Ae^0 + Be^0 \rightarrow 12 = 4 + A + B \rightarrow A + B = 8$$

$$\frac{dv}{dt} = -2Ae^{-2t} - 3Be^{-3t} \rightarrow -12 = -2A - 3B \rightarrow 2A + 3B = 12$$

$$A = 12, B = -4 \quad v(t) = 4 + 12e^{-2t} - 4e^{-3t} \text{ V}, \quad t > 0$$



$$(KCL): i = \frac{v}{2} + \frac{1}{2} \frac{dv}{dt} = 2 + 6e^{-2t} - 2e^{-3t} - 12e^{-2t} + 6e^{-3t} = 2 - 6e^{-2t} + 4e^{-3t} \text{ A}, \quad t > 0$$

*notice:  $i(0) = 0$  as expected*

# General Second-Order Circuits

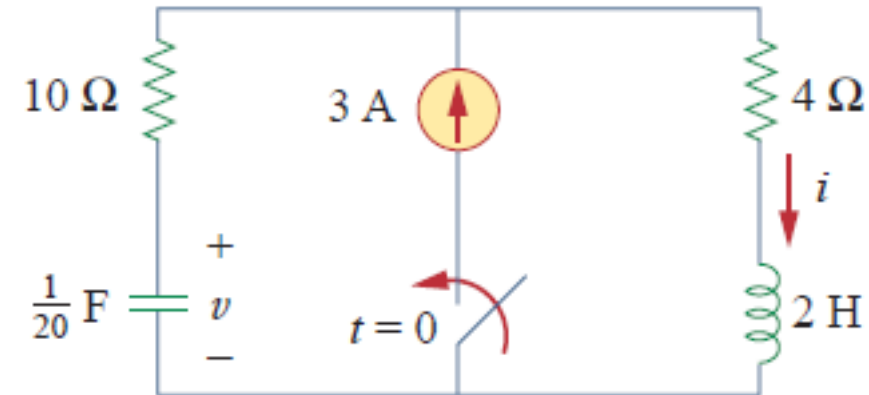


## SOC.17

Find the complete response  $v(t)$  and then  $i(t)$  for  $t > 0$ .

*(Comment: OC series with an ideal current source creates an infinite voltage at the current source terminals. That is impossible → in practice - shunt resistor in parallel with the source. Devices act like ideal current sources have operating range.)*

**Solution**  $v(t) = 12(1 - e^{-5t}) V$ ,  $i(t) = 3(1 - e^{-5t}) A$



# Questions

