DR. GYURCSEK ISTVÁN

Fourier Transform - Examples

Sources and additional materials (recommended)

- *Dr. Gyurcsek – Dr. Elmer: Theories in Electric Circuits, GlobeEdit, 2016, ISBN:978-3-330-71341-3*
- *Ch. Alexander, M. Sadiku: Fundamentals of Electric Circuits, 6th Ed., McGraw Hill NY 2016, ISBN: 978-0078028229*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 1. MK Budapest 2002, TK szám: 49203/I*
- *Dr. Selmeczi K. – Schnöller A.: Villamosságtan 2. TK Budapest 2002, ISBN:9631026043*

Definition of Fourier Transform

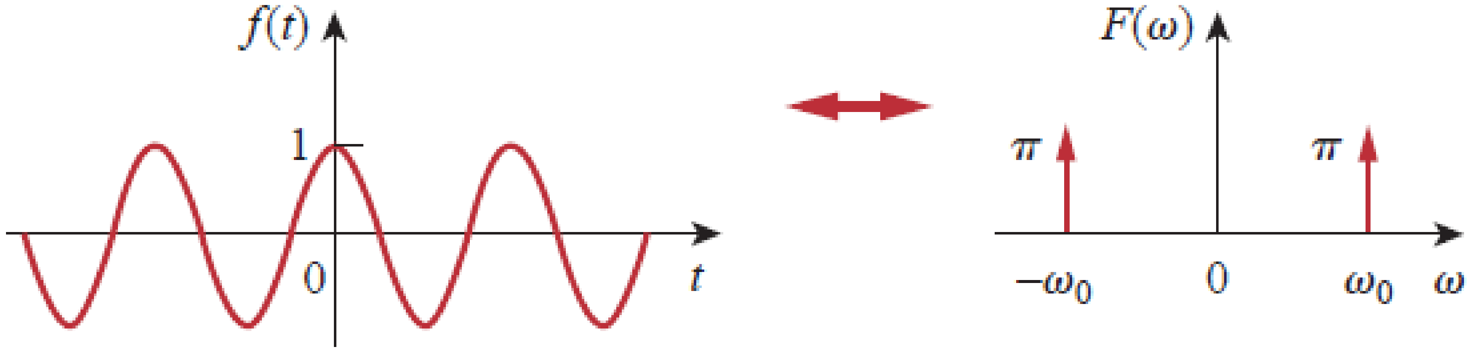
FRT.01 – Find the Fourier transform of the following functions $\delta(t-t_0)$, $e^{j\omega_0 t}$, $\cos\omega_0 t$

Solution

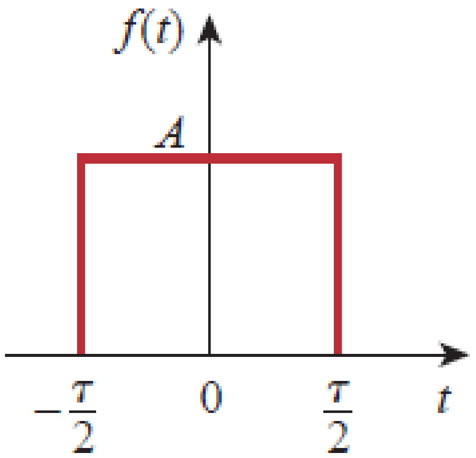
$$F(\omega)=\mathcal{F}[\delta(t-t_0)]=\int_{-\infty}^{\infty}\delta(t-t_0)e^{-j\omega t}dt=e^{-j\omega t_0}\leftarrow shifting\ property\quad spec\ case\colon\ t_0=0\rightarrow\mathcal{F}[\delta(t)]=1$$

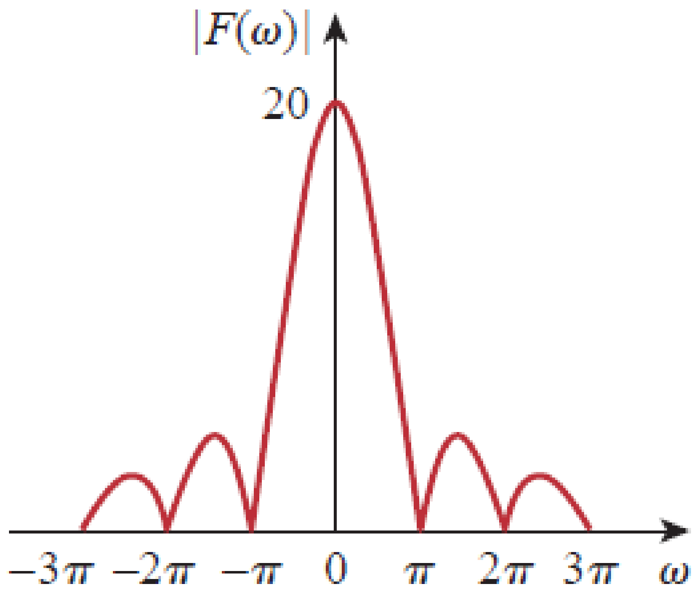
$$F(\omega)=\mathcal{F}\left[e^{j\omega_0 t}\right]=2\pi\delta(\omega-\omega_0)\leftarrow table!\,(to\ cut\ the\ long\ story\ short\ \ldots)$$

$$F(\omega)=\mathcal{F}[\cos\omega_0 t]=\mathcal{F}\left[\frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}\right]=\frac{1}{2}\mathcal{F}\left[e^{j\omega_0 t}\right]+\frac{1}{2}\mathcal{F}\left[e^{-j\omega_0 t}\right]=\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)$$



FRT.02 – Find the Fourier transform of the SINGLE rectangular pulse *(nonperiodic!)*.





Solution $$F(\omega) = \int_{-\tau/2}^{\tau/2} Ae^{-j\omega t}dt = -\frac{A}{j\omega}\bigg|_{-\tau/2}^{+\tau/2} = \frac{2A}{\omega}\left(\frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{2j}\right)$$

$$= A\tau\frac{\sin\omega\tau/2}{\omega\tau/2} = A\tau\,\mathrm{sinc}\,\omega\tau/2 \qquad A = 10, \tau = 2 \rightarrow F(\omega) = 20\,\mathrm{sinc}\,\omega$$

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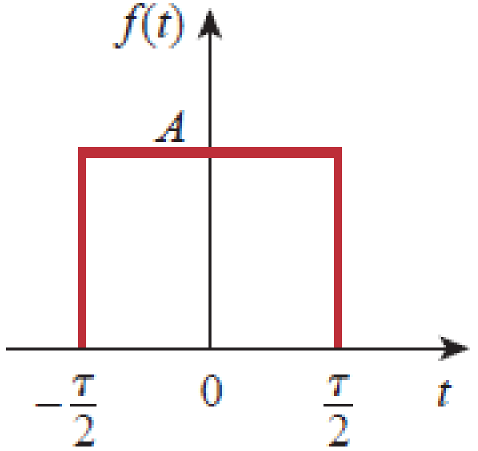
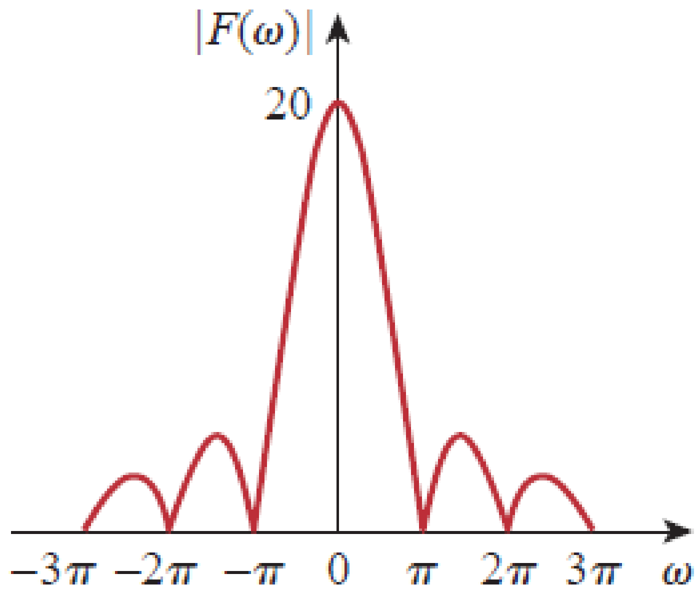
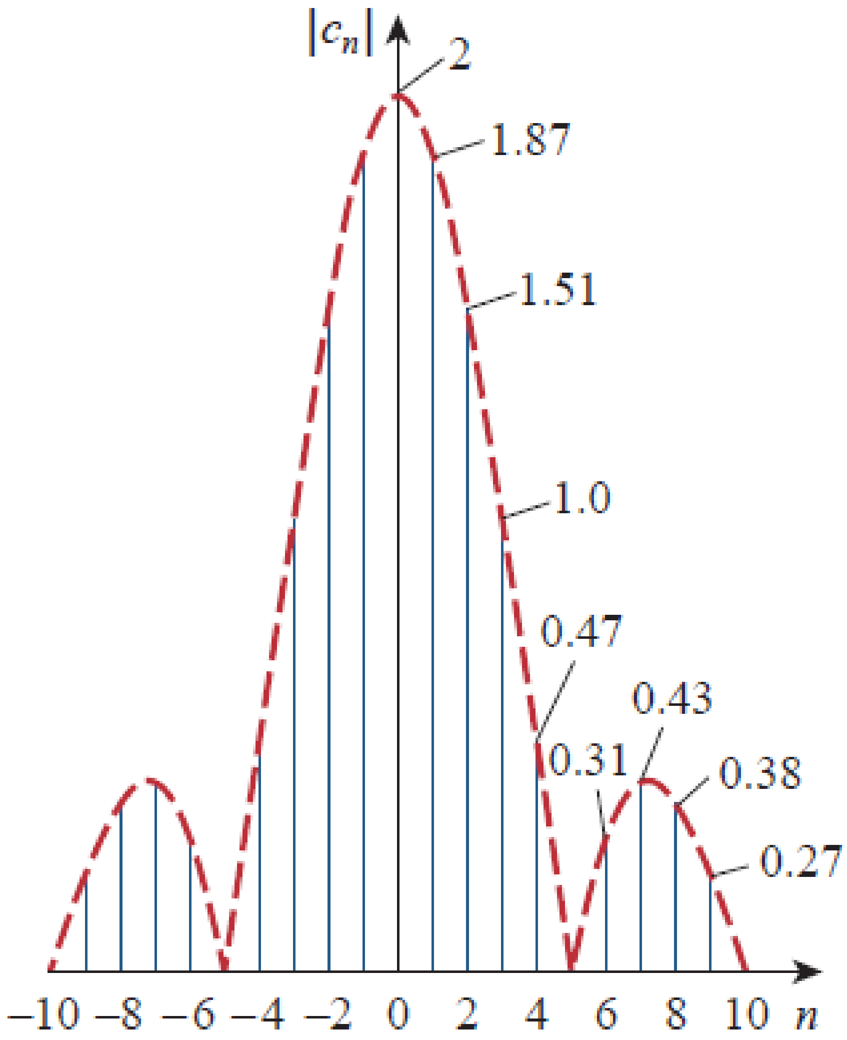
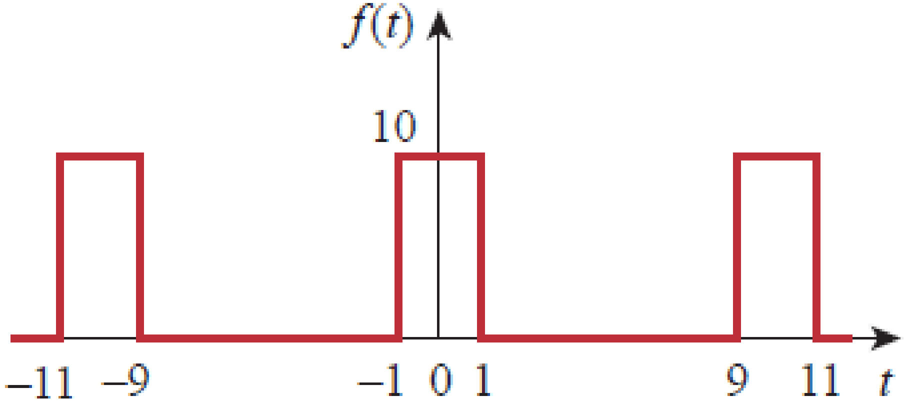
Definition of Fourier Transform







‚Similar’ to the discrete spectrum (<u>Fourier series</u>) of a periodic pulse train



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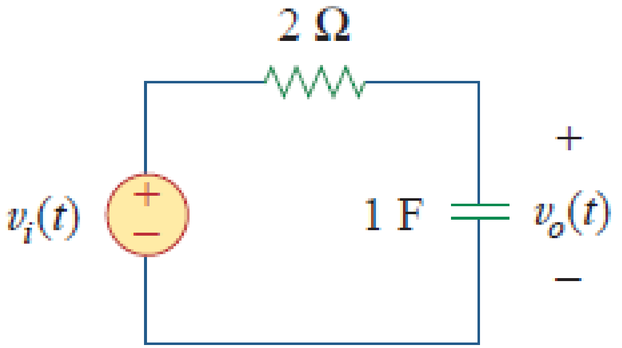
FRT.03 – Find $v_0(t)$ for $v_i(t) = 2e^{-3t}u(t)\ V$



Solution $V_i(\omega) = \dfrac{2}{3 + j\omega}$

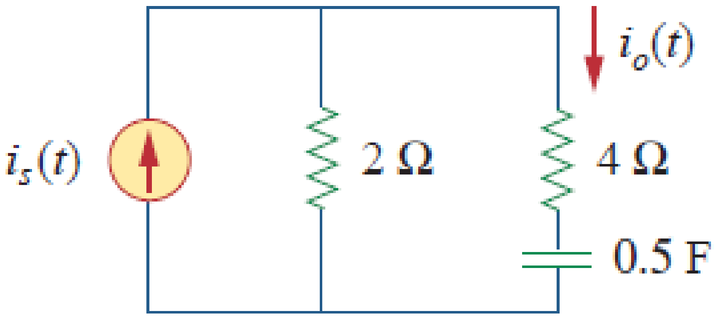
$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{1/j\omega}{2 + 1/j\omega} = \frac{1}{1 + j2\omega}$$

$$V_o(\omega) = V_i(\omega)H(\omega) = \frac{2}{(3 + j\omega)(1 + j2\omega)} = \frac{1}{(3 + j\omega)(0.5 + j\omega)}$$

$$partial\ fractions \rightarrow V_o(\omega) = \frac{-0.4}{3 + j\omega} + \frac{0.4}{0.5 + j\omega}$$

$$\mathcal{F}^{-1}[V_o(\omega)] = 0.4(e^{-0.5t} - e^{-3t})u(t)$$

FRT.04 – Find $i_0(t)$ for $i_S(t) = 10\sin 2t\ \ A$



Solution $I_S(\omega) = j\pi 10[\delta(\omega+2) - \delta(\omega-2)]$

$$H(\omega) = \frac{I_o(\omega)}{I_S(\omega)} = \frac{2}{2+4+2/j\omega} = \frac{j\omega}{1+j\omega 3}$$

$$I_0(\omega) = H(\omega) I_S(\omega) = \frac{10\pi\omega[\delta(\omega-2) - \delta(\omega+2)]}{1+j\omega 3}$$

$$i_0(t) = \mathcal{F}^{-1}[I_o(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{10\pi\omega[\delta(\omega-2) - \delta(\omega+2)]}{1+j\omega 3} e^{j\omega t} d\omega$$

$$shifting\ property \rightarrow \delta(\omega-\omega_0) f(\omega) = f(\omega_0)\ or\ \int_{-\infty}^{\infty} \delta(\omega-\omega_0) f(\omega) d\omega = f(\omega_0)$$

$$i_0(t) = \frac{10\pi}{2\pi}\left[\frac{2}{1+j6} e^{j2t} - \frac{-2}{1-j6} e^{-j2t}\right] = 10\left[\frac{e^{j2t}}{6.082\ e^{j80.54°}} - \frac{e^{-j2t}}{6.082\ e^{-j80.54°}}\right] = 1.644\left[e^{j(2t-80.54°)} + e^{-j(2t-80.54°)}\right]$$

$$i_0(t) = 3.288\cos(2t - 80.54°)\ \ A$$

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Parseval's Theorem

FRT.05 – The voltage across a 10-Ω resistor is $v(t) = 5e^{-3t}u(t)\ V$. Find the total energy dissipated in the resistor

Solution 1 (in time domain) $$W_{10\Omega} = \frac{1}{10}\int_{-\infty}^{\infty} v^2(t)\,dt = 0.1\int_{0}^{\infty} 25\,e^{-6t}dt = 2.5\left.\frac{e^{-6t}}{-6}\right|_0^{\infty} = \frac{2.5}{6} = 416.7\ mJ$$

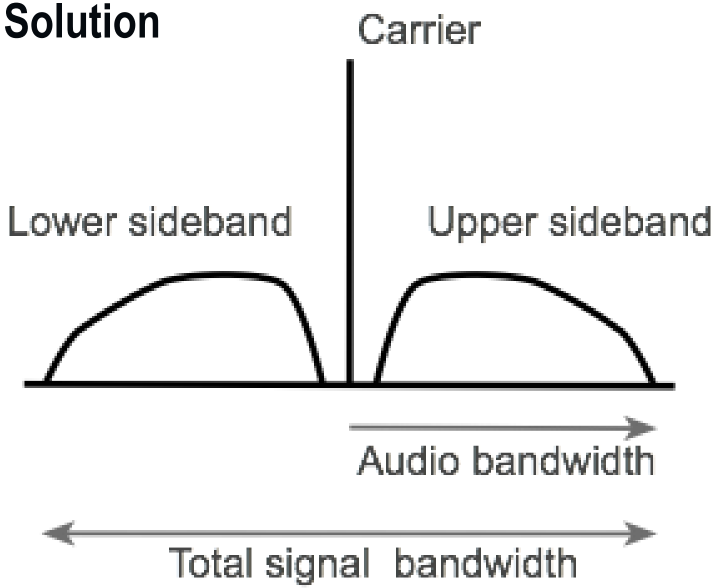
Solution 2 (in frequency domain) $$V(\omega) = \frac{5}{3+j\omega} \to |V(\omega)|^2 = V(\omega)V(\omega)^* = \frac{25}{9+\omega^2}$$

$$W_{10\Omega} = \frac{0.1}{2\pi}\int_{-\infty}^{\infty}|V(\omega)|^2 d\omega = \frac{0.1}{\pi}\int_{0}^{\infty}\frac{25}{9+\omega^2}d\omega = \frac{2.5}{\pi}\left.\left(\frac{1}{3}\tan^{-1}\frac{\omega}{3}\right)\right|_0^{\infty} = \frac{2.5}{\pi}\cdot\frac{1}{3}\cdot\frac{\pi}{2} = \frac{2.5}{6} = 416.7\ mJ$$

Amplitude Modulation

FRT.06 – A music signal has frequency components from 15 Hz to 30 kHz and this signal is used to amplitude modulate a 1.2-MHz carrier. Find the range of frequencies for the lower and upper sidebands.

Solution



$$f_{LSB1} = 1{,}200{,}000 - 30{,}000 = 1{,}1700{,}000\ Hz$$

$$f_{LSB2} = 1{,}200{,}000 - 15 = 1{,}199{,}995\ Hz$$

$$f_{USB3} = 1{,}200{,}000 + 15 = 1{,}200{,}015\ Hz$$

$$f_{USB4} = 1{,}200{,}000 + 30{,}000 = 1{,}230{,}000\ Hz$$

Sampling

FRT.07 – A telephone signal with a cutoff frequency of 5 kHz is sampled at a rate 60 percent higher than the minimum allowed rate. Find the sampling rate.

Solution $Nyquist\ rate \rightarrow f_{S\,min} = 2 \cdot f_{cut-off} = 2 \cdot 5 = 10\ kHz$

$$f_S = 1.6 \cdot f_{S\,min} = 1.6 \cdot 10 = 16\ kHz$$

Questions