DR. GYURCSEK ISTVÁN

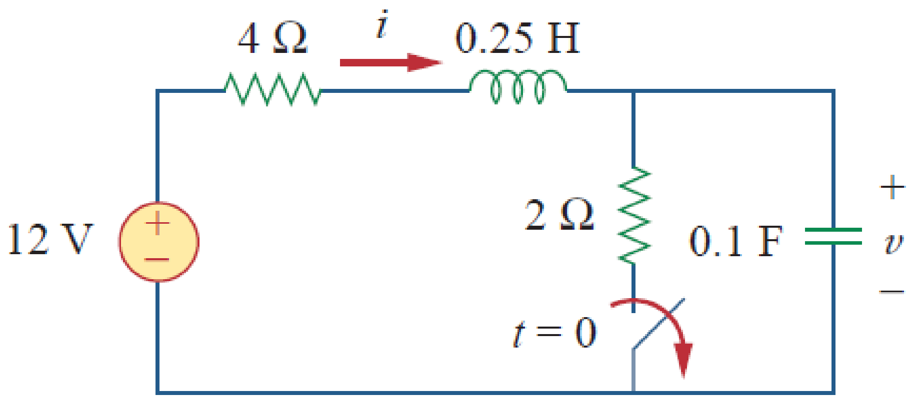
Second-Order Circuits - Examples

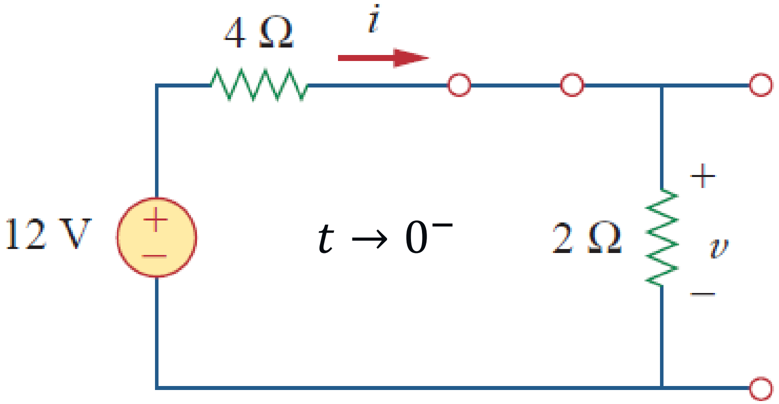
Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Simonyi K.: Villamosságtan. AK Budapest 1983, ISBN:9630534134*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

SOC.01 – Find initial and steady-state values. *(key to work)*

$(\boldsymbol{a})$: $i(0^+) =?$, $v(0^+) =?$, $(\boldsymbol{b})$: $\dfrac{di(0^+)}{dt} =?$, $\dfrac{dv(0^+)}{dt} =?$, $(\boldsymbol{c})$: $i(\infty) =?$, $v(\infty) =?$



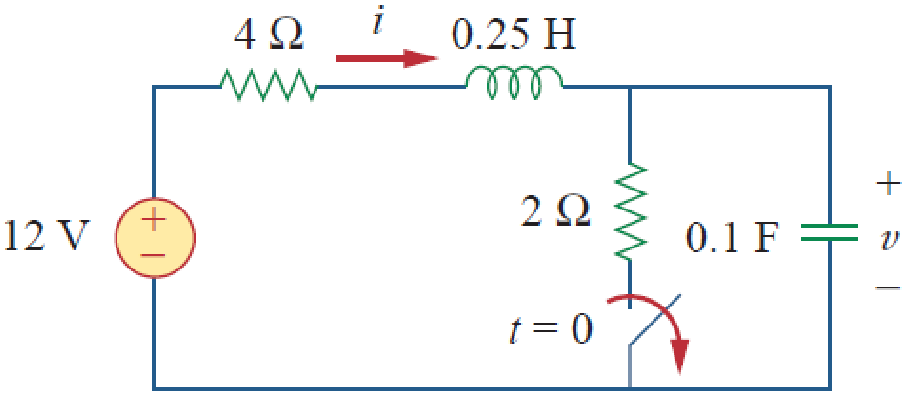


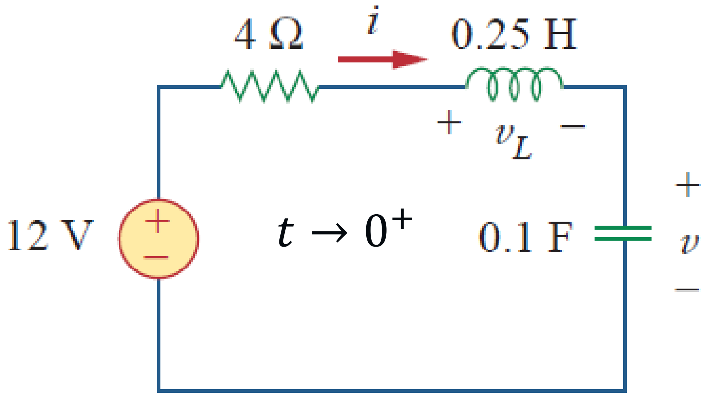
Solution $(\boldsymbol{a})$: $i(0^+) =?$, $v(0^+) =?$

$$i(0^-) = \frac{12}{4+2} = 2\ A, \qquad v(0^-) = 2i = 4\ V$$

$$i(0^+) = i(0^-) = 2\ A, \qquad v(0^+) = v(0^-) = 4\ V$$

Initial and Final Conditions





$$(\boldsymbol{b}): \frac{dv(0^+)}{dt} = ?, \qquad \frac{di(0^+)}{dt} = ?$$

$$i(0^+) = i_C(0^+) = 2\ A$$

$$i_C = C\frac{dv}{dt} \to \frac{dv}{dt} = \frac{i_C}{C}$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{2}{0.1} = 20\frac{V}{s}$$

$$v_L = L\frac{di}{dt} \to \frac{di}{dt} = \frac{v_L}{L}$$

$$(\boldsymbol{KVL}): -12 + 4i(0^+) + v_L(0^+) + v(0^+) = 0$$

$$v_L(0^+) = 12 - 8 - 4 = 0$$

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{0}{0.25} = 0\frac{\mathrm{A}}{\mathrm{s}}$$

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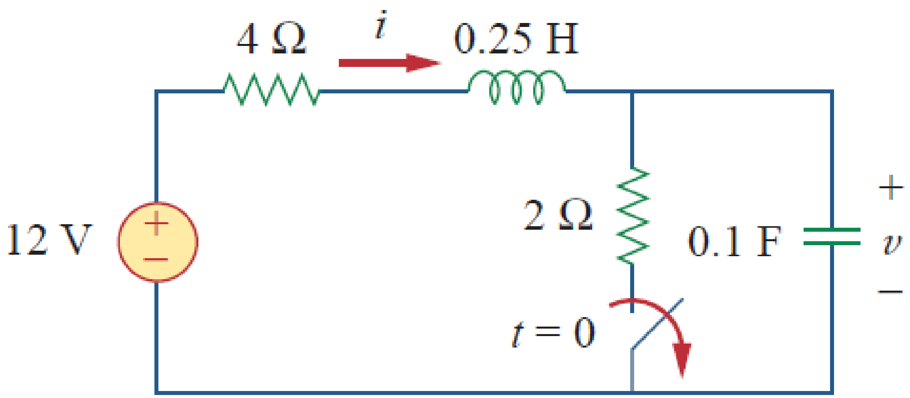
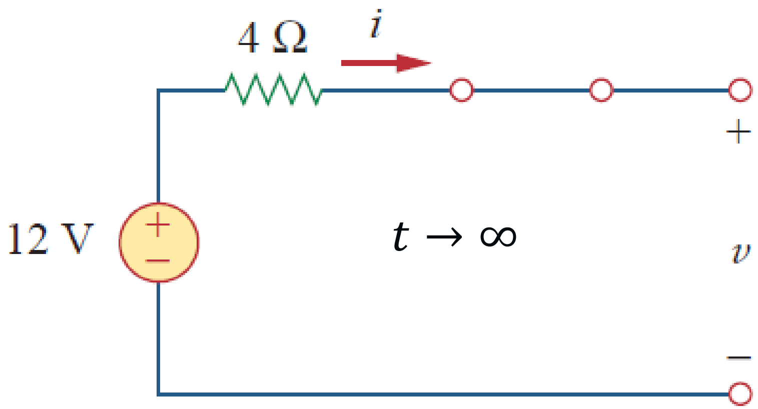
Initial and Final Conditions



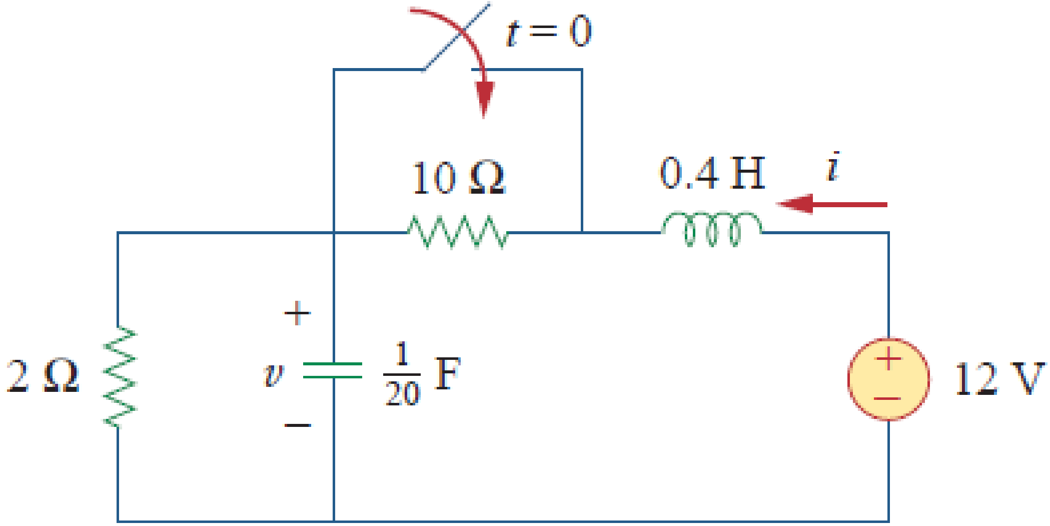


$(\boldsymbol{c})$: $i(\infty) = ?, v(\infty) = ?$

$i(\infty) = 0, \qquad v(\infty) = 12\ V$

SOC.02 – The switch was open for a long time but closed at t = 0.

$$(\boldsymbol{a}):\ i(0^+) =?,\qquad v(0^+) =?,\qquad (\boldsymbol{b}):\ \frac{di(0^+)}{dt} =?,\qquad \frac{dv(0^+)}{dt} =?,\qquad (\boldsymbol{c}):\ i(\infty) =?,\qquad v(\infty) =?$$



Solution $(\boldsymbol{a}):\ i(0^+) = 1\ A, v(0^+) = 2\ V, (\boldsymbol{b}):\ \frac{di(0^+)}{dt} = 25\frac{A}{s}, \frac{dv(0^+)}{dt} = 0\frac{V}{s}, (\boldsymbol{c}):\ i(\infty) = 6\ A, v(\infty) = 12\ V$

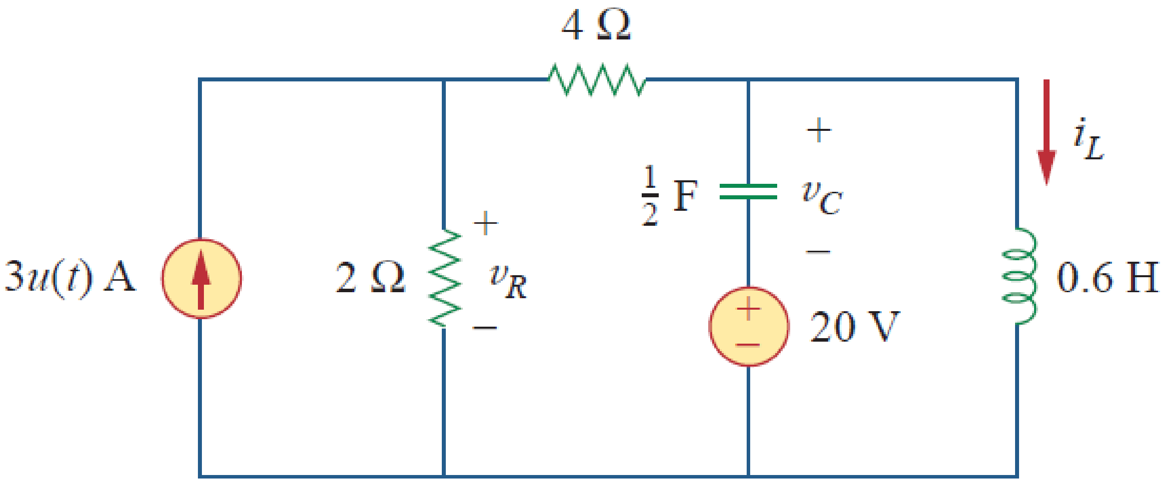
Initial and Final Conditions

SOC.03

$(\boldsymbol{a})$: $i_L(0^+) =?\,, v_C(0^+) =?\,, v_R(0^+) =?$

$(\boldsymbol{b})$: $\dfrac{di_L(0^+)}{dt} =?\,, \dfrac{dv_C(0^+)}{dt} =?\,, \dfrac{dv_R(0^+)}{dt} =?$

$(\boldsymbol{c})$: $i_L(\infty) =?\,, v_C(\infty) =?\,, v_R(\infty) =?$

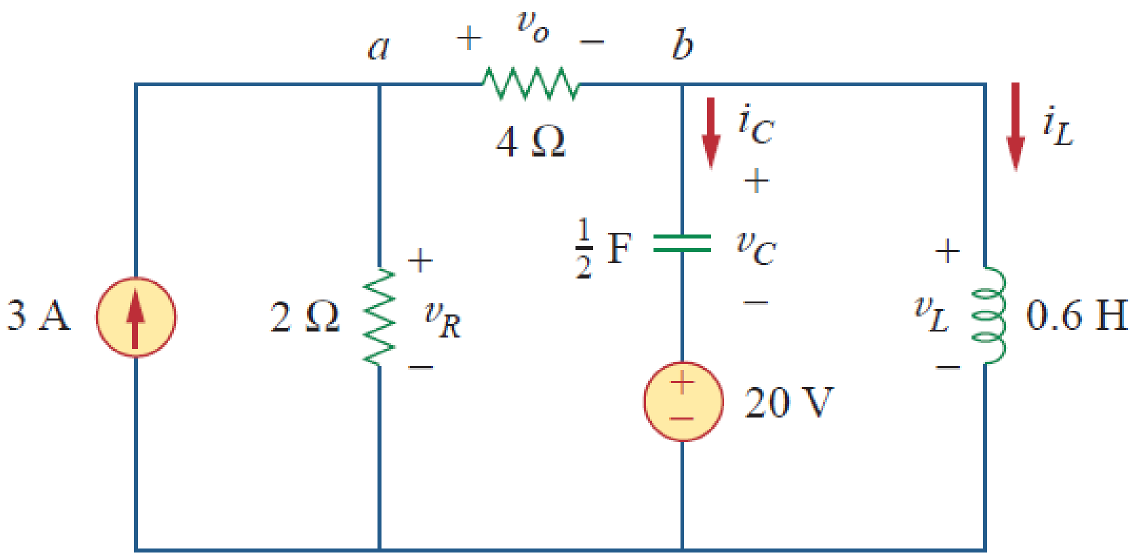


Solution

$i_L(0^-) = 0, v_C(0^-) = -20\ V, v_R(0^-) = 0$

$i_L(0^+) = i_L(0^-) = 0, v_C(0^+) = v_C(0^-) = -20\ V$

Initial and Final Conditions



$v_R(0^+) = ?$

$(\boldsymbol{Node\ a})$: $3 = \dfrac{v_R(0^+)}{2} + \dfrac{v_0(0^+)}{4}$

$(\boldsymbol{KVL2})$: $-v_R(0^+) + v_0(0^+) + v_C(0^+) + 20 = 0$

$(\boldsymbol{but})$: $v_C(0^+) = -20\ V \rightarrow v_R(0^+) = v_0(0^+)$

$(\boldsymbol{but} \rightarrow \boldsymbol{Node\ a})$: $v_R(0^+) = v_0(0^+) = 4\ V$

$\dfrac{di_L(0^+)}{dt} = \dfrac{v_L(0^+)}{L}$ $(\boldsymbol{KVL3})$: $v_L(0^+) = v_C(0^+) + 20 = 0 \rightarrow \dfrac{di_L(0^+)}{dt} = 0$

$\dfrac{dv_C(0^+)}{dt} = \dfrac{i_C(0^+)}{C}$ $(\boldsymbol{Node\ b})$: $\dfrac{v_0(0^+)}{4} = i_C(0^+) + i_L(0^+) \rightarrow \dfrac{4}{4} = i_C(0^+) + 0 \rightarrow i_C(0^+) = 1\ A \rightarrow \dfrac{dv_C(0^+)}{dt} = \dfrac{1}{0.5} = 2\dfrac{V}{s}$

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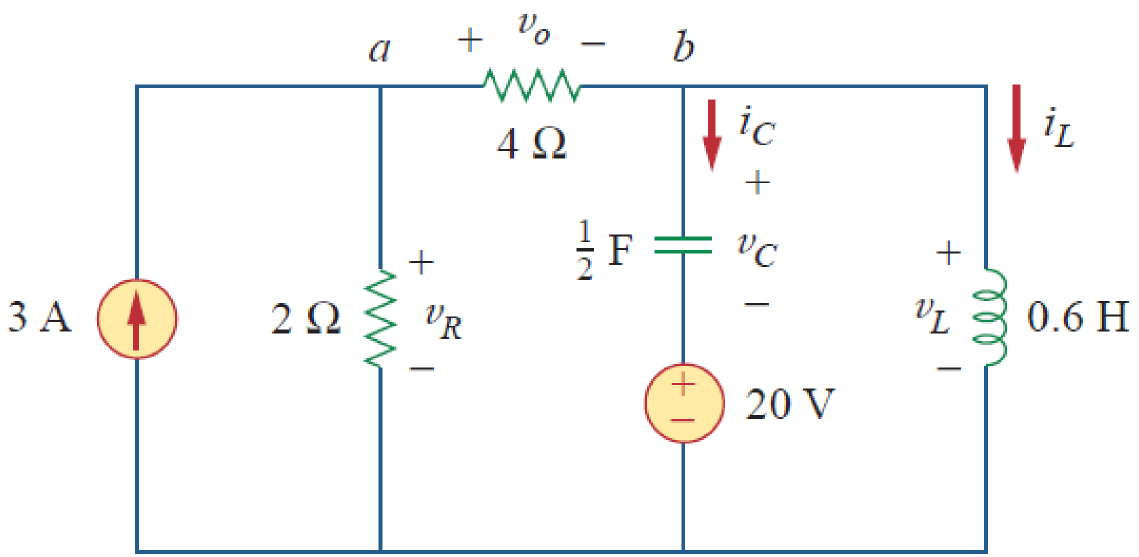
Initial and Final Conditions

$$\frac{dv_R(0^+)}{dt} = ? \qquad (\boldsymbol{Node\ a}):\ 3 = \frac{v_R(0^+)}{2} + \frac{v_0(0^+)}{4} \ \left(\ldots \leftarrow \frac{d}{dt}\right)$$

$$(\mathbf{1}):\ 0 = 2\frac{dv_R(0^+)}{dt} + \frac{dv_0(0^+)}{dt}$$

$$(\boldsymbol{KVL2}):\ -v_R(0^+) + v_0(0^+) + v_C(0^+) + 20 = 0 \ \left(\ldots \leftarrow \frac{d}{dt}\right)$$

$$-\frac{dv_R(0^+)}{dt} + \frac{dv_0(0^+)}{dt} + \frac{dv_C(0^+)}{dt} = 0$$



$$\frac{dv_C(0^+)}{dt} = 2 \rightarrow (\mathbf{2}):\ \frac{dv_R(0^+)}{dt} = 2 + \frac{dv_0(0^+)}{dt}$$

$$(\mathbf{1} \rightarrow \mathbf{2}):\ \frac{dv_R(0^+)}{dt} = \frac{2}{3}\frac{V}{s}$$

$$\frac{di_R(0^+)}{dt} = ?$$

$$i_R(0^+) = \frac{v_R(0^+)}{2} \rightarrow \frac{di_R(0^+)}{dt} = \frac{1}{2}\frac{dv_R(0^+)}{dt} = \frac{1}{3}\frac{A}{s}$$

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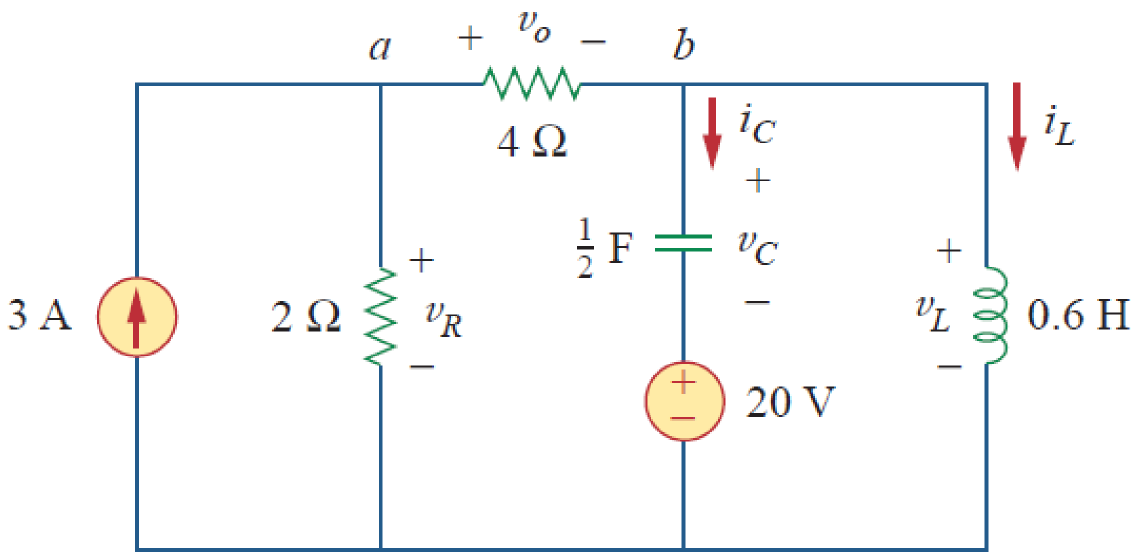


$$i_L(\infty) = 3 \cdot \frac{2}{2+4} = 1\ A$$

$$v_C(\infty) = -20\ V$$

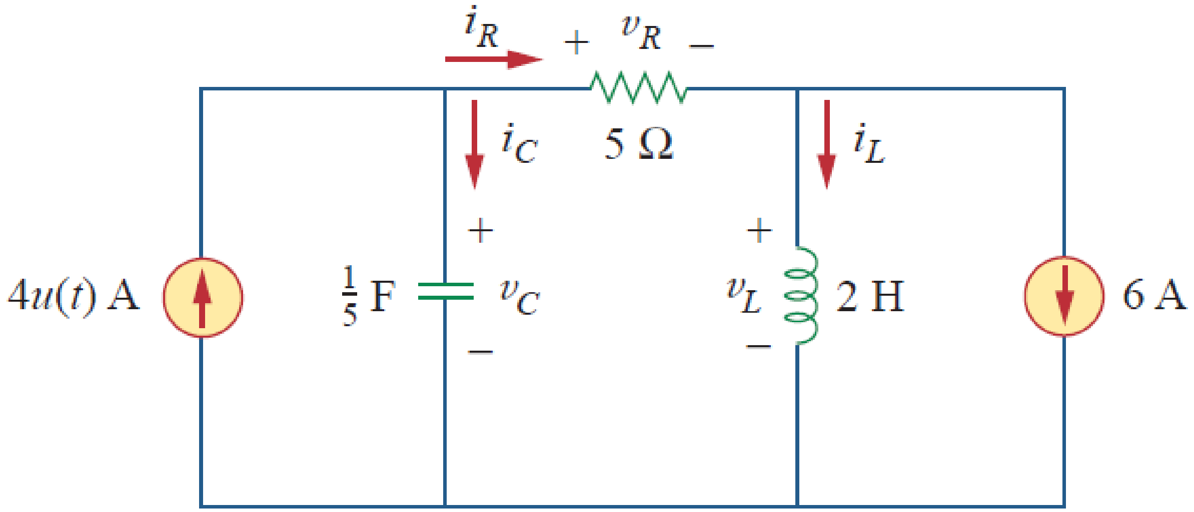
$$v_R(\infty) = 3 \cdot \frac{4}{2+4} \cdot 2 = 4\ V$$

SOC.04

$(\boldsymbol{a})$: $i_L(0^+) = ?,\, v_C(0^+) = ?,\, v_R(0^+) = ?$

$(\boldsymbol{b})$: $\dfrac{di_L(0^+)}{dt} = ?,\, \dfrac{dv_C(0^+)}{dt} = ?,\, \dfrac{dv_R(0^+)}{dt} = ?$

$(\boldsymbol{c})$: $i_L(\infty) = ?,\, v_C(\infty) = ?,\, v_R(\infty) = ?$



Solution (a) -6 A, 0,0, (b) 0, 20 V/s, 0, (c) -2 A, 20 V, 20 V

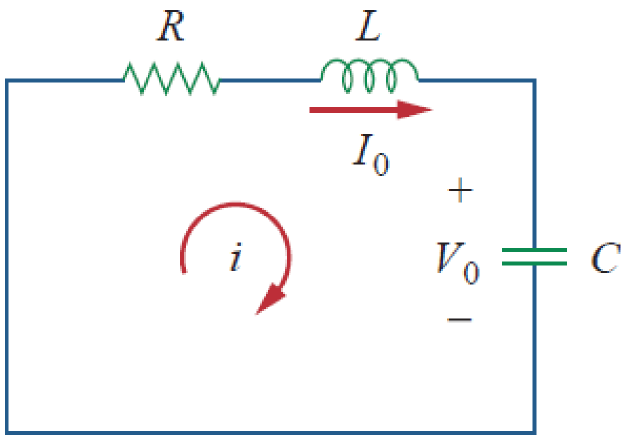
SOC.05 – Calculate the characteristic roots of the circuit. $R = 40\ \Omega, L = 4\ H, C = 1/4\ F$.
Is the natural response overdamped, underdamped, or critically damped?

Solution

$$\delta = \frac{R}{2L} = 5, \omega_0 = \frac{1}{\sqrt{LC}} = 1$$

$$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -5 \pm \sqrt{15 - 1}$$

$$s_1 = -0.101, \qquad s_2 = -9.899$$



$$\delta > \omega \rightarrow overdumped\ (real\ negative\ roots)$$

SOC.06 – Calculate the characteristic roots of the circuit. $R = 10\ \Omega, L = 5\ H, C = 2\ mF$.
Is the natural response overdamped, underdamped, or critically damped?

Solution $\delta = 1, \omega_0 = 10, s_{1,2} = -1 \pm j9.95, underdamped$

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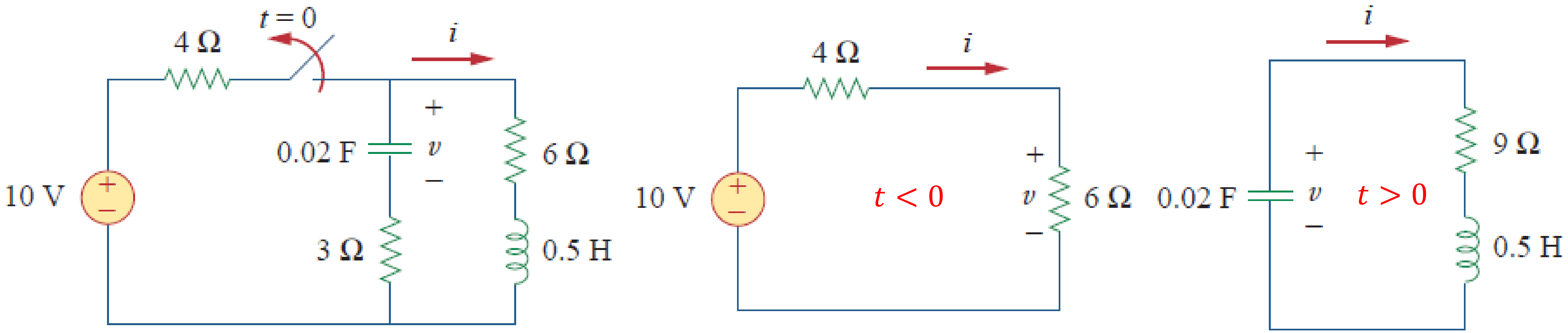
The Source-Free Series RLC Circuit

SOC.07 – Find *i(t)* in the circuit for $t > 0$. (Steady-state has been reached at $\mathrm{t} = 0^{-}$.)



Solution $i(0) = \dfrac{10}{4+6} = 1\ A,\qquad v(0) = 6i = 6\ V$

$$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -9 \pm \sqrt{81 - 100}$$

$$\delta = \frac{R}{2L} = \frac{9}{2 \cdot 0.5} = 9, \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5 \cdot 0.02}} = 10$$

$$s_{1,2} = -9 \pm j4.359, \qquad \delta < \omega \rightarrow underdumped$$

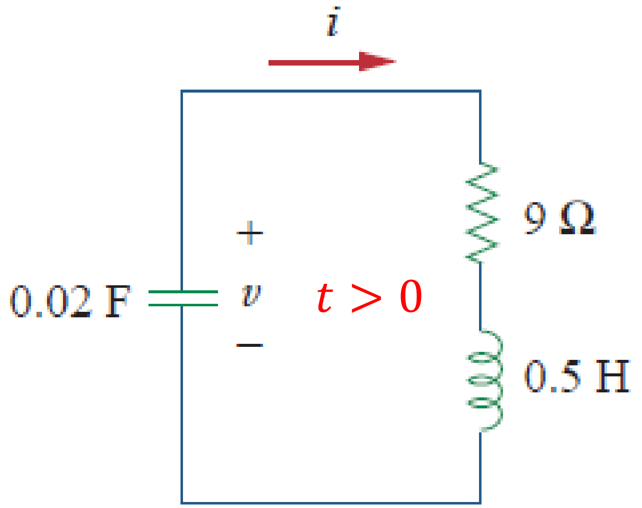
$$s_{1,2} = -9 \pm j4.359, \delta < \omega \rightarrow underdumped$$

$$i(t) = e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t)$$

$$(A_1 = ?, \qquad A_2 = ?) \leftarrow from\ initial\ conditions$$

$$i(0) = 1 = e^0(A_1 \cos 0 + A_2 \sin 0) = A_1$$

$$(KVL): \frac{di(0)}{dt} = -\frac{1}{L}[Ri(0) + v(0)] = -2[9 \cdot 1 - 6] = -6\frac{A}{s}$$

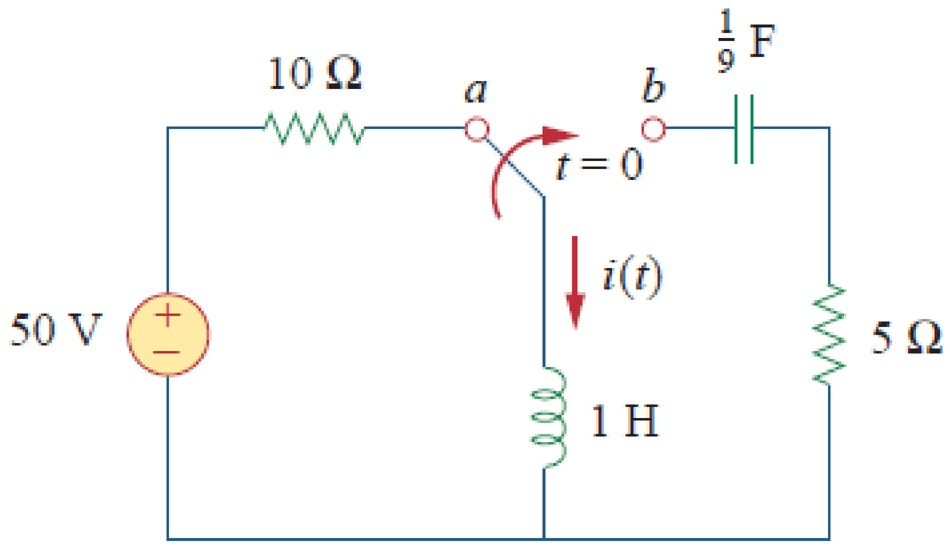


$$\frac{di(t)}{dt} = -9e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t) + e^{-9t}4.359(-A_1 \sin 4.359t + A_2 \cos 4.359t)$$

$$t = 0 \rightarrow -6 = -9(A_1 \cos 0 + A_2 \sin 0) + 4.359(-A_1 \sin 0 + A_2 \cos 0) = -9A_1 + 4.359A_2$$

$$A_1 = 1 \rightarrow -6 = -9 + 4.359A_2 \rightarrow A_2 = 0.6882 \qquad i(t) = e^{-9t}(\cos 4.359t + 0.6882 \sin 4.359t)\ A$$

SOC.08 – Find *i(t)* in the circuit for $t > 0$. (Steady-state has been reached at $\mathrm{t} = 0^{-}$.)



Solution $i(t) = e^{-2.5t}(5\cos 1.6583t - 7.5378\sin 1.6583)\ A$

SOC.09

In a parallel RLC circuit $R = 500\ \Omega, C = 1\ \mu F, L = 200\ mH$. The initial conditions are $I_0 = 50\ mA, V_0 = 0$.
Find the zero-input response of inductor current, resistor current, and capacitor voltage.

Solution

$$LCs^2 + GLs + 1 = 2 \cdot 10^{-7} s^2 + 4 \cdot 10^{-4} s + 1 = 0 \qquad \delta = \frac{G}{2C}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}, \qquad s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$$

$$s_{1,2} = \frac{4 \cdot 10^{-4} \pm \sqrt{16 \cdot 10^{-8} - 8 \cdot 10^{-7}}}{4 \cdot 10^{-7}} = -1000 \pm j2000 \leftarrow underdamped$$

$$i_L(t) = K_1 e^{-1000t} \cos 2000t + K_2 e^{-1000t} \, sin\, 2000t \ , t \geq 0$$

$$i_L(0) = I_0 = K_1 e^0 \cos 0 + K_2 e^0 \, sin\, 0 \ , t \geq 0 \qquad v_C(t) = L \frac{di_L}{dt}$$

$$\frac{di_L}{dt} = -2000 K_1 e^{-1000t} \, sin\, 2000t - 1000 K_1 e^{-1000t} \, cos\, 2000t - 1000 K_2 e^{-1000t} \sin 2000t + 2000 K_2 e^{-1000t} \, cos\, 2000t$$

The Source-Free Parallel RLC Circuit

$$\left.\frac{di_L}{dt}\right|_{t=0} = -2000K_1e^0\,sin\,0 - 1000K_1e^0\,cos\,0 - 1000K_2e^0\sin 0 + 2000K_2e^0\,cos\,0 = -1000K_1 + 2000K_2 = 0$$

$$i_L(t) = 50e^{-1000t}\cos 2000t + 25e^{-1000t}\,sin\,2000t\;\; mA\,, t \geq 0$$

$$v_C(t) = v_L(t) = L\frac{di_L}{dt} = -25e^{-1000t}\sin 2000t\,V, t \geq 0$$

$$i_R(t) = \frac{v_L(t)}{R} = -50e^{-1000t}\,sin\,2000t\;\; mA\,, t \geq 0$$

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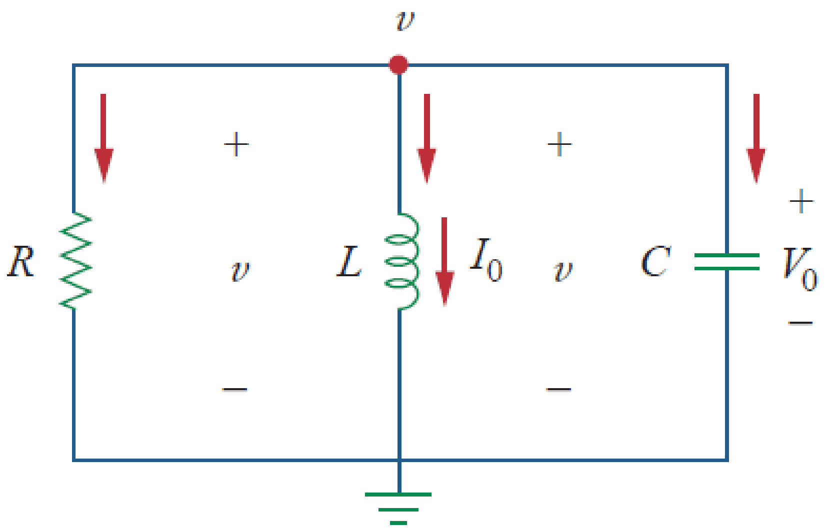
SOC.10

In a parallel RLC circuit $R = 2\ \Omega, C = 25\ mF, L = 400\ mH$. The initial conditions are $I_0 = 10\ mA, V_0 = 0$. Find the zero-input response of capacitor voltage.

Solution

$$v(t) = -400\ t\ e^{-105t}\ u(t)\ mV$$

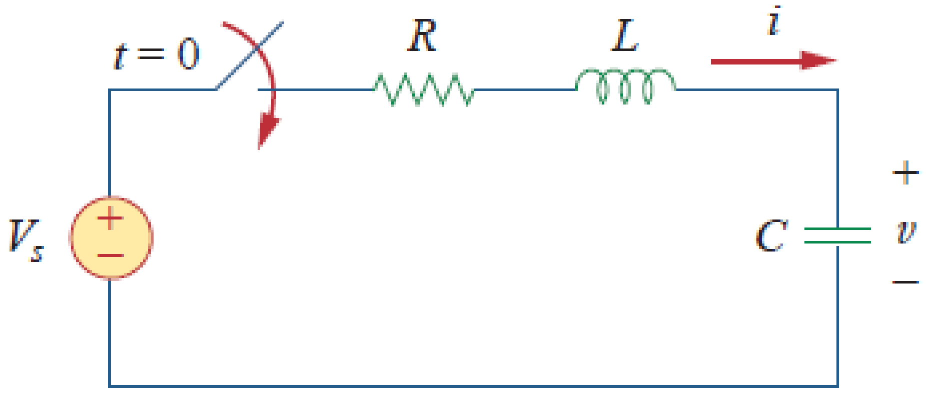


Step Response of a Series RLC Circuit

SOC.11 – The circuit is in the zero state at t = 0. Find the capacitor voltage for t ≥ 0.

$V_S = 10\ V\ , \qquad C = 500\ nF\ , \qquad L = 2\ H\ , \qquad R = 1\ k\Omega$

Solution $\quad LC\dfrac{d^2v}{dt^2} + RC\dfrac{dv}{dt} + v = V_S$



$$10^{-6}\frac{d^2v}{dt^2} + 0.5\cdot 10^{-3}\frac{dv}{dt} + v = 10, t \geq 0$$

$$\frac{d^2v}{dt^2} + 500\frac{dv}{dt} + 10^6\cdot v = 0 \rightarrow$$

$$\delta = \frac{R}{2L} = 250\frac{rad}{s}, \omega_0 = \frac{1000rad}{s}\ (\rightarrow underdamped!), \omega_d = \sqrt{\omega_0^2 - \delta^2} = 968\ rad/s$$

$$v_{tr}(t) = K_1e^{-250t}\cos 986t + K_2e^{-250t}\sin 986t\ , t \geq 0 \qquad v(t) = v_{ss}(t) + v_{tr}(t)$$

$$v(\infty) = V_S = 10\ V \rightarrow v(t) = 10 + K_1e^{-250t}\cos 968t + K_2e^{-250t}\sin 968t\ , t \geq 0$$

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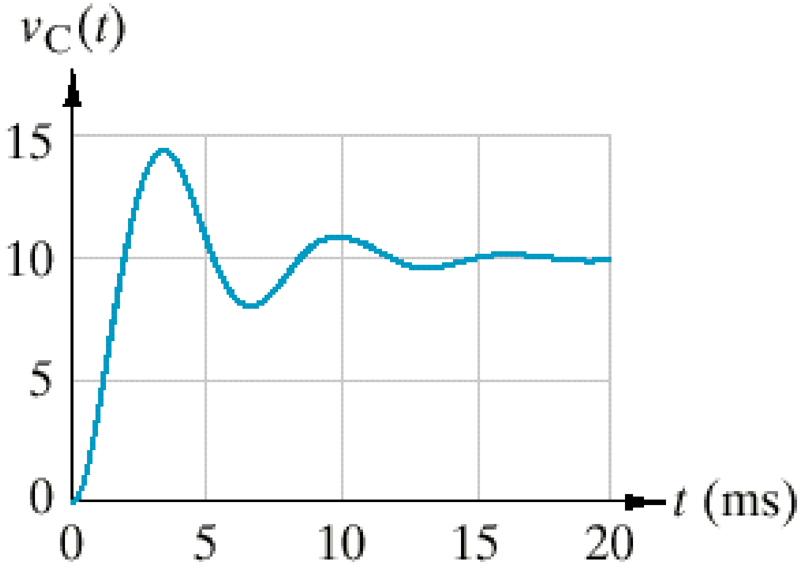
The constants (K_1 and K_2) are determined by the initial conditions.

$$v_C(0) = 10 + K_1 = 0$$

$$\left.\frac{dv_C}{dt}\right|_{t=0} = -250K_1 + 968K_2 = 0$$

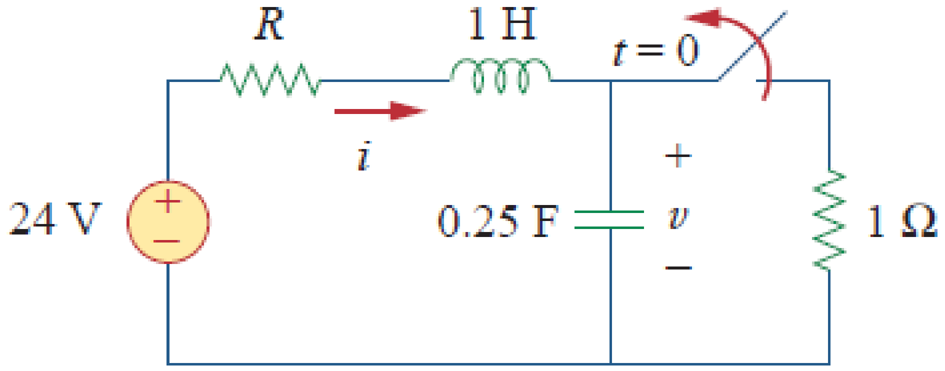
$$K_1 = -10\,, \qquad K_2 = -2.58$$

$$v(t) = 10 - 10e^{-250t}\cos 986t - 2.58e^{-250t}\sin 986t\,, t \geq 0$$



SOC.12 – Find $v(t)$ and $i(t)$ for t > 0. Consider the cases $R = 5\ \Omega, R = 4\ \Omega,\ R = 1\ \Omega$.

Case $R = 5\ \Omega$



$$i(0) = \frac{24}{5+1} = 4\ A, v(0) = 1 \cdot i = 4\ V$$

$$\delta = \frac{R}{2L} = \frac{5}{2 \cdot 1} = 2.5, \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \cdot 0.25}} = 2$$

$$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -1, -4\ (\leftarrow overdamped\ natural\ response)$$

$$v(t) = v_{SS} + (A_1 e^{-t} + A_2 e^{-4t})\ (\leftarrow total\ response)\ v_{SS} = 24\ V\ (\leftarrow steady - state\ response)$$

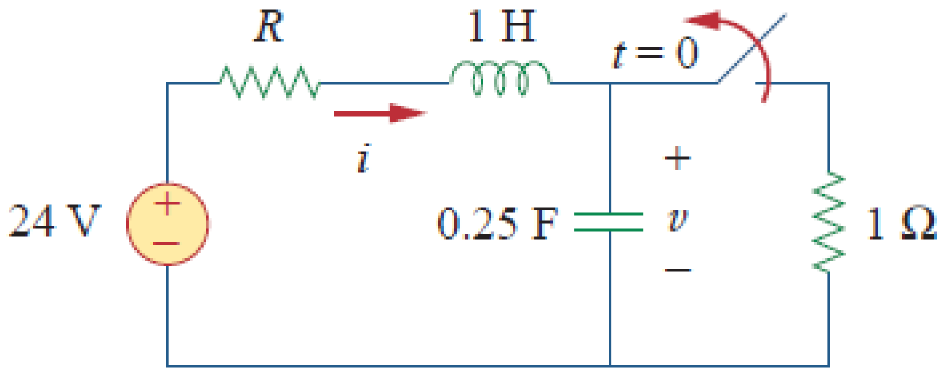
$$t = 0 \rightarrow v(0) = 4 = 24 + (A_1 e^0 + A_2 e^0) \rightarrow A_1 + A_2 = -20$$

Step Response of a Series RLC Circuit

$$i(0) = C\frac{dv(0)}{dt} = 4 \rightarrow \frac{dv(0)}{dt} = \frac{4}{C} = \frac{4}{0.25} = 16$$

$$\frac{dv}{dt} = -A_1e^{-t} - 4A_2e^{-4t} \rightarrow \frac{dv(0)}{dt} = 16 = -A_1 - 4A_2$$

$$\left.\begin{array}{l} A_1 + A_2 = -20 \\ 16 = -A_1 - 4A_2 \end{array}\right\} \rightarrow \left\{\begin{array}{l} A_1 = -64/3 \\ A_2 = 4/3 \end{array}\right.$$



$$v(t) = 24 + \frac{4}{3}(-16e^{-t} + e^{-4t})\ V$$

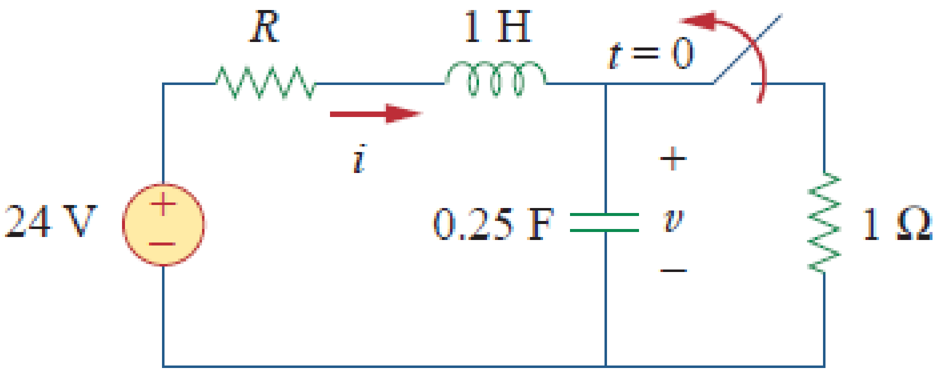
$$i(t) = C\frac{dv(t)}{dt} = \frac{4}{3}(4e^{-t} - e^{-4t})\ A$$

$Note\!: \ i(0) = 4\ A\ as\ expected.$

Step Response of a Series RLC Circuit

Case $R = 4\,\Omega$

$$i(0) = \frac{24}{4+1} = 4.8\,A, v(0) = 1 \cdot i = 4.8\,V$$



$$\delta = \frac{R}{2L} = \frac{4}{2 \cdot 1} = 2, \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \cdot 0.25}} = 2$$

$$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = 2\ (\leftarrow critically\ damped\ natural\ response)$$

$$v(t) = v_{SS} + (A_1 + A_2 t)e^{-2t} (\leftarrow total\ response)\ v_{SS} = 24\,V\ (\leftarrow steady - state\ response)$$

$$t = 0 \rightarrow v(0) = 4.8 = 24 + A_1 \rightarrow A_1 = -19.2$$

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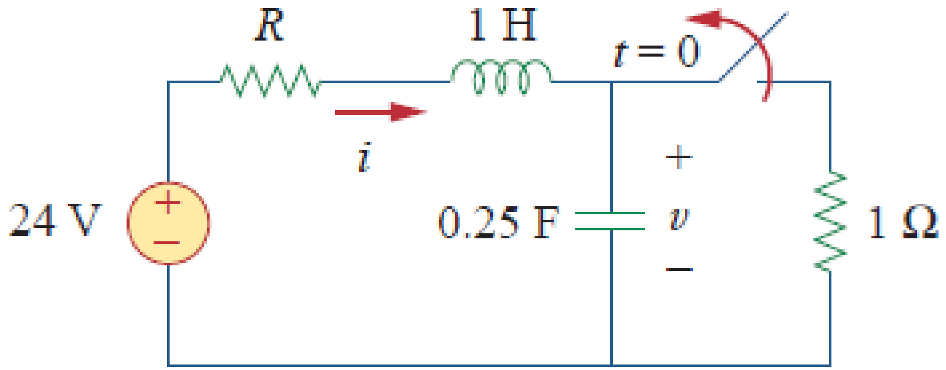
Step Response of a Series RLC Circuit

$$i(0) = C\frac{dv(0)}{dt} = 4.8 \rightarrow \frac{dv(0)}{dt} = \frac{4.8}{C} = \frac{4,8}{0.25} = 19.2$$

$$\frac{dv}{dt} = (-2A_1 - 2tA_2 + A_2)e^{-2t} \rightarrow \frac{dv(0)}{dt} = 19.2 = -2A_1 + A_2$$

$$\left.\begin{array}{r} A_1 = -19.2 \\ 19.2 = -2A_1 + A_2 \end{array}\right\} \rightarrow \left\{\begin{array}{l} A_1 = -19.2 \\ A_2 = -19.2 \end{array}\right.$$



$$v(t) = 24 - 19.2(1 + t)e^{-2t}\ V$$

$$i(t) = C\frac{dv(t)}{dt} = (4.8 + 9.6t)e^{-2t}\ A$$

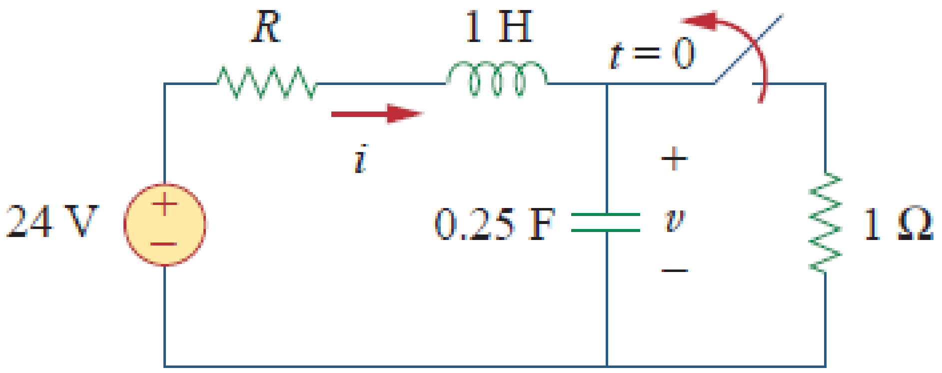
$Note\!: \ i(0) = 4.8\ A\ as\ expected.$

Step Response of a Series RLC Circuit

Case $R = 1\,\Omega$

$$i(0) = \frac{24}{1+1} = 12\,A, v(0) = 1 \cdot i = 12\,V$$

$$\delta = \frac{R}{2L} = \frac{1}{2 \cdot 1} = 0.5, \qquad \omega_0 = 2$$



$$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = -0.5 \pm j1.936\ (\leftarrow \textit{underdamped natural response})$$

$$v(t) = 24 + (A_1 \cos 1.936t + A_2 \sin 1.936t)e^{-0.5t} (\leftarrow \textit{total response})$$

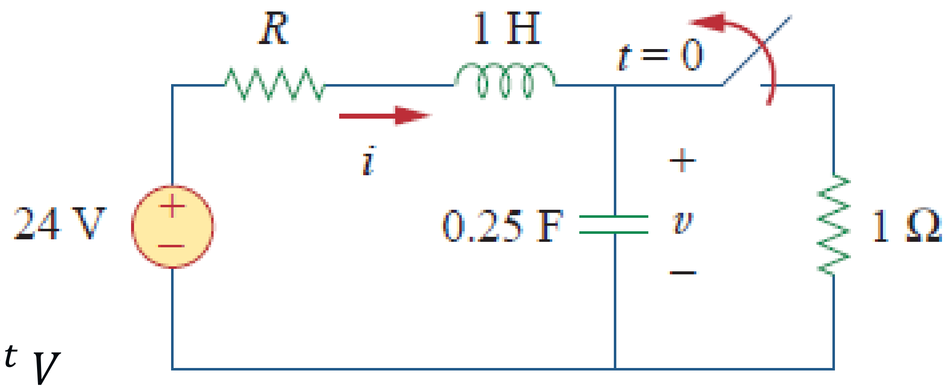
$$t = 0 \rightarrow v(0) = 12 = 24 + A_1 \rightarrow A_1 = -12$$

$$i(0) = C\frac{dv(0)}{dt} = 12 \rightarrow \frac{dv(0)}{dt} = \frac{12}{C} = \frac{12}{0.25} = 48$$

$$\frac{dv}{dt} = e^{-0.5t}(-1.936A_1 \sin 1.936t + 1.936A_2 \cos 1.936t) - 0.5e^{-0.5t}(A_1 \cos 1.936t + A_2 \sin 1.936t)$$

$$\rightarrow \frac{dv(0)}{dt} = 48 = (-0 + 1.936A_2) - 0.5(A_1 + 0)$$

$$\left.\begin{matrix} A_1 = -12 \\ 48 = 1.936A_2 - 0.5A_1 \end{matrix}\right\} \rightarrow \left\{\begin{matrix} A_1 = -12 \\ A_2 = +21.694 \end{matrix}\right.$$

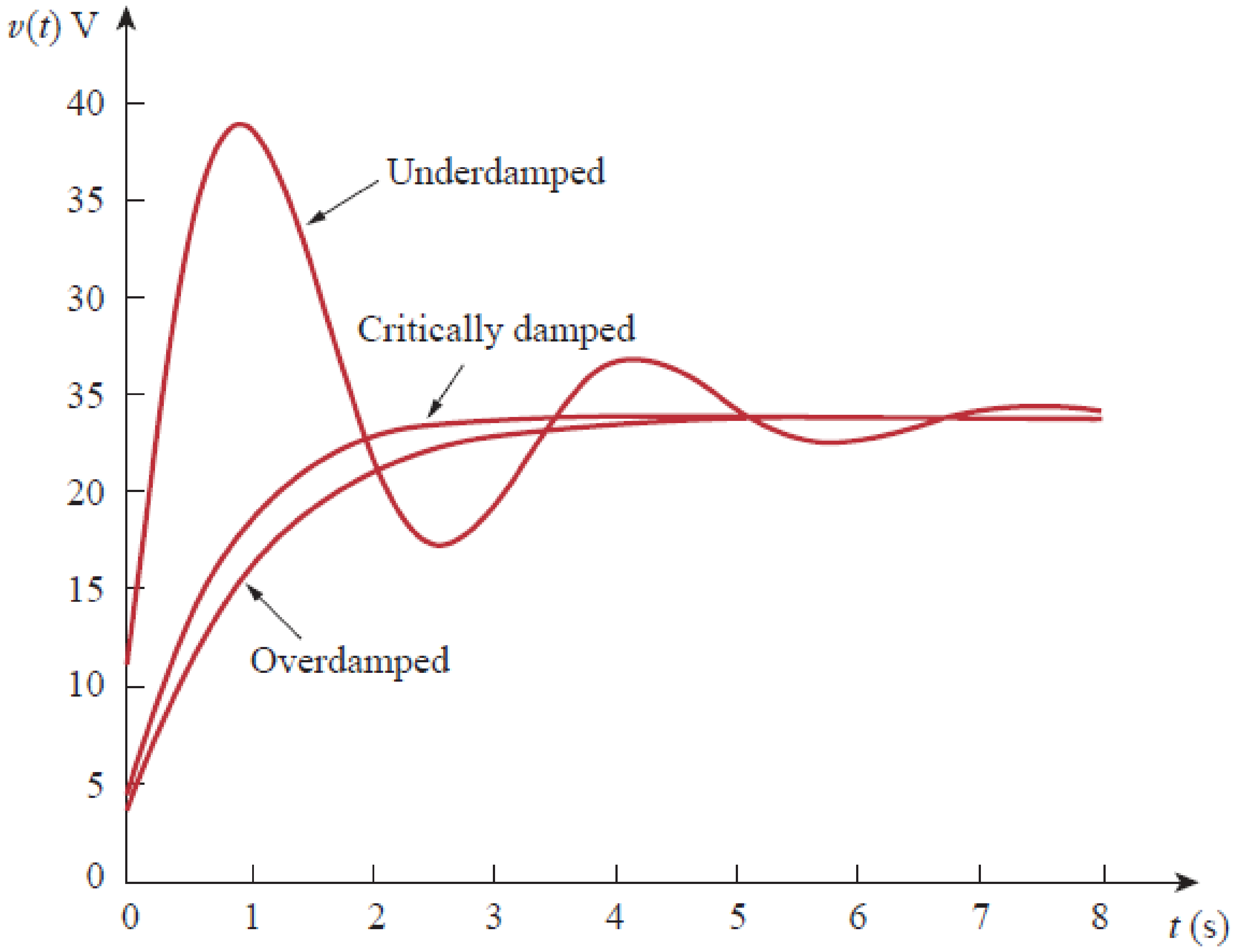


$$v(t) = 24 + (21.694 \sin 1.936t - 12 \cos 1.936t)e^{-0.5t}\ V$$

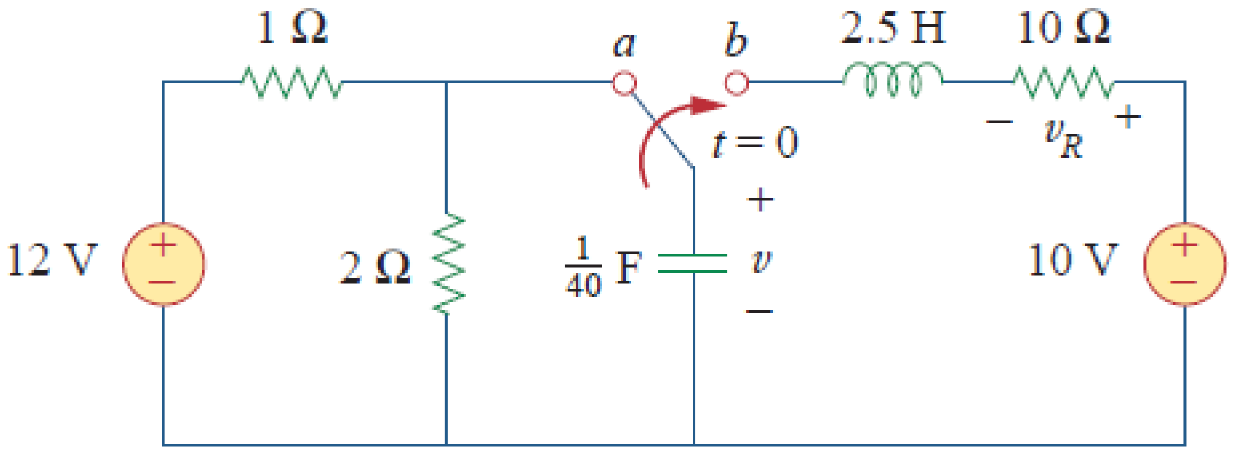
$$i(t) = C\frac{dv(t)}{dt} = (3.1 \sin 1.936t + 12 \cos 1.936t)e^{-0.5t}\ A$$

$Note:\ i(0) = 12\ A\ as\ expected.$

Step Response of a Series RLC Circuit



SOC.13 – Find $v(t)$ and $v_R(t)$ for t > 0.

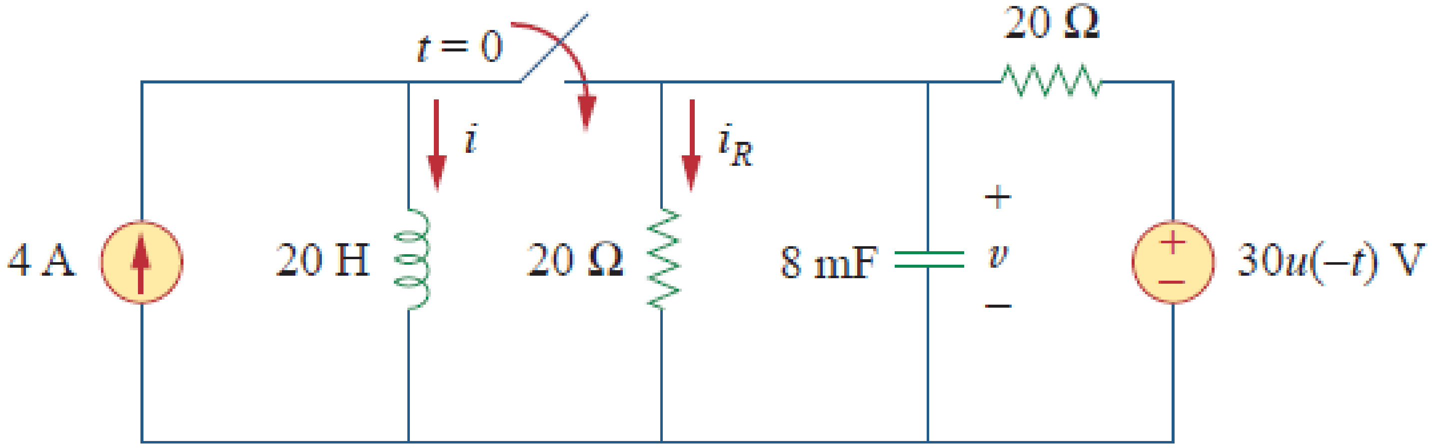


Solution $v(t) = 10 - (1.1547 \sin 3.464t + 2 \cos 3.464t)e^{-2t}\ V$

$v_R(t) = 2.31e^{-2t} \sin 3.464t\ \ V$

Step Response of a Parallel RLC Circuit

SOC.14 – Find $i(t)$ and $i_R(t)$ for t > 0.



Solution $t < 0 \rightarrow i(0) = 4\ A$

$$t < 0 \rightarrow v(0) = 30 \cdot \frac{20}{20+20} = 15\ V$$

$$t > 0 \rightarrow R = 20 \times 20 = 10\ \Omega$$

$$\delta = \frac{1}{2RC} = \cdots = 6.25$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \cdots = 2.5$$

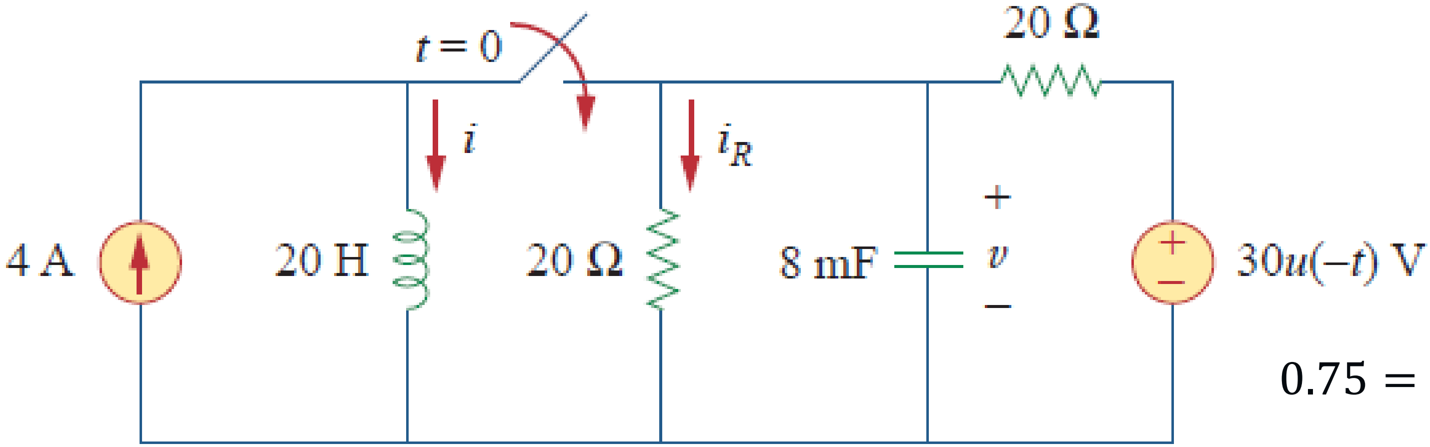
$$s_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2} = \begin{cases} -11.978 \\ -0.5218 \end{cases}$$

$$\delta > \omega \rightarrow overdumped\ (real\ negative\ roots)$$

$$i(t) = I_S + A_1 e^{-11.978t} + A_2 e^{-0.5218t}$$

$$i(0) = 4 = 4 + A_1 e^0 + A_2 e^0 \rightarrow A_2 = -A_1$$

Step Response of a Parallel RLC Circuit



$$(but):\ \frac{di(0)}{dt} = \frac{V(0)}{L} = \frac{15}{20} = 0.75$$

$$0.75 = (11.978 - 0.5218)A_2 \rightarrow A_2 = 0.0655$$

$$i(0) == 4 + A_1e^0 + A_2e^0 \rightarrow A_2 = -A_2$$

$$A_1 = -0.0655$$

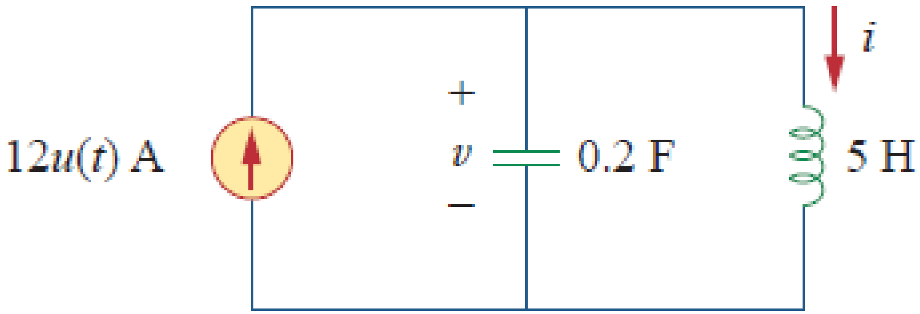
$$\frac{di}{dt} = -11.978A_1e^{-11.978t} - 0.5218A_2e^{-0.5218t}$$

$$i(t) = 4 + 0.0655(e^{-0.5218t} - e^{-11.978t})\ A$$

$$\frac{di(0)}{dt} = -11.978A_1e^0 - 0.5218A_2e^0$$

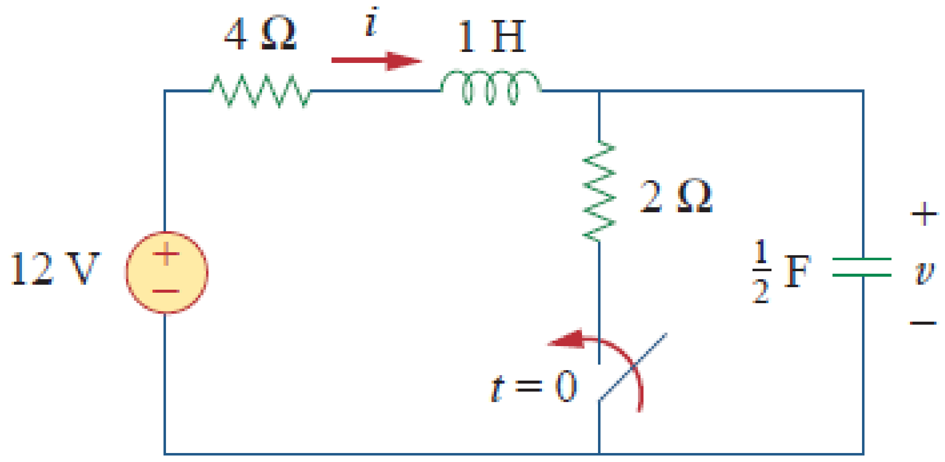
$$i_R(t) = \frac{v(t)}{20} = L\frac{di}{dt} = 0.785e^{-11.978t} - 0.0342e^{-0.5218t}\ A$$

SOC.15 – Find $i(t)$ and $v(t)$ for t > 0.



Solution $12(1-\cos t)\ A, 60 \sin t\ V$

SOC.16 – Find the complete response $v(t)$ and then $i(t)$ for t > 0.



(1) Initial and final conditions

$$v(0^+) = v(0^-) = 12\ V, \qquad i(0^+) = i(0^-) = 0$$

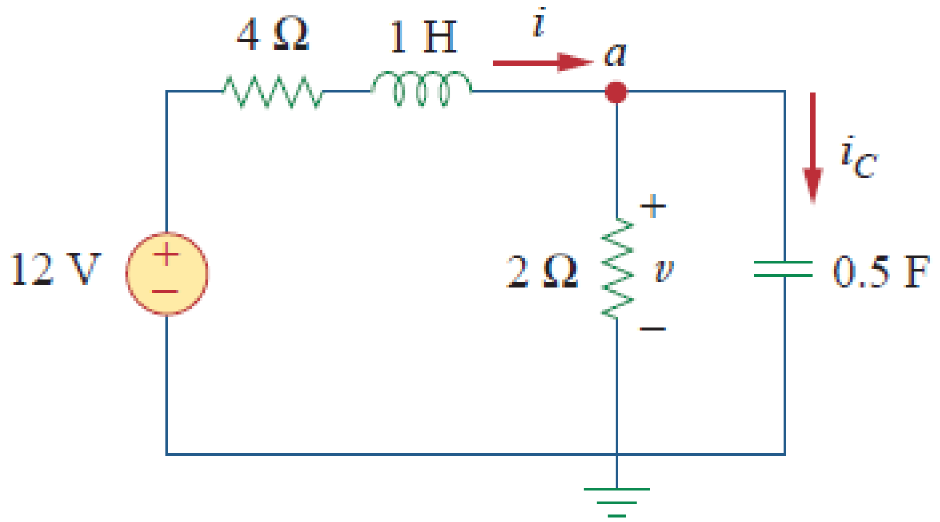
$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C}$$

$$(\boldsymbol{node\ a}):\ i(0^+) = i_C(0^+) + \frac{v(0^+)}{2}$$

$$0 = i_C(0^+) + \frac{12}{2} \to i_C(0^+) = -6\ A$$

$$\frac{dv(0^+)}{dt} = \frac{-6}{0.5} = -12\frac{V}{s}$$

$$i(\infty) = \frac{12}{4+2} = 2\ A, \qquad v(\infty) = 2i(\infty) = 4\ V$$



(2) Transient response (source-free)

$$(\boldsymbol{KCL}):\ i = \frac{v}{2} + \frac{1}{2}\frac{dv}{dt}$$

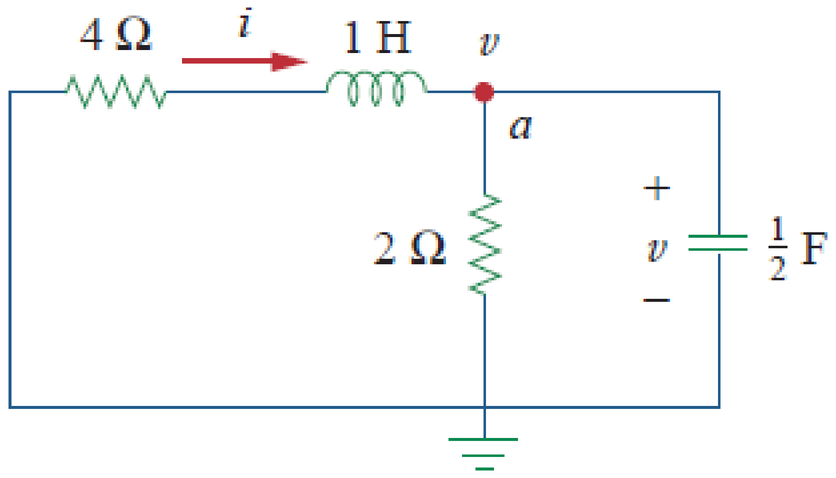
$$(\boldsymbol{KVL}):\ 4i + 1\frac{di}{dt} + v = 0$$

$$(\boldsymbol{KCL} \rightarrow \boldsymbol{KVL}): 2v + 2\frac{dv}{dt} + \frac{1}{2}\frac{dv}{dt} + \frac{1}{2}\frac{d^2v}{dt^2} + v = 0$$

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 0 \rightarrow characteristic\ eq.$$

$$s^2 + 5s + 6 = 0 \rightarrow s_{1,2} = -2, \qquad -3\ (overdamped)$$

$$v_{tr}(t) = Ae^{-2t} + Be^{-3t}$$

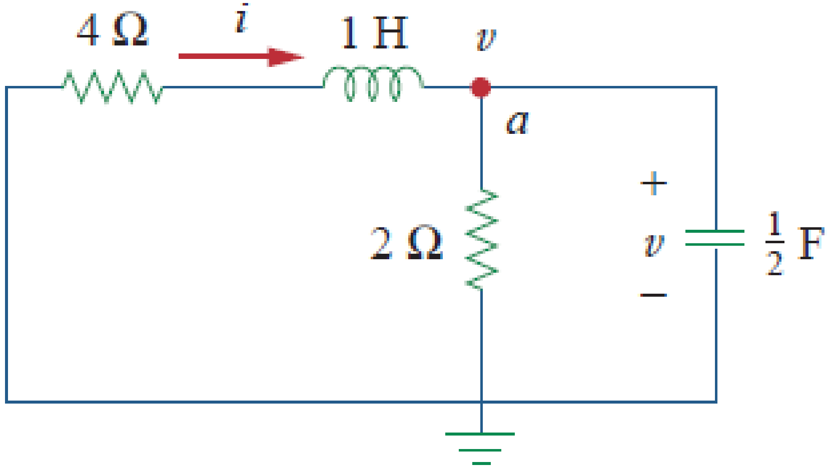


(3) Steady-state response $v_{ss}(t) = v(\infty) = 4$

(4) Complete response

$$v(t) = v_{ss}(t) + v_{tr}(t) = 4 + Ae^{-2t} + Be^{-3t}$$

(5) Find A, B using initial values $v(t) = 4 + Ae^{-2t} + Be^{-3t}$



$$v(0) = 12 = 4 + Ae^{0} + Be^{0} \rightarrow 12 = 4 + A + B \rightarrow A + B = 8$$

$$\frac{dv}{dt} = -2Ae^{-2t} - 3Be^{-3t} \rightarrow -12 = -2A - 3B \rightarrow 2A + 3B = 12$$

$$A = 12, B = -4 \qquad v(t) = 4 + 12e^{-2t} - 4e^{-3t}\, V, \qquad t > 0$$

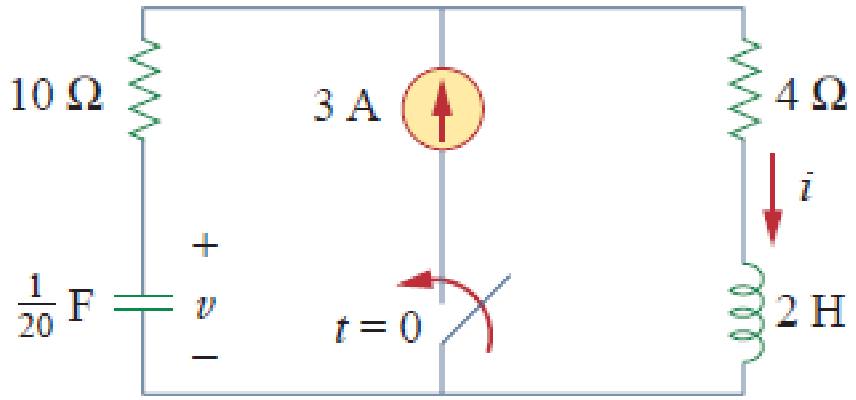
$$(\boldsymbol{KCL}):\ i = \frac{v}{2} + \frac{1}{2}\frac{dv}{dt} = 2 + 6e^{-2t} - 2e^{-3t} - 12e^{-2t} + 6e^{-3t} = 2 - 6e^{-2t} + 4e^{-3t}\, A, \qquad \mathrm{t} > 0$$

$$notice:\ i(0) = 0\ as\ expected$$

SOC.17

Find the complete response $v(t)$ and then $i(t)$ for $\mathrm{t} > 0$.

(Comment: OC series with an ideal current source creates an infinite voltage at the current source terminals. That is impossible → in practice - shunt resistor in parallel with the source. Devices act like ideal current sources have operating range.)



Solution $v(t) = 12(1 - e^{-5t})\ V$, $i(t) = 3(1 - e^{-5t})\ A$

Questions